Vehicle Path Planning Based on Gradient Statistical Mutation Quantum Genetic Algorithm

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***Abstract*—In the field of vehicle path planning, traditional intelligent optimization algorithms have the disadvantages of slow convergence, poor stability and a tendency to fall into local extremes. Therefore, a gradient statistical mutation quantum genetic algorithm (GSM-QGA) is proposed. Based on the dynamic rotation angle adjustment by the chromosome fitness value, the quantum rotation gate adjustment strategy is improved by introducing the idea of gradient descent. According to the statistical properties of chromosomal change trends, the gradient-based mutation operator is designed to realize the mutation operation. The shortest path is used as the metric to build the vehicle path planning model, and the effectiveness of the modified algorithm in vehicle path planning is demonstrated by simulation experiments. Compared with other optimization algorithms, the path length planned by the improved algorithm is shorter and the search stability is better. The algorithm can be effectively controlled to fall into local optimums.**

***Keywords*—Quantum genetic algorithm; path planning; gradient descent; adaptive mutation operator; quantum rotation gate**

I. Introduction

With the continuous development of artificial intelligence, automation technology [1] has shown strong applicability. Autonomous vehicles have become the future direction of the vehicular sector. Its core is autonomous driving technology. Autonomous driving technologies mainly include environment sensing, path planning, behavioral decision-making, and tracking control. Path planning [2] [3], as one of the key aspects of autonomous driving technology, has become a hot research topic in the field of autonomous driving. It has important value in engineering applications [4]. Path planning is mainly to plan a drivable path avoiding obstacles from the starting point to the target one based on the road environment [5]. Planning the shortest path is an NP-hard problem [6]. Thus, the path planning problem has high computational complexity. From the development of path planning algorithms, there are traditional algorithms represented by Dijkstra's algorithm [7], A* algorithm [8], artificial potential field (APF) method [9], and dynamic window algorithm (DWA) [10]. As well as genetic algorithm (GA) [11], ant colony optimization (ACO) [12], and particle swarm optimization (PSO) [13] are as the representative of intelligent optimization algorithms. The high computational cost of traditional algorithms makes it difficult to further improve the efficiency of path search, leading to a gradual decline in utilization [14].

ZHU [7] studied the path planning problem considering intersection properties and proposed a reverse labeling Dijkstra algorithm (RLDA) with minimizing travel time from the origin to the terminus as the optimization objective. The RLDA algorithm has low polynomial time complexity. The convergence efficiency and computational speed of the proposed algorithm are improved. LI [8] introduced a bidirectional alternating search strategy in the A* algorithm and weighted the heuristic function with an exponential decay to improve the search efficiency of the algorithm. In addition, a path node filtering function was introduced to effectively reduce the turning angle. LI [15] proposed a path planning method combining an APF and a dynamic enhanced fireworks algorithm for autonomous vehicles. This real-time path planning method effectively improved smoothness and safety of paths. Hou [16] proposed an enhanced ant colony algorithm with a communication mechanism for path planning. which accelerates the integration of historical paths through direct communication between individuals, and improved the path selection rules and heuristic functions to increase the convergence speed and search efficiency. LIU [17] proposed a path planning method based on the improved gray wolf algorithm, introducing interference factors and dynamic weights based on the lion optimization algorithm to avoid the loss of diversity. However, the ability to jump out of the local optimum needs to be enhanced. Kumar [18] proposed a path planning method combining artificial bee colony and evolutionary planning algorithms, using an artificial bee colony algorithm to perform an initial search based on an improved strategy, followed by an evolutionary algorithm to refine the obtained feasible paths and reduce the search cost. Martinez [19] proposed the integration an autonomous motion planning strategy for a differential robot. It combined the PSO with a Proportional-Integral-Derivative controller to ensure the stability of a differential robot path planning in complex environments.

Intelligent optimization algorithms have become one of the mainstream methods for solving path planning problems due to their better search capabilities and higher computational efficiency compared to traditional path planning algorithms. The GA has stronger global search capabilities than other intelligent optimization algorithms, as well as the ability to easily extend other algorithms. Although genetic algorithms have the above characteristics, there is a problem with early convergence [20] [21] due to high chromosome similarity in the later stages. In response to the GA problem, many researchers have proposed different modified algorithms. HE [22] proposed a GA to improve the fitness function. It added the knowledge in the problem domain as guiding information to the search process of the algorithm and took full advantage of the trend of the function to improve the convergence rate of the algorithm. XU [11] introduced a disaster strategy and a dynamic mutation operator embedded in the A* algorithm into the GA to reduce prematureness and improve the local search ability of the algorithm at later stages. The fitness function with multiple constraints enhanced the smoothness of the planned path. However, the initialization of the population with each catastrophe reduces the computational efficiency. ZHANG [23] proposed a hybrid initialized genetic algorithm, where a portion of individuals use a greedy algorithm to acquire paths, introducing deletion operations and reversal operations to prevent the algorithm from falling into local optimums.

