Comparative Analysis of DIDIM and IV Approaches using Double Least Squares Method

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***Abstract*—Usually, identifying dynamic parameters for robots involves utilizing the Inverse Dynamic Model (IDM) which is linear in relation to the parameters being identified, alongside Linear Least Squares (LLS) methods. To implement this approach, precise measurements of both torque and position must be obtained at a high frequency. Additionally, velocities and accelerations must be estimated by implementing a band-pass filtering technique on the position data. Given the presence of noise in the observation matrix and the closed-loop nature of the identification process, we have modified the Instrumental Variable (IV) method to address the issue of noisy observations. A novel identification technique, named (Direct and Inverse Dynamic Identification Model) DIDIM, which requires only torque measurements as input variables, has recently been successfully applied to a 6-degree-of-freedom industrial robot. DIDIM employs a closed-loop output error approach that utilizes closed-loop simulations of the robot. The experimental results reveal that the IV and DIDIM methods exhibit numerical equivalence. In this paper, we conduct a comparison of these two methods using a double step least squares (2SLS) analysis. We experimentally validate this study using a 2-degree-of-freedom planar robot.**

Keywords*—*Identification; double least squares; instrumental variable; DIDIM method; robotics dynamics

I. Introduction

The identification process for robots usually relies on the use of the inverse dynamic model and linear least squares methods. To create an overdetermined system, the IDM is sampled during the robot's motion with exciting trajectories. This technique has proven successful in identifying many robots and prototypes [1], [2], [3]. However, it requires precise measurements of joint positions and torques at a high sampling frequency (above 100Hz). Moreover, the identification is done in closed-loop due to the unstable nature of the double integrator, resulting in a noisy observation matrix. Consequently, in theory, the LLS estimator can present a bias [4].

To tackle this problem, we have adjusted the instrumental variable (IV) technique [5], taking inspiration from Hugues Garnier's team's research [6], [7], [8]. Recently, a new identification method was introduced and validated on a 2-degree-of-freedom robot [9]. This approach only requires joint torques as input parameters. The robot is simulated in closed loop, assuming the same control structure and exciting trajectories. The best-fit parameters minimize the squared difference between the simulated and measured torques. Experimental findings indicate that the results from the IV method match numerically with those from the DIDIM method, indicating a strong connection between the two approaches.

The objective of this paper is to compare the methods using the double least squares technique and to experience the method of DIDIM with IV if it is perfect to our robot or not. The paper is organized as follows: Section II covers the modeling and identification of robots, Section III presents the identified prototype, Section IV explains the IV and DIDIM methods, Section V discusses the double least squares method, and finally, Section VI analyzes the results of the experiments.

II. Modeling and Identification of Robots

A. Modelisation

The expression for the inverse dynamic model of a robot with n degrees of freedom is given as [10]:

$$\tau_{idm}=M(\theta)\ddot{\theta}+N(\theta,\dot{\theta}) \quad (1)$$

Where $\boldsymbol{\theta}$ represents the (n x 1) vector of joint positions, $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$ are its temporal derivatives, τ_{idm} denotes the (n x 1) vector of joint torques, $M(\boldsymbol{\theta})$ stands for the (n x n) symmetric inertia matrix, and $N(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ represents the (n x 1) vector that combines centrifugal, Coriolis, gravitational, and friction forces. By employing the modified Denavit and Hartenberg geometric description (DHM), we can derive a linear inverse dynamic model in terms of standard dynamic parameters [10]:

$$\tau_{idm}=IDM_{STD}(\theta,\dot{\theta},\ddot{\theta})\chi_{STD} \quad (2)$$

$\mathbf{IDM}_{STD}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}})$ represents the standard linear regressor of size (n x c), where χ_{STD} is the c x 1 column vector of standard dynamic parameters. These parameters include the inertia tensor coefficients XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j of body ${}^jJ_j$, its mass denoted m_j, the first moment vector around the origin of body j denoted ${}^jM_j$, = [MX_j MY_j MZ_j], the Coulomb and viscous friction parameters denoted respectively as Fs_j and Fv_j, and the actuator inertia Ia_j.

One crucial stage in the identification process is to identify the basic parameters. This is because some standard parameters combine in the inverse dynamic model's expression, and only their combination or grouping can be identified. The search for basic parameters involves determining the rank of $\mathbf{IDM}_{STD}$ and identifying linear combinations among its columns. Two primary methods can be used to calculate the minimum inertial parameters: a literal method that involves energy calculations [10] and a numerical method based on QR decomposition [11].

The general relation for the minimal system is as follows:

$$\tau_{idm} = IDM(\theta, \dot{\theta}, \ddot{\theta})\chi \tag{3}$$

IDM ($\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}}$) is the minimal linear regressor of dimension n x b, where χ is the column vector of base parameters of dimension (b x 1). However, due to noise and model errors, the actual torque τ deviates from τ_{idm} and can be expressed as:

$$\tau = \tau_{idm} + e = IDM(\theta, \dot{\theta}, \ddot{\theta})\chi + e \tag{4}$$

B. Identification

The inverse dynamic model is sampled while the robot is being actuated to obtain an overdetermined system, which can be expressed as [12]:

$$Y(\tau)=W(\theta,\dot{\theta},\ddot{\theta})\chi+\rho \tag{5}$$

Or: $\mathbf{Y(\tau)} = \left[\left(\mathbf{Y^1(\tau)}\right)^T \ldots \left(\mathbf{Y^n(\tau)}\right)^T\right]^T$, $\mathbf{Y^j(\tau)} = \left[\tau^j(1) \ldots \tau^j(n_e)\right]^T$

$$\mathbf{W}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}}) = \begin{bmatrix} \mathbf{W^1} \\ \ldots \\ \mathbf{W^n} \end{bmatrix}, \quad \mathbf{W^j} = \begin{bmatrix} \mathbf{IDM^j}(\theta_1, \dot{\theta}_2, \ddot{\theta}_3) \\ \ldots \\ \mathbf{IDM^j}(\theta_{ne}, \dot{\theta}_{ne}, \ddot{\theta}_{ne}) \end{bmatrix}$$

$Y(\tau)$ is the measurement vector with a dimension of (r x 1), and W is the observation matrix with a dimension of (r x b), where $r = n_e x n$, and n_e is the number of recovered samples.

