Design of Orthonormal Filter Banks based on Meyer Wavelet

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***Abstract*—A new design method for orthonormal FIR filter banks, which can be constructed using the generalized Meyer wavelet by taking into account the effect of time-shift factor, is proposed in this paper. These generalized Meyer wavelets are proved to be of the same basic properties and the time-frequency localization characteristics as the classical Meyer wavelet, furthermore some performances of the Meyer wavelets are improved by change of time-shift factor, which can better satisfy requirements of constructing orthonormal filter banks. The simulation shows that design of orthonormal filter banks based on the generalized Meyer wavelets with maximal symmetrical index is rational and effective.**

***Keywords*—Meyer wavelet; Time-shift factor; orthonormal FIR filter banks; Symmetrical Index**

I. Introduction

The Wavelets transform serves as an effective tool for Multi-resolution signal analysis that have found applications in data compression, image processing and information extraction. The time-frequency character of wavelet transform can be widely utilized to perform fine temporal analysis and fine spectrum analysis in high-frequency and low-frequency respectively. As is well known, the discrete wavelet transform is obtained by repeated sampling and filtering with low and high-pass finite impulse response (FIR) so that if the wavelet is orthonormal, the inverse problem, e.g. signal de-noising and perfect reconstruction , can be realized easily. In fact, from a traditional signal processing point of view, a wavelet is a band-pass filter, and therefore the wavelet transform can be interpreted as a constant-Q filtering with a set of filter banks. In terms of the investigation by Daubechies ,as in [1][2], when the wavelet bases are orthonormal, the scaling function $\phi(t)$ and wavelet $\psi(t)$ obey two-scale difference equations as:

$$\begin{aligned} &< \phi(x+l), \phi(x+l) >= \delta_{kl} \\ &< \phi(x+l), \psi(x+l) >= \delta_{kl} \end{aligned}, \tag{1}$$

which means the wavelet $\psi(t)$ bases are a group of perfect reconstruction FIR filter banks. Thus, these orthogonal FIR filter banks with orthogonal impulse responses can be designed by means of compactly supported wavelet bases. The Meyer wavelet is one of the earlier classical orthonormal wavelets bases, as in [3][4], which has good properties, such as fast convergence on frequency domain, regularity, localization in time domain as well as infinitely differentiable, so the Meyer wavelet can be introduced to design such orthonormal perfect reconstruction FIR filter banks, as in [5][6][7]. In this paper, Start with scaling functions $\phi(t)$ of the (classical) Meyer wavelet, the (classical) Meyer wavelets base is extended to the generalized Meyer wavelet bases to design the orthonormal filter banks, then analyze characteristics of the generalized Meyer wavelet including periodicity, symmetry and compact support conditions, finally, the simulation results show that these new wavelet bases are not only provide new approach for design of orthonormal filter banks and the construction of complex wavelets , but also develop and enrich the (classical) Meyer wavelet and others applications.

II. Theoretical Background

A. The Meyer wavelt

The Meyer wavelet function $\psi_M(t)$ and scaling function $\phi_M(t)$ have better localization characteristics, which are defined in the frequency domain, as in [2]. Usually, the wavelet function can be obtained by scaling function as follows:

$$\hat{\phi}_M(\omega) = \begin{cases} 1 & , \ |\omega| \le 2\pi/3 \\ \cos\left[\frac{\pi}{2} v\left(\frac{3}{2\pi}|\omega| - 1\right)\right] & , \frac{2\pi}{3} \le |\omega| \le \frac{4\pi}{3} \\ 0 & , \text{others} \end{cases} \tag{2}$$

where v function satisfy $v(x) + v(1-x) = 1$.

