Adaptive Outlier-tolerant Exponential Smoothing Prediction Algorithms with Applications to Predict the Temperature in Spacecraft

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***Abstract*—The exponential smoothing prediction algorithm is widely used in spaceflight control and in process monitoring as well as in economical prediction. There are two key conundrums which are open: one is about the selective rule of the parameter in the exponential smoothing prediction, and the other is how to improve the bad influence of outliers on prediction. In this paper a new practical outlier-tolerant algorithm is built to select adaptively proper parameter, and the exponential smoothing prediction algorithm is modified to prevent any bad influence from outliers in sampling data. These two new algorithms are valid and effective to overcome the two open conundrums stated above. Simulation and practical results of sampling data from temperature sensors in a spacecraft show that this new adaptive outlier-tolerant exponential smoothing prediction algorithm has the power to eliminate bad infection of outliers on prediction of process state in future.**

Keywords-Exponential prediction; Adaptive smoothing prediction; Outlier-tolerance smoothing prediction.

I. INTRODUCTION

The exponential smoothing prediction algorithm was first suggested by Brown as an operation research analyst during World War II, when he worked on submarine locating and tracking model for antisubmarine warfare. He thought the trend of time series was stability or regularity, so the future state of time series could be deduced reasonably [1]. After more than half a century, exponential smoothing forecasting method has become a common forecasting method as well as the most commonly used forecasting method in short-term economic development trend forecast and many other areas in [1,2]. Exponential smoothing prediction method has obvious advantages: one is that the algorithm is simple, the predicted value at time t_{k+1} can be gained by actual measured value at time t_k and predicted value at time t_k, it is simple iterative relationship without complex operation; and the other is that it takes advantages from full-term average as well as moving average and doesn't abandon the old data. However, according to how long the sampling time of data away from the present moment, different weights are assigned to historical data and the influence of historical data on the current forecast gradually abates.

From the perspective of application, the most difficult point of exponential smoothing prediction algorithm is how to choose the parameter. On the one hand, the exponential smoothing prediction algorithm is very convenient for use because it need only one parameter; on the other hand, the accuracy and reliability of prediction results are changed when we set different value for the smooth parameter. In order to improve quality of prediction algorithm and prediction result, people did lots of explorations and researches from different angles and suggested many kinds of improved algorithms in the past half of the century. For example, some modification algorithms were suggested, such as the double, triple and multiple exponential smoothing prediction algorithms [1]. In recent years, people come up with a variety of adaptive algorithms and selection criteria [3-5] so as to find out some available selection method about smoothing parameter. In order to establish a new collaborative forecasting model, literature [6] also combined the exponential smoothing forecasting algorithm with neural network.

What is more, a lot of practice in using the exponential smoothing prediction show that the exponential smoothing prediction algorithm is lack of outlier-tolerant ability [7-8], because the exponential smoothing prediction with constant parameter is virtually linear prediction algorithm. In fact, the prediction results would inevitably produce serious distortion due to the impact of outliers when there are outliers in the sample sequences.

So, in order to improve the prediction quality and to make sure reliability of prediction results, there are two "bottleneck" problems which are open: one is how to select adaptively the smoothing parameter and the other is how to prevent bad influence of outliers on the prediction algorithm. In this article we build firstly an useful online adaptive smoothing parameter estimation and then make further outlier-tolerant modification on prediction algorithm. At the end of this paper, a practical application is given to show that new algorithms are available.

II. Adaptive Fault-Tolerant Design Of Parameters

The exponential smoothing prediction algorithm is based on the actual value $y(t_k)$ of the dynamic time-series in the t_k period and prediction value $\hat{y}(t_k)$ predicted at time t_k.The equations to calculate an exponential smoothing algorithm can be expressed as follows:

$$\begin{cases} \hat{y}(t_{k+1}) = (1-\alpha)y(t_k) + \alpha\hat{y}(t_k) \\ \hat{y}(t_1) = y(t_1) \end{cases} \tag{1}$$

where α is a smoothing constant parameter, $0 \le \alpha \le 1$.