The estimation theory provides a broad range of methods. Classical methods can be employed to solve overdetermined systems, as long as the elements of W are handled appropriately to obtain good results.

$$\hat{\chi} = \min_{\hat{\chi}} \|\rho\|^2 \tag{6}$$

Since we are considering both base parameters and exciting trajectories, W has a maximum rank, leading to an explicit and unique solution for $\hat{\chi}$.

$$\hat{\chi}=\left(\left(W^T W\right)^{-1} W^T\right)Y=W+Y \tag{7}$$

In practice, the identified values are estimated with their standard deviation by assuming that W is deterministic and that ρ is a centered random vector with independent components, standard deviation σ_ρ, and covariance matrix C_ρ given by:

$$C_\rho = E\left(\rho\rho^T\right) = \sigma_\rho^2 I_r \tag{8}$$

where, I_r is the r-dimensional identity matrix. Assuming that the error vector is centered and has independent components with equal variances, the standard deviation σ_ρ can be calculated using the following unbiased estimator:

$$\sigma_\rho^2 = \|Y - W\hat{\chi}\|^2 / (r-b) \tag{9}$$

The expression for the covariance matrix of the estimation error is:

$$C_{\hat{\chi}} = \sigma_\rho^2 (W^T W)^{-1} \tag{10}$$

We deduce the standard deviation:

$$\sigma_{\hat{\chi}_j} = \sqrt{\mathbf{C}_{\hat{\chi}}(\mathbf{j}, \mathbf{j})} \tag{11}$$

The relative standard deviation is estimated by:

$$\sigma_{\hat{\chi}_{jr\%}} = 100\sigma_{\hat{\chi}_j} / |\hat{\chi}_j| \tag{12}$$

While [13] has used this interpretation, it should be approached with caution in our case as the assumption of a deterministic W is not satisfied. The proposed model is not perfect, and the measurements are noisy, requiring preprocessing.

Although this criterion can be used to evaluate the quality of identification, the fact that W is not deterministic, and the experimental data is noisy poses a challenge. To overcome this, [13] suggests filtering both Y and the columns of W.

C. Conclusion

The LLS approach is particularly advantageous because it avoids the need to integrate a differential system and eliminates issues with initial conditions. However, calculating velocities and accelerations via bandpass filtering of position is required. Lastly, the direct dynamic model (DDM) provided below must be validated through post-simulation.

$$M(\theta)\ddot{\theta}=\tau-N(\theta,\dot{\theta}) \tag{13}$$

Given that M(q) is a positive definite square matrix, the accelerations can be expressed as:

$$\ddot{\theta}=M^{-1}(\theta)(\tau-N(\theta,\dot{\theta})) \tag{14}$$

III. Method IV and DIDIM

A. Method IV

The statistical assumptions necessary for the LLS estimator to work efficiently are not met in practical applications. Equation (5) involves constructing the observation matrix W using joint positions $\boldsymbol{\theta}$, as well as numerically computed derivatives $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$, making W noisy. Additionally, the identification process is performed in a closed loop, further violating the assumptions. As a result, the LLS estimator may be inconsistent.

The IV method addresses this issue [5]. By constructing an instrumental matrix V, the IV method proposes a consistent estimator that satisfies:

$$V^T Y = V^T W_\chi + V^T \rho_{iv} = V^T W \hat{\chi}_{iv} \tag{15}$$

The solution in the sense of the instrumental variable is:

$$\hat{\chi}_{iv} = (V^T W)^{-1} V^T Y \tag{16}$$

Later on, V is computed as a function of $\hat{\chi}^k$ (since only the IV method is discussed in this subsection, we use $\hat{\chi}^k$ instead of $\hat{\chi}_{iv}^k$ as there is no ambiguity). This defines an iterative procedure as follows:

$$\hat{\chi}^{k+1} = (V_k^T W)^{-1} V_k^T Y \tag{17}$$

where,

$$V_k = V(\hat{\chi}^k) \tag{18}$$

Given that V_k^T W matrix is invertible, the following assumptions are made:

$$\lim_{m\to\infty}(V_k^T W / m) \tag{19}$$

$$\lim_{m\to\infty}(V_k^T \rho_{iv} / m) = E(V_k^T \rho_{iv}) \tag{20}$$

Each value of $\hat{\chi}^k$ converges to χ with m, where m denotes the number of repetitions of W and ρ_{iv}.

One of the primary challenges is to determine an instrumental matrix V. One possible solution involves constructing an observation matrix using simulated data rather than measured data. These simulated data, known as instruments, are the outputs of an auxiliary system that approximates the noise-free model of the system to be identified [6][8]. In robotics, this auxiliary model is the robot's DDM, given by equation (13) [14]. The IV method adapted for identifying a robot's dynamics can be outlined by the following algorithm, as illustrated in Fig. 1:

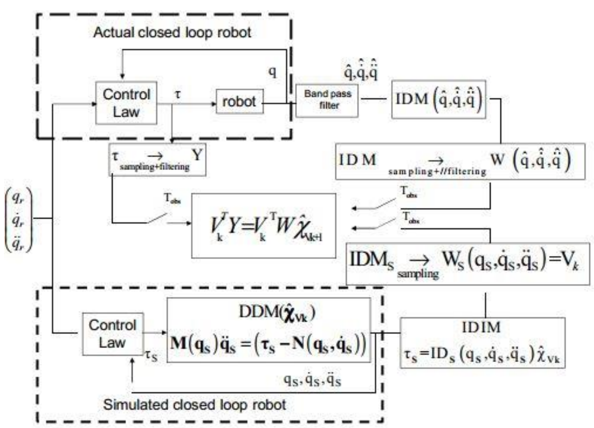

Fig. 1. Instrumental variable procedure.

*During each iteration, θ_S, $\dot{\theta}_S$, and $\ddot{\theta}_S$ are calculated by simulating and integrating the robot's DDM with the parameters identified in the previous iteration. The same control structure and exciting trajectories used for the real robot are applied. W_S is obtained by sampling IDM (θ_S, $\dot{\theta}_S$, $\ddot{\theta}_S$), and the instrumental matrix is selected as follows:

$$V(\hat{\chi}^k) = W_s\left(\theta_s(\hat{\chi}^k), \dot{\theta}_s(\hat{\chi}^k), \ddot{\theta}_s(\hat{\chi}^k)\right) \tag{21}$$

*Y(τ) and W(θ,$\dot{\theta}$,$\ddot{\theta}$) they are constructed as in (5).