The quantum genetic algorithm [24] (QGA) is an emerging intelligent optimization algorithm arising from the GA combined with the quantum computing. Depending on the superposition and entanglement of quantum states, quantum coding and quantum rotation gate update operations are introduced to enable better population diversity and convergence speed of the QGA compared to the GA. However, the QGA mainly relies on the quantum rotation gate for population updating. When solving combinatorial optimization problems [25] [26], it has problems such as low stability, poor convergence, and difficulty in jumping out of local optimums. In recent years, researchers have proposed many improvement strategies. WANG [27] introduced the quantum NOT gate mutation and quantum catastrophe operation and proposed an adaptive rotation angle strategy based on genetic algebra. However, the quantum NOT gate mutation operation is prone to population turbulence, and the randomness of the quantum catastrophe operation may cause the algorithm to fail to converge. XIAO [28] introduced the grouping optimization strategy of hybrid frog-jumping algorithm to divide the population and given the acceptance probability of feasible solutions using simulated annealing reception criterion. The search probability is somewhat improved. ZHANG [29] proposed an adaptive rotation angle strategy based on fitness-based values, and also introduced a quantum NOT gate mutation operation. CHENG [30] proposed an improved double-linked quantum genetic algorithm that uses an inverse sine function to construct the corner step function. The search accuracy of the algorithm is improved.

In this paper, we propose a vehicle path planning method based on a gradient statistical mutation quantum genetic algorithm. A dynamically adjusted quantum rotation gate strategy is used to improve the convergence of the algorithm and the stability of the global search by introducing the idea of gradient descent based on dynamically adjusting the rotation angle according to the fitness value of the chromosomes. Based on the statistical properties of the trend of chromosome change, the mutation operator is designed to implement the mutation operation instead of the quantum NOT gate. An adaptive mutation strategy based on the quantum bit probability density is proposed to improve the ability of the algorithm to jump out of the local optimum. The effectiveness of the proposed algorithm is demonstrated through experimental analysis of path planning simulations.

The rest of the paper is structured as follows: In Section II, the vehicle path planning problem is formulated and the cost function for path planning is described. In Section III, the main steps of QGA are introduced and the principles of chromosome update and mutation operations in the GSM-QGA are presented. The GSM-QGA is applied to path planning. Section IV presents the simulation of global path planning for vehicles using the GSM-QGA.

II. Problem Statement

For the vehicle path planning problem, the main objective in this paper is to obtain a feasible path with the shortest distance based on avoiding static obstacles.

A. Assumptions

The following assumptions are made：

- The vehicle moves from the starting point to the target one in a finite plane space at a uniform speed.
- The shape and size of obstacles and their geographic locations never vary during vehicle movement.
- The vehicle can be considered a mass point concerning static obstacles in the environment map [31].

B. Cost Function of Path Planning

The path planning mainly considers safety and path cost. In this work, the shortest path is used as the path cost in vehicle path planning. To facilitate the computation of path planning, a sequence of spatial location points is often used to represent the travel path of a vehicle, and this representation needs to only take into account the feasibility of each spatial location point. Therefore, two cost functions are constructed: The path point cost function (*point_fit*) and the path cost function (*way_fit*). The path point quantum state updating is determined with *point_fit*, and the path selection is determined based on *way_fit*. The Euclidean distance is used to construct *point_fit* [32]. The cost of a path can be estimated by calculating the sum of distances from its points to both the original position and the goal one. Notably, the smaller such the distance is, the lower the overall cost will be.

$$\min point\_fit = \sqrt{\left(x_{ir}-x_s\right)^2+\left(y_{ir}-y_s\right)^2}+\sqrt{\left(x_{ir}-x_g\right)^2+\left(y_{ir}-y_g\right)^2} \quad (1)$$

Where (x_{ir}, y_{ir}) denotes the coordinates of the *r*th path point in the *i*th drivable path. (x_s, y_s) indicates the coordinates of the starting point. (x_g, y_g) indicates the coordinates of the target one.

To calculate the total path length of all path points connected in sequence, *way_fit* is defined as follows.

$$\min way\_fit = \sum_{r=1}^{M-1}\sqrt{\left(x_{ir}-x_{ir+1}\right)^2+\left(y_r-y_{ir+1}\right)^2} \quad (2)$$

Where M is the number of all path points in a feasible path.

III. Optimization Algorithm

In path planning problems, an improved algorithm is needed for problems where the QGA is not sufficiently stable and tends to fall into a local minima or maxima. In this section, the GSM-QGA for vehicle path planning will be introduced. The key process of the QGA will be described. Principles of chromosome updating and mutation operations in the GSM-QGA are presented along with the vehicle path planning process.

A. Quantum Genetic Algorithm

A quantum bit (qubit) is the smallest information unit of a quantum computer. In a two-state quantum system, the state of a qubit can be described as [33].

