Fuzzy Pi Adaptive Learning Controller for Controlling the Angle of Attack of an Aircraft

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Abstract—In this paper, a Fuzzy PI Adaptive Learning controller is proposed for a flight control system to control the angle of attack of an aircraft. The proposed controller tracks the reference angle as desired by the pilot of the aircraft. The performance indices are evaluated and the corresponding value is compared with that for the conventional controllers obtained from Zigler Nichols (ZN), Tyreus Luyben (TL) and Extended Skogestad Internal Model Controller (ESIMC). The performance indices such as Mean Square Error (MSE), Integral Absolute Error (IAE) and Integral Absolute Time Error (IAITE) are evaluated to verify superiority of one over another.

Keywords—Angle of Attack; Interpolation Rule; Performance Indices; Fuzzy PI Adaptive Learning Controller

I. INTRODUCTION

An aircraft flies in a 3D space controlled by its control surfaces such as aileron, rudder and elevator. Generally the motion of aircraft is changed by these control surfaces, but the angle of attack of the aircraft is controlled by the deflection of the elevator. Since to control the angle of attack of an aircraft is very crucial, therefore fuzzy controllers are frequently used to offer better and accurate output as compared to conventional controllers ZN, TL and ESIMC (Interpolation Rule).


In this paper, an adaptive fuzzy PI controller is implemented for controlling the angle of attack of an aircraft. Then the performance indices (MSE, ISE & ITAE) of the aircraft are evaluated and the results are compared with the conventional Zeigler Nichols, Tyreus Luyben and Skogestad Internal Model Control techniques and it was established that the adaptive fuzzy PI controller gives excellent results which improves the performance indices and reduces the error.

II. ANGLE OF ATTACK CONTROL SYSTEM

Figure 1 below depicts the block diagram representation of the angle of attack with disturbance and controller. In this diagram input is the elevator deflection and output is the angle of attack.

\[ G(s) = \text{Transfer function of angle of attack} \]
\[ C(s) = \text{Fuzzy PI controller Transfer Function} \]
\[ G(s) = G_i(s) = \text{Disturbance} \]

where,
\[ \delta_i(s) = \text{The elevator deflection} \]
\[ \alpha = \text{The angle of attack} \]
Angle of attack is defined as the angle between the chord line of the wing and the relative motion between aircraft and atmosphere. It is controlled by the elevator deflection. Figure 2 below illustrates the angle of attack and the direction of relative wind.

![Fig. 2. Angle of attack and the direction of relative wind](image)

Considering the short period approximation (speed of the aircraft u=constant) the longitudinal dynamics [5] of the aircraft reduces to elevator deflection, then using vector matrix notation, Equation (1) and Equation (2) may be written as

\[ \dot{x} = Ax + Bu \]

where,

\[ A = \begin{bmatrix} Z_u & U_0 \\ \left(M_u + M_e Z_u\right) & \left(M_u + U_0 M_u\right) \end{bmatrix} \]

\[ B = \begin{bmatrix} Z_u \\ M_u + Z_u M_u \end{bmatrix} \]

Now

\[ \begin{bmatrix} sI - A \\ \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \end{bmatrix} \begin{bmatrix} Z_u & U_0 \\ \left(M_u + M_e Z_u\right) & \left(M_u + U_0 M_u\right) \end{bmatrix} \begin{bmatrix} \left(M_u + M_e Z_u\right) & \left(M_u + U_0 M_u\right) \end{bmatrix} \]

\[ = \begin{bmatrix} \left(s - M_u - U_0 M_u\right) s \end{bmatrix} \]

Again, \( \Delta_e(s) = \text{det}[sI - A] = s^2 + \left(Z_u + M_u + U_0 M_u\right)s + \left(Z_u M_u - U_0 M_u\right) \)

The transfer function is given by

\[ \frac{w(s)}{\delta_e(s)} = \frac{\left(U_0 M_u + M_u Z_u\right)(1+sT_i)}{\left(U_0 M_u M_u + M_u Z_u\right)(1+sT_i)} \]

where, \( K_u = \frac{U_0 M_u + M_u Z_u}{T_i} \)

Again,

From Equation (6) and Equation (7), the transfer functions for angle of attack is given by

\[ \frac{\alpha(s)}{\delta_e(s)} = \frac{\left(U_0 M_u + M_u Z_u\right)(1+sT_i)}{U_0 M_u Z_u - M_u Z_u} \]

\[ \Rightarrow \alpha(s) = K_u \left(1+sT_i\right) \]

The above values are the stability derivatives [5] of longitudinal dynamics of FOXTROT aircraft as shown in “Table 1” below.

