An MILP-based Lexicographic Approach for Robust Selective Full Truckload Vehicle Routing Problem

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***Abstract*—Full truckload (FTL) shipment is one of the largest trucking modes. It is an essential part of the transportation industry, where the carriers are required to move FTL transportation demands (orders) at a minimal cost between pairs of locations using a certain number of trucks available at the depots. The drivers who pick up and deliver these orders must return to their home depots within a given time. In practice, satisfying those orders within a given time frame (e.g., one day) could be impossible while adhering to all operational constraints. As a result, the investigated problem is distinguished by the selective aspect, in which only a subset of transportation demands is serviced. Furthermore, travel times between nodes can be uncertain and vary depending on various possible scenarios. The robustness subsequently consists of identifying a feasible solution in all scenarios. Therefore, this study introduces an MILP-based lexicographic approach to solve a robust selective full truckload vehicle routing problem (RSFTVRP). We demonstrated the proposed method's efficiency through experimental results on newly generated instances for the considered problem.**

Keywords—Vehicle routing problem; full truckload; robust optimization; MILP-based lexicographic approach; uncertain travel time

I. Introduction

The vehicle routing problem (VRP) is one of the various investigated combinatorial optimization problems [1]. Its tendency is due to its concrete application in the logistics and transportation domains. These fields play an essential role in the modern market economy by assuring the movement of goods from factories to customers. The FTVRP is a variation of the VRP that has previously garnered limited scientific interest. This problem has a significant application to the truckload industry, where the carriers must service FTL transportation demands (known in the literature as orders or commodities) at a minimal cost between pairs of locations utilizing an available fleet of trucks. They are given a network of sites with FTL orders to be shipped between some pairs of locations. These orders must be shipped using a certain number of trucks available at the depots. The drivers who pick up and deliver these orders must return to their home depots (domiciles) within a given time. The problem is determining the least-cost truck routes so that every order is picked up at its source and shipped to its destination. A typical truck would leave the depot, pick up a commodity, deliver the commodity, travel empty, pick up another commodity, deliver the commodity, and so on. Finally, after picking up and delivering some orders, the truck returns to its domicile. Moreover, each order must be picked between certain hours only. In other words, every pickup must only be made between a specific pickup time window. Once the order is picked up, it must then be delivered to its destination. Depending on the specific problem, the delivery can be made at any time or within a specified delivery time window only. If a driver reaches a pickup location early, he usually has to wait until the open time. In some instances, the driver may be paid based on a certain hourly rate for staying. The integration of the time constraints in the FTL transportation problem gives rise to the FTVRP with time windows (FTVRPTW). Furthermore, trucking companies can service their clients through numerous depots, with each client assigned preferentially to one depot (multi-depot FTVRP, FTMDVRP).

On the one hand, meeting all of the aforementioned attributes may prevent the trucks from honoring all orders. Therefore, the selective feature of the problem is introduced by relaxing the requirement of servicing all transportation demands within a limited time (selective FTVRP, SFTVRP). The goal may be the maximization of the collected total profit, in which a profit is associated with each order, the minimization of the whole travelling costs, in which the assignment covers as many feasible orders as possible, or the optimization of a combination of both.

On the other hand, travel between a pair of locations might be made via multiple paths. However, the optimal one with the shortest travel time will usually be traversed. In practice, the travel time is uncertain and dependent on specific circumstances (peak traffic hours, weather conditions, accidents, and so on). As a result, this paper investigates a robust selective full truckload multi-depot vehicle routing problem with time windows (RSFTMDVRPTW) under uncertainty in transportation time as a set of discrete scenarios. Each one illustrates an eventual situation that could occur throughout the shipping period. In each scenario, we consider that a fixed number of edges connecting locations are perturbed, and the travel times along those edges differ from the ideal ones. In this study, we formulate the RSFTMDVRPTW as a mixed-integer linear programming (MILP) model under uncertainty and the multi-objective facet, which addresses two functions: an economic component to be maximized and a component related to the worst observation of the total travel time over all scenarios to be minimized. These two objectives are conflicting in that increasing profit necessitates servicing more commodities, resulting in a longer transit time.

Many papers in the literature have been devoted to introducing, formulating, and solving FTVRP variants. Among them, the SFTVRP was solved using mathematical models, exact solvers, meta-heuristics, and hybrid methods. The novelty of the problem under consideration is that the SFTVRP is treated in a robust backhaul trucking context while considering two objectives. As a result, combining the robust and selective aspects helps to bring the model closer to reality.

The remainder of this study is organized as follows: Section II describes some of the related works. Section III then formulates the RSFTMDVRPTW, while Section IV describes our MILP-based lexicographic approach. Section V presents the experimental findings. Finally, Section VI concludes the paper and suggests some future research directions.

