

Binary Operating Antenna Array Elements by using Hausdorff and Euclidean Metric Distance

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Abstract—In this paper, the linear antenna array is designed by the usage of Woodward–Lawson method. The design procedure fits antenna array radiation pattern to a predefined/required radiation mask. In this study will be investigated the possibility of powering off some antenna elements without modifying the behavior and power ratio of the elements which remains on. The aim of powering off antenna elements is to reduce power consumption of the designed array antenna and to reduce power dissipation problems on modern full digital beam forming architecture. The choice of binary operation of antenna element (on/off) reduces computational effort required for complex beam forming techniques. The results can be stored on look-up tables, in order to be recalled on demand by antenna operator. There are used two different metrics to identify how close, to the required design, is the modified antenna pattern. Euclidean and Hausdorff distances are both used as score of the modified array performance. The obtained solutions shows the applicability of binary operations on existing antenna array and the metric can be effectively used as ranking solutions.

Keywords—Antenna array; Hausdorff distance; Woodward–Lawson

I. INTRODUCTION

Array antennas are a set of two or more elements (with element we mean individual antenna) spread in one, two or three direction in the space. Through modifying the feed in amplitude and/or in phase can be achieved different radiation models [1][2].

The array antennas are of a great usage in nowadays technology. Used in fields like wireless communication, satellite communications, military, radar communications, astronomical studies, naval usage, telecommunication technologies etc., they have changed the way these fields have improved over the years.

The telecommunication technologies, in particular, have benefitted from their usages in 2G and 3G/B3G by increasing the system capacity. In the latest system, 4G LTE and 5G ratio, they have given their impact through increasing of the spectral efficiency (SE), antenna diversity and multiplexing gain, interference suppression, etc.[3][4].

In this paper we will use a linear antenna array as one of the 4G antennas configuration, which have equal distance throughout the N elements (of the same type). We will get the array factor (AF) using the Woodward–Lawson method, and we will compare it with the defined pattern mask.

The proposed analysis is based on changing the feed status of some antenna elements (powering on or off). The modification of the array elements behavior will reflect also in the array pattern. The aim of this work is to demonstrate that the modified pattern is also a valid trade-off between number of elements powered off and the requirements from relative array pattern mask. To score the obtained pattern output, will be used two different metrics: Euclidean and Hausdorff distance. Both metrics permits to score how close to the desired radiation, is the modified array.

Powering off a part of array elements will reduce the relative power consumption and thus, due to electronic efficiency that is always lower than 1 (digital beam forming network), there will be a reduction in the power dissipation issues. From the antenna operator perspective, reducing power consumption will increase antenna life and will decrease its relative OPEX (Operating Expenses).

A preliminary linear array will be designed to demonstrate the possibility of powering off part of array elements, and the relative pattern feet to a predefined mask. The methodology used for the preliminary array design is based on Woodward–Lawson method. Our aim is to get this array factor by not feeding (powering off) some elements of the antenna and to compare the obtained pattern with the mask using both methods.

The possibility of powering off array elements and their influence to multiple beam array behavior will be investigated in future works [5][6][7].

This paper is organized as follows: Section 2 describes mathematical definitions for linear antenna array design based on W-L method, followed by definitions of both metrics (Euclidean and Hausdorff) which are used to score how close to required pattern is the designed and modified antenna pattern. Section 3 describes the antenna array design and setup; Section 4 describes test case analysis, different array alternatives analysis and numerical useful data extraction. Conclusions and recommendations for future work will be emphasized in Section 5.

II. MATHEMATICAL DEFINITIONS

A. Woodward – Lawson Method

The method that we are going to use for the preliminary antenna design is the Woodward- Lawson method. It is a popular antenna pattern synthesis method used for beam shaping. It was introduced by Woodward and Lawson from

which it took also its name (in year 1947). Even though in the late '80 it was a debate related to its use after those years [8], the time has shown that it is the base for a lot of nowadays researches.

The Woodward- Lawson method is linear and the most important thing is the fact that in defined directions, responsible for the radiation, the virtual feed has the maximum of beam (in that direction). So if we don't want radiation in a specified direction, we can easily turn off the corresponding virtual array [9]. Woodward-Lawson method is possible if there are maintained two assumptions related to the feed and the phase:

- The formula of array factor based on the elements feed is a linear function.
- Through operating only in the phase of a set of elements with uniform feed (when possible), it is possibly to change the direction of maximal radiation of the array factor.

