BRIQA: Framework for the Blind and Referenced Visual Image Quality Assessment

Jaime Moreno‡, Oswaldo Morales*, Ricardo Tejeida*, Eduardo García†, and Yasser Sánchez†.
Escuela Superior de Ingeniería Mecánica y Eléctrica, Campus Zacatenco
Instituto Politécnico Nacional
07738, Mexico.
*Research professors (Docente Investigador).
†Thesis students of degree level (Tesistas de Nivel Licenciatura).

Abstract—Our proposal is to present a Blind and Referenced Image Quality Assessment or BRIQA. Thus, the main proposal of this paper is to propose an Interface, which contains not only a Full-Referenced Image Quality Assessment (IQA) but also a No-Referenced or Blind IQA applying perceptual concepts by means of Contrast Band-Pass Filtering (CBPF). Then, this proposal consists in contrast a degraded input image with the filtered versions of several distances by a CBPF, which computes some of the Human Visual System (HVS) variables. If BRIQA detects only one input, it performs a Blind Image Quality Assessment, on the contrary if BRIQA detects two inputs, it considers that a Referenced Image Quality Assessment will be computed. Thus, we first define a Full-Reference IQA and then a No-Reference IQA, which correlation is important when is contrasted with the psychophysical results performed by several observers. BRIQA weights the Peak Signal-to-Noise Ratio by using an algorithm that estimates some properties of the Human Visual System. Then, we compare BRIQA algorithm not only with the mainstream estimator in IQA, PSNR, but also state-of-the-art IQA algorithms, such as Structural SIMilarity (SSIM), Mean Structural SIMilarity (MSSIM), Visual Information Fidelity (VIF), etc. Our experiments show that the correlation of BRIQA correlated with PSNR is important, but this proposal does not need imperatively the reference image in order to estimate the quality of the recovered image.

Keywords—Image Quality Assessment; Contrast Band-Pass Filtering; Peak Signal-to-Noise Ratio.

I. INTRODUCTION

The evolution of sophisticated Models and applications of Processing of Digital Images gives as a result of extensive literature describing these models. A significant number of this research is dedicated to Algorithms for improving only the image appearance. However, we consider that the digital image quality is distantly perfect. Images are presumably distorted during the whole process of compression or representation. Thus, it is important in the coding process of any image to improve image quality in order to identify and quantify the degree of degradation of a digital image.

Today, MSE or Mean Square Error is yet the most used quantitative metrics, since many other algorithms which evaluate image quality are based on it, Peak Signal-to-Noise Ratio (PSNR), for instance. Some authors as Wang and Bovik in [1], [2] mention that MSE is a poor assessment to be used in systems that predict image quality or fidelity. So, we want to expose what is wrong regarding MSE estimations, in order to propose new algorithm that makes use of some properties of the human eye, also our proposal tries to maintain the best properties of the MSE.

By one hand, let us define \( f(i, j) \) and \( \hat{f}(i, j) \) as the couple of images compared, which size is the amount of pixels inside them. Being \( f(i, j) \) the original image, considered with best possible quality or fidelity, and \( \hat{f}(i, j) \) a possible degraded version of \( f(i, j) \), whose quality we want to estimate. By the other hand, let us define the of the MSE and the PSNR in Equations 1 and 2, respectively.

\[
MSE = \frac{1}{l \times m} \sum_{i=1}^{l} \sum_{j=1}^{m} [f(i, j) - \hat{f}(i, j)]^2 \tag{1}
\]

and

\[
PSNR = 10 \log_{10} \left( \frac{\alpha^2}{MSE} \right) \tag{2}
\]

where \( \alpha \) is the hight value in terms of intensity inside \( f(i, j) \), size=\( l \times m \). Thus, for images witch contains only one channel, namely 8 bits per pixel (bpp) \( \alpha = 2^8 - 1 = 255 \). For chromatic images, Equation 2 also defines the estimation of PSNR, but for color images the MSE is separately computed of every component and then individual results are averaged.

Both MSE and PSNR are widely used in the field of image processing, as these algorithms have favorable features:

1) Convenient for the purpose of optimizing a certain algorithm that needs to improve quality. For instance in JPEG2000, MSE is employed both in Optimal Rate Allocation Methodology [3], [4] and Region of Interest Algorithms[5], [4]. Also, MSE is differentiable and integrable, so its employment could solve these kind of problems in terms of optimization, when it is use along with linear algebra, for instance.