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{3}$$

Where the state of a qubit $|\varphi\rangle$ is the superposition of uncertainty between the state of qubit $|0\rangle$ and qubit $|1\rangle$. α and β are the probability amplitudes. They satisfy the normalization conditions as follows [33].

$$|\alpha|^2 + |\beta|^2 = 1 \tag{4}$$

The use of qubit coding for population initialization enables the inclusion of complex population information at a small population size. An initialized quantum population is represented as follows.

$$Q(0) = \{q_i(0)\},\ (i = 1, 2, \cdots, m) \tag{5}$$

Where *m* is the population size. An individual is defined by one chromosome, denoted as $q_i(0)$. In addition, $q_i(0)$ is represented as a feasible solution too. Its coded form is expressed as follows.

$$q_i(0) = \begin{vmatrix} \alpha_{i1}^1(0)\ \alpha_{i1}^2(0)\ \cdots\ \alpha_{i1}^k(0) \cdots \alpha_{in}^1(0)\ \alpha_{in}^2(0) \cdots \alpha_{in}^k(0) \\ \beta_{i1}^1(0)\ \beta_{i1}^2(0)\ \cdots\ \beta_{i1}^k(0) \cdots\ \beta_{in}^1(0)\ \beta_{in}^2(0) \cdots\ \beta_{in}^k(0) \end{vmatrix} \tag{6}$$

Where $q_i(0)$ denotes the *i*th individual in the initialized population. *n* is the number of gene points contained in a feasible solution, and *k* is the number of qubits contained in gene coding. When the population is initialized, in order to ensure the equilibrium of the population distribution, the probability magnitude of each qubit in an individual is expressed as (7).

$$\alpha_{ir}^j(0) = \beta_{ir}^j(0) = \frac{1}{\sqrt{2}} \tag{7}$$

Where $i = 1, 2, \cdots m$; $r = 1, 2, \cdots n$; $j = 1, 2, \cdots, k$.

QGA uses a quantum rotation gate to update the probability amplitude of the qubit in order to search for the optimal solution of the problem. The quantum rotation gate is commonly adapted as follows [34].

$$U(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{8}$$

Where θ is the rotation angle, obtained by looking up the table.

The updated state $|\varphi'\rangle$ is denoted as

$$|\varphi'\rangle = \begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = U(\theta) \times |\varphi\rangle = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{9}$$

Where α' and β' denote the probability magnitudes after updating of states $|0\rangle$ and $|1\rangle$.

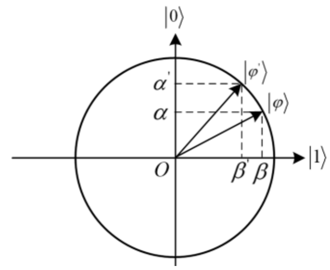

Fig. 1. The Update Of Qubits Probability Amplitudes.

In the two-state quantum system, the update of qubits probability amplitudes is shown in Fig. 1. The probability amplitudes of qubits are taken with continuity. In this way, the QGA has continuous spatial search capability.

B. Gradient Statistical Mutation Quantum Genetic Algorithm

1) Adaptive quantum rotation gate: The rotation angle of the quantum rotation gate plays a key role in chromosome renewal. The size and direction of θ determine the speed and direction of individual evolution. In the QGA, θ is obtained by looking up the table. This approach does not give a basis for the choice of rotation angle depending on the specific problem to be solved. Moreover, θ obtained in this way fails to consider not only the differences between chromosomes in the population, but also the trends in the search points. In the GSM-QGA, differences between different chromosomes in a population of the same generation are taken into account, and trends in chromosomal gene points between populations of different generations are considered to influence population evolution. In this paper, we relate the magnitude of the rotation angle to the chromosome fitness value. At the same time, the idea of gradient descent was introduced to study the trend of chromosomal gene points. Thus, a strategy for adaptive adjustment of the rotation angle is proposed as follows.

$$\theta_{ir}^j = -\mathrm{sgn}(A) \cdot \theta_0 \cdot \exp\left(a \cdot \frac{|fit_{best} - fit_i|}{fit_{best}} + (a-1) \cdot \frac{|\nabla f(X_{ir}) - \nabla f_{r\min}|}{\nabla f_{r\max} - \nabla f_{r\min}}\right) \tag{10}$$

$$\nabla f_{r\max} = \max\left\{\left|\frac{\partial f(X_i)}{\partial X_{ir}}\right|\right\} \quad (i = 1, 2, \mathrm{L}\ , m) \tag{11}$$

$$\nabla f_{r\min} = \min\left\{\left|\frac{\partial f(X_i)}{\partial X_{ir}}\right|\right\} \quad (i = 1, 2, \mathrm{L}\ , m) \tag{12}$$

Where θ_{ir}^j denotes the rotation angle of the *j*th qubit in the *r*th gene point of the *i*th chromosome. $-\mathrm{sgn}(A)$ indicates the direction of rotation angle. θ_0 denotes the initial rotation angle step. The weight $a \in (0,1)$ is used to reflect the effect of fitness function values and gene point gradients on the degree of chromosome evolution. fit_i is the fitness value of the *i*th chromosome in the current generation. $\nabla f(X_{ir})$ is the gradient at the *r*th gene point of the *i*th chromosome. $\nabla f_{r\min}$ and $\nabla f_{r\max}$ are the minimum and maximum values of the gradient at the *r*th gene point in the current population. *A* is

$$A = \begin{vmatrix} \alpha_0 & \alpha_1 \\ \beta_0 & \beta_1 \end{vmatrix} \tag{13}$$

Where $(\alpha_0, \beta_0)^{\mathrm{T}}$ is the probability amplitude of the corresponding qubit in the current optimal chromosome and $(\alpha_1, \beta_1)^{\mathrm{T}}$ is the probability amplitude of the corresponding qubit in the current chromosome.