If this set is chosen carefully it will be possibly to reaches, not only the maximum radiation in different directions, but also the nulls of radiation to be in the same directions where the other arrays have their maximums. By combining also the appropriate feed of virtual array, we can achieve (in that direction) a radiation amplitude like the one needed by the radiation mask. The overlay of all the virtual arrays can create the desired array factor (AF) which is an interpolation of the factor of the virtual arrays as in Fig. 1.

The complex antenna array can be analyzed as an overlay of different virtual, independent and superposed arrays. An example feeding function can be expressed like (1):

$$I_n = I_n^{(1)} + I_n^{(2)} + I_n^{(3)} + \dots + I_n^{(N)}; \quad n = 0, \dots, N-1 \quad (1)$$

The feed for each of them is expressed as $I_n = a_n e^{j\beta_n}$. We can chose a set of virtual array that have the maximum of radiation in different directions.

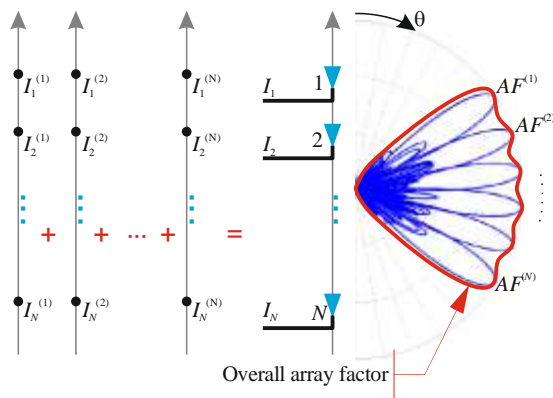


Fig. 1. Concept of Virtual Array Feeding and Beam Forming.

The realization of the pattern through the use of the chosen method follows the upcoming steps:

- The first function produced is a pattern in which the position of the beam is set on by the value of uniform progressive phase.
- The same concept is used also for the creation of second function but in this case a uniform progressive phase is adjusted in order that the maximum main lobe coordinates with the deepest null of the first step function. So an excitation in amplitude of this function determines the filling-in of the deepest null of the second function. The third function part of the main function is altered so the maximum of main lobe which occurs at the second deepest null of the first function and so on. In this way are created one after other all the functions part of the sum.

The array factor projected through the Woodward- Lawson method will be used as the original model in all the arrays. Based in this project, we will do the analysis of powering off some random elements to see the changes in the radiation model and how much it diverges from the desired model showed from the mask.

Based on this method, in this paper, we will focus on finding the distance in two different forms: Euclidean and Hausdorff distances as per below.

B. Euclidean Distance

The Euclidean distance can be expressed as the straight line between two given points (of course that points can be vector or matrix). The length of the straight line represents the shortest distance between the two defined points. It is also used for higher dimensional problems [10].

Through this paper we will use the sum of all Euclidean distances (E) between array factor (AF) and the mask (M, the desired model) as given in equation (2) and shown in Fig. 2.

$$E = \sum_{\theta=0^\circ}^{180^\circ} \|AF(\theta) - M(\theta)\| \quad (2)$$

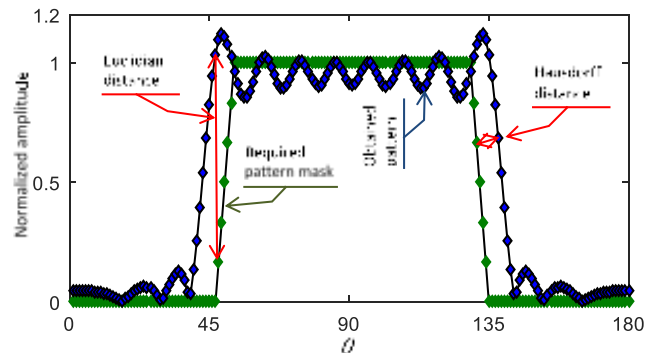


Fig. 2. Differences between Hausdorff and Euclidean Distances, Applied among the Antenna Pattern and Required Pattern Mask.

