

Towards an Improvement of Fourier Transform

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Abstract—With the development of information technology and the coming period of large data, the image signals play an increasingly more significant role in our life because of the phenomenal development of system correspondence innovation, and the comparing high proficiency image handling strategies are requested earnestly. The Fourier transform is an important image processing tool, which is used in a wide range of applications, such as image filtering, image analysis, image compression and image reconstruction. It's the simplest among the other transformation method used in mathematics. The real time consumption is lesser due to this method. It has a vast use in image processing, particularly object 2D, 3D and other representation. This paper proposes a new Fourier transform which is called Non Uniform Fourier Transform (NUFT). The proposed descriptor takes into consideration the change of point index. Also, an application is made on 2D set of points and a real image. The main advantages of the proposed transform are invariance under change of index point and robustness to noise. Also, the extraction of invariant under rotation and affinity is immediate because the linearity is assured. The proposed descriptor is tested on MPEG 7 database and compared with the normal Fourier transform to shows its efficiency. The experimental results prove the effectiveness of the proposed descriptor.

Keywords—Fourier transform; NUFT; noise; invariant

I. INTRODUCTION

Fourier transform is an interesting image processing tool which used to decompose an image into sine and cosine components. Fourier transform is used in several fields such as image processing and filters, transformation, representation, etc. Historically, one of the most widely used shape description methods is Fourier descriptors (FD) [1], [2], [3], [4]. The discrete Fourier transform (DFT) is one of the most fundamental and important numerical algorithms which plays a central role in the image processing area, including image denoising [5], image feature extraction [6], and compressed sensing [7]. The Fast Fourier Transform (FFT) [8] which computes the DFT of an n-size signal in $O(n \log n)$ time greatly simplifies the complexity of DFT and gets a wide range of applications. In [9], [10] and [11], [12] are implementations of the discrete Fourier transform (DFT). Fourier transform hypotheses of central importance in a vast range of applications in engineering, applied mathematics, and physical science. In addition, Fourier transform is a mathematical concept which suited extremely well for signal analysis. In [13], [14] authors proposed sparse fast Fourier transform for one-dimension (1D-SFFT) signal which is faster than traditional DFT. However, the two-dimensional image signal is more broadly used, and a two-dimensional sparse

Fourier transform cannot simply be constructed with a one-dimensional sparse Fourier transform. Sheng Shi et al in [15] proposed a new fast two-dimensional Fourier transform (2D-SFFT) that takes advantage of image sparsity.

In this work, the advantage of our method is that it is systematic and it allows obtaining the analytical form of all invariant polynomials of a given order, which was not the case using the Suck method and Flusser [16].

The rest of this paper will be organized as follows: Section II provides an overview of related work. Section III describes normal Fourier descriptors and the proposed Fourier transform. In Section IV the experiments are presented. The results and discussions are presented in Section V. Section VI contains the conclusion and the future work highlighted in Section VII.

II. RELATED WORKS

Fourier transform is one of the oldest and well-known methods in the field of the mathematics. It used in a wide range of applications, such as image filtering, image analysis, image reconstruction, and image compression. In the literature, several papers have described methods for approximating one-dimensional Non uniform Fast Fourier Transforms by interpolating an oversampled Fast Fourier transforms, start with [17] and including [18], [19], [20], [21], [22], [23], [24], [25], [26], [27]. In [28] the fundamentals of Fourier transform, Fourier series, discrete Fourier transform and fast Fourier transform with simple examples and review of Fourier transform to supply a clear understanding of its applications in power quality issues. In [29], the author presents a novel method for improving the Fast Fourier Transform (FFT) based spectral estimation for the diagnosis of faults in induction motors. In [30], the authors propose a new method for optical image encryption using fractional Fourier transform (FRFT). Two-dimensional fast Fourier transform (2D-FFT) is successfully applied to analyse images [31]. Image compression technique that uses real Fourier transform is proposed by Kekre et al in [32]. Their technique is applied on the image in three ways: Row transform, Column transform, and Full transform. Aznag et al. in [33] applied this new descriptor for 3D parameterized point set and 3D curve. Previous research has demonstrated that there is a non-uniform Fourier transform [16], [34], but the problem treated in this new work is not the spacing between samples, but simply the change in order of points in storage or manipulation of those points.

