Identification and Nonlinear PID Control of Hammerstein Model using Polynomial Structures

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Abstract—In this paper, a new nonlinear discrete-time PID is proposed to control Hammerstein model. This model is composed by a static nonlinearity gain associated to a linear dynamic sub-system. Nonlinear polynomial structures are used to identify and to control this class of systems. The determination of parameters is based on the use of RLS algorithm. A coupled two-tank process is given to illustrate the effectiveness of the proposed approach.

Keywords—Parametric identification; Hammerstein model; RLS algorithm; Polynomial structure; Nonlinear PID controller

I. INTRODUCTION

There are well-developed theories for the control and identification of linear time invariant (LTI) systems. In modern applications, physical systems are nonlinear. This drives an increasing need for modeling techniques able to adequately describe these systems behavior. Nonlinear system identification is an important tool which can be used to improve control performance. Indeed, there are several types of models that describe perfectly this process such as Hammerstein model [1], polynomial structures [2] [3], Volterra [4], NARMAX [5], etc.

Hammerstein model is consisted of a static nonlinearity followed by a linear dynamic system. Many chemical processes have been modeled with it, for examples, pH neutralization processes [6], distillation columns [7] [8], polymerization reactor [9] [10] and dryer process [11].

Polynomial models are possibly the most attractive of all nonlinear representations due to the inherent simplicity of the model structure and because they revealed the dynamical properties of the underlying system is a very straightforward manner [12].

Serval nonlinear predictive control algorithms are existed based on PID [13], neural networks [14], B-spline neural networks [15], Fuzzy logic [16] adaptive predictive control [17] [18]. In most algorithms for nonlinear predictive control, their performance functions are minimized using nonlinear programming techniques to compute the future manipulated variables in on-line optimization. This can make the realization of the algorithms very difficult for real-time control.

An important advantage of block-structured models is that they allow the use of standard linear controller design methods. This is possible because static nonlinearity in the process can be negated by inserting the nonlinear inverse of static nonlinearity at the appropriate place in the loop [19] [20] [21].

For the Hammerstein model, reverse nonlinear tuning must be placed at the output of the controller, wish only sees the linear dynamic part of the process and conventional linear controller methods can be used. Often static nonlinearity may be non-invertible this present a limit for this method.

In this work, a polynomial structure is employed to describe the nonlinear static function of Hammerstein model. Recursive least squares RLS algorithm is used to estimate the unknown parameters. A new nonlinear discrete PID is proposed. It is composed by a linear controller associated with the inverse of the nonlinearity wish is obtained by an approximation using polynomial structure.

The remainder of this paper is organized as follows: first, a Parametric identification of the Hammerstein model is defined. Second, a proposed nonlinear polynomial structure of Hammerstein model is described. Third, a method to control the model is presented. After that, the proposed identification and control method are applied to a coupled two-tank system.

II. PARAMETRIC IDENTIFICATION OF MODEL HAMMERSTEIN

Assume that the Hammerstein model of Fig. 1 is composed of a nonlinear block $f(.)$ associated with a linear sub-system $B(q^{-1})/A(q^{-1})$. It is described by:

\[
\begin{align*}
q_k &= B(q^{-1})u_k \\
q_k &= f(u_k)
\end{align*}
\]  

(1)

with:

\[
A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-nA}
\]

\[
B(q^{-1}) = b_1 q^{-1} + \ldots + b_n q^{-nB}
\]

\[
v_k = d_1 u_k + d_2 u_k^2 + \ldots + d_N u_k^N
\]

$q^{-1}$ delay operator, $u_k$ input of the system, $y_k$ output, $v_k$ the unmeasurable internal signal and $w_k$ represents the modeling error, external disturbances, etc.