* $\hat{\chi}^{k+1}$ is given by (17). The algorithm stops when the relative errors become negligible:

$$\left(\left\|\rho_{iv}^{k+1}\right\| - \left\|\rho_{iv}^{k}\right\|\right) / \left\|\rho_{iv}^{k}\right\| \le tol_1 \tag{22}$$

$$\max_{1,\ldots,b}\left|\left(\hat{\chi}^{k+1} - \hat{\chi}^k\right) / \hat{\chi}^k\right| \le tol_2 \tag{23}$$

Where tol_1 is an ideally small value set by the user. Typically, there is a trade-off between accuracy and convergence speed.

It has been demonstrated in [15][16] that applying a filter F(s) to the columns of V is not mandatory. However, to reduce the sizes of Y, W, and V, we use a sub-sampling filter and isolate the frequency range of interest. Typically, the cutoff frequency of this filter is set to 10 times the closed-loop system's bandwidth value to achieve a balance between precision and convergence speed.

B. DIDIM Method

DIDIM is a closed-loop output error (CLOE) identification method that does not require position measurement [9]. The output y = τ is the actual torque τ. The simulated output $y_s = \tau_{ddm}$ is the simulated torque of the DDM given by (13).

The signal $\theta_{ddm}(t, \chi)$ is the result of integrating the DDM. The optimal solution $\hat{\chi}$ minimizes the quadratic criterion J(χ) = $\|Y_s - Y\|^2$. The vectors Y(τ) and $Y_S = Y(\tau_{ddm})$ are obtained by sampling the vectors τ and τ_{ddm} respectively.

The solution to this nonlinear problem is obtained through the application of Gauss-Newton regression. This approach relies on a Taylor series expansion of y_s around the current estimate of parameters at time k, represented by $\hat{\chi}^k$ (given that this subsection only pertains to the DIDIM method, we use the notation $\hat{\chi}^k$ instead of $\hat{\chi}^k_{didim}$ for clarity):

$$y_s(\chi^{k+1}) = y_s(\chi^k) + \left(\partial(y_s(\chi))/\partial\chi\right)_{\hat{\chi}^k}\left(\chi^{k+1} - \hat{\chi}^k\right) + o \tag{24}$$

Where $\left(\partial(y_s(\chi))/\partial\chi\right)_{\hat{\chi}^k}$ is the Jacobian matrix of dimension (nxb). The MDD input torque τ_{ddm} can be calculated analytically with the MDI expression (3) such that:

$$\begin{aligned} y_s(\chi) &= \tau_{ddm}(\chi) = \tau_{idm}(\chi) = \\ & IDM\left(\theta_{ddm}(\chi), \dot{\theta}_{ddm}(\chi), \ddot{\theta}_{ddm}(\chi)\right)\chi \end{aligned} \tag{25}$$

In this case, the Jacobian matrix is given by:

$$\begin{aligned} \left(\frac{\partial(y_s)}{\partial\chi}\right)_{\hat{\chi}^k} &= \left(\frac{\partial(\tau_{ddm})}{\partial\chi}\right)_{\hat{\chi}^k} = \left(\frac{\partial(\tau_{idm})}{\partial\chi}\right)_{\hat{\chi}^k} \\ &= \frac{\partial}{\partial\chi}\left(IDM\left(\theta_{ddm}(\hat{\chi}^k), \dot{\theta}_{ddm}(\hat{\chi}^k), \ddot{\theta}_{ddm}(\hat{\chi}^k)\right)\hat{\chi}^k\right) \end{aligned} \tag{26}$$

As we use the same control for both the simulation and the real robot, the simulated states (positions, velocities, and accelerations) are minimally dependent on χ. At each $\hat{\chi}^k$ value, the Jacobian matrix (26) can be approximated as:

$$\left(\frac{\partial(y_s)}{\partial\chi}\right)_{\hat{\chi}^k} = IDM\left(\theta_{ddm}(\hat{\chi}^k),\dot{\theta}_{ddm}(\hat{\chi}^k),\ddot{\theta}_{ddm}(\hat{\chi}^k)\right) \tag{27}$$

Taking into account (27), the Taylor expansion becomes:

$$y = \tau = IDM\left(\theta_{ddm}(\hat{\chi}^k),\dot{\theta}_{ddm}(\hat{\chi}^k),\ddot{\theta}_{ddm}(\hat{\chi}^k)\right)\chi^{k+1} + (o+e) \tag{28}$$

The MDI (3) estimates the states $(\theta, \dot{\theta}, \ddot{\theta})$ using $(\theta_{ddm}, \dot{\theta}_{ddm}, \ddot{\theta}_{ddm})$ obtained by simulating and integrating (13). At each iteration k, we sample (28) to obtain an over-determined linear system given by:

$$Y = W_{ddm}\left(\theta_{ddm}(\hat{\chi}^k),\dot{\theta}_{ddm}(\hat{\chi}^k),\ddot{\theta}_{ddm}(\hat{\chi}^k)\right)\chi^{k+1} + \rho_{didim} \tag{29}$$

The MCLs are then applied to obtain the estimate at k+1 denoted $\hat{\chi}^{k+1}$.

$$\hat{\chi}^{k+1} = (W_{ddm}^T W_{ddm})^{-1} W_{ddm}^T Y \tag{30}$$

This algorithm is iterated until:

$$\left(\left\|\rho_{didim}^{k+1}\right\| - \left\|\rho_{didim}^{k}\right\|\right) / \left\|\rho_{didim}^{k}\right\| \leq tol_1 \tag{31}$$

$$\max_{1,\ldots,b}\left|\left(\hat{\chi}^{k+1} - \hat{\chi}^k\right) / \hat{\chi}^k\right| \leq tol_2 \tag{32}$$

Where tol_1 is an ideally small value set by the user. Generally, a compromise is made between accuracy and speed of convergence.