II. Related Works

Over the last two decades, many researchers have studied various FTVRP variants by adding constraints to the fundamental problem to better match real-world applications. Different approaches are used to solve these variants. Interested readers are referred to [2] for a detailed review of FTVRPs. To position our study in relation to the literature, we classified the various contributions of selective FTVRP (SFTVRP) variants based on whether the routes primarily contain the FTL shipment, transportation demand selection, multiple depots, time window constraints (TWs), and uncertainty. Table I outlines the most critical SFTVRP-related works and the current study features.

Ball et al. [3] were the first to introduce the multi-depot SFTVRP (SFTMDPDP). The problem consists of constructing routes for private vehicles and subcontracting chemical product commodities to common carriers with the goal of decreasing total cost while meeting a maximum time limit on truck routes. For resolving the FTMDPDP, three heuristics are proposed: a greedy insertion strategy (GI) and two algorithms based on the route-first, cluster-second (RF-CS) technique. Wang and Regan [4] investigated an SFTVRP with time windows (SFTPDPTW) considering only loading TWs, in which the objective is to minimize the empty travelling cost while serving the maximum number of orders within their time constraints. They devised an iterative strategy for resolving the problem by employing the window-partition-based (WPB) algorithm. Miori [5] proposed a TS algorithm for solving a similar SFTVRPT without TWs and with the same later goal. Li and Lu [6] presented a hybrid GA based on improved savings for an SFTVRP with split orders and the objective of maximizing the total profit. Liu et al. [7] developed a memetic algorithm to solve an SFTVRP in collaborative transportation. Another SFTVRPT in the collaborative logistics context was presented in [8]. The authors formulated the problem as an MILP model with the objective function of minimizing the total cost. They proposed a branch-and-cut-and-price-based heuristic to solve the model. Wang et al. [9] considered an SFTVRP with heterogeneous fleet application in the petrochemical industry that involves rich features, including multiple loading locations, optional orders, and loading dock capacity limitations. They presented an MILP mathematical model for the problem, which is solved using the commercial solver Gurobi.

Yang et al. [10], Tjokroamidjojo et al. [11], and Zolfagharinia and Haughton [12] used some rolling horizon approaches (variants of re-optimization or heuristics) to deploy dynamic SFTVRP variants. A fraction of orders to be carried in a given day become known only a short time before service is needed, truck movements are added to the system as the day advances, and orders must periodically be reassigned. Li et al. [13] addressed a dynamic SFTVRP in a collaborative context in which the carrier can dynamically select its collaborative requests based on the surplus of its transport capacity in the collaborative process. The authors proposed a mixed integer programming (MIP) model for the problem, aiming to maximize the carrier's total profits after outsourcing requests. The model is solved through CPLEX software. Annouch and Bellabdaoui [14] proposed an adaptive GA to solve the open FTMDVRPTW with split delivery (FTOVRPTWSD) in the liquefied petroleum gas (LPG) distribution industry. The FTVRP variant addressed in this study is not selective, and the robustness aspect is not considered.

In our previous studies, we investigated a mono-objective variant of the selective FTMDVRPTW (SFTMDVRPTW) in an empty return context. The objective is to maximize the total profit of selective routes. The resolution of this problem is based on the development of mathematical models, exact solvers, meta-heuristics, and hybrid methods. We described a mathematical formulation of the SFTMDVRPTW as an MILP model [15]. Numerical results on small and medium-size instances are presented using the CPLEX solver. To solve larger instances, we developed, adapted, and applied some heuristic methods: an ant colony system (ACS) [16], a genetic algorithm (GA) [17-18], and a reactive tabu search (RTS) [19].

To the best of our knowledge, while uncertainty is present and relevant, it is rarely addressed for SFTVRP variants. Hammami et al. [20] investigated an SFTVRP with uncertain clearing prices. They developed an exact non-enumerative algorithm to obtain optimal solutions for small instances and a two-phase hybrid heuristic to solve larger instances.

The main contributions of this paper can be summarized as follows:

- Formulate the RSFTMDVRPTW as an MILP model under uncertainty and the multi-objective facet, which addresses two conflicting functions: an economic component to be maximized and a component related to the worst observation of total operation time to be minimized.
- Introduce a MILP-based lexicographic method for solving the RSFTMDVRPTW.