In order to have a unique parameterization of the Hammerstein model structure, the first coefficient of the nonlinear function $f(.)$ equals to 1, $d_1 = 1$ [22] [23].
The output $y_k$ is given by:

$$y_k = -\sum_{i=1}^{n_a} a_i y_{k-i} + \sum_{i=1}^{n_b} b_i \left( u_{k-i} + \sum_{p=2}^{N} d_p u^p_{k-i} \right)$$

Eq. 2 can be put in the following form:

$$y_k = \Phi_k^T \theta_k$$

with:

$$\Phi_k = \begin{pmatrix} Y_k \\ U_k \end{pmatrix}, \quad \theta_k = \begin{pmatrix} a_k \\ b_k \\ s_k \end{pmatrix},$$

$$Y_k = \begin{pmatrix} -y_{k-1} \\ -y_{k-2} \\ \vdots \\ -y_{k-n_a} \end{pmatrix}, \quad U_k = \begin{pmatrix} U_{1k} \\ U_{2k} \\ \vdots \\ U_{Nk} \end{pmatrix},$$

$$U_{jk} = \begin{pmatrix} u_{j,k}^{l-1} \\ u_{j,k}^{l-2} \\ \vdots \\ u_{j,k}^{l-k_n} \end{pmatrix}, \quad \text{for } j = 1, 2, \ldots, N$$

$$a_k = \begin{pmatrix} a_{1k} \\ a_{2k} \\ \vdots \\ a_{nk} \end{pmatrix}, \quad b_k = \begin{pmatrix} b_{1k} \\ b_{2k} \\ \vdots \\ b_{nk} \end{pmatrix}, \quad s_k = \begin{pmatrix} d_{2k} b_k \\ d_{3k} b_k \\ \vdots \\ d_{Nk} b_k \end{pmatrix},$$

$\Phi_k$ and $\theta_k \in R^{n u}$ where $n_R = n_a + N n_b$, $Y_k \in R^{n_a}$, $U_k \in R^{n_u}$, $a_k \in R^{n_u}$, $b_k \in R^{n_u}$ and $s_k \in R^{N n_b}$.

The parameter vector $\theta_k$ can be estimated using the RLS algorithm. It is described by the following equations:

$$\begin{align*}
\dot{\theta}_k &= \dot{\theta}_{k-1} + P_k \Phi_k \varepsilon_k \\
P_k &= P_{k-1} - \frac{P_{k-1} \Phi_k \Phi_k^T P_{k-1}}{1 + \Phi_k^T P_{k-1} \Phi_k} \\
\varepsilon_k &= y_k - \dot{\theta}_k^T \Phi_k
\end{align*}$$

III. PROPOSED NONLINEAR POLYNOMIAL STRUCTURE OF THE HAMMERSTEIN MODEL

We propose a nonlinear polynomial structure of $f(\cdot)$. It's described by:

$$v_k = f(u_k) = \sum_{i=1}^{r} w_{i,k} U_k^{[i]} = \sum_{i=1}^{r} \tilde{w}_{i,k} \tilde{U}_k^{[i]}$$

with $w_{i,k}, \tilde{w}_{i,k} \in R^{1 \times n_i}$ (resp. $\tilde{w}_{i,k} \in R^{1 \times n_i}$) are variable vector and $U_k = (u_k, u_{k-1}, \ldots, u_{k-n_a+1})^T \in R^{n_u}$ where $n_a \leq n_A$. $U_k^{[i]}$ is the Kronecker power of the vector $U_k$ defined as [12]:

$$
\begin{align*}
U_k^{[0]} &= 1, \\
U_k^{[i]} &= U_k^{[i-1]} \otimes U_k = U_k \otimes U_k^{[i-1]}, \quad \text{for } i \geq 1
\end{align*}
\ni \otimes \text{designates the symbol of the Kronecker product,}
\tilde{U}_k^{[i]} \in R^{n_i}, \quad \text{for } i = 1, 2, \ldots, r \quad \text{and } n_i = \left( \frac{n + i - 1}{i} \right),
$$

is the non-redundant. It’s defined as:

$$\tilde{U}_k^{[i]} = U_k^{[i]} = U_k$$

$$\begin{align*}
\tilde{U}_k^{[i]} &= \begin{pmatrix} u_{i,k}^{1} \\ u_{i,k}^{1} u_{i,k}^{0-1} u_{i,k}^{0-1} \\ \vdots \\ u_{i,k}^{i-3} u_{i,k}^{1} \\ u_{i,k}^{i-3} u_{i,k}^{1} \\ \vdots \\ u_{i,k}^{0} \end{pmatrix}, \quad \text{for } i \geq 2
\end{align*}$$

when the repeated components of the redundant $(i)$th - power $U_k^{[i]}$ are omitted and $r$ is the polynomial order.

In this work, we have modeled $v_k = f(u_k)$ by a nonlinear polynomial structure as:

$$v_k = \tilde{w}_{1,k}^{1} U_k + \tilde{w}_{2,k}^{1} \tilde{U}_k^{[2]} + \tilde{w}_{3,k}^{1} \tilde{U}_k^{[3]} + O(U_k^{[4]})$$

with $U_k = \begin{pmatrix} u_k \\ u_{k-1} \\ \vdots \\ u_{k-n_a+1} \end{pmatrix}$, $\tilde{w}_{i,k}^{1} = \begin{pmatrix} \alpha_{i,k} \\ \alpha_{i,k} \alpha_{i,k} \end{pmatrix}$ and $\tilde{w}_{j,k}^{1} = \begin{pmatrix} \alpha_{j,k} \alpha_{j,k} \alpha_{j,k} \alpha_{j,k} \end{pmatrix}$

and the linear dynamic system by:

$$y_k = -a_{1,k} y_{k-1} - a_{2,k} y_{k-2} + b_{1,k} v_{k-1} + b_{2,k} v_{k-2}$$

Equations 8 and 9 give:

$$y_k = -a_{1,k} y_{k-1} - a_{2,k} y_{k-2} + b_{1,k} v_{k-1} + b_{2,k} v_{k-2} + \tilde{w}_{2,k}^{1} \tilde{U}_k^{[2]} + \tilde{w}_{3,k}^{1} \tilde{U}_k^{[3]} + O(U_k^{[4]})$$

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The parameters $\alpha_{i,k}, \ i = 1, 2, \ldots, 7,$ will be successively estimated by the RLS algorithm.

IV. NONLINEAR PID CONTROLLER OF THE HAMMERSTEIN MODEL

In this section, the control of the Hammerstein model with a nonlinear PID will be discussed. Firstly, we presented a nonlinear PID based on the exact inverse of $f(.)$. After that, we proposed a method to determine the inverse of the nonlinearity using the polynomial structure which will be used to control the Hammerstein model.

A. Nonlinear PID controller using the exact inverse

The design strategy discrete-time control is implemented by introducing the inverse function of Hammerstein model [20]. Fig. 2, illustrates the control of Hammerstein model. It is based PID regulator as [24]:

$$u_k = K_p \varepsilon_k + K_i \sum_{j=0}^{k} \varepsilon_j + K_d \frac{\varepsilon_k - \varepsilon_{k-1}}{T_e} \quad (11)$$

where $\varepsilon_k = y^m_k - y^m_{\text{set}}$ is the error, $y^m_{\text{set}}$ is the set point, $y^m_k$ is the response of the model, $T_e$ is the sampling period and $K_p, K_i$ and $K_d$ are the proportional, integral and derivative controller gains, respectively.

![Fig. 2 – Nonlinear PID controller of a Hammerstein model [21].](image)

This technique is valid only if the nonlinear function $f(.)$ is invertible.