III. METHODOLOGY

This section discusses the proposed descriptor. We first give an overview of the normal Fourier descriptor. Then, we describe more in detail our proposed Fourier descriptor.

A. Normal Fourier Descriptor

1) *Fourier descriptor for two-dimensional indexed point set*: To define Fourier descriptor (FD) for 2D indexed point set; Let $X(x(n), y(n))$ $n=1 \dots N$, denotes a closed contour with N is the number of points on the normalized contour and 2π as length, then the Fourier descriptors are given by equations. (1), (2) and (3).

$$Z(k) = \begin{pmatrix} u(k) \\ v(k) \end{pmatrix} \quad (1)$$

With

$$u(k) = \frac{1}{N} \sum_{n=1}^N x(n) e^{-j \frac{2\pi kn}{N}} \quad (2)$$

$$v(k) = \frac{1}{N} \sum_{n=1}^N y(n) e^{-j \frac{2\pi kn}{N}} \quad (3)$$

For $k = 1 \dots N$

Where u and v represent the Fourier descriptors of x and y respectively.

2) *Shift theorem and transformation effect*: Let X and \tilde{X} are two objects having the same shape but with a shift in starting points, then (Equations. (4), (5) and (6)):

$$\tilde{Z}(k) = e^{jk l_0} AZ(k) + B \delta_k \quad (4)$$

For all integer k ,

$$Z(k) = (u(k), v(k)) \quad (5)$$

and

$$\tilde{Z}(k) = (\tilde{u}(k), \tilde{v}(k)) \quad (6)$$

Are respectively the bi-dimensional Fourier descriptor vectors of X and \tilde{X} .

Where

$$\delta_k = \begin{cases} 1 & \text{if } k = N \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Is the Kronecker symbol (Equation. (7)). The real l_0 denotes the difference between starting points on a contour and its transformed.

B. Proposed Fourier Descriptor

In this section, we present the proposed descriptor. As normal Fourier transform is dependent on point index. So, to solve this problem we define the novel descriptor. We apply the proposed Fourier transform for the 2D parameterized point set and the binary objects. The idea of our descriptor is simple and easy, we replace integer k by parameter τ (see Equation. (16)).

In general, we have two cases:

- A structured set of points, indexed by n integer which represents the order of points.
- Unstructured set of points, in this case the order is not respected and we propose the use of another characteristic of a point, which is independent of the order.

3) *Novel Fourier descriptor for two-dimensional indexed point set*: To define novel Fourier descriptor (FD) for 2D indexed point set; Let $X(\tau) = (x(\tau), y(\tau))$ denotes a 2D set of point having τ as parameter and N is the number of points, then the novel 2D Fourier descriptors is defined by.

$$\begin{cases} u(k) = \frac{1}{N} \sum_{\tau} x(\tau) e^{-j \frac{k\tau}{T}} \\ v(k) = \frac{1}{N} \sum_{\tau} y(\tau) e^{-j \frac{k\tau}{T}} \end{cases} \quad (8)$$

Where

$$T = \int \tau d\tau$$

4) *Linearity*: Let $X = (x(\tau), y(\tau))$ and $\tilde{X} = (\tilde{x}(\tau), \tilde{y}(\tau))$ denote two shapes having the same shape with an affine transform difference i.e (Equations. (9), (10), (11), (12), (13) and (14)):

$$\tilde{X} = AX \quad (9)$$

Where

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (10)$$

So

$$\tilde{x} = a_{11}x(\tau) + a_{12}y(\tau) \quad (11)$$

$$\tilde{y} = a_{21}x(\tau) + a_{22}y(\tau) \quad (12)$$

Then

$$\tilde{u}(\tau) = \frac{1}{N} \sum_{n=1}^N \tilde{x}(\tau) e^{-j \frac{k\tau}{T}} \quad (13)$$

$$= \frac{1}{N} \sum_{n=1}^N (a_{11}x(\tau) + a_{12}y(\tau)) e^{-j \frac{k\tau}{T}}$$

$$= a_{11}u(k) + a_{12}v(k)$$

The same for:

$$\tilde{v}(k) = a_{21}u(k) + a_{22}v(k) \quad (14)$$

In case of rotation transformation, a 2×2 transformation matrix is defined by

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad (15)$$

Where θ is the angle of rotation. This principle of linearity allows us to extract invariant easily from shapes X and \tilde{X} .