How to select the parameter α is important because different values of α may bring on different predicting value of a dynamic process. For example, figure 1(a) is a series of simulation sampling data, the solid line and the dotted line in figure 1(b) are one-step prediction plots of figure 1(a) with two different parameter (α=0.2 and α=0.8) respectively. Comparing these two prediction curves shown in figure 1(b) clearly shown that the value of α has significant effect on smoothing prediction results.

How to choose the smoothing parameter α to get reliable prediction effect? An intuitive idea is to seek such a parameter estimator of α which satisfying the following relation:

$$S_k(\alpha) = \sum_{i=1}^{k}(y(t_i) - \hat{y}(t_i))^2 \xrightarrow{\alpha} \min \tag{2}$$

By the recursive formula (1), right side of the expression (2) can be written as follows

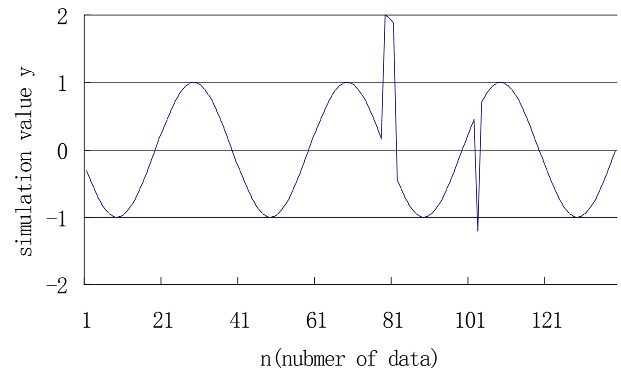
$$S_k(\alpha) = \sum_{i=1}^{k}\{[y(t_i) - y(t_{i-1})] + \alpha[y(t_{i-1}) - \hat{y}(t_{i-1})]\}^2$$

then the minimum point series of the parameter α in the function $S_k(\alpha)$ can be obtained as follows:

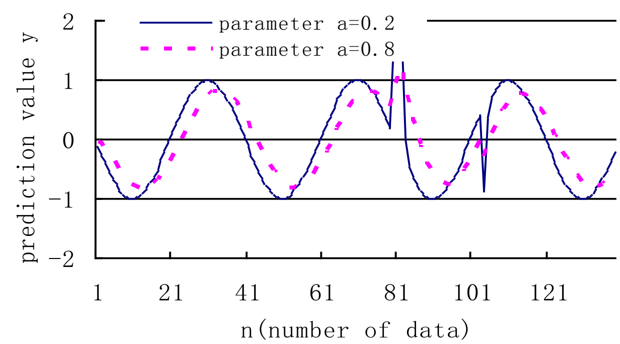
$$\hat{\alpha}_k^{(0)} = -\frac{\sum_{i=1}^{k}[y(t_i) - y(t_{i-1})][y(t_{i-1}) - \hat{y}(t_{i-1})]}{\sum_{i=1}^{k}[y(t_{i-1}) - \hat{y}(t_{i-1})]^2} \tag{3}$$

In order to ensure that design value of the smoothing parameter meets the constraint condition that the parameter α is a nonnegative real value and $0 \le \alpha \le 1$, we can change formula (3) into formula (4):

$$\hat{\alpha}_k = \max\{0, \min\{\hat{\alpha}_k^{(0)}, 1\}\} \tag{4}$$



(a) Smulation data plot



(b) Exponential smoothing prediction value plot

Figure 1. Simulation and exponential prediction plots

We can get online optimal estimation of smoothing coefficient from formula (3). Therefore, taking formula (1) and (3) into consideration, we can build a new form of adaptive exponential smoothing prediction algorithm as follows

$$\hat{y}(t_{k+1}) = y(t_k) + \frac{\sum_{i=1}^{k}[y(t_i) - y(t_{i-1})][y(t_{i-1}) - \hat{y}(t_{i-1})]}{\sum_{i=1}^{k}[y(t_{i-1}) - \hat{y}(t_{i-1})]^2}(y(t_k) - \hat{y}(t_k)) \tag{5}$$

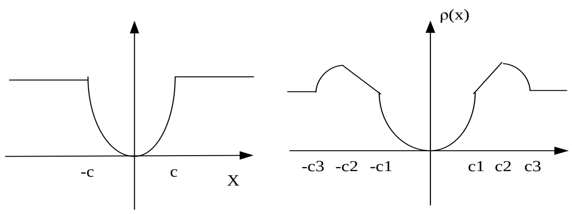
with the beginning value $\hat{y}(t_1) = y(t_1)$.

