Behavior of the Minimum Euclidean Distance Optimization Precoders with Soft Maximum Likelihood Detector for High Data Rate MIMO Transmission

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Abstract—The linear closed loop Multiple-input Multiple-output (CL-MIMO) precoding techniques characterized by the channel state information knowledge (CSI), at both sides of the link, aims to improve information throughput and reduce the bit error rate in the communication system. The processing involves multiplying a signal by a precoding matrix, computing from the CSI with some optimized criteria. In this paper, we proposed a new concatenation of the precoders optimizing the minimal Euclidean distance with soft Maximum Likelihood (soft-ML) detection. We analyze the performance in terms of bit error rate (BER) for the proposed association with the three well-known quantized precoders: Maximum of minimum Euclidean distance (Max-dmin) precoder, Orthogonalized Spatial Multiplexing precoder (POSM), and Orthogonalized Spatial Multiplexing (OSM) based on the same criteria, in coded MIMO system over a Rayleigh fading channel, using Quadrature Amplitude Modulation (QAM). Simulations show the interest of the proposed association of the dmin-based precoder with a soft - ML detector, and the best result is achieved for Max-dmin precoder.

Keywords—MIMO; max-dmin; POSM; singular values decomposition (SVD); soft-ML detector

I. INTRODUCTION

Modern wireless communications [1], especially fifth-generation (5G) cellular networks [2], [3], require high data throughput with low transmission latency. For example, high-speed coding, high-order modulation, and Multiple-Input Multiple-Output (MIMO) technology are essential tools for achieving high data rates. MIMO technology not only offer the diversity and capacity gains, but also achieve higher link reliability comparable with single antenna systems (SISO). The advantages of using Multiple antennas at the transmitter and receiver of the wireless MIMO system have been well exploited in the recent years [4]. The benefits of MIMO communication are generally ensured by both open-loop and closed-loop MIMO techniques. Open loop techniques, such as spatial coding (STC) and spatial multiplexing (SM) [5], [6], are used without the need for Channel State Information (CSI) at the transmitter. In order to overcome the multipath effect and to improve the robustness of spatial multiplexing systems, closed loop linear pre-coding techniques [7] may be used at the transmitter. The principle of precoding techniques is that, when the channel knowledge is available to the transmitter, the transmit signal is pre-multiplied by a precoding matrix so that the inter-symbol interference (ISI) in the receiver is greatly reduced. This knowledge of the characteristics of the channel makes it possible to anticipate any damage caused by the propagation, in order to obtain a "global" transmission channel favorable to communication. This technique is used in particular in WLAN networks (IEEE 802.11n standard) and mobile networks (LTE standard of the 3GPP project) [8]. Several types of precoders have been proposed in the literature, they have been designed according to various criteria. We can sites output capacity maximization [9], the BER minimization [10], signal-to noise ratio (SNR) maximization [11], mean square error (MSE) minimization [12] and minimum singular value maximization [13], this provides diagonal precoders and focus on power allocation schemes. Recently others precoders completely optimize the precoding matrix for a very specific purpose such as maximizing the minimal Euclidean distance between the constellations received referred to as Max-dmin precoder [14], whose principle is basing on the maximization of the minimum Euclidean distance between the received symbols. It has demonstrated its ability to improve both the spectral efficiency and the robustness of the transmission, and outperforms other kinds of MIMO precoders in terms of BER performance [15], particularly in correlated propagation scenarios, Like the Max-SNR, channel information is required for transmission.

In the literature, MIMO pre-encoding (or pre-equalization, or channel formation) consists of pre-mixing the signals prior to the channel, choosing the precoder according to the available CSIT, so to obtain a "global" transmission channel favorable to communication. Research conducted at Bretagne Telecom has made it possible to determine the pre-mix for globally optimizing a MIMO / OFDM system using criteria based on the minimum distance (Dmin) [15], [16].

The results make it possible to obtain a robust transmission with respect to the fading of the channel, which is the main cause of error in the estimation of the symbols. In addition, the proposed MIMO precoding in [15]-[17] has the advantage of preserving a maximum transmission rate comparable to the spatial multiplexing technique where the data is transmitting...
independently over several transmitting antennas and in the same frequency band. This excellent flow / performance compromise is a crucial asset for future MIMO systems.