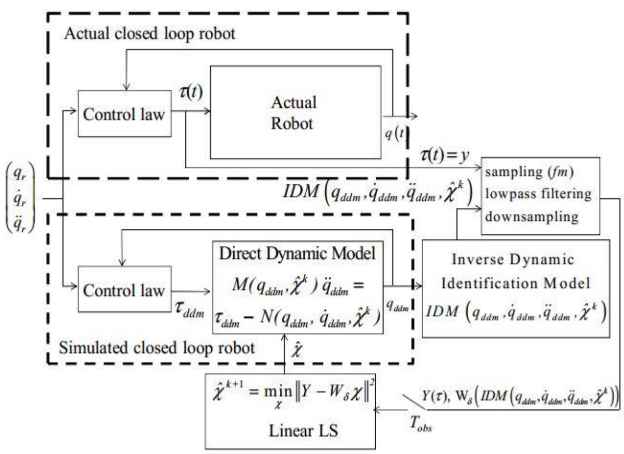

Fig. 2. Procedure of the DIDIM method.

The DIDIM method is named after its use of both direct and inverse dynamic models. As shown in Fig. 2, this approach combines these models to achieve dynamic identification.

C. MDD Simulation and Integration

According to [8], the instrumental matrix must be close to the W_{nf} matrix defined by:

$$W_{nf} = W(\theta_{nf}, \dot{\theta}_{nf}, \ddot{\theta}_{nf}) \tag{33}$$

where, $\theta_{nf}, \dot{\theta}_{nf}, \ddot{\theta}_{nf}$ are the noiseless values of $\theta, \dot{\theta}, \ddot{\theta}$.

Assuming model errors are negligible, it is necessary to have a well-tuned control loop that keeps θ_{nf}, $\dot{\theta}_{nf}$ and $\ddot{\theta}_{nf}$ close to the reference states θ_r, $\dot{\theta}_r$ and $\ddot{\theta}_r$. According to (33), the simulated states θ_s, $\dot{\theta}_s$ and $\ddot{\theta}_s$ must remain close to θ_r, $\dot{\theta}_r$ and $\ddot{\theta}_r$ at every iteration of the algorithm. To achieve this, we use the same control structure for simulation as we do for the robot and adjust the control gains at each iteration to maintain the closed-loop bandwidth. This ensures that the bandwidth remains constant, regardless of the estimate $\hat{\chi}^k$. Therefore, we obtain:

$$(\theta_{nf}, \dot{\theta}_{nf}, \ddot{\theta}_{nf}) = (\theta_s, \dot{\theta}_s, \ddot{\theta}_s) = (\theta_r, \dot{\theta}_r, \ddot{\theta}_r) \forall \hat{\chi}_{iv}^k \tag{34}$$

Adapting the control gains at each iteration in the simulator allows us to ensure the approximation (27). Thus, (34) becomes:

$$(\theta_{nf}, \dot{\theta}_{nf}, \ddot{\theta}_{nf}) = (\theta_{ddm}, \dot{\theta}_{ddm}, \ddot{\theta}_{ddm}) = (\theta_r, \dot{\theta}_r, \ddot{\theta}_r) \forall \hat{\chi}_{didim}^k \tag{35}$$

This relation allows us to write that we have:

$$(\theta_{ddm}, \dot{\theta}_{ddm}, \ddot{\theta}_{ddm}) = (\theta_s, \dot{\theta}_s, \ddot{\theta}_s) \forall \hat{\chi}_{iv}^k \, and \, \forall \hat{\chi}_{didim}^k \tag{36}$$

We arrive at the following results: the matrix W_{ddm} is precisely our instrumental matrix V, and (30) can be expressed in this way as well:

$$\hat{\chi}_{didim}^k = (V_k^T V_k)^{-1} V_k^T Y \tag{37}$$

The article does not provide an explanation of gain modulation at each iteration as it is beyond the scope of the paper. A brief overview can be found in [15],[16].

D. Initialization of Algorithms

There are multiple methods to initialize the algorithm, such as using the CAD values or the identified LS values. However, since we ensure (34) and (35) in simulation, the easiest approach is to set $Z\hat{Z}_{j/0}$ and the other parameters to 0, which is called regular initialization [9]. This method results in an invertible initial mass matrix.

IV. Prototype to Identify and Results

We employ the IV and DIDIM methods on a two-degree-of-freedom planar robot depicted in Fig. 3, utilizing the modified Denavit-Hartenberg notation for geometric representation. The robot operates without a reducer (direct drive) and is actuated using DC motors. The inverse dynamic model is dependent on 8 minimal parameters $\chi = [ZZ_{1R}\ fv_1\ fs_1\ ZZ_2\ MX_2\ MY_2\ fv_2\ fs_2]^T$. Position control of the robot is achieved through a PD controller, with a closed-loop bandwidth of 2Hz, and an acquisition frequency of 200Hz. The torque is obtained from the current reference vir, with the current loop having a broad bandwidth (1KHz):

$$\tau_j = gt_j v_{irj} \tag{38}$$

gt_j being the j-axis actuation gain.

Fig. 3. Prototype 2 DDL plan to identify.

Experimental application of the IV and DIDIM methods is carried out on the 2-degree-of-freedom planar prototype. In the IV method, the current command image and the position of each motor are measured, while only the current command image of each motor is measured in the DIDIM method.

In both methods, the columns of V are filtered using an under-sampling filter with a cutoff frequency of 20Hz, as the closed-loop bandwidth is 2Hz. All information regarding the rigid model is preserved, and the MATLAB decimate command is utilized for filter implementation.

Both methods use identical exciting trajectories and control structures as those used on the actual robot for simulation. The control gains are adjusted for every iteration.

In the case of the IV method, a fourth-order bidirectional Butterworth filter is used to filter the position, with a cutoff frequency of 20Hz. The velocity and acceleration are obtained through a centered difference calculation, ensuring that there is no phase distortion. Lastly, each column of W is filtered using the same under sampling filter as that used for filtering the columns of V.

Both methods are initialized with regular initialization. The simulation and MDD integration are carried out on the same MATLAB-SIMULINK platform for both methods. The platform is run on a laptop equipped with an INTEL Pentium 4 single-core processor operating on WINDOWS XP. Each iteration, which includes MDD integration and optimal solution calculation, takes approximately thirty seconds.

The experimental results are reported in Table I for the IV method and in Table II for the DIDIM method. Table III shows the convergence of the parameters, with both algorithms converging in just three iterations.