2) By definition MSE compares the square difference of two images, giving as a result a clear meaning of leak of energy.

However, in some cases MSE estimates image quality with a low relation with quality given by an observer. A clear example is depicted by Figure 1, where both (a) Baboon and (b) Splash are coded and decoded by JPEG2000 compression...
Figure 1: Patches with size=256 × 256 of recovered images compressed by JPEG2000, PSNR=32dB.

with PSNR=32 dB. Figures 1(a) and 1(b) have very different visual quality. Then, either MSE or PSNR do not correlates with Human Visual System (HVS).

II. DEFINITION OF IMAGE QUALITY ASSESSMENT

In this section we outline of IQA definition, thus, we divide the IQA algorithms in two: Referenced and Non-Referenced approaches, the latter is known as Blind IQA. Thus, Referenced IQA Metrics can be divided in Bottom-Up and Top-Down Approaches.

Bottom-up approaches for evaluating image quality are methods that try to simulate well modeled features of HVS, and integrate them into the design of algorithms quality evaluation, hopefully, perform similar to HVS in the evaluation the image quality.

Moreover, the bottom-up attempt to simulate functional features in HVS that are important for the evaluation of image quality approaching. The main objective is to build algorithms that work alike HVS, at least for assessing of image quality evaluation.

On the contrary, the top-down systems simulate HVS differently. Top-down algorithms see HVS as a black box, and only the input-output task is cause for concern. A system for evaluating image quality from top to bottom can operate quite differently, since it predicts the behavior evaluation of image quality of an average human observer correctly.

An obvious task for the construction of a methodology of this type top-down approach is to formulate the problem of automatic supervised learning, as illustrated in Figure 2. Thus, HVS is blindly treated in order to learn its behavior. Training data is obtained through subjective experiments, where are viewed and evaluated by human subjects a large number of test images. The main objective is to model the system algorithm, so as to minimize the error between the desired output (subjective assessment) and the model prediction. This is generally a problem of regression or an approximation function.

By the other hand, No-reference or Blind image quality evaluation is a very difficult task in this field of image coding, but the conceptualization of the problem is very simple.

Somehow, an objective model should assess the quality of any real-world image, without reference to an original image. Thus, this looks like very difficult mission. The quality of an image can be judged quantitatively without having a objective algorithm of what a good/poor image quality is supposed to be similar. Then, surprisingly, this is a fairly easy assignment for human observers. HSV can easily recognize images with high quality when they are contrasted with low-quality images, and also our eye can identify what of these two images is good or bad without watching the reference image. In addition, humans observers tend to agree with each other to a very high degree. Example of this behavior when the human eye assesses image quality without seeing the reference image, it is very probable that says that the image is noisy, fuzzy, or compress by any image coder. In this way, Figure 1 shows an example of JPEG2000 compression, where the recovered images have lower quality than moved and stretched luminance contrast images.

III. BRIQA ALGORITHM

A. Contrast Band-Pass Filtering

The Contrast Band-Pass Filtering (CBPF) approximately estimates the image seen by a human observer with a δ separation by filtering some frequencies which are important or irrelevant for HVS. So, first of all let us define \( f(i,j) \) as the mathematical representation of the reference Image and δ as the separation between observer and the screen. Then CBPF estimates a filtered image \( \hat{f}(i,j) \), when \( f(i,j) \) is seen from δ centimeters. CBPF is founded on three main features: frequency of the pixel, spatial scales and surround filtering.

The CBPF methodology decomposes reference image \( f(i,j) \) into a set of wavelet planes \( \omega(s,o) \) of different spatial scales \( s \) (i.e., frequency of the pixel \( \nu \)) and spatial scales as:

\[
f(i,j) = \sum_{n=1}^{N} \omega(s,o) + c_n \quad (3)
\]
where $n$ is the amount of wavelet decompositions, $c_n$ is the plane in the pixel domain and $o$ spatial scale either vertical, horizontal or diagonal.