IV. EXPERIMENTS

A. Application on Sets of Points

In these experiments, we have four sets of 2D points with x and y coordinates shown in Fig. 1. We present the experimental test of our proposed descriptor in four shapes that are shown in Fig. 1(a), 1(b), 1(c) and 1(d). Using Equation (16), the parameter τ used in this experiment is defined by

$$\tau = \sqrt{x^2 + y^2} \tag{16}$$

B. Application on Real Image

To test our approach; we apply our method on the MPEG7 database [35]. In this database there are 70 classes of shapes, each one has 20 members as shown in Fig. 6. In this section we applied our approach to a real image, an MPEG7 image of size 750×531 is presented in Fig. 2. To obtain contour points we can browse edge image using row by row or column by column. So, the coordinates of contours have not the same index using previous browses. Starting points on each contour are presented by a circle (as shown in Fig. 3). Not only the starting points are different but also the order of other points. Fig. 4 shows results with contours of an elephant.

C. Noise Effect

In order to test our new Fourier transform for noise, we add noise to the Elephant image (see Fig. 5). The percentage of added noise is 10% and we take the first 10 coefficients (development into Novel transformed of coordinates x).

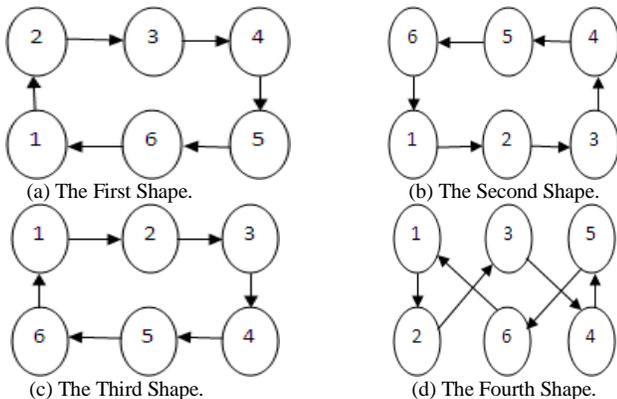


Fig. 1. A Set of Points with Associated 2D Shapes.



Fig. 2. Elephant Image.

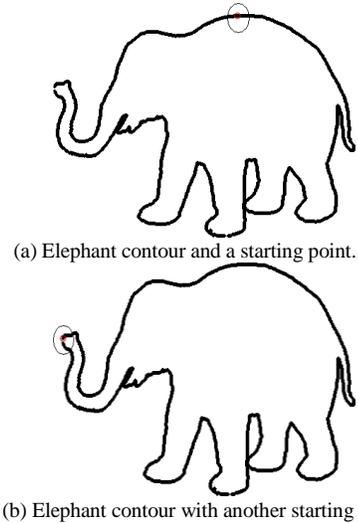


Fig. 3. The Contours of Elephant Represent the Same Content but the Order of Points is not the Same.