The key finding is that the IV method provides the same numerical estimation as the DIDIM method. The only difference is in the values of the parameters Fv1 and Fv2, which have minimal impact on the robot's dynamics. As these parameters have a high relative standard deviation, their removal from the model results in little variation in the identified values of the other parameters and the residual norm.

From these experimental results, we can conclude that numerically $\hat{\chi}_{IV} = \hat{\chi}_{DIDIM}$. This can be explained by the following intuitive reasoning: the instruments are constructed from a simulation and integration of the MDD. Therefore, if the instruments are representative of the model to be identified, then we can assume that W = V + w where w is a noise matrix of dimension (rxb). Furthermore, if each column of w denoted by wk is orthogonal to the space spanned by the columns of V, then we have $V^T w$ = zeros(b,b). With these conditions, we do indeed obtain $\hat{\chi}_{IV} = \hat{\chi}_{DIDIM}$.

The next section will present the method of least squares, which we will use to prove this statement.

TABLE I. IDENTIFICATION WITH IV

Parameters	$\hat{\chi}^0$	$\hat{\chi}^3$	$2\sigma_{\hat{\chi}}$	$\%\sigma_{\hat{\chi}}$
ZZ_{1R}	1.0	3.45	0.036	0.52
F_{v1}	0.0	0.04	0.032	40.0
F_{c1}	0.0	0.82	0.05	3.0
ZZ_2	1.0	0.061	0.0006	0.49
LMX_2	0.0	0.124	0.0013	0.52
LMY_2	0.0	0.0065	0.0005	3.5
F_{v2}	0.0	0.013	0.0084	30.0
F_{c2}	0.0	0.137	0.008	3.0

TABLE II. IDENTIFICATION WITH DIDIM

Parameters	$\hat{\chi}^0$	$\hat{\chi}^3$	$2\sigma_{\hat{\chi}}$	$\%\sigma_{\hat{\chi}}$
ZZ_{1R}	1.0	3.45	0.036	0.52
F_{v1}	0.0	0.03	0.030	40.0
F_{c1}	0.0	0.82	0.05	3.0
ZZ_2	1.0	0.061	0.0006	0.49
LMX_2	0.0	0.124	0.0013	0.52
LMY_2	0.0	0.0067	0.0005	3.5
F_{v2}	0.0	0.015	0.0084	30.0
F_{c2}	0.0	0.137	0.008	3.0

TABLE III. CONVERGENCE OF VALUES FOR THE TWO METHODS

Parameters	$\hat{\chi}^0$	$\hat{\chi}^1$	$\hat{\chi}^2$	$\hat{\chi}^3$
ZZ_{1R}	1.0	3.46	3.45	3.45
F_{v1}	0.0	0.04	0.02	0.03
F_{c1}	0.0	0.82	0.85	0.82
ZZ_2	1.0	0.06	0.061	0.061
LMX_2	0.0	0.122	0.124	0.124
LMY_2	0.0	0.05	0.068	0.067
F_{v2}	0.0	0.005	0.014	0.015
F_{c2}	0.0	0.135	0.137	0.137

V. DOUBLE LEAST SQUARES METHOD

A. General Idea

As stated in [4], Theil introduced the method of two-stage least squares in 1953, and independently, Basmann also introduced it in 1957 for simultaneous equation modeling. The Two-Stage Least Squares (2SLS) approach involves estimation in two stages:

- During the first stage, we carry out a regression of each column of W, denoted by $W_{:,k}$, on V, to separate the part of $W_{:,k}$ that is correlated with ρ from the part that is correlated with the model. This leads to an estimation of $W_{:,k}$, denoted b $\hat{W}_{:,k}$. By concatenating the estimated columns $\hat{W}_{:,k}$, we obtain an estimate of the matrix W, denoted by $\hat{W}$.

- To perform the second stage, we regress Y on $\hat{W}$. Thus, we obtain the following general solution:

$$\hat{\chi}_{2SLS}=(\hat{W}^T\hat{W})^{-1}\hat{W}^TY \tag{39}$$

This is the 2SLS solution.

B. Implementation

Typically, when using 2SLS, the first regression stage for each column of W is defined as follows:

$$W_{:,k}=V\prod_k+w_k \tag{40}$$

Where Π_k is a coefficient vector of dimension (bx1) and where w_k is a noise vector of dimension (rx1). We obtain the estimate of each column $W_{:,k}$ as:

$$\hat{W}_{:,k}=V\hat{\prod}_k \tag{41}$$

After performing this regression for each column of W and concatenating the estimated columns, we obtain a matrix equation in the form:

$$\hat{W}=V\hat{\prod} \tag{42}$$

Where $\hat{\prod}=\left\lfloor\hat{\prod}_1\ldots\hat{\prod}_k\ldots\hat{\prod}_b\right\rfloor$ is a matrix of constant coefficients of dimension (bxb). This relationship also involves:

$$W=\hat{W}+w=V\hat{\prod}+w \tag{43}$$

The second step is the regression of Y on $\hat{W}$. We have:

$$\hat{W}=V\hat{\prod}=V(V^TV)^{-1}V^TW \tag{44}$$

By incorporating this relation into (39) and assuming that relation (19) holds, we obtain:

$$\hat{\chi}_{2SLS}=(V^TW)^{-1}V^TY \tag{45}$$

$$\hat{\chi}_{2SLS}=\hat{\chi}_{IV} \tag{46}$$

Therefore, the 2SLS solution is equivalent to the instrumental variable solution.

C. Using 2SLS to Compare IV and DIDIM

Using the 2SLS algorithm to identify the dynamic parameters of robots is feasible, given our knowledge on constructing the instrumental matrix V. However, our objective is to establish a link between the IV method and the DIDIM method.

By utilizing 2SLS, we can examine the projection of each column $W_{:,k}$ of W (and thus, W as a whole) onto the space formed by the columns of V. Equation (44) denotes the orthogonal projection of each column $W_{:,k}$ onto the space generated by the columns of V. To conduct the first regression of each column $W_{:,k}$ on V, we express it as follows:

$$W_{:,k}=\left(\text{ones}(r,1)\ V\right)\prod_k+w_k \tag{47}$$

Where

$$V=\left[V_{:,1}\ldots V_{:,b}\right] \tag{48}$$

$$\prod_k=\left[\pi_{0,k}\pi_{1,k}\ldots\pi_{b,k}\right]^T$$

An offset term denoted as $\pi_{0,k}$ is deliberately introduced to estimate the average value of the residual wk. The estimated value of this term, denoted as $\hat{\pi}_{0,k}$, is expected to be zero in theory.