The filtered image $\hat{f}(i,j)$ is recovered by scaling these $\omega(s,o)$ wavelet coefficients employing Contrast Band-Pass Filtering function, which is at the same time an approximation Contrast Sensitivity Function(CSF, Figure 3). The CSF tries to approximate some psychophysical features [6], considering surround filtering information (denoted by $r$), perceptual frequency denoted by $\nu$, which is the gain of frequency either positive or negative depending on $d$. Filtered image $f(i,j)$ is defined by Equation 4.

$$\hat{f}(i,j) = \sum_{s=1}^{n} \beta(\nu,r)\omega(s,o) + c_n$$  \hspace{1cm} (4)

where $\beta(\nu,r)$ is the CBPF weighting function reproduce some properties of the HVS. The term $\beta(\nu,r)\omega(s,o) = \omega_{s,o,p,\delta}$ is the filtered wavelet coefficients of image $f(i,j)$ when it is watch at $\delta$ centimeters and is written as:

$$\beta(\nu,r) = z_{ctr} \cdot C_p(s) + C_{min}(s)$$ \hspace{1cm} (5)

Figure 4 depicts some examples of filtered images of Lenna, estimated by Equation 4 for a 19 inch monitor in the diagonal and 1280 pixels in columns, at $\delta = \{30, 100, 200\}$ cm.

**B. General Methodology**

Algorithm 1 shows the main methodology of this work. Thus, BRIQA Algorithm estimates the referenced visual quality of the distorted image $\hat{f}(i,j)$ regarding $f(i,j)$ the original reference image, if it exists, otherwise BRIQA estimates a blind visual image quality. Both algorithms need the definition of the Observational Distance $d$ given by the observer, so if $d$ is not defined, we estimate the distance $d$ from the actual observer by means of 3D/stereoscopic methodology, Algorithm 3.

<table>
<thead>
<tr>
<th>Algorithm 1: BRIQA: Framework to assess the quality of a digital image.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> $f(i,j)$, $\hat{f}(i,j)$, and $\delta$</td>
</tr>
<tr>
<td><strong>Output:</strong> ImageQuality</td>
</tr>
<tr>
<td>1 <strong>if</strong> $d$ does not exist <strong>then</strong></td>
</tr>
<tr>
<td>2 $d$=Compute Observational Distance by means of 3D/stereoscopic approach in Algorithm 3.</td>
</tr>
<tr>
<td>3 <strong>if</strong> $f(i,j)$ exists <strong>then</strong></td>
</tr>
<tr>
<td>4 ImageQuality = Algorithm 2($f(i,j)$, $\hat{f}(i,j)$, $\delta$), Referenced-IQA</td>
</tr>
<tr>
<td>5 <strong>else</strong></td>
</tr>
<tr>
<td>6 Estimate $f(i,j)$ from a pattern of the same size of $f(i,j)$, Figure 5</td>
</tr>
<tr>
<td>7 ImageQuality = Algorithm 2($f(i,j)$, $\hat{f}(i,j)$, $\delta$), Blind-IQA</td>
</tr>
</tbody>
</table>

Then a Full-reference image quality metric is performed, there is an reference image $f(i,j)$ and a recovered presumably distorted version $\hat{f}(i,j) = \theta[f(i,j)]$ that is contrasted against $f(i,j)$. It is important to mention $\theta$ is the algorithm that distorts the reference image and henceforth we refer the Full-reference image quality algorithm in BRIQA as RIQA. Otherwise, in the no-referenced image quality issue we refer BRIQA as BIQA. Furthermore, it is important to mention that BIQA only processes a degraded version of $\hat{f}(i,j)$. Thus from Figures 5(a) and 5(b), we compare $f(i,j)$ against a repetitive pattern $\Upsilon([0,1;1,0])$. Then, we perform the same algorithm in RIQA.

![Figure 5: (a) Primary Pattern [0,1;1,0] or $\Upsilon$. (b) Sixteenth Pattern or $\Upsilon^6$.](image)

Since both $f(i,j)$ and $\hat{f}(i,j)$ are observed at the same time at an observational distance $\delta$, if the similarity between $f(i,j)$ and $\hat{f}(i,j)$ appears to be better perceived is because $\delta$ tends to $0$. In contrast, if the observer judges $f(i,j)$ and $\hat{f}(i,j)$ when $\delta$ tends to $\infty$ the correlation between reference and distorted image would be the same. As any algorithm we
need to approximate the $\delta = \infty$, namely where similarity is so big that the observer confuse both images, we propose a no-linear regression for approximating $\infty$ to $\delta = \Delta$.