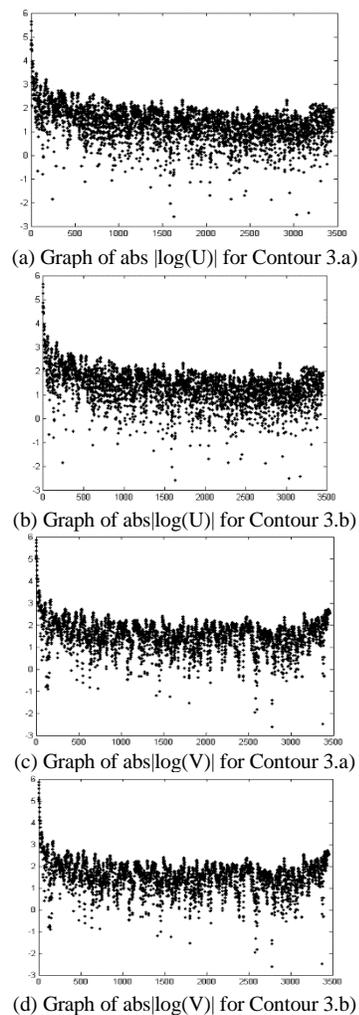


Fig. 4. Graphs Represent Abs $|\log(U)|$ (a) and (b), Graphs Represent Abs $|\log(V)|$ (c) and (d).

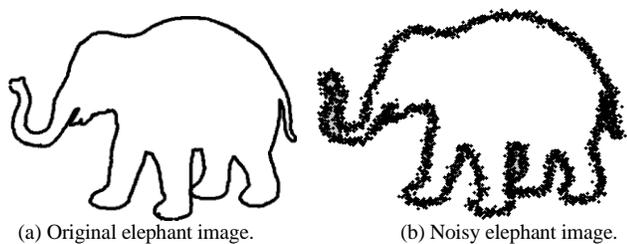


Fig. 5. Original and Noisy Elephant Images.

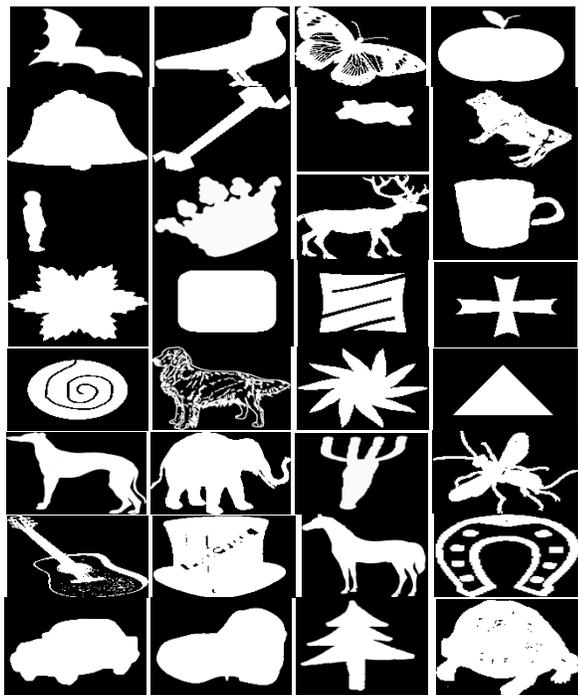


Fig. 6. Example of MPEG7 Database.

V. RESULTS AND DISCUSSION

The problem treated in this work is not the spacing between samples, but simply the change in the order of points in storage or manipulation of those points. We notice that the Normal Fourier transform requires $O(N \log N)$ to compute N Fourier modes from N data points. Also, novel transform achieves the same $O(N \log N)$ computational complexity.

Tables I, II, III and IV we present the x and y coordinates of the points with different indices of the four shapes given in Fig. 1. The polar radius for each point is shown in Table V. In Tables VI, VII, VIII, and IX we present the normal Fourier transform for each shape given in Fig. 1, we can show no equality of coefficients. In Tables X, XI, XII, and XIII we present the novel Fourier transform for each shape. We see that the novel two components U and V are the same. It's clear from Tables VI, VII, VIII, IX, X, XI, XII, and XIII which the proposed Fourier transform is efficient.