The direction of the rotation angle is chosen as follows: When $A \neq 0$, the direction of the rotation angle is $-\mathrm{sgn}(A)$; when $A = 0$, the direction is chosen randomly [35].

The dynamic adjustment strategy of a quantum rotation gate considers both the fitness values of chromosomes and the trends of chromosome loci. When the chromosome fitness value is far from the optimal chromosome and the quantum position gradient changes weekly, the rotation angle is increased to expedite convergence. Conversely, in order to prevent missing the optimal chromosome, the rotation angle must be diminished, thereby enhancing both the speed of convergence in the algorithm and the stability of the global search.

2) Quantum mutation: Despite the strong global search capability of the QGA, it is easy to get trapped in local optima by updating the population only through a single quantum rotation gate. Therefore, a certain perturbation operation is needed to reduce the occurrence of "premature" population. Improved QGA [36] usually uses the quantum NOT gate to perform variation on the probability magnitudes of individual qubits in the population, which can avoid the local optimum to a certain extent. When a chromosome performing the mutation is very close to the optimal chromosome, the quantum NOT gate mutation will cause the reversal of the direction of qubit update, which may cause the population turbulence and the loss of excellent chromosome information. In addition, this operator fails to consider the effect of chromosomal information contained in the population and perturbative factors such as external environment on chromosomal gene mutations, resulting in a lack of population perception.

Therefore, in this paper, we propose a mutation operator that includes the past information of individuals in the population. It enables the quantum mutation operation to impose reasonable perturbations during population evolution to avoid premature convergence of the population. Genetic information decreases with increasing number of generations, and current chromosomal gene points are most affected by paternal chromosomes. As a result, we only consider the effect of paternal chromosomes on the current chromosomal gene point. The gradient (∇Z_{ir}) of statistical past chromosomal gene point fitness values is expressed as (14).

$$\nabla Z_{ir} = \nabla f(X_{ir}) + o(\nabla f(X_{ir})) \tag{14}$$

Where $o(\nabla f(X_{ir}))$ is the higher order infinitesimal of the gradient of the offspring and parent.

The probability density function is designed according to the trend of chromosomal gene points as follows.

$$f(\nabla Z_{ir}) = \sum_{l}^{5} b_{l1} \cdot \exp\left(-((\nabla Z_{ir} - b_{l2}) / b_{l3})^2\right) \tag{15}$$

Where b_{l1}, b_{l2} and b_{l3} are Gaussian mixture distribution parameters, respectively.

Transforming (15) into a probability distribution function as follows.

$$F(\nabla Z_{ir}) = \sum_{l}^{5} b_{l1} \cdot \int_{-\infty}^{Z_{ir}} \exp\left(-((\nabla Z_{ir} - b_{l2}) / b_{l3})^2\right) \tag{16}$$

The mutation operation is performed on the qubit probability amplitude, and the probability distribution of the qubit gradient is used as the mutation operator.

$$\begin{cases} \alpha_{ir}^{j}(t) = \sqrt{1 - F(\nabla Z_{ir})} \\ \beta_{ir}^{j}(t) = \sqrt{1 - \alpha_{ir}^{j}(t)^2} \end{cases} \tag{17}$$

Where $\alpha_{ir}^{j}(t)$ and $\beta_{ir}^{j}(t)$ denote the probability amplitude of the state $|0\rangle$ and $|1\rangle$ at the *j*th gene point of the *r*th chromosome in the *t*th generation, separately.

In addition, a reasonable mutation probability is beneficial for improving diversity and stability of population evolution. Thus, according to the evolutionary trend of chromosomes in the population, an adaptive mutation mechanism based on the qubit probability density is proposed as follows: The mutation probability of the current qubit is determined based on the gradient of gene points. When chromosome evolution is relatively "flat", a large perturbation probability is given to make chromosomes jump out of the local optimum and increase population diversity. On the contrary, a small perturbation probability is given to avoid destroying individuals with good genes and to improve the stability of the population. Random selection of qubits in an individual based on adaptive mutation probability is used to apply mutation operations. The adaptive mutation probability (p_m) is expressed as:

$$\begin{aligned} p_m &= P(\nabla Z \geq \nabla Z_{ir}) \\ &= 1 - F(\nabla Z_{ir}) \\ &= 1 - \sum_{l}^{5} b_{l1} \cdot \int_{-\infty}^{Z_{ir}} \exp(-((\nabla Z_{lr} - b_{l2}) / b_{l3})^2) \end{aligned} \tag{18}$$

The flowchart of the GSM-QGA is shown in Fig. 2.