Either Reference Assessment or Blind Assessment, our proposal is based on Algorithm 2.

**Algorithm 2:** Algorithm for estimating Visual Image Quality Assessment.

- **Input:** $f(i,j)$, $\tilde{f}(i,j)$, and $\delta$
- **Output:** ImageQuality

1. Direct Wavelet Transformation of images $f(i,j)$ and $\tilde{f}(i,j)$
2. Estimation of Distance $\Delta$ (Equation 6), The distance where the observer cannot distinguish any difference in terms of quality between $f(i,j)$ and $\tilde{f}(i,j)$
3. Compute $f_s(i,j)$ and $\tilde{f}_s(i,j)$, namely contrast band-pass filtered wavelet coefficients at a distance $\Delta$. Where $\omega_{i,n,p,\Delta} = CBPF(f(i,j), \Delta)$ and $\tilde{\omega}_{i,n,p,\Delta} = CBPF(\tilde{f}(i,j), \Delta)$
4. Inverse Wavelet Transformation of $\omega_{i,n,p,\Delta}$ and $\tilde{\omega}_{i,n,p,\Delta}$ obtaining the contrast band-pass filtered images $f_s(i,j)$ and $\tilde{f}_s(i,j)$, respectively.
5. $ImageQuality =PSNR$ between contrast band-pass filtered images $f_s(i,j)$ and $\tilde{f}_s(i,j)$.

$nP$ and $emL$ are two features involved in the evaluation of Distance $\Delta$. Equation 6 show the estimation of $\Delta$, besides these two parameters it is important to know or estimate also $\delta$ in order to figure out the $nP$ and $emL$ distances. Furthermore Figure 6 depicts the Wavelet Energy Loss or $\varepsilon R$ (b), which shows not only the behavior of the relative energy but also the significance of $\Delta$, $nP$ and $emL$ inside an $\varepsilon R$ chart (a).

$\Delta = nP + emL$

Furthermore Figure 6(b) also show that the pinnacle inside teh function is $nP$, which is describe for the eye specialist as Near Point, which is between 15 to 20 centimeters for an adult. Thereby, $nP$ also can be defined as the distance where human eye can evaluate a pair of images $f(i,j)$ and $\tilde{f}(i,j)$. From this point $nP$, fewer the differences are perceived by the observer, until these differences disappear in the $\infty$. We find $\Delta$ by projecting the points $(nP, \varepsilon R(nP))$ and $(\Delta, \varepsilon R(\Delta))$ to $(\Delta, 0)$.

**C. Estimation of the Observational Distance $\delta$**

Estimation of the Observational Distance $\delta$ is based on Algorithm 3, which divided in six steps and is described as follows:

**Step 1:** Camera calibration by means of Function Stereo Calibration. Calibration Results are stored in a structure, which is defined as stereoParams.

**Step 2:** Ones both left and right cameras are calibrated, we take two images $I_l$ and $I_r$.

**Step 3:** With the parameters defined in stereoParams, we calibrate both $I_l$ and $I_r$ images using undistortImage function, giving as a result $I_{Cl}$ and $I_{Cr}$.

**Step 4:** In both $I_{Cl}$ and $I_{Cr}$ images we estimate two human characteristics: face and eyes detection. This detection is made by means of the function vision.CascadeObjectDetector. If in both $I_{Cl}$ and $I_{Cr}$ images are detected faces, then we detect eyes. This procedure increases the probability to find the head of the observer in the stereo-pair.

### IV. EXPERIMENTAL RESULTS

**A. Referenced Image Quality Assessment**

MSE[7], PSNR[7], SSIM[8], MSSIM[9], VSNR[10], VIF[11], VIPF[8], UQI[12], IFC[13], NQM[14], WSNR[15] and SNR are compared against the performance of BRIQA.
for JPEG2000 compression distortion. We chose for evaluating these assessments the implementation provided in [16], since it is based on the parameters proposed by the author of each indicator.

Table I shows the performance of RIQA and the other twelve image quality assessments across the set of images from TID2008, LIVE, CSIQ and IVC image databases employing Kendall Rank-Order Correlation Coefficient (KROCC) for testing the distortion produced by a JPEG2000 compression.

Table I: KROCC of RIQA and other quality assessment algorithms on multiple image databases using JPEG2000 distortion. The higher the KROCC the more accurate image assessment. Bold and italicized entries represent the best and the second-best performers in the database, respectively. The last column shows the KROCC average of all image databases.