Once we have obtained the estimates, we can interpret the results as follows:

$$\hat{\pi}_{j,k}=1 \quad \text{pour } j=k \tag{49}$$

$$\hat{\pi}_{j,k}=0 \quad \text{pour } j\neq k \tag{50}$$

In terms of physicality, this indicates that the instruments are reliable (i.e., adequately reflecting the model to be identified) and that we have effectively separated the portion of $W_{:,k}$ that correlates with ρ and the portion that corresponds to the model. Through concatenation, we achieve:

$$\hat{W}=\left[\text{ones}(r,1)\ V\right]\hat{\Pi} \tag{51}$$

With:

$$\hat{\Pi}=\begin{bmatrix}\text{zeros}(1,b)\\ I_{b,b}\end{bmatrix} \tag{52}$$

$I_{b,b}$ represents the identity matrix with dimensions (bxb). Now, let's delve into the details of this relationship:

$$\hat{W}=\text{zeros}(r,b)+VI_{b,b}=V \tag{53}$$

So, with (39) we get:

$$\hat{\chi}_{2SLS}=(V^TV)^{-1}V^TY=\hat{\chi}_{didim} \tag{54}$$

Or, according to (46):

$$\hat{\chi}_{2SLS}=\hat{\chi}_{iv}=\hat{\chi}_{didim} \tag{55}$$

Another way to arrive at this result is by expressing that we have, using equations (43) and (53):

$$W=V+w \tag{56}$$

Alternatively, using geometric construction, we can establish that every residual wk is perpendicular to the subspace formed by the columns of V (normal equation). Consequently, this leads to the following implication:

$$V^Tw=\text{zeros}(b,b) \tag{57}$$

So by incorporating (56), (57) into (16), we get:

$$\hat{\chi}_{iv}=(V^TV)^{-1}V^TY \tag{58}$$

And we find well (55).

D. Conclusion

Through the utilization of 2SLS, we have established that the solution yielded by the IV method is equivalent to the solution obtained through DIDIM, given that relations (49) and (50) are upheld. These two relationships hold great significance as they imply the following physical interpretations:

- The instruments demonstrate high quality, effectively capturing the essence of the physical model under consideration. This has allowed us to successfully separate the component of W that correlates with the model from the component that correlates with ρ.
- Moreover, the model error remains significantly minimal in comparison to other sources of noise.

During the experimental phase, we conduct the dual regression at every iteration of the IV method. Furthermore, we specify the necessary conditions to ensure the continuous validation of both relationships (49) and (50).

VI. Experimental Results and Analysis

The experimental trials are carried out using identical conditions as outlined in Section IV. The dual regression is executed for every iteration and each column of W. The resulting estimated vectors $\hat{\Pi}_k$ are stored in an array for subsequent analysis of the estimations.

The determined coefficient values for each $\hat{\Pi}_k$ at every iteration are consolidated in Table IV. To improve clarity, the results for each column are presented individually, Tables IV to XI.

TABLE IV. Estimates of ΠJ, J = 1

Parameter	$\hat{\Pi}_1^0$	$\hat{\Pi}_1^1$	$\hat{\Pi}_1^2$	$\hat{\Pi}_1^3$
$\pi_{0,1}$	0.0	0.0	0.0	0.0
$\pi_{1,1}$	1.0	1.0	1.0	1.0
$\pi_{2,1}$	0.001	0.0	-0.001	0.001
$\pi_{3,1}$	0.0	0.0	0.0	0.0
$\pi_{4,1}$	0.0	0.0	0.0	0.0
$\pi_{5,1}$	0.0	0.0	0.0	0.0
$\pi_{6,1}$	0.002	0.001	0.002	-0.001
$\pi_{7,1}$	-0.001	0.001	0.002	0.001
$\pi_{8,1}$	0.0	0.0	0.0	0.0

TABLE V. Estimates of ΠJ, J = 2

Parameter	$\hat{\Pi}_2^0$	$\hat{\Pi}_2^1$	$\hat{\Pi}_2^2$	$\hat{\Pi}_2^3$
$\pi_{0,2}$	0.001	0.0	-0.001	0.001
$\pi_{1,2}$	0.002	0.002	-0.001	-0.001
$\pi_{2,2}$	1.001	1.002	1.001	1.001
$\pi_{3,2}$	0.001	0.002	0.001	-0.002
$\pi_{4,2}$	-0.002	-0.002	0.002	0.002
$\pi_{5,2}$	-0.002	-0.001	0.0	-0.001
$\pi_{6,2}$	0.001	0.001	0.002	-0.001
$\pi_{7,2}$	0.001	0.001	0.002	0.001
$\pi_{8,2}$	-0.002	-0.002	-0.002	-0.002

TABLE VI. Estimates of ΠJ, J = 3

Parameter	$\hat{\Pi}_3^0$	$\hat{\Pi}_3^1$	$\hat{\Pi}_3^2$	$\hat{\Pi}_3^3$
$\pi_{0,3}$	0.0	0.0	0.0	0.0
$\pi_{1,3}$	0.0	0.0	0.0	0.0
$\pi_{2,3}$	-0.001	0.0	-0.001	-0.001
$\pi_{3,3}$	1.0	1.0	1.0	1.0
$\pi_{4,3}$	0.0	0.0	0.0	0.0
$\pi_{5,3}$	0.0	0.0	0.0	0.0
$\pi_{6,3}$	0.001	0.001	0.002	-0.001
$\pi_{7,3}$	0.001	0.001	-0.002	0.001
$\pi_{8,3}$	0.0	0.0	0.0	0.0

TABLE VII. Estimates of ΠJ, J = 4

Parameter	$\hat{\Pi}_4^0$	$\hat{\Pi}_4^1$	$\hat{\Pi}_4^2$	$\hat{\Pi}_4^3$
$\pi_{0,4}$	0.0	0.0	0.0	0.0
$\pi_{1,4}$	0.0	0.0	0.0	0.0
$\pi_{2,4}$	0.0	0.0	-0.001	-0.001
$\pi_{3,4}$	0.0	0.0	0.0	0.0
$\pi_{4,4}$	1.0	1.0	1.0	1.0
$\pi_{5,4}$	0.0	0.0	0.0	0.0
$\pi_{6,4}$	0.002	0.0	0.002	-0.001
$\pi_{7,4}$	0.0	0.001	-0.002	-0.002
$\pi_{8,4}$	0.0	0.0	0.0	0.0