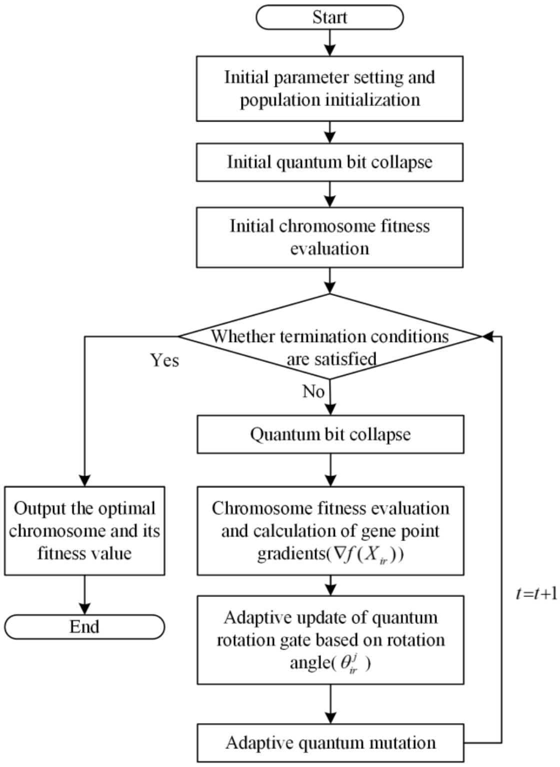

Fig. 2. The flowchart of the GSM-QGA

C. Vehicle Path Planning Based on Gradient Statistical Mutation Quantum Genetic Algorithm

To plan a vehicle path based on the GSM-QGA, the path is first encoded, where path points on the same parallel line are encoded with qubits in a grid map. Then, the quantum encoded path points are arranged to form a feasible path, which constitutes a chromosome. The encoding form of the chromosome is shown in Fig. 3.

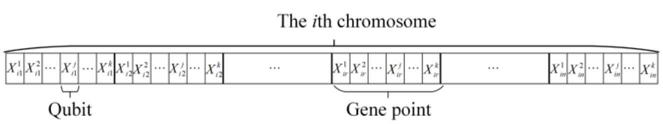

Fig. 3. Chromosome coding

Fitness values at the path points are discretize. Thus, the gradient of path points in the chromosome is represented using the first-order difference between two adjacent generations as follows.

$$\nabla f_{r\max} = \max\left\{\left| f\left[X_{ir}(t-1)\right] - f\left[X_{ir}(t)\right]\right|\right\} \tag{19}$$

$$\nabla f_{r\min} = \min\left\{\left| f\left[X_{ir}(t-1)\right] - f\left[X_{ir}(t)\right]\right|\right\} \tag{20}$$

Where $i = 1, 2, \cdots, m$.

By statistically analyzing the gradient information of path points in past chromosomes, we obtain parameter values of formula (15).

$$(\boldsymbol{b}_1\, \boldsymbol{b}_2\, \boldsymbol{b}_3)^{\mathrm{T}} = \begin{bmatrix} 0.7521 & 0.1421 & 0.1519 & -0.6852 & 0.04352 \\ 0.5204 & 2.072 & 0.1908 & 0.4204 & 5.743 \\ 0.9951 & 2.126 & 0.6099 & 1.01 & 2.565 \end{bmatrix} \tag{21}$$

Here are the steps of vehicle path planning based on the GSM-QGA.

Step 1: The environmental map construction. Building a grid map based on a known planning space.

Step 2: Initial parameters configuration and the population initialization. The population size (*popsize*) is m. The number of genetic generations is t and the initial value of the rotation angle is θ_0. The starting point and ending one must be defined prior to initialization. After quantum encoding of path points, the initialized population ($Q(0)$) is generated. All probability amplitudes of path point qubits in the primitive chromosome are represented by $\alpha_{ir}^j(0) = \beta_{ir}^j(0) = 1/\sqrt{2}$.

Step 3: Initial qubits collapse. All $Q(0)$ undergo measurement, causing their respective qubits to collapse into a predetermined state ($p(0)$). The resulting $p(0)$ set of collapsed qubits represents the desired path points for the vehicle.

Step 4: Initial path adaptation evaluation. $point\_fit_r(0)$ and $way\_fit_i(0)$ are computed. $way\_fit_i(0)$ in the population are compared, and the chromosome indicating the current shortest path (q_{best}) and its corresponding fitness value (way_fit_{best}) are recorded.

Step 5: Qubits collapse. All $Q(t)$ undergo measurement, causing their respective qubits to collapse into a predetermined state ($p(t)$).

Step 6: Path adaptation evaluation. $point\_fit_r(t)$ and $way\_fit_i(t)$ are calculated, as well as $\nabla f(X_{ir})$. $way\_fit_i(t)$ in the population are compared, and the chromosome that represents the current q_{best} and its corresponding way_fit_{best} are recorded.

Step 7: The quantum rotation gate adaptive updating. θ_{ir}^j is obtained by (10) to (13), and the chromosome is updated adaptively using a quantum rotation gate.

Step 8: Quantum mutation operation. p_m is determined by an adaptive mutation mechanism. Qubits in a chromosome are chosen randomly by p_m, and an adaptive mutation operator is used to apply mutation operations to these qubits.