In light of theory of multi-resolution, since the $\hat{\phi}_M(\omega)$ defined by (2) is compact support, its support interval is $[-4\pi/3, 4\pi/3] \subset (-2\pi, 2\pi)$ and

$\hat{\phi}_{\mathrm{M}}(\omega)=1, \omega \in[-2\pi/3, 2\pi/3]$, Meyer scaling coefficients h_{M} expression in frequency-domain are given ,as in [3][4]:

$$\hat{h}_{\mathrm{M}}(\omega)=\sqrt{2}\frac{\hat{\phi}_{\mathrm{M}}(2\omega)}{\hat{\phi}_{\mathrm{M}}(\omega)}=\sqrt{2}\hat{\phi}_{\mathrm{M}}(2\omega)$$
$$=\sqrt{2}\cos\left[\frac{\pi}{2}v(\frac{3}{2\pi}|\omega|-1)\right]$$

(2)

Clearly, $\hat{h}_{\mathrm{M}}(\omega)$ is real and odd function, at the same time, its support interval is $[-2\pi/3, 2\pi/3]$, so the scaling coefficients function can be found as follows:

$$h_{\mathrm{M}}(t)=\frac{\sqrt{2}}{2\pi}\int_{-2\pi/3}^{2\pi/3}\cos[\frac{\pi}{2}v_m(\frac{3}{\pi}|\omega|-1)]\cos(\omega t)\,\mathrm{d}\,\omega$$

(3)

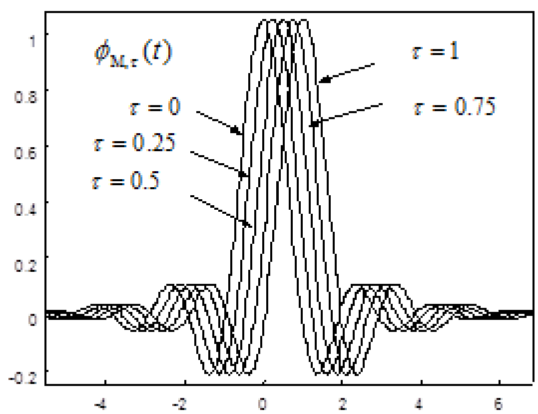

Fig. 1. The generalized Meyer scaling function $\phi_{M,\tau}(t)$

B. The Generilized Meyer Wavelet

In equation (3), the sampling sequence of the $h_{\mathrm{M}}(t)$ at the integral point $t = k, k \in Z$ is just the scaling coefficient filter h_{M} of the classical Meyer scaling function. However, according to the Nyquist sampling theorem, one can also get a new kind of time sequence which depends on the sampling values at the non-integral points ($t \neq k, k \in \mathrm{Z}$)of $h_{\mathrm{M}}(t)$ with the same sampling step $\Delta=1$, as in[3][4]. that is

$$h_{\mathrm{M}\tau}: h_k = h_{\mathrm{M}}(\Delta(k-\tau) \quad \tau \in R, k \in Z) \tag{4a}$$

using these sampling values, $h_{\mathrm{M}}(t)$ also can be reconstructed completely. In (4) τ be called a time-shift factor. In contrast to the classical Meyer scaling function coefficients h_{M}, these new sequences $h_{\mathrm{M},\tau}$ originating from different time-shift factor τ have the same magnitude spectrum: $|H_{\mathrm{M},\tau}(\omega)|=|H_{\mathrm{M}}(\omega)|$, which independent on the time-shift factor τ , such that they may reserve many similar basic characteristics (when $\tau=0$, $h_{\mathrm{M}}=h_{\mathrm{M},0}$). Similarly, the corresponding wavelets coefficients sequence can also be calculated as follows :

$$g_{\mathrm{M},\tau}: g_k = (-1)^k h_{\mathrm{M},\tau}(1-k) \quad \tau \in R, k \in Z$$

(4b)

Therefore, the $h_{\mathrm{M},\tau}, g_{\mathrm{M},\tau}$ $\tau \in R$ be called as generalized Meyer scaling coefficients sequences and wavelets coefficients sequence respectively. The corresponding scaling function $\psi_{\mathrm{M},\tau}(t)$ and wavelet $\phi_{\mathrm{M},\tau}(t)$ are named for the generalized Meyer scaling function and the generalized Meyer wavelet respectively in this paper. All of them constitute the generalized Meyer bases. The waveform of generalized Meyer $\psi_{\mathrm{M},\tau}(t)$ and $\phi_{\mathrm{M},\tau}(t)$ are shown in Fig.1 and Fig.2 when τ =0. 25, 0. 5,0. 75, 1.