TABLE VIII. Estimates of ΠJ, J = 5

Parameter	$\hat{\Pi}_5^0$	$\hat{\Pi}_5^1$	$\hat{\Pi}_5^2$	$\hat{\Pi}_5^3$
$\pi_{0,5}$	0.0	0.0	0.0	0.0
$\pi_{1,5}$	0.0	0.0	0.0	0.0
$\pi_{2,5}$	0.001	0.001	-0.001	-0.001
$\pi_{3,5}$	0.0	0.0	0.0	0.0
$\pi_{4,5}$	0.0	0.0	0.0	0.0
$\pi_{5,5}$	1.0	1.0	1.0	1.0
$\pi_{6,5}$	0.0	0.0	0.001	-0.001
$\pi_{7,5}$	0.0	0.001	0.0	0.001
$\pi_{8,5}$	0.0	0.0	0.0	0.0

TABLE IX. Estimates of ΠJ, J = 6

Parameter	$\hat{\Pi}_6^0$	$\hat{\Pi}_6^1$	$\hat{\Pi}_6^2$	$\hat{\Pi}_6^3$
$\pi_{0,6}$	0.002	0.001	0.002	0.001
$\pi_{1,6}$	-0.001	0.001	-0.001	0.002
$\pi_{2,6}$	-0.001	-0.001	-0.001	-0.001
$\pi_{3,6}$	0.001	0.002	0.001	0.002
$\pi_{4,6}$	-0.001	-0.002	-0.002	-0.001
$\pi_{5,6}$	-0.001	-0.001	-0.001	0.001
$\pi_{6,6}$	0.998	0.999	0.998	0.998
$\pi_{7,6}$	0.002	0.001	-0.001	-0.002
$\pi_{8,6}$	0.002	0.002	0.001	0.002

TABLE X. ESTIMATES OF Π_J, J = 7

Parameter	$\hat{\Pi}_7^0$	$\hat{\Pi}_7^1$	$\hat{\Pi}_7^2$	$\hat{\Pi}_7^3$
$\pi_{0,7}$	-0.002	-0.001	0.002	-0.001
$\pi_{1,7}$	-0.001	0.001	-0.001	0.002
$\pi_{2,7}$	0.001	-0.001	-0.002	0.001
$\pi_{3,7}$	-0.002	0.001	-0.001	-0.002
$\pi_{4,7}$	0.001	0.002	-0.001	0.002
$\pi_{5,7}$	-0.002	0.001	-0.001	0.002
$\pi_{6,7}$	0.002	0.002	0.001	0.002
$\pi_{7,7}$	1.002	0.999	1.001	0.998
$\pi_{8,7}$	-0.002	0.002	0.001	-0.002

TABLE XI. ESTIMATES OF Π_J, J = 8

Parameter	$\hat{\Pi}_8^0$	$\hat{\Pi}_8^1$	$\hat{\Pi}_8^2$	$\hat{\Pi}_8^3$
$\pi_{0,8}$	0.0	0.0	0.0	0.0
$\pi_{1,8}$	0.0	0.0	0.0	0.0
$\pi_{2,8}$	-0.001	-0.002	-0.001	0.002
$\pi_{3,8}$	0.0	0.0	0.0	0.0
$\pi_{4,8}$	0.0	0.0	0.0	0.0
$\pi_{5,8}$	0.0	0.0	0.0	0.0
$\pi_{6,8}$	0.001	-0.001	0.001	-0.001
$\pi_{7,8}$	0.002	0.001	-0.002	0.001
$\pi_{8,8}$	1.0	1.0	1.0	1.0

The estimated values obtained through 2SLS are identical to those presented in Table I and Table II. Based on these experimental findings, we can conclude that it is possible to write $\hat{\Pi} = \begin{bmatrix} \text{zeros}(1,b) \\ I_{b,b} \end{bmatrix}$ for each iteration. This suggests that relations (49) and (50) are practically fulfilled. Consequently, we have effectively confirmed through experimentation that we have $\hat{\chi}_{iv} = \hat{\chi}_{didim}$.

The slight discrepancies we observe could be attributed to minor modeling errors.

The crucial aspect of this analysis revolves around relations (36), (49), and (50). Essentially, these relations indicate that we can treat the states simulated by the IV method and the DIDIM method as interchangeable, and that column $W_{:,k}$ projects orthogonally onto its counterpart $V_{:,k}$. Importantly, $W_{:,k}$ is not derived from a linear combination of multiple columns of V, which could potentially be the case in an absolute sense. As a result, we effectively differentiate the component of $W_{:,k}$ that is correlated with the model from the component correlated with ρ. The strong adherence of relations (49) and (50) primarily arises from the careful adjustment of control gains during each iteration in the simulation, aiming to ensure that $(\theta_s, \dot{\theta}_s, \ddot{\theta}_s) = (\theta_{ddm}, \dot{\theta}_{ddm}, \ddot{\theta}_{ddm}) = (\theta_{nf}, \dot{\theta}_{nf}, \ddot{\theta}_{nf})$ closely approximates $(\theta_r, \dot{\theta}_r, \ddot{\theta}_r)$, while keeping modeling errors at a minimum. Experiments were conducted with significant modeling errors. For example, we intentionally omitted the term LMX_2 from the model. As a result, both methods converge to an inaccurate solution. Consequently, relations (49) and (50) are no longer fulfilled. This serves as evidence that our projections are flawed because Y is being regressed on a subspace that no longer accurately represents the model.

A. The Parameters Used

To overcome these issues, an alternative identification approach is proposed. This method relies on a closed-loop simulation, where the direct dynamic model is used with the same control law and reference trajectories as those applied to the real robot. The parameters obtained through this identification method are determined by minimizing the 2-norm of the error between the measured torque and the simulated torque. This results in a nonlinear least squares problem. The analytical expression of the sensitivity functions is greatly simplified by using the inverse model to express the simulated torque, which greatly facilitates the calculation of the solution.