MIMO systems can make the most of the useful information available in the CSI but with CSIT uncertainty robustness. The difficulty lies in the fact that the conventional singular value decomposition (SVD) and water-filling (WF) techniques are sensitive to the CSIT error, while other alternatives available, for example spatio-temporal coding, cannot fully exploit the advantages offered by the CSIT available. The problem remains to design transmission schemes for MIMO channels that can fully exploit the available CSIT benefits while being robust against CSIT uncertainty. In addition, convolutive error correcting codes and soft ML detection is not taken into account for this precoders.

In this context, we investigate the precoder design for convolutive encoded MIMO systems by assuming Soft detection at the receiver. In this paper, we propose firstly a new concatenation of the precoder optimizing the minimal Euclidean distance (Max dmin precoder) with convolutive error correcting code and soft Maximum Likelihood (soft-ML) detection. Thus, as a second contribution, and to see the effective method for our proposed system, we compare the performance of Max-dmin, POSM and OSM procedures using Soft-ML detection. These three techniques optimize the same criterion (minimum Euclidean distance), but each precoder uses a different method, the SDV, the antenna selection and Coding-Orthogonalization respectively.

We simulate the three techniques for a Rayleigh channel model and perfect CSI-T (channel state information at the transmitter). The performance is based on the evaluation of the BER (Binary Error Rate) for different number of antennas at both sides of the link and several modulation profiles. We use the classical Spatial Multiplexing technique as reference for our results.

Section 2 of this paper gives the MIMO close-loop system model (precoders, channel and detection techniques). Section 3 presents the simulation results, and finally Section 4 concludes the paper.

II. SYSTEM MODEL

Let us consider a MIMO system with $n_t$ transmit and $n_r$ receive antennas, i.e. a $(n_t, n_r)$ MIMO system, and $b$ independent data-streams over a Rayleigh fading channel. The basic system model is defined by:

$$y = GHFS + Gn$$

Where $y$ is the $b \times 1$ received symbol vector, $G$ is the $b \times n_r$ linear decoder matrix, $H$ is the $n_r \times n_T$ channel matrix, $F$ is the $n_T \times b$ linear precoder matrix, $S$ the $b \times 1$ transmitted symbol vector, $n$ is the $n_r \times 1$ additive Gaussian noise vector. We assume that $b \leq r = \text{rank}(H) \leq \min(n_T, n_r)$ and $I_b$ (denotes the $b \times b$ identity matrix).

$$E[ss^*] = I_b, E[nn^*] = 0 \text{ and } E[nn^*] = R.$$  With $R$ the noise covariance matrix, and superscript $*$ stands for conjugate transpose. Under the perfect CSI condition at both the transmitter and the receiver, the channel can be diagonalized by using the virtual transformation (Fig. 1) and is decomposing in three steps: noise whitening, channel diagonalization and dimensionality reduction.

Firstly, the precoding and decoding matrix can be written as $F = F_v F_d$ and $G = G_v G_d$. Then, the new decomposition of $F_v$ and $G_v$ matrices into the product of three matrices are considering:

$$F_v = F_1 F_2 F_3 \text{ and } G_v = G_1 G_2 G_3$$

Where the $(F_i, G_i)$ perform the particular operations: noise whitening, channel diagonalization and dimensionality reduction.

Therefore, the input-output relation (1) will be:

$$y = G_dH_vF_dS + G_dn_v$$

![Fig. 1. MIMO channel precoded in virtual channel.](image)

The decoding matrix $G_d$ has no effect on the performance when the ML detection is considered. Therefore, we adopt in this paper that $G_d$ is an $b \times b$ identity matrix.

$$H_v = \text{diag}(\sigma_1, ..., \sigma_b)$$

Were $\sigma_i$ are entries with: $\sigma_1 > \sigma_2 > ... > \sigma_i$ and $i = 1, ..., b$. $n_v = G_d n$ is the $b \times 1$ transformed additive Gaussian noise vector. Now we can write the virtual system model as:

$$y = H_v F_d S + n_v$$

A. The Max-dmin Precoder

The Max-dmin is non-diagonal precoder, which maximize the minimal distance between received constellations [14] affects the system performances, especially with the ML detector. The value of the minimal distance between received constellations is denoted $d_{\text{min}}$ and given by:

$$d_{\text{min}}(F_d) = \min_{\delta_k: x_k \in C, \delta_k \neq \delta_i} \|H_v F_d(s_k - s_i)\|$$

Were $C$ represents the set of complex symbols of the constellation.