It can be seen from Fig.1: (1) Though new sequence $h_{M,\tau}$ have not compact support interval, it decay fast. So the corresponding scaling function $\phi_{\mathrm{M},\tau}(t)$ also decay fast. (2). The generalized Meyer scaling function $\phi_{\mathrm{M},\tau}(t)$ about the parameter variable τ is periodic, the period is 1. Moreover, one finds that the waveform variation tendency of the generalized wavelets function $\psi_{\mathrm{M},\tau}(t)$ is also periodic, equal to 1, as can be seen from Fig.2.

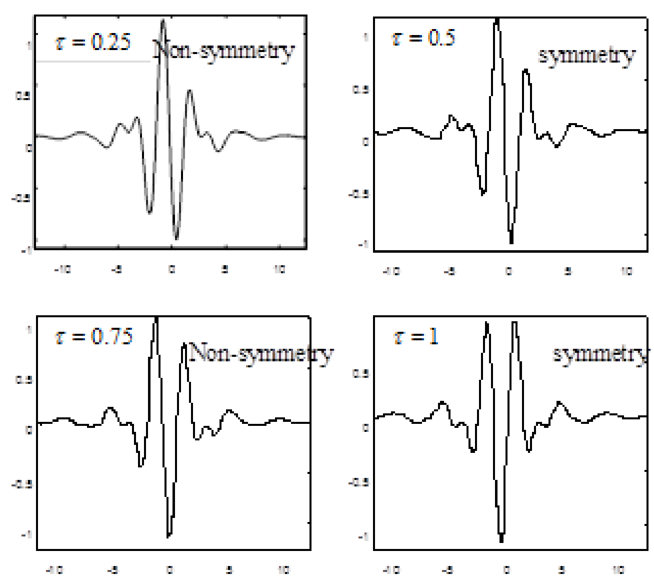

Fig. 2. The generalized Meyer wavelets fucnction $\psi_{M\tau}(t)$

It is ease to verify that new wavelet bases $\psi_{\mathrm{M},\tau}(t)$ which be determined by $h_{\mathrm{M},\tau}$ $\tau \in R$ have the same localization characteristics (which include in time domain, frequency domain and time-frequency domain), regularity and orthonormality as the classical wavelets bases $\psi(t)$, as in[3].

III. SIMULATION AND DISCUSSION

The generalized Meyer bases symmetry, including function scaling $\phi_{M,\tau}(t)$ and wavelets function $\psi_{M,\tau}(t)$, depends on the generalized Meyer scaling function $h_M(t)$ and time-shifting factor τ, in other words, be decided completely by symmetry of sequence $h_{M,\tau}$. In order to study the symmetrical degree of the generalized Meyer bases, the symmetrical index μ can be considered into sequence $h:\{h_k, k \in Z\}$, as following:

$$\mu = \max\left\{\sum_Z |h_k||h_{i-k}|\right\} \Big/ \|h\|_2^2 \,. \quad (5)$$

The symmetrical index μ ($0 < \mu \le 1$) can effectively evaluate symmetrical degree of sequence h and have nothing to do with position of central point. Here, In terms of maximal symmetrical index, the coefficients $h_{M,\tau}$ at $\tau = 0.5$ and $\tau = 1$ will be selected prior to other sequence $h_{M,\tau}$ for designing the orthonormal FIR filter banks, as in [4], which have linear phase or approximate linear phase . According to relation of $\psi(t) \leftrightarrow g \leftrightarrow h$, one can easily obtain a fast algorithm for decomposing real signal $s(t) \in L^2(R)$ and complex signal by a group of filters constructed by the generalized Meyer bases, as in [7][8]. Naturally, the initialized sequence can be given by

$$\begin{aligned} c^0 &: x_k^0 = \sum_n x(n)\phi(n-k) \\ {}_I c^0 &: x_k^0 = \sum_n x(n)\phi_\tau(n-k) \end{aligned} \,. \quad (6)$$