TABLE I. POSITION OF OUR STUDY IN RELATION TO THE FTVRP LITERATURE

Reference		Constraints								Objective fonction	Area	Solution approach	
Authors	Year	TW	Het	MD	Cap	SD	S	D	U			type	Method
Ball et al. [3]	1983			x			x			Min. cost	Academic	H	• RF-CS • Greedy insertion
Wang and Regan [4]	2002	x		x			x			Min. EMV Max. #Ord	Service transportation	H	WPB
Yang et al. [10]	2004	x					x	x		Min. cost	Service transportation	E H	• Re-optimization • RHA
Tjokroamidjojo et al. [11]	2006	x					x	x		Min. cost	Service transportation	E H	• Re-optimization • RHA
Liu et al. [7]	2010						x			Min. cost	Academic	PBM	Memetic Algorithm
Miori [5]	2011	x					x			Min. cost Max. #Ord Min. #Veh	Academic	SSBM	Goal programming with TS
Li and Lu [6]	2014				x	x	x			Max. profit	Service transportation	PBM	Hybrid Genetic Algorithm
Zolfagharinia and Haughton [12]	2014	x		x			x	x		Max. profit	Service transportation	H	RHA
Li et al. [13]	2015			x			x	x		Max. profit	Service transportation	E	CPLEX
El Bouyahyiouy and Bellabdaoui [16]	2017	x	x	x			x			Max. profit	Academic	PBM	Ant Colony System
Hammami et al. [20]	2021						x		x	Max. profit	Service transportation	E H	• Branch-and-Cut • Hybrid heuristic
Wang et al. [9]	2021	x	x		x		x			Min. cost	Service transportation	E	Gurobi
El Bouyahyiouy and Bellabdaoui [17]	2022	x	x	x			x			Max. profit	Academic	E PBM	• CPLEX • Genetic Algorithm
Öner and Kuyzu [8]	2021						x			Min. cost	Academic	E	Branch-and-cut-and-price
This study		**x**	**x**	**x**			**x**		**x**	**Max. profit** **Min. worst TT**	**Academic**	**E**	**MILP-based lexicographic**

Note: Het: Heterogeneous Fleet; MD: Multi-Depot; Cap: Capacitated; SD: Split delivery; S: Selective; D: Dynamic; U: uncertainty; TT: Travel time; #Veh: Number of vehicles; #Ord: Number of served orders; EMV: Empty vehicle movements; E: exact; H: heuristic; SSBM: Single solution-based metaheuristic; PBM: Population based metaheuristic; RHA: Rolling horizon planning approach; RF-CS: Route-first, cluster-second.

III. A MATHEMATICAL FORMULATION OF THE RSFTMDVRPTW

A. Problematic

This study addresses a variant of the full truck vehicle routing problem under uncertainty in transportation time (RSFTMDVRPTW). The position of the problem is as follows. Assume a set of n orders to be served by a fixed fleet of m trucks. Each truck k is characterized by a starting point D_k, an ending point A_k, an earliest service start date D_k^{min} and a latest service end date D_k^{max}. Each order O_i $(i = 1, \dots, n)$ is characterized by a collection point L_i (origin) and a delivery point U_i (destination), a profit p_i (determined on the basis of the distance between origin and destination), a loading time window $[L_i^{min}, L_i^{max}]$, and an unloading time window $[U_i^{min}, U_i^{max}]$. The travel time for each arc $(i,j) \in \{(D_k, LU_i)\} \cup \{(LU_i, LU_j)\} \cup \{(LU_i, A_k)\} \cup \{(D_k, A_k)\}$ is described by a set of NS scenarios t_{ij}^{ξ} $(\xi = 1, \dots, NS)$, where each scenario reflects a potential time requirement for a truck traversing arc (i,j).

The problem consists of selecting a subset of orders to be served and assigning them to trucks, thus finding an optimal sequence of orders assigned to each truck while maximizing total profit, minimizing the worst observation of the total travel time over all scenarios and respecting availability and time window constraints. Fig. 1 depicts a solution representation for an RSFTMDVRPTW instance with two trucks and 14 orders.

B. A discrete Scenario-based MILP Model

In this section, we propose an MILP model of the RSFTMDVRPTW, in which a set of discrete scenarios represents the uncertain travel times. The distribution of the uncertainty parameters is assumed to be unknown. As a result, all scenarios are generated uniformly. Table II defines all data and variable notations. Next, the objectives and constraints are introduced and explained.

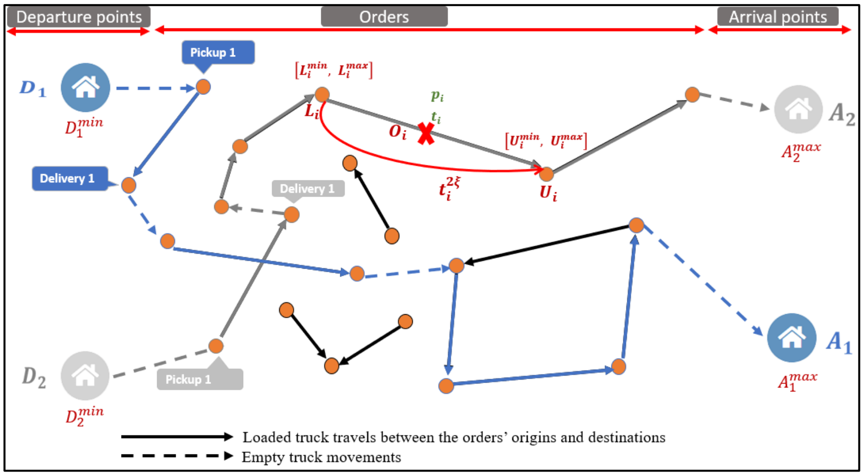

Fig. 1. An illustration of the RSFTMDVRPTW.