$x_k$ and $x_i$ are two transmit signals, and $S$ is the set of all these possible transmit vectors. Let us define $\tilde{x}$ a difference vector as $\tilde{x} = s_k - s_i$, with $s_k \neq s_i$.The Max-dmin solution consists to find the $F_d$ matrix coefficients, which maximize the minimum distance of the received constellation:

$$F_d = \arg\max_{F_d} \{d_{\text{min}}(F_d)\}$$

with

$$\text{trace}(F_d F_d^*) = P_0$$

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This problem may be particularly difficult to solve, because the distance expression takes account of several parameters: the transmission channel $H_v$, the digital modulation and the number of considers channels $b$. In this paper a solution is found for $b = 2$ and BPSK, QAM-4 and QAM-16 modulations. The optimization for the virtual sub-channels proposed in [18] is obtained by a variable change of two channels eigenvalues denoted $\sigma_1$ and $\sigma_2$. It is a simple change of Cartesian coordinates into polar coordinates. The new variables are:

$$\begin{align*}
\sigma_1 &= \rho \cos \gamma \\
\sigma_2 &= \rho \sin \gamma
\end{align*}$$

Where $\rho$ and $\gamma$ represent the channel gain and channel angle, respectively. The virtual channel is then given by:

$$H_v = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} = \rho \begin{pmatrix} \cos \gamma & 0 \\ 0 & \sin \gamma \end{pmatrix}$$

Note that $\sigma_1 \geq \sigma_2 > 0$, so we have $0 < \gamma \leq \pi/4$.

We give here the precoding matrix $F_d$ for a 4 QAM modulation; this solution is relatively simple with two forms of precoder:

- If $0 < \gamma \leq \gamma_0$

$$F_d = F_{rl} = \sqrt{E_s} \begin{pmatrix} \frac{\sqrt{2} + \sqrt{2} e^{j\pi/2}}{2} \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ e^{j\pi} \end{pmatrix}$$

- If $\gamma_0 \leq \gamma \leq \pi/4$

$$F_d = F_{octa} = \sqrt{E_s} \begin{pmatrix} \cos \Psi & 0 \\ 0 & \sin \Psi \end{pmatrix} \begin{pmatrix} 1 \\ e^{j\pi} \end{pmatrix}$$

Where:

$$\Psi = \arctan \sqrt{\frac{1}{\tan \gamma}}$$

$$\gamma_0 = \arctan \sqrt{\frac{3\sqrt{3} - 2\sqrt{2} - 2\sqrt{3} - 3}{3\sqrt{3} - 2\sqrt{2} + 1}} = 17,28^\circ$$

The parameter $\Psi$ in relations is the power allocation on each sub-channel, and the constant threshold $\gamma_0$ allows the precoder to use one or two sub-channels. The value of $\gamma_0$ is obtained when considering that the two precoders give the same minimum Euclidean distance $d_{min}$. This one depends on $\rho$ and $\gamma$ and is expressed in [18].

$$d_{min} = \begin{cases} 
\sqrt{E_s \rho \frac{1 - \frac{1}{\sqrt{3}} \cos \gamma}{1 + \frac{1}{\sqrt{3}} \cos \gamma}} & \text{if } 0 < \gamma \leq \gamma_0 \\
\sqrt{E_s \rho \frac{(1 - 2 \sqrt{2}) \cos \gamma \sin \gamma}{1 + (2 - 2 \sqrt{2}) \cos \gamma}} & \text{if } \gamma_0 < \gamma \leq \frac{\pi}{4}
\end{cases}$$