Step 9: Determine if the maximum number of iterations or convergence condition is satisfied. If it is satisfied, the algorithm ends and the shortest path is output. Otherwise, the number of iterations $t = t + 1$, and return to step 4.

IV. Results and Discussion

This section presents a vehicle path planning simulation based on the GSM-QGA aimed at demonstrating its effectiveness in this field. In this paper, we explore vehicle path planning within an industrial park, which is set against an area of 1×10^{6} m2. The environment map was generated by adopting the grid approach consisting of splitting the industrial park into grids that measure 0.05 km in length, forming a total of 20×20 grids. White grids represent traversable areas for vehicles, whereas black grids indicate obstructions. The starting point of the vehicle is situated at the coordinate (1, 1) and denoted with a pentagram, whereas the destination is marked using a similar symbol at coordinate (20, 20). Finally, initialization parameters were established. The maximum number of genetic generations is $t=50$. The population sizes of the GA are $popsize=20$ and $popsize=100$. The crossover probability and mutation probability of the GA are $p_c=0.8$ and $p_m=0.1$. The population size of the QGA is $popsize=20$. The population size for the Quantum Genetic Algorithm of quantum NOT gate mutation (N-QGA) is $popsize=20$. The mutation probability of the N-QGA is $p_m=0.1$. The population size of the GSM-QGA proposed in this paper is $popsize=20$. The initial rotation angle of the GSM-QGA is $\theta_0=0.1\pi$. The solution accuracy and convergence speed of these four algorithms are compared to verify the performance of our algorithm in path planning.

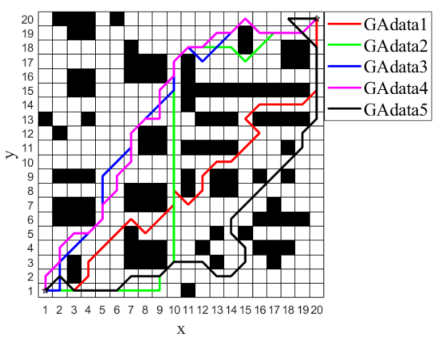

Fig. 4. The path planning simulation results of the GA with *popsize*=20 run 5 times.

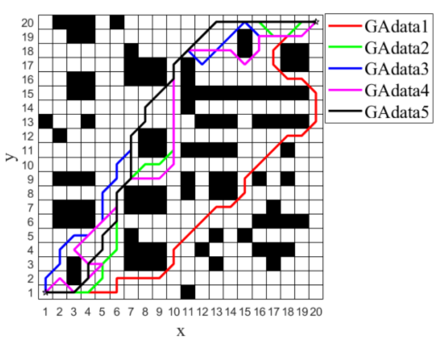

Fig. 5. The path planning simulation results of the GA with popsize=100 run 5 times.

Fig. 4 to 8 show the path planning simulation results of the four algorithms run 5 times in a 20×20 grid map. Fig. 9 to 13 show the path iteration convergence curves for the four algorithms run 5 times.

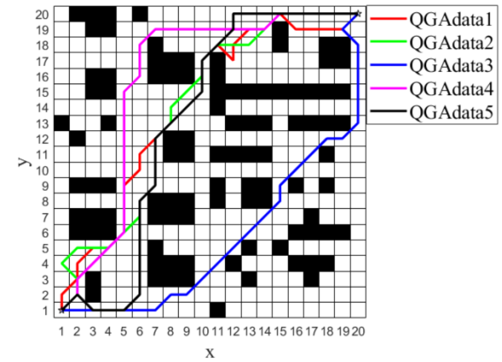

Fig. 6. The path planning simulation results of the QGA run 5 times.

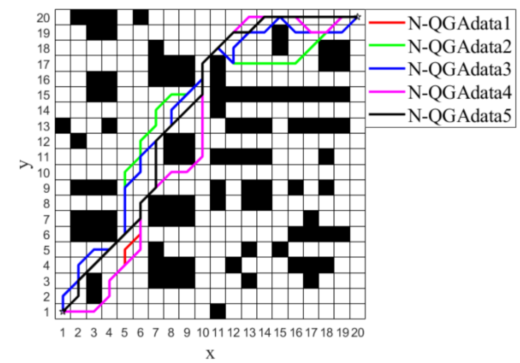

Fig. 7. The path planning simulation results of the N-QGA run 5 times.