<table>
<thead>
<tr>
<th>Metrics</th>
<th>TID2008</th>
<th>LIVE</th>
<th>CSIQ</th>
<th>IVC</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Images</td>
<td>100</td>
<td>228</td>
<td>150</td>
<td>50</td>
<td>528</td>
</tr>
<tr>
<td>IFC</td>
<td>0.7905</td>
<td>0.7936</td>
<td>0.7667</td>
<td>0.7788</td>
<td>0.7824</td>
</tr>
<tr>
<td>MSE</td>
<td>0.6382</td>
<td>0.8249</td>
<td>0.7708</td>
<td>0.7262</td>
<td>0.7400</td>
</tr>
<tr>
<td>MSSIM</td>
<td>0.8656</td>
<td>0.8818</td>
<td>0.8335</td>
<td>0.7821</td>
<td>0.8408</td>
</tr>
<tr>
<td>NQM</td>
<td>0.8034</td>
<td>0.8574</td>
<td>0.8242</td>
<td>0.6801</td>
<td>0.7913</td>
</tr>
<tr>
<td>PSNR</td>
<td>0.6382</td>
<td>0.8249</td>
<td>0.7708</td>
<td>0.7262</td>
<td>0.7400</td>
</tr>
<tr>
<td>SNR</td>
<td>0.5767</td>
<td>0.8055</td>
<td>0.7665</td>
<td>0.6538</td>
<td>0.7006</td>
</tr>
<tr>
<td>SSIM</td>
<td>0.8573</td>
<td>0.8597</td>
<td>0.7592</td>
<td>0.6916</td>
<td>0.7919</td>
</tr>
<tr>
<td>UQI</td>
<td>0.7415</td>
<td>0.7893</td>
<td>0.6995</td>
<td>0.6061</td>
<td>0.6602</td>
</tr>
<tr>
<td>VIF</td>
<td>0.8515</td>
<td>0.8590</td>
<td>0.8301</td>
<td>0.7903</td>
<td>0.8327</td>
</tr>
<tr>
<td>VIFP</td>
<td>0.8215</td>
<td>0.8547</td>
<td>0.8447</td>
<td>0.7229</td>
<td>0.8110</td>
</tr>
<tr>
<td>VSNR</td>
<td>0.8042</td>
<td>0.8472</td>
<td>0.7117</td>
<td>0.6949</td>
<td>0.7645</td>
</tr>
<tr>
<td>WSNR</td>
<td>0.8152</td>
<td>0.8402</td>
<td>0.8362</td>
<td>0.7656</td>
<td>0.8143</td>
</tr>
<tr>
<td>RIQA</td>
<td><strong>0.8718</strong></td>
<td><strong>0.8837</strong></td>
<td><strong>0.8682</strong></td>
<td><strong>0.7981</strong></td>
<td><strong>0.8555</strong></td>
</tr>
</tbody>
</table>

Thus, for JPEG2000 compression distortion, RIQA is getting the best results in all databases. RIQA correlates in 0.8837 for a database of 228 images of the LIVE database. On the average, RIQA algorithm is also correlates in 0.8555, using KROCC. Furthermore, JPEG2000 compression distortion, MSSIM is the second best indicator not only for TID2008, LIVE and IVC image databases but also on the average, since VIFP occupies second place for CSIQ image database. Thus, the correlation between the opinion of observers and the results of MSSIM is 0.0143 less than the ones of RIQA. So in general, we can conclude that PSNR can be improved its performance in 11.5% if it includes four steps of filtering, RIQA.

B. Blind Image Quality Assessment

Some metrics estimate Quality as PSNR does, but some metrics estimates degradation, MSE, for instance. It is important to mention that BIQA estimates the degradation with this degradation tends to zero means that the overall quality is getting better. We already check the behavior of RIQA, so in this section we develop comparisons for verifying the performance BIQA by comparing significance performance of different compress versions of the image Baboon. BIQA is a metric that gives decibels as PSNR does, so instead of employing a Non-Parametric Correlation, we use a parametric correlation coefficient, i.e. Pearson correlation coefficient in order to better compare the results between BIQA and PSNR.