B. The P-OSM Precoder

The technique known as Orthogonalized Spatial Multiplexing (OSM) proposed for closed-loop MIMO systems, associates a symbols coding with an orthogonalization by rotation (Coding-Orthogonalization). In addition, Y. Kim and al propose in [19] a precoding technique for the OSM system, this is referred as the P-OSM precoder. Like the Max-dmin, the P-OSM optimize minimal Euclidean distance of the received constellation. The OSM system transmits $b = 2$ independent informations channels on $n_t = 2$ transmission antennas and if $n_t > 2$, an antenna selection method must be associated. The principle of the OSM consists to precode the transmitted symbols $x_1$ and $x_2$ as:

$$F(x, \theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{pmatrix} s(x)$$

Where $\theta$ is the phase rotation angle applied to the second antenna and $s(x)$:

$$s(x) = \begin{pmatrix} Re[x_1] + j Re[x_2] \\ Im[x_1] + j Im[x_2] \end{pmatrix}$$

In a real representation, the P-OSM system model can be written as:

$$y_r = H_v^\theta s_r(x) + n_r = [h_1^\theta h_2^\theta h_3^\theta h_4^\theta] s_r(x) + n_r$$

The real vector $h_1$ of length $2n_r$ represents the $i$th column of the real matrix $H_v^\theta$. The columns $h_1^\theta$ and $h_2^\theta$ are respectively orthogonal to $h_3^\theta$ and $h_4^\theta$. The angle of rotation $\theta$ necessary to guarantee the orthogonality between $h_1$ and $h_4$ or $h_2$ and $h_3$ is calculated from the original channel matrix as follows:

$$\theta = \tan^{-1} \left( \frac{B}{A} \right) \pm \frac{\pi}{2}$$

Where $A = \sum_{n=1}^{n_r} |h_{rn1}|^2 |h_{rn2}|^2 \sin(\angle h_{rn2} - \angle h_{rn1})$ and $B = \sum_{n=1}^{n_r} |h_{rn1}| |h_{rn2}| \cos(\angle h_{rn2} - \angle h_{rn1})$. $\angle$ is the argument.

At this stage, the OSM system do not optimize any criterion, but enable the orthogonalization of the received symbols. Thus, the P-OSM precoder, which maximizes $d_{min}$, is also given in an actual representation by:

$$y_r = H_v^\theta \hat{P} s_r(x) + n_r = H_v^\theta \hat{P} s_r(x) + n_r$$

Where $\hat{P}$ is an actual precoding matrix of size $2 \times 2$ and the power constraint is:

$$\text{trace}(P_s(x) s_r(x)^* P^*) = P_0$$

For simplicity, the precoder is not optimal. So, the matrix $\hat{P}$ is decomposed as [19]:

$$\hat{P} = R_{\theta_1} \begin{pmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{pmatrix} R_{\theta_2}$$

With

$$R_{\theta_1} = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{pmatrix}$$

The angle $\theta_1$ is directly computed from the matrix $H_v^\theta$.

$$\theta_1 = \tan^{-1} \left( \frac{C + \sqrt{C^2 + 4D^2}}{2D} \right)$$

With

$$C = \|h_2^\theta\|^2 - \|h_4^\theta\|^2$$

$$D = \|h_2^\theta\| \|h_4^\theta\|$$

$$\theta_2$$ and $p$ are chosen according to the modulation under consideration and a parameter $k$ defined as:

$$k = \frac{\|h_1^\theta\|}{\|h_2^\theta\|}$$

$$H_v^\theta = H_v^\theta R_{\theta_1} = [h_1^\theta h_2^\theta h_3^\theta h_4^\theta]$$
\[ \|H_k \| \] and \[ \|H_k^{\perp} \] represent respectively the first and the second singular value (\( \sigma_1 \) and \( \sigma_2 \)) of the channel matrix \( H \). The solution for a 4-QAM is [17]:

- If \( 1 \leq k < 7 \) \( \quad p = \frac{\theta}{\sqrt{k+3}} \approx 1 \) and \( \theta_2 = 45^\circ \)
- If \( k \geq 7 \) \( \quad p = \sqrt{2} \) and \( \theta_2 = 26.5^\circ \)

Fig. 2 shows the entire system block diagram to simulate.