Using the formula (4), if quality of sampling sequence data is reliable, the ordinary exponential smoothing prediction algorithm (1) is changed into a new adaptive prediction algorithm (5), which can be used effectively to solve the open problem state in section I: how to choose the smooth coefficient and ensure that the forecast results can effectively track and early show the change of process.

The adaptive exponential smoothing prediction algorithm (5) is concise and practical. What’s more, it is easy and convenient to choose the smoothing coefficient parameter. From engineering data processing and the actual process prediction viewpoint, the algorithm (5) as well as algorithm (1) does not have the ability of outlier-tolerance for isolated outliers and patchy outliers: when the sampling data series of a dynamic process seriously deviate from the real state trend in a time or time period, subsequent predictions values of local arcs will distort or cause a big prediction deviation. So we will build a group of outlier-tolerant exponential smoothing prediction algorithms to improve outlier–tolerance ability of prediction algorithm for outliers and ensure that the predictions will not cause deviation even there are outliers occasionally in the sampling.

Following the famous research ideas about robust statistics suggested by Huber, we set the appropriate threshold parameter (c,c1,c2,c3) and select Huber-type $\rho-$ function [9] and the re-descending (short as Rd-)-type $\rho-$ function [10] in figure 2, then the formula (2) can be written as follows:

$$S_k(\alpha \mid \rho)=\sum_{i=1}^{k}\rho((y(t_i)-\hat{y}(t_i))^2)\xrightarrow{\alpha}\min \qquad (6)$$



(a) Huber-type $\rho-$ function (b) Rd-type $\rho-$ function

Figure 2. Two kinds of $\rho-$ function with fault-tolerance

Similarly, we also can get an outlier–tolerant algorithm of for the exponential smoothing prediction parameter α:

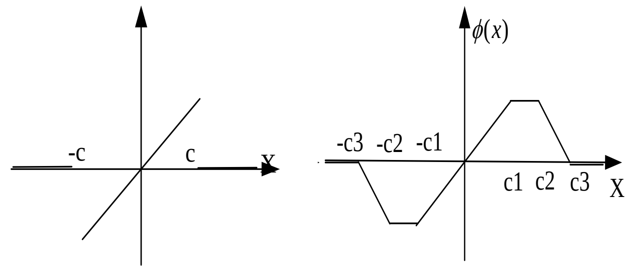
$$\begin{cases}\hat{\alpha}_k^{(0)}(\phi)=-\dfrac{\sum_{i=1}^{k}\phi((y(t_i)-\hat{y}(t_i))^2)(y(t_i)-y(t_{i-1}))(y(t_{i-1})-\hat{y}(t_{i-1}))}{\sum_{i=1}^{k}\phi((y(t_i)-\hat{y}(t_i))^2)(y(t_{i-1})-\hat{y}(t_{i-1}))^2}\\ \hat{\alpha}_k(\phi)=\max\{0,\min\{\hat{\alpha}_k^{(0)}(\phi),1\}\}\end{cases}$$

(7)

Accordingly, algorithm (5) and algorithm (1) can be amended as the following form of adaptive fault-tolerant exponential smoothing prediction algorithm as follows:

$$\begin{cases}\hat{y}(t_{k+1})=\hat{y}(t_k)+(1-\hat{\alpha}_k(\phi))\phi(y(t_k)-\hat{y}(t_k))\\ \hat{y}(t_1)=y(t_1)\end{cases} \qquad (8)$$

In formula (8), the $\phi-$ function $\phi(x)=\dot{\rho}(x)$. If the $\rho-$ function is the Huber-type function or Rd-type function in figure 2, then the $\phi-$ function is shown in figure 3.