TABLE II. DECISION VARIABLES AND PARAMETERS OF THE MILP MODEL OF THE RSFTMDVRPTW

Notation	Meaning
m	Number of trucks
D_k	Departure depot of truck k
D_k^{min}	Earliest service start time of truck k
A_k	Arrival depot of truck k
A_k^{max}	Latest service end time of truck k
n	Number of commodities
$\{O_1, \dots, O_n\}$	Set of commodities
L_i	Collection point (origin) of the commodity O_i
U_i	Delivery point (destination) of the commodity O_i
p_i	profit associated with commodity O_i
L_i^{min}	Earliest time to load the commodity O_i
L_i^{max}	Latest time to load the commodity O_i
U_i^{min}	Earliest time to unload the commodity O_i
U_i^{max}	Latest time to perform the unloading of the commodity O_i
NS	Number of scenarios
Ξ	Set of possible scenarios
t_i^ξ	Travel time between collection and delivery points of the commodity O_i under scenario $\xi \in \Xi$
t_{ij}^ξ	Empty travel time from the collection point of commodity O_i to the delivery point of commodity O_j under scenario $\xi \in \Xi$
$t_{0i}^{k\xi}$	Empty travel time from departure depot D_k to the collection point of commodity O_i under scenario $\xi \in \Xi$
$t_{i,n+1}^{k\xi}$	Empty travel cost from the delivery point of commodity O_i to the arrival depot A_k under scenario $\xi \in \Xi$
M	A big number
Decision variables	
x_{ij}^k	Binary decision variable that indicates whether the truck k visits commodity O_j immediately after commodity O_j
$t_{i,L}^{k\xi}$	Start time of the loading of commodity O_i on truck k under scenario $\xi \in \Xi$
$t_{i,U}^{k\xi}$	Start time of the unloading of commodity O_i from truck k under scenario $\xi \in \Xi$
$a_{i,L}^{k\xi}$	Amount of time to wait before the loading of commodity O_i of truck k under scenario $\xi \in \Xi$
$t_{0,L}^{k\xi}$	Departure time of service of truck k from its starting depot D_k under scenario $\xi \in$
$t_{n+1,U}^{k\xi}$	Arrival time of truck k at its finishing depot A_k under scenario $\xi \in \Xi$

The MILP model of the RSFTMDVRPTW is given as follows:

$$Maximize\ f_1 = \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n+1} p_i x_{ij}^k \tag{1}$$

$$Minimize\ f_2 = TT_{worst} \tag{2}$$

Subject to:

$$\sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n+1} t_i^\xi x_{ij}^k + \sum_{k=1}^{m}\sum_{j=1}^{n+1} t_{0,j}^{k\xi} x_{0,j}^k + \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij}^\xi x_{ij}^k + \sum_{k=1}^{m}\sum_{i=1}^{n} t_{i,n+1}^{k\xi} x_{i,n+1}^k + \sum_{k=1}^{m}\sum_{i=1}^{n} (w_{i,L}^{k\xi} + t_{i,U}^{k\xi} - t_{i,L}^{k\xi} - t_i^\xi) \le TT_{Worst}, \forall \xi = 1,\dots,NS \tag{3}$$

$$\sum_{j=1}^{n+1}\sum_{k=1}^{m} x_{ij}^k \le 1 \qquad \forall i = 1,\dots,n \tag{4}$$

$$\sum_{i=0}^{n}\sum_{k=1}^{m} x_{ij}^k \le 1 \qquad \forall j = 1,\dots,n \tag{5}$$

$$\sum_{j=1}^{n+1} x_{0j}^k = 1 \qquad \forall k = 1,\dots,m \tag{6}$$

$$\sum_{i=0}^{n} x_{ih}^k - \sum_{j=1}^{n+1} x_{hj}^k = 0 \quad \forall h = 1,\dots,n, \ \ \forall k = 1,\dots,m \tag{7}$$

$$\sum_{i=0}^{n} x_{i,n+1}^k = 1 \qquad \forall k = 1,\dots,m \tag{8}$$

$$x_{i,0}^{k} = 0 \qquad \forall k = 1,..,m, \ \forall i = 0,...,n+1 \tag{9}$$

$$x_{n+1,i}^{k} = 0 \qquad \forall k = 1,..,m, \quad \forall i = 0,...,n+1 \tag{10}$$

$$D_k^{\min} \le t_{0,L}^{k\xi}, \qquad \forall k = 1,...,m, \ \ \forall \xi = 1,..., NS \tag{11}$$

$$t_{n+1,U}^{k\xi} \le A_k^{\max} \qquad \forall k = 1,...,m \ , , \ \ \forall \xi = 1,..., NS \tag{12}$$