![Block diagram of precoded MIMO system](image-url)

**C. Detection Techniques**

At receiver, the MIMO detection consists to estimate the symbols generated at the transmitter before the coding channel processes. Several methods are available in the literature for the MIMO systems [20]. The most common approach is the maximum likelihood (ML) detection [21], which achieve optimal performances at the expense of computational complexity. Besides the ML criterion, the zero forcing (ZF) and minimum mean square error (MMSE) [22] detections are linear equalization-based methods. They are low-complexity and simple to implement, but with lower performances. In this work, we compare the ML hard and soft detection performances for different precoding techniques.

In the MIMO case, ML-hard detection consists of finding the most likely transmit symbols vector:

\[
\tilde{s} = \arg \max_{s \in S} f \left( \frac{y}{s} \right)
\]

Where \( S \) is the set of all possible \( M^{NT} \) transmit symbol vector candidates with the modulation order \( M \).

\[
f(y/s) = \frac{1}{\pi^{NR}} \exp(-\|y - Hs\|^2)
\]

So, the ML hard detection is equivalent to finding the transmit symbol vector that minimizes the Euclidean Distance (ED) \( \|y - Hs\|^2 \) between \( y \) and \( Hs \):

\[
\tilde{s} = \arg \min_{s \in S} \|y - Hs\|^2
\]

In this detection technique, there are low-complexity algorithms that try to find optimum \( s \) without calculating all the EDs for all transmit symbols vector. In this case, hard detection ignores a large part of the information contained in the receive vector \( y \).

In ML-soft detection case [23], the information about the decision and its reliability are usually delivered jointly for every bit \( b_{x,n} \) using log likelihood ratios (LLRs) [24]. The n-th bit of x-th stream:

\[
L(b_{x,n}) = \log \left( \frac{P(y|b_{x,n}=1)}{P(y|b_{x,n}=0)} \right)
\]

Using Max-Log-MAP approximation, we calculate the approximate LLR:

\[
L(b_{x,n}) \approx \min_{s \in S_{x,n}} \|y - Hs\|^2 - \min_{s \in S_{x,n}} \|y - Hs\|^2
\]

To conclude this part, the ML-soft demodulation looks similar to the ML-hard detection problem but present more difficult in reality, such as the search space for the transmit symbol vector with the minimum EDs is limited to \( S_{x,n} \), and the partitioning transmit symbol vector candidates into \( S_{x,n} \) is all different depending on \( x \) and \( n \).

**III. RESULTS AND ANALYSIS**

In this section, we present our simulation results of the Bit Error Rate (BER) evaluation for different precoding techniques based on the system model shown in Fig. 2.

For these simulations, we consider the (2x2) and (4x4) MIMO precoding systems. The channel is disrupted by Rayleigh distribution. The transmission structure is a Bit Interleaved Coded Modulation (BICM) type, resulting from the concatenation of a channel encoder, a bit interleave and a bit-to-symbol conversion. The channel coding (Convontial Code) is performed by an encoding rate \( R = 1/2 \), and a constraint length \( K = 7 \). The polynomials generator of the convolutional coder are [133, 177].

The encoded data frame is subsequently interleaved randomly and converted into complex symbols belonging to the constellation alphabet of the modulation, we used the 4QAM and 16QAM modulations. One of the quantified precoding techniques (Max-dmin, OSM and POSM) follows this BICM emission structure. Soft or hard decoding is performed for channel decoding, using the Max-log-MAP algorithm. For this simulation, 10 000 frames of 800 bits each were transmitted. The channel is quasi-statistical, so that the matrix \( H \) is assumed constant during the transmission of 800 bits. The number of transmitted streams is limited to \( b = 2 \). For simplicity, we consider perfect estimation of the CSI. We use the “Spatial Multiplexing (SM)” MIMO system as reference to compare all the precoders techniques.