(a) Huber type $\phi-$ function (b) Rd-type $\phi-$ function

Figure 3. Two kinds of $\phi-$ function with fault-tolerance

Intuitively, if the $\phi-$ function is from figure 3, then the exponential smoothing prediction algorithm (8) is outlier-tolerant. In fact, it can be seen from the figure 3 that so long as the forecast residual $\tilde{y}(t_k)=\tilde{y}(t_k)-\hat{y}(t_k)$ is below the reasonable scope threshold, two kinds of $\phi-$ function can make full use of innovation brought by new samples; However, once one-step predicting residual is beyond fixed threshold, the Huber-type $\phi-$ function has the ability to directly eliminate some negative influence from abnormal innovation on the of subsequent prediction; the Rd-type $\phi-$ function can make full use of the advantages of innovation and gradually reduce its impact on predicting results, according to the unusual degree with innovation.

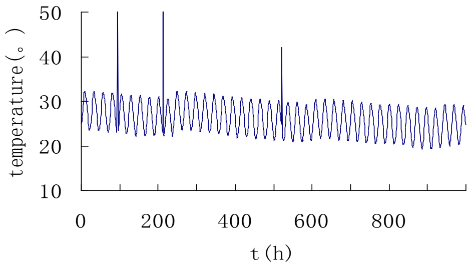
III. Actual Data Based Calculation And Analysis

Using data series as shown in figure 4(a) and the adaptive exponential smoothing prediction algorithm (5), the calculation results were shown in figure 4(b). Comparing the figure 4(b) with the figure 4(a), we may find out that the algorithm (5) can well forecast process variations. However, it is sensitive to outliers emerged in the sampling process, even causing a partial distortion to prediction results.

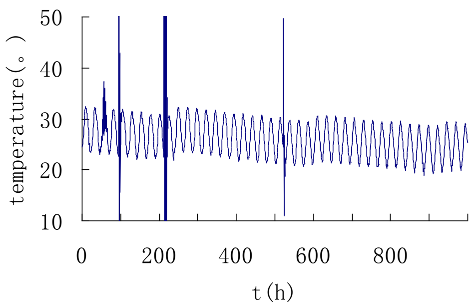
Using the formula (7)~(8) of the adaptive outlier-tolerant exponential smoothing prediction algorithm, and choosing the Rd-type $\phi-$ function $(c_1=3\sigma, c_1=5\sigma, c_1=7\sigma)$,we get one-step ahead prediction plot shown in figure 4(c). As can be clearly seen from these calculation data sequence, using the adaptive fault-tolerant exponential smoothing prediction algorithm can effectively avoid the adverse impact on outliers in sampling sequence and accurately predict the change of process status.

IV. Conclusion

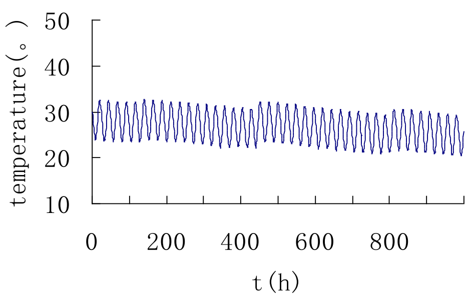
Comparing figure 4(c) with figure 4(a),it is shown that the adaptive outlier-tolerant exponential smoothing prediction algorithm has the ability to forecast process variations accurately under normal conditions for data sampling. Even if there are a few outliers in sampling data, the adaptive outlier-tolerant exponential smoothing prediction algorithm can also do well.



(a) Actual data plot



(b) Adaptive exponential smoothing prediction plot



(c) Adaptive fault-tolerant exponential smoothing prediction plot

Figure 4. Adaptive exponential smoothing prediction and adaptive fault-tolerant exponential smoothing prediction curves of spacecraft sensor temperature

The research contents of this paper and the results obtained have widely application value. The adaptive outlier-tolerant exponential smoothing prediction algorithm can be widely used in many different fields, such as space control and process monitoring and economic forecasting and sensor fault detection.

In fact, a lot of change in dynamic process can be transformed into abnormal changes in measurement data or in system state.

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