B. Proposed PID controller based-on polynomial structure

The proposed method consisted to approximate the inverse nonlinear gain using the polynomial structure, noted $f^{-1}_{\text{app}}(.)$. It eliminated the effect of the nonlinear gain in the Hammerstein model. Hence, a new nonlinear PID, noted $PID^NL$, is obtained which is described as follow:

$$PID^NL = PID f^{-1}_{\text{app}}(.) \quad (12)$$

![Fig. 3 – Proposed nonlinear PID controller of a Hammerstein model.](image)

Noting that $\hat{v}(k)$ is an approximate signal of $u_k$, we have chosen the following structure of $u_k = f^{-1}_{\text{app}}(\hat{v}_k)$:

$$u_k = w^1_k V_k + \bar{w}^2_{1,k} [V^2_k + \bar{w}^3_{3,k} V^3_k] + O(V^4_k) \quad (13)$$

with $V_k = \left( \hat{v}_k, \hat{v}_{k-1} \right)$, $w^1_{1,k} = ( \beta_{1,k} \beta_{2,k} )$, $\bar{w}^2_{2,k} = ( \beta_{3,k} \beta_{4,k} \beta_{5,k} )$ and $\bar{w}^3_{3,k} = ( \beta_{6,k} \beta_{7,k} \beta_{8,k} \beta_{9,k} )$.

By the identification of equations 8 and 13, we obtain:

$$\beta_{1,k} = 1; \beta_{2,k} = -1; \beta_{3,k} = -\alpha_{1,k};$$
$$\beta_{4,k} = 2 \alpha_{1,k} - \alpha_{2,k}; \beta_{5,k} = \alpha_{2,k} - \alpha_{3,k};$$
$$\beta_{6,k} = 2 \alpha_{1,k}^2 - \alpha_{4,k};$$
$$\beta_{7,k} = 3 \alpha_{1,k} \alpha_{3,k} - 6 \alpha_{2,k}^2 + 3 \alpha_{4,k} - \alpha_{5,k};$$
$$\beta_{8,k} = -5 \alpha_{1,k} \alpha_{3,k} + 5 \alpha_{2,k}^2 + 2 \alpha_{1,k} \alpha_{2,k} - 3 \alpha_{4,k} - 2 \alpha_{5,k} - \alpha_{6,k};$$
$$\beta_{9,k} = \alpha_{2,k}^3 + 3 \alpha_{4,k}^2 + \alpha_{1,k} \alpha_{3,k} + \alpha_{2,k} \alpha_{3,k} + 3 \alpha_{5,k} + \alpha_{6,k}.$$
B. Mathematical modeling

The coupled tank system is described by the following nonlinear equations:

\[
\begin{align*}
    h_1 &= K \text{sat}(h_{11}) \\
    h_{11} &= \eta \text{sat}(u) - \frac{a_1}{A} \sqrt{2g \text{sat}(h_{11})} \\
    h_2 &= K \text{sat}(h_{22}) \\
    h_{22} &= \frac{a_2}{A} \sqrt{2g \text{sat}(h_{11})} - \frac{a_1}{A} \sqrt{2g \text{sat}(h_{22})}
\end{align*}
\]

(14)

with:

\[
\text{sat}(h_{ii}) = \begin{cases} 
    h_{ii} & \text{if } 0 \leq h_{ii} \leq 0.3 \\
    0.3 & \text{if } h_{ii} > 0.3 \\
    0 & \text{if } h_{ii} < 0
\end{cases}
\]

\[
\text{sat}(u) = \begin{cases} 
    u & \text{if } 0 \leq u \leq 5 \\
    5 & \text{if } u > 5 \\
    0 & \text{if } u < 0
\end{cases}
\]

\(h_1\) and \(h_2\) denote the water level in the corresponding tank and \(u\) is voltage applied to the pumps. \(a_1\) and \(a_2\) are the outlet area of the tanks, \(\eta\) constant relating the control voltage with the water flow from the pump, \(A\) is the cross-sectional area of the tanks and \(g\) is the gravitational constant. The values of the simulink system parameters are shown in the table I.