### TABLE I. STABILITY DERIVATIVES OF FOXTROT AIRCRAFT

<table>
<thead>
<tr>
<th>Stability Derivatives</th>
<th>Flight Condition (FC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FC-1</td>
</tr>
<tr>
<td>( U_0 ) ( (ms^{-1}) )</td>
<td>70</td>
</tr>
<tr>
<td>( Z_u )</td>
<td>-0.117</td>
</tr>
<tr>
<td>( Z_w )</td>
<td>-0.452</td>
</tr>
<tr>
<td>( Z_q )</td>
<td>-0.76</td>
</tr>
<tr>
<td>( M_u )</td>
<td>0.0024</td>
</tr>
<tr>
<td>( M_w )</td>
<td>-0.006</td>
</tr>
<tr>
<td>( M_q )</td>
<td>-0.002</td>
</tr>
<tr>
<td>( M_y )</td>
<td>-0.317</td>
</tr>
<tr>
<td>( X \delta_e )</td>
<td>1.83</td>
</tr>
<tr>
<td>( Z \delta_e )</td>
<td>-2.03</td>
</tr>
<tr>
<td>( M \delta_e )</td>
<td>-1.46</td>
</tr>
</tbody>
</table>

The transfer function for both the flight conditions (Flight Condition-1 and Flight Condition-2) are obtained after substituting the values of the stability derivatives mentioned in “Table 1” above. Now the transfer functions are given by

Flight Condition-1

\[ G_1(s) = \frac{2.0302s + 102.8}{s^2 + 0.901s + 0.5633} = \frac{3.604s + 182.5}{1.7752s^2 + 1.5798s + 1} \]

Flight Condition-2

\[ G_2(s) = \frac{15.11s + 0.003027}{s^2 + 1.2989s + 8.216} = \frac{1.84s + 368.5}{0.1217s^2 + 0.1581s + 1} \]

### III. CONVENTIONAL PI CONTROLLERS

The transfer function for PI controller \( C(s) \) is given by \( C(s) = K_p(s) + K_i(s) \) and the values of \( K_p \) and \( K_i \) are determined by various types of conventional PI controllers, such as Zeigler Nichols, Tyreus Luyben and Extended Skogestad Internal Model Controller and are discussed as follows:

#### A. Zeigler-Nichols (ZN) PI controller

In this method, the PI controller parameters \( K_p \) and \( T_i \) depend on the value of ultimate gain \( K_u \) and ultimate period

\[ K_p = \frac{U_0 M_u + M_u Z_u}{T_i} \]

\[ T_i = \frac{Z_u}{K_u} \]
\( P_u \) for sustained oscillations. The value of PI controller parameters is shown in Table 2 below.

<table>
<thead>
<tr>
<th>PID Type</th>
<th>( K_p )</th>
<th>( T_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>( k_u )</td>
<td>( 2.2 )</td>
</tr>
</tbody>
</table>

### B. Tyreus-Luyben(TL) PI Controller

In this type of controller [3], the oscillations are minor and the controller is robust unlike Ziegler Nichols and the tuning parameters \( K_p \) and \( T_i \) are illustrated in the Table 3 below.

<table>
<thead>
<tr>
<th>PID Type</th>
<th>( K_p )</th>
<th>( T_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>0.45( k_u )</td>
<td>( \frac{P}{1.2} )</td>
</tr>
</tbody>
</table>

### C. Extended Skogestad Internal Model (ESIMC) PI Controller (Interpolation Rule)

In this type of controller [2], the values of \( K_p \) and \( K_i \) for proportional and integral controller are given by

\[
K_p = \max \{ A, X \},
\]

where, \( X = B \) for \( \zeta \geq 1 \) and \( X = \zeta B + (1-\zeta) C \) for \( \zeta < 1 \)

\[
K_i = \max \{ A, X \},
\]

where, \( X = B \) for \( \zeta \geq 1 \) and \( X = \zeta B + (1-\zeta) C \) for \( \zeta < 1 \)

The values of \( A, B, B' \) and \( C \) for proportional and integral controllers are given in Table 4 below.