TABLE. I. COORDINATES X AND Y FOR SHAPE (FIG.1.A)

Index	1	2	3	4	5	6
Coordinates x	1	1	1.5	2	2	1.5
Coordinates y	1	2	2	2	1	1

TABLE. II. COORDINATES X AND Y FOR SHAPE (FIG.1.B)

Index	1	2	3	4	5	6
Coordinates x	1	1.5	2	2	1.5	1
Coordinates y	1	1	1	2	2	2

TABLE. III. COORDINATES X AND Y FOR SHAPE (FIG.1.C)

Index	1	2	3	4	5	6
Coordinates x	1	1.5	2	2	1.5	1
Coordinates y	2	2	2	1	1	1

TABLE. IV. COORDINATES X AND Y FOR SHAPE (FIG.1.D)

Index	1	2	3	4	5	6
Coordinates x	1	1	1.5	2	2	1.5
Coordinates y	2	1	2	1	2	1

TABLE. V. RADIUS OF SHAPES (FIG.1.A), (FIG.1.B), (FIG.1.C) AND (FIG.1.D)

τ for shape 1.a)	1.41	2.23	2.50	2.82	2.23	1.80
τ for shape 1.b)	1.41	1.80	2.23	2.82	2.50	2.23
τ for shape 1.c)	2.23	2.50	2.82	2.23	1.80	1.41
τ for shape 1.d)	2.23	1.41	2.23	2.23	2.82	1.80

TABLE. VI. NORMAL FOURIER TRANSFORM OF SHAPE (FIG.1.A)

U	V
-0.0000 + 0.2887i	-0.3333 - 0.0000i
0.0000 + 0.0000i	0.0000 - 0.0000i
0 - 0.0000i	0.1667 - 0.0000i
-0.0000 - 0.0000i	-0.0000 - 0.0000i
-0.0000 - 0.2887i	-0.3333 - 0.0000i
1.5000 + 0.0000i	1.5000 + 0.0000i

TABLE. VII. NORMAL FOURIER TRANSFORM OF SHAPE (FIG.1.B)

U	V
-0.2500 + 0.1443i	0.1667 + 0.2887i
0.0000 - 0.0000i	0.0000 - 0.0000i
0.0000 - 0.0000i	0.1667 - 0.0000i
-0.0000 - 0.0000i	-0.0000 - 0.0000i
-0.2500 - 0.1443i	0.1667 - 0.2887i
1.5000 + 0.0000i	1.5000 + 0.0000i

TABLE. VIII. NORMAL FOURIER TRANSFORM OF SHAPE (FIG.1.C)

U	V
-0.2500 + 0.1443i	-0.1667 - 0.2887i
0.0000 - 0.0000i	0.0000 + 0.0000i
0.0000 - 0.0000i	-0.1667 - 0.0000i
-0.0000 - 0.0000i	-0.0000 - 0.0000i
-0.2500 - 0.1443i	-0.1667 + 0.2887i
1.5000 + 0.0000i	1.5000 + 0.0000i

TABLE. IX. NORMAL FOURIER TRANSFORM OF SHAPE (FIG.1.D)

U	V
-0.0000 + 0.2887i	0.0000 - 0.0000i
0.0000 + 0.0000i	0.0000 + 0.0000i
0 - 0.0000i	-0.5000 - 0.0000i
-0.0000 - 0.0000i	-0.0000 + 0.0000i
-0.0000 - 0.2887i	-0.0000 - 0.0000i
1.5000 + 0.0000i	1.5000 + 0.0000i

TABLE. X. NOVEL FOURIER TRANSFORM OF SHAPE (FIG.1.A)

U	V
-0.9558 - 0.9468i	-1.0020 - 0.9093i
0.0571 + 0.9518i	0.1659 + 0.9746i
0.2418 - 0.4530i	0.1739 - 0.5847i
-0.1428 + 0.2268i	-0.2092 + 0.3785i
0.1581 - 0.1539i	0.3136 - 0.1968i
-0.1102 - 0.0536i	-0.2207 - 0.1324i

TABLE. XI. NOVEL FOURIER TRANSFORM OF SHAPE (FIG.1.B)

U	V
-0.9558 - 0.9468i	-1.0020 - 0.9093i
0.0571 + 0.9518i	0.1659 + 0.9746i
0.2418 - 0.4530i	0.1739 - 0.5847i
-0.1428 + 0.2268i	-0.2092 + 0.3785i
0.1581 - 0.1539i	0.3136 - 0.1968i
-0.1102 - 0.0536i	-0.2207 - 0.1324i