C. Hausdorff Distance

This distance is named after Felix Hausdorff (first introduced in 1905). It is often referred as Pompeiu–Hausdorff distance. Hausdorff distance can be expressed as the maximum distance of a set to the nearest point in the other set [11]. Through this definition it is easy to understand that the distance expresses the longest distance that it might be forced to travel between two sets of points. Given 2 sets of points AF and M , the Hausdorff distance between them can be defined as in (3):

$$H(AF, M) = \max \{h(AF, M), h(M, AF)\} \quad (3)$$

Where $h(AF, M)$ is called direct Hausdorff distance and defined as in (4):

$$h(AF, M) = \max_{AF(\theta_i) \in AF} \min_{M(\theta_j) \in M} \|AF(\theta_i) - M(\theta_j)\| \quad (4)$$

It identifies the point in AF that is the farthest from any point in M (*max* definition) and measures the Euclidean distance from that point to the nearest neighbor in M (*min* definition).

Likewise, the definition for $h(M, AF)$ is in (5):

$$h(M, AF) = \max_{M(\theta_i) \in M} \min_{AF(\theta_j) \in AF} \|M(\theta_i) - AF(\theta_j)\| \quad (5)$$

Hausdorff and Euclidean distances applied to the actual antenna array design, are shown in Fig. 2.

III. ANTENNA ARRAY SETUP

By using Woodward-Lawson method, we will build the standard linear antenna array which will have $N = 21$ elements, an equidistance between elements of $d = \lambda/2$ and will take in consideration a rectangular mask (M) of unit amplitude (linear scale) from 45° to 135° and zero elsewhere (Fig. 3). For mobile operators the design of the array at a given center frequency of 2100MHz, gives an antenna with a high around 220 cm.

The Woodward-Lawson procedure does not specify how to generate the superposed sample beams or the required linear array complex element excitations. In 5G or in latest developments for 6G system the complex feed can be achieved through full digital beam forming network. In this case each antenna element of the array is directly connected to a dedicated RF chain (RF: Radio Frequency) comprising PA/LNA (Power Amplifier/Low Noise Amplifier) as in Fig. 4. One of the biggest drawbacks of this architecture, despite its cost, is the power consumption and high heat dissipation requirements [3][4].

To improve power consumption and extend antenna's life span, through decreasing its demand for power heat dissipation, in this paper we are investigating the possibility of intentionally turning off part of antenna elements (corresponding RF chain). Turning off some of antenna elements will inevitably also change the original radiation pattern. To understand how far from the desired radiation mask, the modified pattern is, it will be used a procedure of

measuring Euclidean and Hausdorff distances referring to the required mask.

Our intention is to compare both Euclidean and Hausdorff distances as an effective metric for antenna pattern deviation. But what is more important, to turn off part of RF chain network, which will bring more control to the mobile antenna operator, having so the possibility to decide whether to turn on or off part of antenna elements. This can be useful in case of low traffic or lower number of users connected for example by night or on low demand traffic hours. So the operator can reduce power consumption and heating dissipation, prolonging antenna life span without deteriorating the communication capabilities.

Instead of designing antenna array which focus in finding the best power feeding distribution network ratio for each element (which can be done with different methods) [12][13][14][15], in this paper, we will choose powering on or off (binary operation) selected antenna element and as well will be traced and scored modifications to the radiation pattern. In this case of all antenna elements powered on, its power ratio is defined by designing through W-L method. This choice of binary operation of antenna element (on/off) reduces computational effort required to complex beam forming techniques. The results can be stored on look-up tables to be recalled on demand. In this case, each RF chain operates as usual and can be on two states (on or off) based on mobile operator trade-offs.

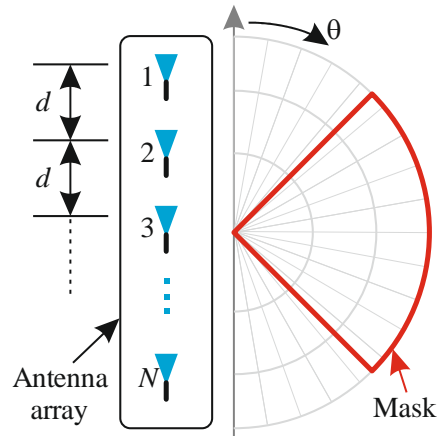


Fig. 3. Problem Defining Geometry.

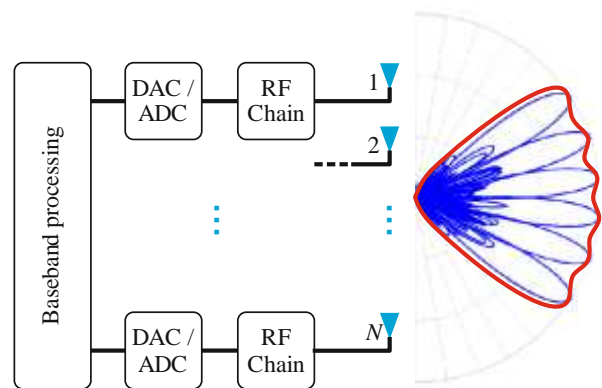


Fig. 4. Full Digital Beam Forming Architecture.