In the robot identification procedure, the inverse dynamic model (IDM) and the linear least squares (LLS) method are commonly used. To create an overdetermined system, the IDM is sampled during the robot's motion using stimulating trajectories. This approach has proven to be effective in identifying numerous robots and prototypes [17], [20], [3].

However, it requires accurate measurements of joint positions and torques at a high sampling frequency (greater than 100 Hz). Furthermore, the identification is performed in a closed loop due to the inherent instability of the double integrator, resulting in a noisy observation matrix. Consequently, in theory, the linear least squares estimator may exhibit bias [19].

To address this issue, we have adapted the instrumental variables (IV) technique, building upon the research work conducted by Hugues Garnier's team [6], [9], [8]. Recently, a new identification method has been proposed and validated on a two-degree-of-freedom robot [18]. This approach only requires the joint torques as input parameters. The robot is simulated in a closed loop, assuming the same control structure and stimulating trajectories.

The optimal parameters are determined by minimizing the squared difference between the simulated and measured torques. Experimental results demonstrate a significant correlation between the outcomes obtained using the instrumental variables (IV) method and those of the DIDIM method, indicating a strong agreement between these two approaches.

VII. CONCLUSION

This paper presents a comparative analysis of the IV and DIDIM methods using 2SLS. The theoretical framework is substantiated by experimental findings. The results of this study demonstrate that under specific conditions, we achieve a numerical equality of $\hat{\chi}_{iv} = \hat{\chi}_{didim}$.

From our standpoint, this outcome holds significance as it indicates that the IV method, as employed in robot identification and widely used in various applications, tends to converge towards the model-based approach. This observation provides a possible explanation for the IV method's resilience to assumptions made about noise. However, it would be overly simplistic, and possibly incorrect, to equate the IV method with the model-based method. The outcome relies on the construction of our instruments and how they are implemented.

In the end, 2SLS could serve as a diagnostic tool, enabling us to examine the projection of each regressor column onto the space formed by the instrumental matrix columns. The MDI and DIDIM methods will be implemented on cable-driven parallel structure interfaces, such as the VIRTUOSE robots developed by HAPTION; also, will explore other methods for robot enhancement.

REFERENCES

[1] A. Janot, C. Bidard, F. Gosselin, M. Gautier, D. Keller, and Y. Perrot, “Modeling and identification of a 3 DOF haptic interface,” Proc. of 2007 IEEE International Conference on Robotics and Automation, 2007, pp. 4949-4955.

[2] G. Venture, P. Riper, W. Khalil, M. Gautier, and P. Bodson, “Modeling and identification of passenger car dynamics using robotics formalism,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 7, Sep. 2006, pp. 349-355.

[3] C. Lemaire, P. Vandanjon, M. Gautier, and C. Lemaire, “Dynamic identification of a vibratory asphalt compactor for contact efforts estimation,” *14th IFAC Symposium on System Identification, 2006 System Identification*, B. Ninness, Ed., Australia: 2006, pp. 973-978.

[4] R. Davidson and J. Mackinnon, *Estimation and Inference in Econometrics*, Oxford University Press Inc, 1993.

[5] P. Young and A. Jakeman, “Refined instrumental variable methods of time-series analysis: Part 1, SISO systems,” *International Journal of Control*, vol. 29, 1979, pp. 1-30.

[6] P. Young, H. Garnier, and M. Gilson, “Simple Refined IV Methods of Closed-Loop System Identification,” *Proc. of 15th IFAC Symposium on System Identification, SYSID 2009*, Saint-Malo (France): 2009, pp. 1151-1156.

[7] H. Garnier, M. Gilson, and P. Cervellin, “Latest developments for the matlab contsid toolbox,” *14th IFAC Symposium on System Identification, SYSID-2006*, Newcastle, Australia: 2006.

[8] H. Garnier and L. Wang (Eds), *Identification of Continuous-time Models from Sampled Data*, Springer, 2008.

[9] M. Gautier, A. Janot, and P. Vandanjon, “A new closed-loop output error method for parameter identification of robot dynamics,” *http://hal.archives-ouvertes.fr/hal-00520258/fr/*, Sep. 2010.

[10] W. Khalil and E. Dombre, *Modélisation identification et commande* des robots, Hermès, 1999.

[11] M. Gautier, A. Janot, and P. Vandanjon, “A new closed-loop output error method for parameter identification of robot dynamics,” http://hal.archives-ouvertes.fr/hal-00520258/fr/, Sep. 2010.

[12] M. Gautier and W. Khalil, “Exciting trajectories for the identification of the inertial parameters of robots,” *International Journal of Robotics Research*, vol. 11, Aug. 1992, pp. 362-375.

[13] M. Gautier, “Dynamic Identification of Robots with Power Model,” *Proc. of IEEE International Conference on Robotics and Automation*, Albuquerque, USA: 1997, pp. 1922-1927.

[14] A. Janot, P. Vandanjon, and M. Gautier, “Refined Instrumental Variable method for non-linear dynamic identification of robots,” *http://hal.archives-ouvertes.fr/hal-00520261/fr/*, Sep. 2010.

[15] A. Janot, P. Vandanjon, and M. Gautier, “Identification des paramètres dynamiques des robots avec la méthode de la variable instrumentale,” *Pro. de la Sixième Conférence Internationale Francophone d'Automatique*, Nancy, France: 2010.

[16] A. Janot, P. Vandanjon, and M. Gautier, “Using the Instrumental Variable Method for Robots Identification,” *Proc. of 15th IFAC Symposium on System Identification*, É. Walter, Ed., Saint-Malo (France): 2009, pp. 480-485.

[17] Janot, Alexandre, Peter C. Young, and Maxime Gautier. "Identification and control of electro-mechanical systems using state-dependent parameter estimation." International Journal of Control 90.4 (2017): 643-660.

[18] M. Gautier, A. Janot, and P. Vandanjon, “A new closed-loop output error method for parameter identification of robot dynamics,” http://hal.archives-ouvertes.fr/hal-00520258/fr/, Sep. 2010.

[19] Davidson, Richard J. "Affective neuroscience and psychophysiology: Toward a synthesis." Psychophysiology 40.5 (2003): 655-665.

[20] Bonnet, Vincent, et al. "Optimal exciting dance for identifying inertial parameters of an anthropomorphic structure." IEEE Transactions on Robotics 32.4 (2016): 823-836.