<table>
<thead>
<tr>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u)</td>
<td>0 - 5 V</td>
<td>Voltage level of pump</td>
</tr>
<tr>
<td>(A)</td>
<td>0.01389 m(^2)</td>
<td>Cross-sectional area</td>
</tr>
<tr>
<td>(a_i)</td>
<td>0.26.5 (10^{-6}) m(^2)</td>
<td>Outlet area of tank (i)</td>
</tr>
<tr>
<td>(\eta)</td>
<td>2.4 (10^{-3}) m(^3)/s</td>
<td>Water level of tank (i)</td>
</tr>
<tr>
<td>(g)</td>
<td>9.81</td>
<td>m/s(^2) Gravitational constant</td>
</tr>
<tr>
<td>(K)</td>
<td>100</td>
<td>constant</td>
</tr>
</tbody>
</table>

\[\text{TABLE I – Parameters of the System [25]}\]

C. Parametric estimation and control result using the exact inverse function of \(f(.)\)

The input of the system is shown in Fig. 5. The signal is set to \([0 \ldots +5V]\). It’s a pseudo-random binary sequence SBPA. The value of the sampling period is \(T_e = 1s\).

\[\text{Fig. 5 – SBPA signal } u_k.\]

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simulation system parameters are shown in the table I.
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```

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```

```plaintext
\[\text{Fig. 5 – SBPA signal } u_k.\]
```
• The inverse of the nonlinearity $f^{-1}(\tilde{v}_k)$:
  $$u_k = w_{1,k}^3 V_k + \tilde{w}_{2,k}^3 \hat{V}_k^2 + \tilde{w}_{3,k}^3 \hat{V}_k^3 + O(V_k^4)$$
  with:
  $$w_{1,k}^3 = (1 -1)$$
  $$\tilde{w}_{2,k}^3 = (-5 10 -5)$$
  $$\tilde{w}_{3,k}^3 = (25 75 76 25)$$

• The PID regulator:
  $$(K_p, K_i, K_d) = (0.45, 0.7, 0)$$

D. Parametric estimation and control result using polynomial structures

We used the signal shown in Fig. 5 as input to estimate the model described by:

$$\begin{align*}
y_k &= \frac{B(q^{-1})}{A(q^{-1})} v_k = 10^{-3} \frac{3.7657q^{-1} + 5.1097q^{-2}}{1 - 1.9518q^{-1} + 0.9527q^{-2}} v_k \\
v_k &= \tilde{w}_{1,k}^1 U_k + \tilde{w}_{2,k}^1 \hat{U}_k^2 + \tilde{w}_{3,k}^1 \hat{U}_k^3 + O(U_k^4)
\end{align*}$$

(16)

Fig. 10 shows the diagram block of the proposed nonlinear PID controller of a two-tank system. It consists of:

Fig. 11 presents the responses of estimated and real output $h_2$. Simulation results demonstrate that the proposed structure describe very well the system behavior.

Fig. 10 – Responses of estimated and real output $h_2$. Simulation results demonstrate that the proposed structure describe very well the system behavior.

![Fig. 9 – Responses of $u$ and $h_2$ of system.](image)

![Fig. 10 – Responses of the real (solid line) and estimated (dotted line) output $h_2$ using the proposed approach.](image)

![Fig. 11 – Proposed nonlinear PID controller of a two-tank system.](image)

![Fig. 12 – Responses of $u$ and $h_2$ using the proposed method of identified model.](image)

![Fig. 13 – Responses of $u$ and $h_2$ using the proposed method of system.](image)
The control signals and the results of the regulator are presented in figures 12 and 13. They proved that the $PID^{NL}$ had achieved a satisfactory performance in tracking the reference signal.

VI. CONCLUSION

In this paper, a new strategy to identify Hammerstein model has been proposed. A polynomial structure is used to model the nonlinear static function. This structure provides a good identification results. It has the advantage to approximate the inverse of the nonlinear part of Hammerstein model. A new design of $PID^{NL}$ controller is successfully elaborated. It is composed by a PID associated with the inverted of the identified nonlinearity. A two-tank system is presented to illustrate the effectiveness of the proposed approach.

REFERENCES