TABLE. XII. NOVEL FOURIER TRANSFORM OF SHAPE (FIG.1.C)

U	V
-0.9558 - 0.9468i	-1.0020 - 0.9093i
0.0571 + 0.9518i	0.1659 + 0.9746i
0.2418 - 0.4530i	0.1739 - 0.5847i
-0.1428 + 0.2268i	-0.2092 + 0.3785i
0.1581 - 0.1539i	0.3136 - 0.1968i
-0.1102 - 0.0536i	-0.2207 - 0.1324i

TABLE. XIII. NOVEL FOURIER TRANSFORM OF SHAPE (FIG.1.D)

U	V
-0.9558 - 0.9468i	-1.0020 - 0.9093i
0.0571 + 0.9518i	0.1659 + 0.9746i
0.2418 - 0.4530i	0.1739 - 0.5847i
-0.1428 + 0.2268i	-0.2092 + 0.3785i
0.1581 - 0.1539i	0.3136 - 0.1968i
-0.1102 - 0.0536i	-0.2207 - 0.1324i

In Tables XIV and XV, we see that coefficients U and V are the same for two contours, only the first 10 coefficients using novel Fourier transform are presented. Graphic representation of $abs|\log(U)|$ and $abs|\log(V)|$ for all coefficients is given in Fig. 4. We note that coefficients are always in the same order of magnitude (see Table XVI).

TABLE. XIV. NOVEL FOURIER TRANSFORM OF COORDINATES X FOR TWO CONTOURS 4(A) AND 4(B)

U	U
1.5477 - 2.3974i	1.5477 - 2.3974i
-1.0556 - 2.2168i	-1.0556 - 2.2168i
-1.8839 - 0.1026i	-1.8839 - 0.1026i
-0.5257 + 1.1775i	-0.5257 + 1.1775i
0.7773 + 0.4266i	0.7773 + 0.4266i
0.3836 - 0.7878i	0.3836 - 0.7878i
-0.8005 - 0.6661i	-0.8005 - 0.6661i
-1.0159 + 0.4648i	-1.0159 + 0.4648i
-0.1105 + 1.0289i	-0.1105 + 1.0289i
0.6326 + 0.5173i	0.6326 + 0.5173i

TABLE. XV. NOVEL FOURIER TRANSFORM OF COORDINATES Y FOR TWO CONTOURS 4(A) AND 4(B)

V	V
1.5950 - 3.0604i	1.5950 - 3.0604i
-1.8277 - 2.4957i	-1.8277 - 2.4957i
-2.5307 + 0.5763i	-2.5307 + 0.5763i
-0.2078 + 2.0807i	-0.2078 + 2.0807i
1.6373 + 0.5444i	1.6373 + 0.5444i
0.7498 - 1.3746i	0.7498 - 1.3746i
-1.1345 - 1.0216i	-1.1345 - 1.0216i
-1.2702 + 0.7405i	-1.2702 + 0.7405i
0.2403 + 1.3063i	0.2403 + 1.3063i
1.0990 + 0.1632i	1.0990 + 0.1632i

TABLE. XVI. RESULT $abs|\log(U)|$ FOR FIG. 5(A) AND $abs|\log(U)|$ FOR FIG. 5(B)

U	U
5.6537	5.6538
5.5034	5.5035
5.2400	5.2399
4.8595	4.8591
4.4849	4.4839
4.4731	4.4717
4.6457	4.6445
4.7160	4.7150
4.6394	4.6388
4.4033	4.4035

VI. CONCLUSION

This paper presents a new Fourier transform to solve the problem of index of a point. The radius parameter is used in the development. The experimental results show that NUFT presents substantial advantages than normal Fourier transform. Shift theorem is available only where the Shift is linear, but when the order of point is randomly this theorem is not valid. The advantage of our transform is invariance under change of index point, and especially robustness to noise.

VII. FUTURE WORK

In the future, the authors are interested to implement our descriptor on speech signal and for 3D objects (mesh, surfaces) and using neural network.

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