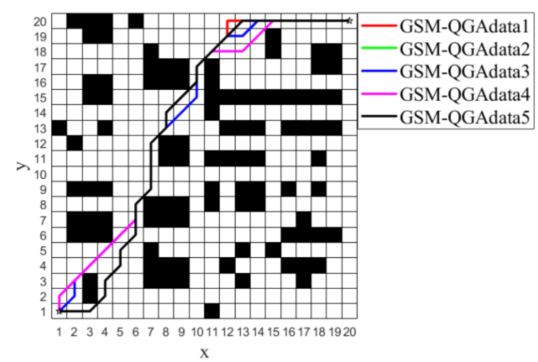

Fig. 8. The path planning simulation results of the GSM-QGA run 5 times

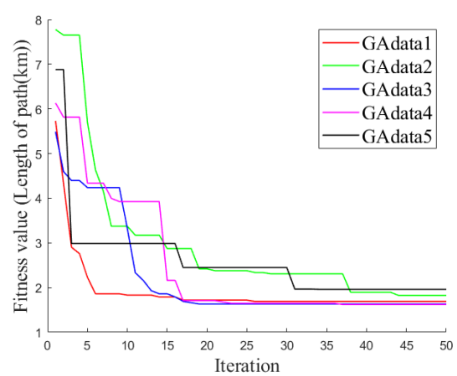

Fig. 9. The path iteration convergence curves for the GA with popsize=20 run 5 times.

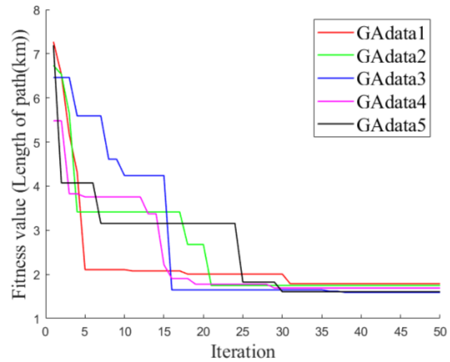

Fig. 10. The path iteration convergence curves for the GA with popsize=100 run 5 times.

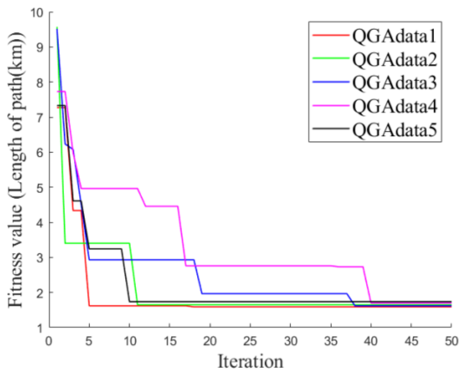

Fig. 11. The path iteration convergence curves for the QGA run 5 times.

Since these algorithms have a certain degree of search randomness, we count the results gathered from five runs of each algorithm, as displayed in Table I. "Length of path" in the Table I indicates the path distance from the start point to the target one. "Number of iterations" in Table I expresses the number of generations for population update when converging to the optimal solution. "Optimum path length", "Worst path length", and "Average path length" in Table I represent the minimum, maximum, and average values of the path distance among the five runs of the algorithm, respectively. With the condition of keeping population sizes and iteration times constant, as can be seen visually in Fig. 4 and Fig. 9, When the population size is small, the paths planned by the GA are too long and the optimal paths obtained differ significantly each time. The difference between the optimal path and the worst path is 0.3415 km. The GA converges slowly, with an average number of 32 iterations, and is easily trapped in a local optimum. By increasing the population size to 100, the GA method yields improved path length and convergence speed. The optimal path length is 1.5899 km. However, it is clear from Fig. 5 that there is still significant redundancy in the path distance. While both the QGA and the N-QGA are capable of further optimizing the path distance, it can be shown from Fig. 6 and Fig. 7 that the searched paths still vary considerably and the stability of the algorithm is not good. The difference between the optimal path and the worst path is 0.1475 km and 0.1293 km, respectively. As shown in Fig. 11 and Fig. 12, the overall convergence speed still requires further enhancement. As illustrated in Fig. 8 and Fig. 13, the GMS-QGA proposed in this paper can break away from local optimization. The obtained optimal path is 1.5485 km and an average number of iterations is 15.8. Meanwhile, the results obtained from 5 runs confirm the good stability of the algorithm. The difference between the optimal path and the worst path is only 0.0586 km.

Fig. 14 and Fig. 15 provide a more intuitive comparison of the four path planning algorithms in terms of solution accuracy and convergence speed. Evidently, the GA exhibits slow convergence and susceptibility to premature optimization. Its planned paths exhibit a higher degree of redundancy. The QGA converges in the 18th iteration with an optimal path of 1.5889 km. While it converges quickly, it struggles to break away from local optima. By leveraging mutation operators based on the quantum NOT gate's local perturbation, the N-QGA effectively escapes local optima. It obtains an optimal path of 1.5485 km. However, the magnitude of its mutations is large, which can easily lead to population turbulence. The N-QGA has slow convergence and poor algorithmic stability. In contrast, the GSM-QGA updates quantum coding path points using adaptive quantum rotation gate, exhibiting faster path convergence. The optimal path of 1.5485 km is obtained by the 16th iteration. Furthermore, through a mutation operator and variation strategy designed based on past path data, this algorithm can better perturb the path planning process while mitigating premature optimization, thereby achieving shorter planned path distances. The average path obtained by GSM-QGA is 1.56608 km. From the calculation of the data in Table I, compared to the other three algorithms, the GSM-QGA averages 10.26%, 7.06%, 5.52%, and 2.99% reduction in length while increasing the average speed of convergence by 50.63%, 46.98%, 32.48%, and 26.85%, all while maintaining superior algorithm stability.