<table>
<thead>
<tr>
<th>Calculation of ( K_C )</th>
<th>Calculation of ( K_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) ( = \frac{2\zeta}{\sqrt{\zeta^2 - 1}} )</td>
<td>( \frac{1}{k' (\tau_c + \theta) \tau_0} )</td>
</tr>
<tr>
<td>( B ) ( = \frac{1 + 4(\tau_c + \theta) + (\zeta + \sqrt{\zeta^2 - 1})}{k' (\tau_c + \theta) \tau_0} )</td>
<td>( \frac{1}{k' (\tau_c + \theta)^2 \tau_0} )</td>
</tr>
<tr>
<td>( B' ) ( = \frac{1 + 4(\tau_c + \theta) + \zeta}{k' (\tau_c + \theta) \tau_0} )</td>
<td>( \frac{\zeta}{k' (\tau_c + \theta) \tau_0} )</td>
</tr>
<tr>
<td>( C ) ( = \frac{1}{2k' (\tau_c + \theta)^2} )</td>
<td>( \frac{1}{16k' (\tau_c + \theta)^3} )</td>
</tr>
</tbody>
</table>

In Table 4 above,

\[
k' = \frac{k}{\tau_0^2}
\]

\( k \) = The gain

\[
\tau_0 = \frac{1}{\omega_n}
\]

\( \omega_n \) = Natural frequency of oscillation

\( \zeta \) = Damping ratio

\( \tau_c \) = The controller tuning parameter

\( \theta = \tau_0 (1.5 + 0.5\zeta)(0.6)^2 \) = the delay angle

\( a = \tau_0^2 \)

\( B \) is obtained by setting \( \sqrt{\zeta - 1} = 0 \) in \( B \).

### D. Result Analysis for Conventional Controllers

The simulations for above three controllers are done by the help of Matlab 7.1. The step response of controller output 'u' and the system output (angle of attack) 'y' for three controllers for Flight Condition-1 and Flight Condition-2 with set-point and disturbances are shown in Figures 3 to 6, respectively.

![Fig. 3. Step response of ‘u’ with set-point and disturbance for flight condition-1](https://example.com/fig3)

![Fig. 4. Step response of ‘y’ with set-point and disturbance for flight condition-1](https://example.com/fig4)

![Fig. 5. Step response of ‘u’ with set-point and disturbance for flight condition-2](https://example.com/fig5)
IV. ADAPTIVE FUZZY LEARNING CONTROLLER (AFLC)

An adaptive Fuzzy PI Controller [6] utilises a learning mechanism for controlling the angle of attack and adjusts the rule base such that the overall system behaves like a reference model. The fuzzy controller improves the stability of a time-variant non-linear system by tuning controller parameters. Figure 7 below shows functional block diagram of the controller.

Fig. 7. Functional Block Diagram of Fuzzy Learning Controller

A. Fuzzy Rule Base

It is nothing but a set of if-then rules according to which the Fuzzy Controller operates to control the angle of attack of an aircraft. The rule base for the present work is shown in Table 5 below.

B. Fuzzy Membership Functions

The membership functions characterise the situations for application of the fuzzy rules. In this work the membership functions for input and output are taken into consideration. The membership functions input universe of discourse is assumed to be constant and are not tuned by adaptive controller whereas that for output universe of course are known.

In this work the tuning parameters \( g_c = 2/\pi \), \( g_u = 250 \) and \( g_a = 8\pi/18 \) for an output universe of discourse \([-1, 1]\] are triangular in shape with base widths of \( 0.4g_u \) and centres at zero are chosen. This choice represents that the fuzzy controller initially knows nothing about how to control the plant so it inputs \( u = 0 \) to the plant initially. Fuzzy controller input and output membership functions are depicted in following Figures 8 and 9, respectively.

![Figure 8](image_url)  
**Fig. 8.** Fuzzy controller input Membership Function

![Figure 9](image_url)  
**Fig. 9.** Fuzzy controller output Membership Function

<table>
<thead>
<tr>
<th>Elevator Deflection</th>
<th>Change in Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>-5</td>
</tr>
<tr>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>-4</td>
<td>5</td>
</tr>
<tr>
<td>-3</td>
<td>5</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

![Table 5](image_url)  
**Table V. Rule Base for the Angle of Attack Fuzzy Model**
C. The Learning Mechanism

The rule base of the fuzzy controller is tuned by the learning mechanism to make the close loop system a reference model. The modification of rule base is done according to the error between the actual and desired values. The fuzzy inverse model maps with the change in input required to force the output to zero. In this paper, membership functions for the input universes of discourse are symmetrical triangular-shaped.