In order to evaluate behavior of our proposed association (Max-dmin with soft ML detection), we simulated four different scenarios. The first one is a (2x2) MIMO transmission without any precoder; this is “Spatial Multiplexing MIMO (SM-MIMO)” system. The second one, consists in the same MIMO scheme with OSM precoder.
The third one consists in a MIMO scheme with the POSM precoder (POSM-MIMO system). The last scenario is the MIMO scheme with Max-dmin precoder (Max-dmin–MIMO system).

Fig. 3, give performances comparison between two conventional detection algorithms, the MMSE and the maximum likelihood structures, for the different systems cited above. In this case, we use 2x2 MIMO system with 4QAM modulation.

As shown in Fig. 3, the maximum likelihood (ML) receiver is more efficient than MMSE for SNRs greater than 6dB. The ML receiver archive BER around $10^{-2}$, whereas MMSE does not exceed $10^{-2}$ for SM-MIMO system. We can also see from this curves that the Max-dmin -MIMO and POSM-MIMO have closely performances and exceed the SM-MIMO and OSM-MIMO systems performances respectively. For example, POSM-MIMO and Max-dmin-MIMO provide 6dB SNR gain for $10^{-4}$ BER compared to SM-MIMO and OSM-MIMO.

Fig. 4 show the performances of the four simulated systems in terms of BER for 2x2 MIMO system with 4QAM modulation. The POSM–MIMO transmission with ML-soft decision has the best performance compared to Max-dmin–MIMO, SM-MIMO and OSM- MIMO transmissions with ML-Soft and ML-hard detectors respectively. The SNR gains of the POSM-MIMO-ML-soft are from 1dB to 2.5dB, compared to the other precoders.

In Fig. 5, we illustrate the simulation of BER for the POSM and max-dmin for 2x2 MIMO system with 4QAM and 16 QAM modulation. The result allows to verify that the modulation order has a big impact on the performance of the MIMO precoders systems. Since the ML detector estimates the Euclidean distance between the constellation points, the expansion of this constellation increases the number of errors on the transmit symbols. We achieve the best performance for 4-QAM modulation.

Fig. 6 shows the simulation of BER for $n_t=n_r=4$ using 4QAM modulation. The curve shows that the max-dmin-MIMO with ML Soft system remains the best BER precoder with a SNR gain of 1.5 dB and 2 dB at BER of $10^{-4}$ compared to the POSM-ML Soft and OSM-ML Soft systems respectively. For the same SNR the Max-dmin-MIMO with ML Soft system has a gain of 5dB on the SM-ML soft-MIMO system. These results show clearly that the SVD method is more efficient than antenna selection specifically when the receive antennas is equal or greater than four elements.
In this paper, we study the behavior of the concatenation of the Soft-ML detector with the precoder based on the minimum Euclidian Distance criterion. Given perfect CSI at the transmitter, we provided simulations results that demonstrate the efficiency of the proposed concatenation. The simulations show that the Soft ML detector concatenated with precoder based on the minimum Euclidian distance criterion using convolutive error correcting code provide better performance than the hard-ML and MMSE detectors in terms of BER. In addition, we conclude from the results that Max-dmin and P-OSM precoders in MIMO systems have the best BER performance with Soft-ML detector compared to the other systems. When two transmitted antennas are used, the two precoders are equivalent in terms of BER performances. When $n_t > 2$, the POSM is limited and should be associated with the antenna selection algorithm, in this case the Max-dmin based on the SVD method outperforms POSM with a significant SNR gain. Finally, our proposed system presents a real interest in the evolution of transmission techniques in general and those of LTE and LTE-A more precisely. The optimization of MIMO transmitters and receivers proposed in this work will have an impact on the energy consumption of these systems, a major challenge of our century.

As a future work, we want to implement non-binary LDPC codes (NB-LDPC) in precoded MIMO systems, The use of non-binary LDPC codes (NB-LDPC) is a promising solution because it allows to obtain excellent performance in error correction in the case of short frames and / or large size constellations. These properties are particularly well suited to use NB-LDPC codes with precoded MIMO systems.

REFERENCES


