Fuzzy Soft Sets Supporting Multi-Criteria Decision Processes

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***Abstract*—Students experience various types of difficulties when it comes to examinations, where some of them are subject related while others are more of a psychological character. A number of factors influencing academic success or failure of undergraduate students are identified in various research studies. One of the many important questions related to that is how to select individuals endangered to be unable to complete a particular study program or a subject. The intention of this work is to develop an approach for early discovery of students who could face serious difficulties through their studies.**

***Keywords*—Soft sets; Uncertainties; Decision making**

I. Introduction

Exam failure is a serious problem for both students and the respective educational institutions where these students are enrolled inn. One of the important questions arising in such cases is related to early identification of students who are potentially in danger of exam failure.

Students experience various types of difficulties when it comes to examinations, where some of them are subject related while others are more of a psychological character. The former are usually more specific while the latter are more general. Some examples of the latter include anxiety, low level of concentration, increased stress level and sleep disorders, [14], [15]. A large number of factors influencing academic success or failure of university students is listed in [5]. Our intention is to identify students who might be in danger of not being able to complete a particular course at a very early stage of their enrollment and consequently provide them with individual recommendations. Since such processes are usually described in uncertain and unprecised ways handling them with methods from fuzzy soft set theory is proposed in this work. In the soft set theory [12], the initial description of the object has an approximate nature, [13]. Very useful group decision making methods based on intuitionistic fuzzy soft matrices are presented in [10]. In this paper one of their approaches is expended in a way that allows obtaining a set of interesting items that differ from the one with the highest score.

The rest of this work goes as follows. Definitions and statements are placed in Section II. The main results are presented in Section III, and a conclusion can be found in Section IV.

II. Soft Sets

Let U be an initial universe set and E_U be the set of all possible parameters under consideration with respect to U. The power set of U (i.e., the set of all subsets of U) is denoted by $P(U)$ and $A \subseteq E$, [1]. A soft set is defined in the following way:

Definition 1: [12] A pair (F, A) is called a soft set over U, where F is a mapping given by

$$F : A \to P(U).$$

Definition 2: [3] Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and X be a fuzzy set over E. An FP-soft set F_X on the universe U is defined by the set of ordered pairs

$$F_X = (\mu_X(x)/x, f_X(x)) :$$
$$x \in E, f_X(x) \in P(U), \mu_X(x) \in [0,1]\},$$

where the function $f_X : E \to P(U)$ is called approximate function such that $f_X(x) = \emptyset$ if $\mu_X(x) = 0$, and the function $\mu_X : E \to [0,1]$ is called membership function of FP-soft set F_X. The value of $\mu_X(x)$ is the degree of importance of the parameter x, and depends on the decision makers requirements.

Definition 3: [3] Let $F_X \in FPS(U)$, where $FPS(U)$ stands for the sets of all FP-soft sets over U. Then a fuzzy decision set of F_X, denoted by F_X^d, is defined by

$$F_X^d = \mu_{F_X^d}(u)/u : u \in U$$

which is a fuzzy set over U, its membership function $\mu_{F_X^d}$ is defined by $\mu_{F_X^d} : U \to [0,1]$,

$$\mu_{F_X^d}(u) = \frac{1}{|supp(X)|} \sum_{x \in supp(X)} \mu_X(x)\chi_{f_X(x)}(u)$$

where $supp(X)$ is the support set of X, $f_X(x)$ is the crisp subset determined by the parameter x and

$$\chi_{f_X(x)}(u) = \begin{cases} 1, & u \in f_X(x), \\ 0, & u \notin f_X(x). \end{cases}$$

Definition 4: [9] The union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall\ e \in C$,

$$H(e) = \begin{cases} F(e), & if \ \ e \in A - B, \\ G(e), & if \ \ e \in B - A, \\ F(e) \cup G(e), & if \ \ e \in A \cap B. \end{cases}$$

It is denoted as $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 5: [9] The intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$, and $\forall\ e \in C$, $H(e) = F(e)$ or $G(e)$ (as both are same set). It is denoted as $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Definition 6: [9] Let (F, A) and (G, B) be soft sets over a common universe set U. Then

(a) $(F, A) \wedge (G, B)$ is a soft set defined by

$$(F, A) \wedge (G, B) = (H, A \times B),$$

where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall(\alpha, \beta) \in A \times B$, and $\cap$ is the intersection operation of sets.

(b) $(F, A) \vee (G, B)$ is a soft set defined by

$$(F, A) \vee (G, B) = (K, A \times B),$$

where $K(\alpha, \beta) = F(\alpha) \cup G(\beta)$, $\forall(\alpha, \beta) \in A \times B$, and $\cup$ is the union operation of sets.

Soft set relations and functions are well presented in [2]. An intuitionsitic fuzzy soft sets based decision making is discussed in [8].

III. Attribute Selection

Suppose three advisors are forming a committee that has to select attributes indicating potential exam failure. Advisors' opinions are to be taken with weights 0.5, 0.3 and 0.2 respectively, as in [10]. Weight distributions can also be determined by a decision making body that is in charge of that project. It is worth mentioning that in case the three advisors are assumed to have different influence, then there are not that many weight combinations that can actually effect attribute choice. Thus, if the lowest weight is 0.1 then the highest has to be at least 0.5 and if the lowest weight is 0.2 then the highest can be 0.4 (this implies two advisers with equal weight 0.4), 0.5 or 0.6 (this implies two advisers with equal weight 0.2).