Figures 7(a), 7(b), and 7(c) depict three JPEG2000 compression of the image Baboon with 0.05, 0.50, and 1.00 bits per pixel, respectively. Thereby PSNR estimates 18.55dB for Figure 7(a), 23.05dB for Figure 7(b), and 25.11dB Figure 7(c). While BIQA computes 43.49, 30.07 and 28.71 dB, respectively. Thus for the 0.05 bpp (Figure 7(a)), higher distortion is estimated both PSNR and BIQA.

Figure 8) depicts multiple JPEG2000 decoded images from 0.05 bpp to 3.00bpp, the increments of varies every 0.05 bpp. With the later data we can found that PSNR and BIQA between them is 0.9695, namely, for image Baboon for every 1,000 tests BIQA estimates in a wrong way only 20 assessments.

| P | g | e |

- **Figure 7:** JPEG2000 Distorted versions of color image Baboon at different bit rates expressed in bits per pixel (bpp). (a) High Distortion, (b) Medium Distortion and (c) Low Distortion.

- **Figure 8:** Comparison of PSNR and BIQA for the JPEG2000 distorted versions of image Baboon.

C. BRIQA Interface

In Figure 9(a) the graphic interface is shown that allows upload pictures and calculate their quality by using methods with and without reference which are selectable via a drop-down menu. The observer can also select the type of distance with and without reference which are selectable via a drop-down menu. The observer can also select the type of distance used the code which is the distance from the screen to the face of the observer.

In the Figure 9(b), it shows that selecting the metric referenced by the drop-down menu, you can load the original images (without compression) and distorted (noisy) using the buttons to load image which display a window that lets you explore folders and select the image. Pressing the button Calculate the Legend RIQA method allowing the return a numeric result associated with the image quality is applied.
Figure 9: Estimation of distance: Static Distance

Figure 9(c) shows that selecting the metric without reference automatically displays a window with the caption: No original image and the button to load the original image now is disabled.
By selecting the metric without reference Calculate button, the algorithm automatically switches to a method without reference to return a numeric value associated with the image quality, namely BIQA.

When selecting the type of distance in static mode, it is possible to move the slider that lets you change the value
of the distance used in the algorithms metrics with and without reference. By selecting the distance of dynamic type green buttons (enabled preview), show preview (blue) stop preview (red) are enabled allowing handling. Otherwise slider is disabled, Figure 10(a).

When the Preview button is pressed stereo cameras transmit the images to the computer and it is displayed on the screen, Figure 10(b).

Thus, When Measure distance button is pressed, it takes an arrangement of pictures from stereo-cameras. With this arrangement our algorithm automatically tries to detect the face and eyes of the observer if BRIQA finds them it estimates the observational distance $\delta$, Figure 10(c).

V. CONCLUSIONS AND FUTURE WORK

BRIQA is a metric divided in two algorithms full-reference (RIQA) and non-reference (BIQA) image quality assessments based on filtered weighting of PSNR by using a model that tries to simulate some features of the Human Visual System (CBPF model). Both proposed metrics in BRIQA are based on five steps.

When we compared RIQA Image Quality Assessment against several state-of-the-art metrics our experiments gave us a result that RIQA was the best-ranked image quality algorithm in the well-know image databases such as TID2008, LIVE, CSIQ and IVC, JPEG2000 compression algorithm is used as a method of distorting the cited image databases. Thus, it is 2.5% and 1.5% better than the second best performing method, MSSIM. On average, RIQA improves the results of PSNR in 14% and 11.5% for MSE.

In the Blind Image Quality Assessment, BIQA assessment correlates almost perfect for JPEG2000 distortions, since difference between BIQA and PSNR, on the average is only 0.0187.

Combine both RIQA and BIQA in the same interface was the main contribution of this work. Thus, a expert or non-expert in the quality images assessment field can perform its own experiments. These experiments could include dynamic quality estimations or static ones. As a future work of this paper could be to include a set of quality images estimators including RIQA and BIQA.

ACKNOWLEDGMENT

This work is supported by National Polytechnic Institute of Mexico (Instituto Politecnico Nacional, Mexico) by means of Project No. SIP-20160786, the Academic Secretary and the Committee of Operation and Promotion of Academic Activities (COFAA) and National Council of Science and Technology of Mexico (CONACyT) by means of grant No. 204151/2013.

It is important to mention that Sections III and IV are part of the degree thesis supported by Eduardo García and Yasser Sánchez.

REFERENCES