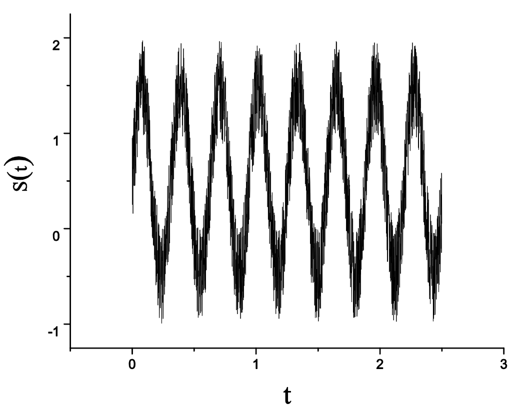

Fig. 3. The original signal $S(t)$ polluted by gauss white noise

Assuming the polluted signal is modeled as s(t) = sin(20t) + n(t), where n(t) denotes gauss white noise. It can been seen from Fig.3 that the distortions happened at peak value of the primitive signal sin(20t), and thereby the signal s(t) is composed of multiple frequency components. According to the above Generalizing method, the wavelet decomposition coefficients $h_{M,0.5}$ can be easily obtained as:

$$h_{M,0.5} = \{-0.1403\ 0.1686\ 0.6668\ 0.6668\ 0.1686\ -0.1403\ -0.0404\ 0.0664\}$$

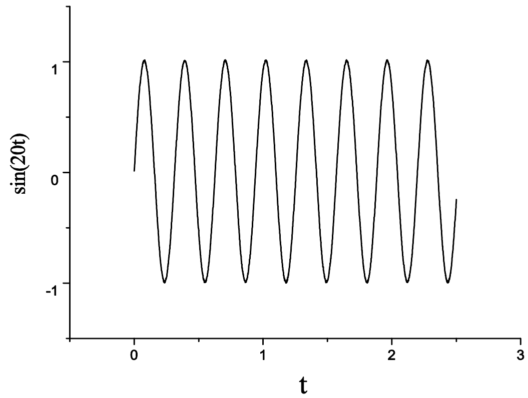

Fig. 4. The Reconstruction signal by orthonormal filter banks

Here, these generalized Meyer wavelet coefficients can be used to construct the orthonormal filter banks, as in [5][6], and such filter banks can be used for de-noising signals, which be called signal reconstruction. As shown in Fig.4, the primitive sine function are perfectly reconstructed and the noise is also reduced completely. These results show the generalized Meyer wavelets controlled by time shift τ really produce different properties of filter banks to satisfy such requirements as orthonormality, symmetry and regularity, as in [9][10].

IV. CONCLUSION

In this paper, the time shift τ can be considered into the scale function of the Meyer wavelet so as to produce a group of new wavelet bases, called the generalized Meyer wavelets. Because all generalized Meyer bases inherit many good properties of the original Meyer bases, a set of perfect reconstruction FIR filter banks designed by the generalized Meyer wavelet are regular and orthonormal accordingly. In addition, FIR filter banks with linear phase also might be constructed on the basis of symmetrical index, and has better smoothness for extracting signal envelope and construction of signal. As the time shift only affects the phase of filter function, it is also convenient to construct the complex analytical signal with desired phase for digital signal processing. In the subsequent work, the fast algorithm of these generalized Meyer wavelet coefficients will be discussed more in-depth and the analytical relation between the time shift and the phase change of will be derived for applications.

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