In our case the set of attributes to be considered contains the following elements

A 1 - health related issues,

A 2 - last education relevant to this study has been obtained at least five years ago,

A 3 - time consuming obligations outside of the study,

A 4 - preliminary test results,

A 5 - amount of time a student can devote to study that subject weekly,

A 6 - student absences from classes, tutorials, etc.,

A 7 - insufficient preliminary knowledge,

A 8 - opportunities to work together with other students.

TABLE I: Attributes significance

	O 1	O 2	O 3
A 1	(0.83, 0.1)	(0.6, 0.2)	(0.6, 0.1)
A 2	(0.3, 0.51)	(0.58, 0.4)	(0.8, 0.1)
A 3	*(0.6, 0.18)*	*(0.71, 0.24)*	(0.31, 0.5)
A 4	(0.9, 0.05)	(0.8, 0.13)	(0.4, 0.52)
A 5	(0.55, 0.3)	(0.9, 0.01)	(0.5, 0.36)
A 6	(0.47, 0.21)	(0.66, 0.3)	(0.7, 0.22)
A 7	(0.8, 0.08)	(0.8, 0.15)	(0.83, 0.1)
A 8	(0.58, 0.12)	(0.3, 0.64)	*(0.69, 0.3)*
λ_{med} (E)	(0.6, 0.18)	(0.71, 0.24)	(0.69, 0.3)

TABLE II: Values for all attributes

	O 1	O 2	O 3	Attributes values
A 1	1	0	0	1
A 2	0	0	1	1
A 3	1	1	0	2
A 4	1	1	0	2
A 5	0	1	0	1
A 6	0	0	1	1
A 7	1	1	1	3
A 8	0	0	1	1

Each attribute is rated applying values from the set $0.1, ..., 1.0$, where 1.0 is the most important. Notations in Table I are as follows: O 1, O 2, O 3 represent opinions of first, second and third advisor with respect to attributes A 1, A 2, ..., A 8. A number in the first position of each couple describes a degree to which that attribute is important and the second number describes a degree to which that attribute is not important. A threshold vector λ_{med} (E) is based on median. Values for threshold vectors in Table I are emphasized.

The paper continues following mainly the work presented in [3]. Due to the specific nature of this investigation it is needed to tune a bit their approach. Thus instead of applying predefined degrees of attributes' importance values from Table II are taken. In [4], [8], and [10] they were referred to as choice values. For our study this seems more reasonable. Otherwise it will be necessary to ask every student to supply such degrees of importance. Most students would find such requests difficult and very few would be able to provide meaningful responses.

Next important difference is that the goal here is to identify all students in danger to fail their exam while following [3] one would find one student who seems to have more problems than the rest of his classmates. To achieve this the fuzzy decision set F_X^d is calculated and all students within the last quartile are selected.

To avoid confusions the terms 'classical set' and 'fuzzy soft set' are used in every single case without assuming that one or the other is understood by convention.

The classical set of proposed attributes is $\{A1, A2, ..., A8\}$ as described above. Students who answered a Web based inquiry are denoted by $\{St1, ..., St20\}$. Their responses are assumed to be binary in this case, see Table III. Nonbinary scale can be used if a finer grading is found to be more beneficial. Once again, the idea is to keep it simple. Students should not be overload with too many questions and too many

TABLE III: Responses from students

	A 1	A 2	A 3	A 4	A 5	A 6	A 7	A 8	F^d_{St}
St 1	×	×		×	×			×	0.75
St 2	×		×		×	×		×	0.75
St 3		×	×		×	×	×		1
St 4	×			×		×	×	×	1.25
St 5		×	×		×			×	0.625
St 6		×	×	×		×	×		1.25
St 7	×	×		×	×	×		×	1
St 8	×		×	×	×		×		1.25
St 9		×	×		×			×	0.625
St 10	×	×	×		×	×		×	0.875
St 11		×			×	×	×	×	0.875
St 12	×		×	×		×			0.75
St 13	×	×	×		×			×	0.75
St 14		×	×	×	×	×			0.875
St 15	×	×					×	×	0.75
St 16	×	×		×	×		×		1
St 17	×			×		×			0.5
St 18		×	×		×	×		×	0.875

options to choose from.

Members of the fuzzy decision set F^d_{St} are shown in Table III. After working on 4-quantiles of F^d_{St} we believe that students belonging to the forth quartile should be the ones to begin with. In other words, students $St3, St4, St6, St7, St8, St16$ should receive personal advises on what ought to be done in order to avoid exam failure.

Initially experience from previous courses is used. The intention is to tune the system after some time when new data has been collected. When it comes to handling situations requiring aggregation methods the approach in [10] is suggested. In case different datasets are to be used for drawing conclusions, applying statements presented in [9] seems to be quite approporiate.

A. Discussion

Another method that can be used involves formal concept analysis, [6], [7]. This is a method supporting data analysis among many other things. Once the sets of attributes and objects, and their relations are well presented in an information table, a corresponding concept lattice can be depicted. Each node in that lattice contains all the students that share the same attributes. A fuzzy function indicating degrees to which each node contents reflect danger in exam failure has to be build.

It seems that fuzzy soft sets are well equipped to handle problems presented in this work because every student is treated individually. The outcomes of formal concept analysis studies are beneficial for group of students and as a result some details concerning individuals might be omitted. Formal concept analysis based methods can be very helpful while dealing with new students and/or new advisors.

IV. Conclusion

Exam failure is most of the time a result of internal and external factors. Among the internal ones are lack of commitment and motivation, fear of exams, personal or financial problems, etc. The external ones are related to overloaded study programmes, supervision quality, inadequate requirements and so on. To determine which factors are of the highest importance one should study particular educational institutions and students groups. More research has to be done in order to determine the correct value of a proper q-quantile, as well.

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