$$L_i^{\min} \le t_{i,L}^{k\xi} \le L_i^{\max}, \quad \forall i = 1,...,n, \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{13}$$

$$U_i^{\min} * \sum_{j=1}^{n+1} x_{ij}^{k} \ \ \le t_{i,U}^{k\xi} \le U_i^{\max}, \quad \forall i = 1,...,n, \ \ \forall k = 1,...,m \ , \forall \xi = 1,..., NS \tag{14}$$

$$t_{i,L}^{k\xi} + t_i^{\xi} \le t_{i,U}^{k\xi}, \ \forall i = 1,...,n, \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{15}$$

$$t_{0,L}^{k\xi} + t_{0,i}^{k\xi} + w_{i,L}^{k\xi} \le t_{i,L}^{k\xi} + \mathrm{M} * \left(1 - x_{0i}^{k}\right) \quad \forall i = 1,...,n, \ \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{16}$$

$$t_{0,L}^{k\xi} + t_{0,i}^{k\xi} + w_{i,L}^{k\xi} \ge t_{i,L}^{k\xi} \text{-} \mathrm{M} * \left(1 - x_{0i}^{k}\right) \quad \forall i = 1,...,n, \ \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{17}$$

$$t_{i,U}^{k\xi} + t_{i,j}^{\xi} + w_{j,L}^{k\xi} \le t_{j,L}^{k\xi} + \mathrm{M} * \left(1 - x_{ij}^{k}\right) \quad \forall i, j = 1,...,n, \ \forall k = 1,...,m \ , \forall \xi = 1,..., NS \tag{18}$$

$$t_{i,U}^{k\xi} + t_{i,j}^{\xi} + w_{j,L}^{k\xi} \ge t_{j,L}^{k\xi} \text{-} \mathrm{M} * \left(1 - x_{ij}^{k}\right) \quad \forall i, j = 1,...,n, \ \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{19}$$

$$t_{i,U}^{k\xi} + t_{i,n+1}^{k\xi} \le t_{n+1,U}^{k\xi} + \mathrm{M} * \left(1 - x_{i,n+1}^{k}\right) \quad \forall i = 1,...,n, \ \ \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{20}$$

$$t_{i,U}^{k\xi} + t_{i,n+1}^{k\xi} \ge t_{n+1,U}^{k\xi} \text{ - } \mathrm{M} * \left(1 - x_{i,n+1}^{k}\right) \quad \forall i = 1,...,n, \ \forall k = 1,...,m \ , \forall \xi = 1,..., NS \tag{21}$$

$$x_{ij}^{k} \in \{0,1\} \qquad \forall i, j = 0,...,n+1, \ \ \forall k = 1,...,m \tag{22}$$

$$t_{i,L}^{k\xi} \ge 0, \forall i = 0,...,n, \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{23}$$

$$t_{i,U}^{k\xi} \ge 0, \forall i = 1,...,n+1, \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{24}$$

$$w_{i,L}^{k\xi} \ge 0, \forall i = 1,...,n, \forall k = 1,...,m, \forall \xi = 1,..., NS \tag{25}$$

$$TT_{worst} \ge 0 \tag{26}$$

The problem represents a bi-objective optimization problem. The first objective function (1) seeks to maximize the total profit obtained from the selected commodities and the second objective function (2) aims to minimize the worst total operation time of all trucks over all considered scenarios.

Constraints (3) ensure that for all scenarios, the total operation time (empty travel time (first term), full travel time (second, third, and fourth terms), and waiting time before loading and unloading commodities (fifth term)) needed by all trucks does not exceed TT_{worst}. Constraints (4) and (5) imply that each collection and delivery location can be visited at most once. Constraint (6) guarantee that each truck must begin its journey from its starting depot. Constraints (7) ensure the conservation of flow; once a truck has packed an order, it must unload it at the corresponding delivery location. Constraint (8) guarantee that each truck finishes its route at the arrival depot. Constraints (9) and (10) ensure that each truck cannot return to its departure depot and cannot visit any point after its arrival depot. Inequalities (11)-(21) are used to compute the truck start time, the start time of the loading/unloading of commodities, and the time to wait at loading points in all scenarios. The time window constraints are respected using Inequalities (11)-(14). Constraints (15) require, at a commodity level, that the unloading time be greater than the sum of the loading time and the time from the commodity's collection location to its delivery location. Constraints (16) and (17) impose that loading a commodity onto a truck can only start after the truck has left its departure depot. Constraints (18) and (19) ensure that a truck can only pick up the next commodity after unloading the previous one, and displacement occurs. Constraints (20) and (21) guarantee that a truck can only unload a commodity if it can arrive at the arrival depot before the latest service end time. Finally, constraints (22)-(26) specify the appropriate values for decision variables.