D. Rule Base Modifier

The rule base of the fuzzy controller can be changed by a rule base modifier to force the error of the control action to zero. The input to the fuzzy controller is the error signal and the change in error signal. The rule base can be changed by shifting the centres of the membership functions as depicted in Figure 10 below.

![Fig. 10. Shifting of Centers of Membership Functions](Image)

E. Simulation Results of the Adaptive Fuzzy Learning Controller (AFLC)

The simulation is done by using Matlab 7.1. The simulation is done by taking two cases into consideration.

1) Case-I: Simulation without Sensor Noise
2) Case-II: Simulation with Sensor Noise

1) Case-I: Simulation without Sensor Noise: In this case the reference signal is applied for a duration of 40 seconds out of which the first 25 seconds is for FC-1 with a speed of 70m/s and the next 15 seconds for FC-2 with a speed of 265m/sec. Initially, AFLC has no adaptation as the flight proceeds the controller gets adapted with changing the centre of membership function.

Figure 11-a depicts the angle of attack and desired angle of attack whereas Figure 11-b shows the elevator deflection i.e. input to the aircraft which is output from the fuzzy controller. Similarly, Figure 11-c depicts the Fuzzy inverse model output in which the non-zero values indicate the adaptation. Again, Figure 11-d depicts the error between the actual and desired values whereas Figure 11-e depicts the change in error. Figure 11-f shows the error between angle of attack and the reference model and Figure 11-g shows the corresponding change in error.

![Fig. 11-a: Angle of attack (solid) and desired Angle of attack (dashed), deg.](Image)

![Fig. 11-b: Elevator deflection, output of fuzzy controller (input to the aircraft), deg.](Image)

![Fig. 11-c: Fuzzy inverse model output (nonzero values indicate adaptation)](Image)

![Fig. 11-d: Angle of attack error between Angle of attack and desired Angle of attack, deg.](Image)

![Fig. 11-e: Change in Angle of attack error, deg./sec](Image)

![Fig. 11-f: Error between Angle of attack and Reference model Angle of attack, deg.](Image)

![Fig. 11-g: Change in error between output and reference model, deg./sec](Image)

![Fig. 11. Responses without Sensor Noise](Image)
2) **Case-II: Simulation with Sensor Noise:** In this case the pulse duration is also 40 seconds for the reference model. A random noise \(0.01 \times \frac{\pi}{180} (2 \times \text{rand} - 1)\) is added uniformly with the Angle of attack to verify the adaptive nature of the controller. Figure 12 depicts the results of the simulation of all the parameters of Figure 11 in presence of the noise and it is clear that controller is noise adaptive.

**F. Control Surface**

Figures 13 and 14 shows the control surfaces [5] of AFLC without and with sensor noise, respectively. It reveals from figure that the control surface is non-linear in nature. This non-linearity nature of control surface changes with change in system parameters and is indicated by the angle of attack error and change in angle of attack error.

**G. Performance Indices**

The performance indices of the system are given by

\[
\text{IAE} = \int_{t_0}^{t_f} |e(t)| \, dt, \quad \text{MSE} = \frac{1}{T} \int_{t_0}^{t} e^2(t) \, dt, \quad \text{IATE} = \int_{t_0}^{t_f} e(t) \, dt
\]

where, the control error, \(e = \alpha - \delta_{E}\).

The performance indices of Zeigler Nichols Controller, Tyreus Luyben Controller, Extended Skogestad Internal Model Controller and Adaptive Fuzzy Learning Controller are compared to establish the superiority of adaptive fuzzy controller over other three controllers. It was also established that AFLC gives better results as depicted in Table 6 below.
V. CONCLUSION

In this paper, the angle of attack of the aircraft is controlled using various techniques and the results are depicted in Figures 3, 4, 5, 6, 11 and 12. Also the performance indices of the system are compared as shown in Table 5 above. It reveals that AFLC adapts the change in flight conditions from FC-1 to FC-2 and gives excellent results, improves the performance indices and reduces the errors. The performance indices MSE, IAE and IATE are very less as compared to ZN, TL and ESIMC controllers. The proposed controller not only tracks the desired angle of attack but also noise adaptation. In case of noisy input (Figure 12-b) the non-zero values of the controller output indicates that the controller continuously sends the output which nullifies the error to track the desired angle of attack. Therefore, AFLC can also be applied to other dynamic systems for its better performance and output.

REFERENCES