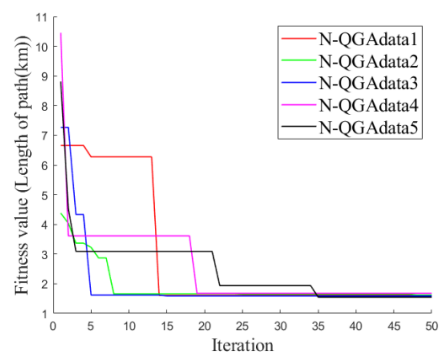

Fig. 12. The path iteration convergence curves for the N-QGA run 5 times.

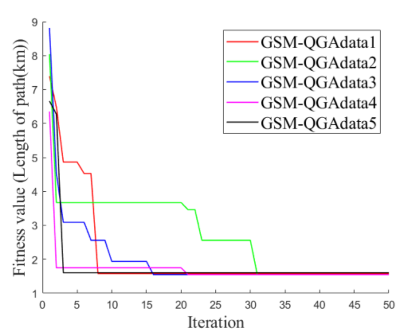

Fig. 13. The path iteration convergence curves for the GSM-QGA run 5 times.

TABLE I. STATISTICAL TABLE OF THE RESULTS FOR THE FOUR ALGORITHMS RUN 5 TIMES

Algorithm	Result statistics of different algorithms					
	Running of *i*th time	**Length of path(km)**	**Number of iterations**	**Optimum path length(km)**	**Worst path length(km)**	**Average path length(km)**
GA (*popsize*=20)	1	1.6899	26	1.6192	1.9607	1.7451
	2	1.8243	44			
	3	1.6314	19			
	4	1.6192	37			
	5	1.9607	34			
GA (*popsize*=100)	1	1.7899	31	1.5899	1.7899	1.68506
	2	1.7485	21			
	3	1.5899	38			
	4	1.6899	29			
	5	1.6071	30			
QGA	1	1.5889	18	1.5889	1.7364	1.6576
	2	1.6485	11			
	3	1.6192	38			
	4	1.695	40			
	5	1.7364	10			
N-QGA	1	1.6071	14	1.5485	1.6778	1.61436
	2	1.6485	25			
	3	1.5899	15			
	4	1.6778	19			
	5	1.5485	35			
GSM-QGA	1	1.5778	8	1.5485	1.6071	1.56608
	2	1.5485	31			
	3	1.5485	16			
	4	1.5485	21			
	5	1.6071	3			

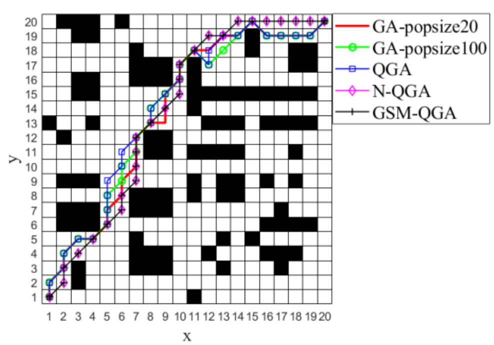

Fig. 14. The path planning simulation results of the four algorithms.

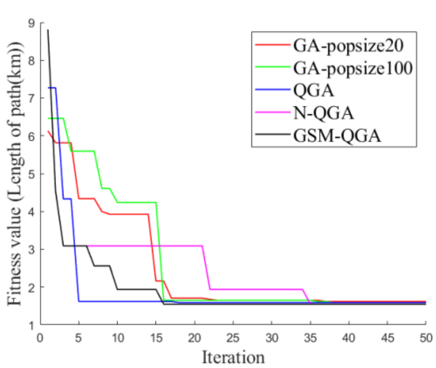

Fig. 15. The optimal path iteration convergence curves for the four algorithms.

V. CONCLUSION

In the vehicle path planning problem, the path planning method based on a gradient statistical mutation quantum genetic algorithm was proposed for the problems that GA is prone to early maturation and slow convergence.

In this paper, we proposed a dynamic adjustment strategy for a quantum rotation gate that considered both the chromosome fitness values and the trend of gene point changes. The convergence speed of the algorithm and the stability of the search for superiority are improved. Moreover, based on the statistical properties of the trend of chromosome change, a mutation operator was designed, and an adaptive mutation strategy based on the qubit probability density was proposed to effectively control the algorithm into a local optimum. Simulation results reveal the superiority of our GSM-QGA over GA, QGA and N-QGA: the average path length is reduced by 10.26%, 7.06%, 5.52%, and 2.99%; the average convergence speed increases by 50.63%, 46.98%, 32.48%, and 26.85%, respectively. The GSM-QGA has advantages over GA, QGA and N-QGA in terms of path length, convergence speed and algorithm stability. The effectiveness of the GSM-QGA in path planning is demonstrated.

Future work includes further extension of the algorithm in combination with other algorithms for application to more complex traffic environments. On the other hand, an attempt is made to design a new quantum gate as a transition matrix for population updating to address the problem that the quantum rotation gate has a finite range of rotation angles in the high-dimensional space.

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