IV. MILP-based Lexicographic Approach for the RSFTMDVRPTW

Multi-objective optimization (MOO) problems involve optimizing more than one objective function simultaneously, which is usually in conflict, so improving one leads to worsening another. The lexicographic approach is a widely used solution method for MOO [21]. Fig. 2 depicts a general example of the lexicographic method in which the decision maker begins by ranking the objective functions in order of importance and then solves sequentially mono-objective problems starting with the most critical function and progressing to the least critical function. The lexicographic approach, similar to other methods (epsilon constraint, weighted sum, and so on), does not require any parameter configurations. Moreover, once the decision maker has prioritized one objective function over the other, it can provide a Pareto-optimal solution for the MOO problem.

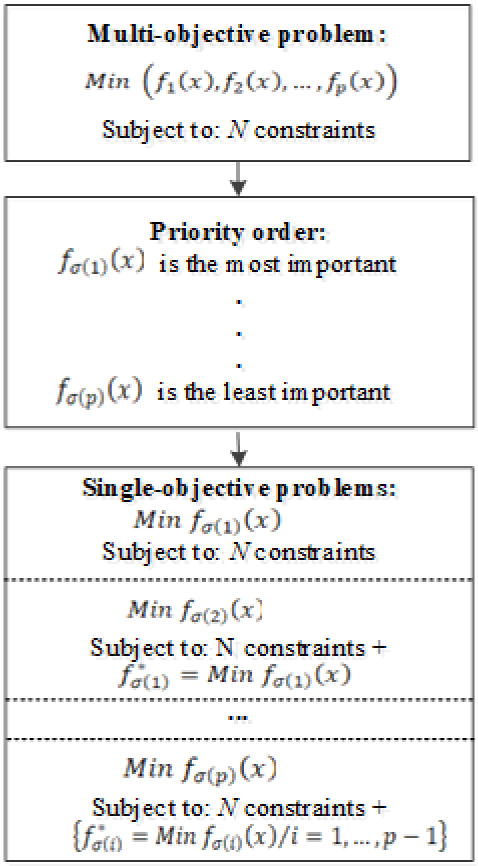

Fig. 2. Principles of the lexicographic optimization approach.

In this study, we use a lexicographic method based on the MILP formulation already defined. The selective aspect facilitates the ranking of the objective functions. This ranking was chosen on the grounds that maximizing the profit can reflect a higher quality of service, as well as on the grounds that beginning with the minimization of total travel time will generate a solution where no delivery commodity is assigned to trucks, where each route of a truck k will be the shortest path from departure point D_k to end depot A_k to obtain the lowest travel time. Therefore, we maximize the first objective f_1 (the collected profit) first and then minimize the second objective f_2 (the total travel time) based on the obtained solution for f_1. As a result, the original MILP model is transformed into two sequential models (P_1) and (P_2) as follows:

$$(P_1):\ \ f_1^* = Min\ f_1$$

$$\text{Subject to: } Constraints\ (4) - (25)$$

$$(P_2):\ \ f_2^* = Min\ f_2$$

$$\text{Subject to : } \begin{cases} Constraints (3) - (26) \\ f_1 \geq f_1^* \end{cases}$$

V. Computational Experiments

The computational experiments were performed using the AMPL programming language with CPLEX solver (version 12.7) on a laptop computer Intel Pentium Core i7- 4790 with 3.6 GHz and 16 GB of RAM memory.

The experiments are conducted across adapted SFTMDVRPTW instances proposed by EL Bouyahyiouy and Bellabdaoui [17], which are generated based on three classes R / C / RC of Solomon's VRPTW benchmark instances [22]. 'C' means that the points are clustered, 'R' indicates that the points are random, and 'RC' denotes that the points are both clustered and random.

In this study, the tests were restricted to the R problem class since it is most relevant to the FTVRP variant and the most difficult to solve. We used eight different instances from the datasets of El Bouyahyiouy and Bellabdaoui [17] with two different types of time windows ($SFT1-4\_R25\_20\_2$ and $SFT1-4\_R100\_50\_5$). We have adapted these instances to the RSFTMDVRPTW by adding a number of scenarios. A fixed number of arbitrary edges is selected in each scenario, and their travelling times are perturbed. Therefore, 96 new instances are generated and solved with the MILP-based lexicographic method.

Each instance is labelled as $Ri\_n\_m\_NS\_l\_p$, where:

- i is the instance *ID*.
- n gives the number of orders, $n \in \{25, 50\}$.
- m gives the number of trucks, $m \in \{2, 5\}$.
- NS denotes the number of scenarios,
- $NS \in \{10, 50, 100\}$.
- l denotes the uncertainty level, $l \in \{50, 100\}$. The travel time on each perturbed edge for each scenario varies on the interval $[d(i,j), (1 + l\,\%)\, d(i,j)]$, where $d(.,.)$ denotes the Euclidean distance between any two points (shortest time)
- p denotes the number of perturbed edges for every scenario, representing 10% or 20% of the total edges for each instance.

Table III summarizes the results performed on the 96 generated instances. TT_{worst} denotes the worst observation of the total travel time over all scenarios, and CPU represents the running time for CPLEX. For each instance, CPLEX is run for two-hour-time limits.

Table III shows the following observations:

- The proposed MILP-based lexicographic method performs well in all instances with 20 commodities where the CPLEX solver can provide optimal solutions in a relatively short time. When the number of commodities is increased to 50, CPLEX is unable to solve some instances optimally within 2 hours.
- As expected, the CPU time is significantly impacted by the number of commodities, the width of the time windows, and, in particular, the selective aspect (i.e.,

the number of unselected commodities in the obtained optimal solution). In all cases, as the number of unselected orders grows, the trucks cannot select some orders, resulting in increased CPU time.

- Furthermore, the number of scenarios and the uncertainty level directly affect the CPU time. As the values of these two parameters grow, so does the difficulty of resolving the instances.
- If the uncertainty level is set to 100, the travel time will likely be doubled. Utilizing many scenarios can diminish the uncertainty of travel time, resulting in more conservative estimates of the total worst-case travel time for all assumed scenarios.
- When comparing two different instances, a larger number of perturbed edges does not always imply a lower profit because those edges are selected randomly (e.g., $R1$_50_5_100_100_10 and $R1$_50_5_100_100_20 have optimal profits of 5852 and 5841, respectively).

The robust aspect can significantly increase the problem's complexity, impacting the CPU time required to obtain a Pareto-optimal solution. Furthermore, the reported solutions are still worse than or equal to the non-robust solutions computed employing just the ideal scenario [17]. However, the feasibility of the obtained solution, over all scenarios, is the main advantage of robust optimization.

TABLE III. RESULTS OF THE PROPOSED MILP-BASED LEXICOGRAPHIC METHOD ON THE 92 GENERATED INSTANCES

Instance	$Profit$	TT_{worst}.	CPU (s)	Instance	$Profit$	TT_{worst}.	CPU (s)
$R1$_20_2_10_50_10	2760	740	99.88	$R1$_50_5_10_50_10	5942*	2175*	5726.6
$R1$_20_2_10_50_20	2743	783	152.52	$R1$_50_5_10_50_20	5925	2218	6277.47
$R1$_20_2_10_100_10	2710	867	175.88	$R1$_50_5_10_100_10	5892	2302	6981,85
$R1$_20_2_10_100_20	2750	920	206.91	$R1$_50_5_10_100_20	5932	2355	6984,030
$R1$_20_2_50_50_10	2760	814	229.75	$R1$_50_5_50_50_10	5939	2249	6425.61
$R1$_20_2_50_50_20	2730	858	292.39	$R1$_50_5_50_50_20	5780	2293	6535.11
$R1$_20_2_50_100_10	2740	907	306.93	$R1$_50_5_50_100_10	5734	2342	7200
$R1$_20_2_50_100_20	2650	959	378.85	$R1$_50_5_50_100_20	5816	2394	6896.57
$R1$_20_2_100_50_10	2710	823	490.02	$R1$_50_5_100_50_10	5822*	2258	7200
$R1$_20_2_100_50_20	2760	863	557.05	$R1$_50_5_100_50_20	5759	2298	6615.23
$R1$_20_2_100_100_10	2740	922	621.41	$R1$_50_5_100_100_10	5841	2357	6841.13
$R1$_20_2_100_100_20	2740	952	687.18	$R1$_50_5_100_100_20	5852	2387	6558.90
$R2$_20_2_10_50_10	1769	735	19.5	$R2$_50_5_10_50_10	5717	2180	6874.16
$R2$_20_2_10_50_20	1757	778	72.14	$R2$_50_5_10_50_20	5700*	2223	6926.8
$R2$_20_2_10_100_10	1753	862	95.5	$R2$_50_5_10_100_10	5667*	2307	6950.16
$R2$_20_2_10_100_20	1747	915	126.53	$R2$_50_5_10_100_20	5707*	2360	6981.19
$R2$_20_2_50_50_10	1744	809	149.37	$R2$_50_5_50_50_10	5717*	2254	7200
$R2$_20_2_50_50_20	1714	853	212.01	$R2$_50_5_50_50_20	5687*	2298	7200
$R2$_20_2_50_100_10	1749	902	226.55	$R2$_50_5_50_100_10	5697*	2347	6236.75
$R2$_20_2_50_100_20	1759	954	298.47	$R2$_50_5_50_100_20	5607*	2399	7200
$R2$_20_2_100_50_10	1759	818	409.64	$R2$_50_5_100_50_10	5667*	2263	7147.10
$R2$_20_2_100_50_20	1755	858	476.67	$R2$_50_5_100_50_20	5717*	2303	7200
$R2$_20_2_100_100_10	1739	917	541.03	$R2$_50_5_100_100_10	5697	2362	7155.41
$R2$_20_2_100_100_20	1724	947	606.8	$R2$_50_5_100_100_20	5667	2392	7182.44
$R3$_20_2_10_50_10	3020	1220	17.7	$R3$_50_5_10_50_10	6008*	3372*	5500.5
$R3$_20_2_10_50_20	3020	1263	70.34	$R3$_50_5_10_50_20	5991	3415	5553.14
$R3$_20_2_10_100_10	3020	1347	93.7	$R3$_50_5_10_100_10	5958	3499	5576.5
$R3$_20_2_10_100_20	3020	1400	124.73	$R3$_50_5_10_100_20	5998	3552	5607.53
$R3$_20_2_50_50_10	3020	1294	147.57	$R3$_50_5_50_50_10	6008	3446	5630.37
$R3$_20_2_50_50_20	3020	1338	210.21	$R3$_50_5_50_50_20	5978	3490	5693.01
$R3$_20_2_50_100_10	3020	1387	224.75	$R3$_50_5_50_100_10	5988	3539	5707.55
$R3$_20_2_50_100_20	3020	1439	296.67	$R3$_50_5_50_100_20	5898	3591	5779.47
$R3$_20_2_100_50_10	3020	1303	407.84	$R3$_50_5_100_50_10	5958	3455	5890.64
$R3$_20_2_100_50_20	3020	1343	474.87	$R3$_50_5_100_50_20	6008	3495	5957.67
$R3$_20_2_100_100_10	3020	1402	539.23	$R3$_50_5_100_100_10	5988	3554	6022.03
$R3$_20_2_100_100_20	2970	1432	605	$R3$_50_5_100_100_20	5988	3584	6087.8
$R4$_20_2_10_50_10	2819	1219	15.9	$R4$_50_5_10_50_10	5840	3530	5099.16
$R4$_20_2_10_50_20	2802	1262	68.54	$R4$_50_5_10_50_20	5820*	3354*	6702.54

$R4$_20_2_10_100_10	2769	1346	91.9	$R4$_50_5_10_100_10	5780*	3438*	6649.9
$R4$_20_2_10_100_20	2809	1399	122.93	$R4$_50_5_10_100_20	5830*	3491	6856.93
$R4$_20_2_50_50_10	2819	1293	128.77	$R4$_50_5_50_50_10	5770*	3385	6779.77
$R4$_20_2_50_50_20	2809	1337	215.41	$R4$_50_5_50_50_20	5740*	3429	7200
$R4$_20_2_50_100_10	2749	1386	310.95	$R4$_50_5_50_100_10	5700*	3311	7200
$R4$_20_2_50_100_20	2799	1438	332.87	$R4$_50_5_50_100_20	5700*	3478	6928.87
$R4$_20_2_100_50_10	2789	1302	424.04	$R4$_50_5_100_50_10	5720*	3394	7165.04
$R4$_20_2_100_50_20	2809	1342	500.07	$R4$_50_5_100_50_20	5710*	3434	6877.07
$R4$_20_2_100_100_10	2759	1401	520.43	$R4$_50_5_100_100_10	5720*	3576	7189.43
$R4$_20_2_100_100_20	2789	1431	560.2	$R4$_50_5_100_100_20	5700*	3523	7190.23

* Indicates a feasible solution

VI. Conclusions and Future Research

In this work, we have studied an essential variant of the full truck vehicle routing problem under uncertainty in transportation time, notably a robust selective full truckload multi-depot vehicle routing problem with time windows (RSFTMDVRPTW), in which a set of discrete scenarios represents uncertain travel times. We have proposed a discrete scenario-based MILP model for the RSFTMDVRPTW under the multi-objective facet, which addresses two conflicting functions: an economic component to be maximized and a component related to the worst observation of total operation time to be minimized. To solve the RSFTMDVRPTW, we have used an MILP-based lexicographic method, which maximizes the collected profit first and then minimizes the worst observation of total travel time based on the obtained solution for the first objective.

The considered approach was solved using CPLEX 12.6 and tested on 96 newly generated instances of up to 50 orders and five trucks adapted from the literature. The encouraging results demonstrate that the proposed lexicographic method provides a plausible Pareto-optimal solution for all instances with 20 commodities within an acceptable computing time. However, when the number of commodities is increased to 50, CPLEX cannot solve some instances optimally within 2 hours. Indeed, we remarked that the selective aspect, the values of the number of scenarios, and the uncertainty level strongly impact the proposed lexicographic method. Furthermore, the reported solutions are still worse than or equal to the nonrobust solutions computed employing only the ideal scenario. However, the feasibility of the solution over all scenarios is the main advantage of robust optimization.

As the problem is quite complex, only small instances can be solved optimally by CPLEX. Therefore, in future works, we will design an efficient metaheuristic algorithm to solve large instances of the problem with a large number of scenarios.

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