IV. ARRAY ANALYSIS AND SIMULATION

The original pattern designed as per W-L method (Fig. 5) is used to analyze the antenna array. The corresponding amplitude and phase of each element is presented in the second part of the same figure. This is a uniformly distributed antenna array, with non-uniform distribution feeding. By using the Linear Euclidean distance, can be easily noticed that: in case of all antennas up and running, Euclidean (E) and Hausdorff (H) distances are not null and are respectively $E = 750.62$, $H = 103.47$.

In the analyzed scenario, since the possibilities for each antenna elements are only two (On or Off) there are a finite number of combinations to be tested ($2^N = 2\ 097\ 152$). With the original antenna array power feeding ratio, designed as by W-L method, analyzing all the above combinations without modifying the feeding ratio between elements but only through changing their state, can be easily done through modern calculators.

Ranking the obtained solutions as for example by Euclidean distance (or the Hausdorff one) is inverse and not linear correlated to sorting solutions based on total number of powered off elements. As previously mentioned, the goal of mobile operators is to reach it without worsening the radiation pattern. These two goals are in contradiction with each other. In this case a more appropriate multiobjective optimization is required as in [16].

Turning off 6 antenna elements as in Fig. 6, the modified pattern shows a Euclidean distance $E = 995.14$ and Hausdorff distance $H = 117.18$. Both distances are increased, but the obtained antenna pattern is still satisfactory in the main lobe.

Increasing the number of elements turned off from 6 to 10, especially those far from the array center, mainly modifies the side lobe level of the original W-L pattern. This can be evaluated by comparing Fig. 5 and Fig. 7 and theoretical analysis [14]. In this case, both metrics used show an increase in their respective values referred to the original W-L designed.

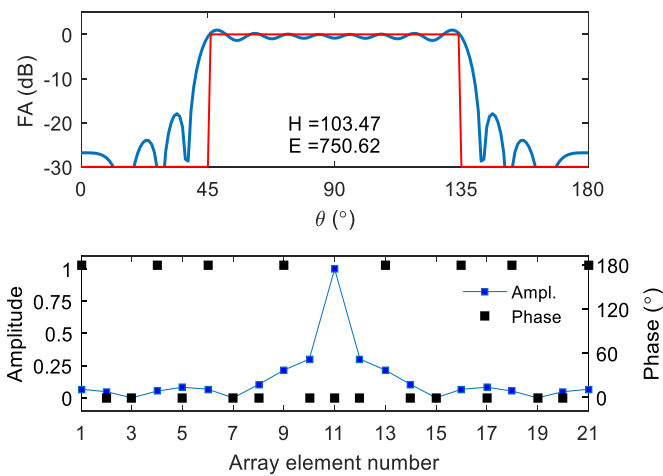


Fig. 5. Array Pattern and Relative Current at each Element as by W-L.

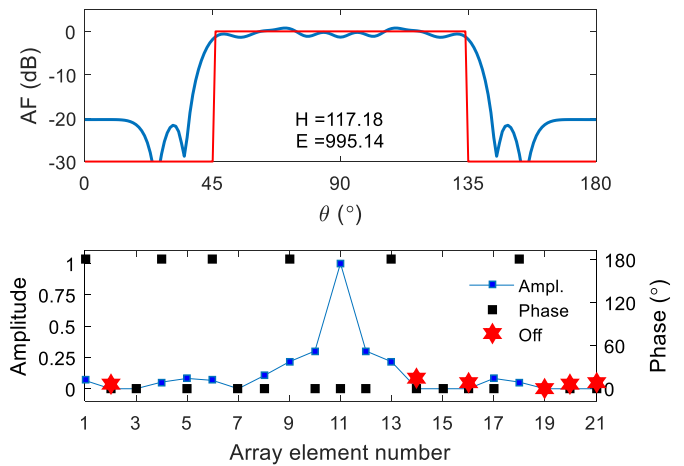


Fig. 6. Array Pattern and Relative Feed Ratio to all 21 Elements with 6 Antenna Elements turned off.

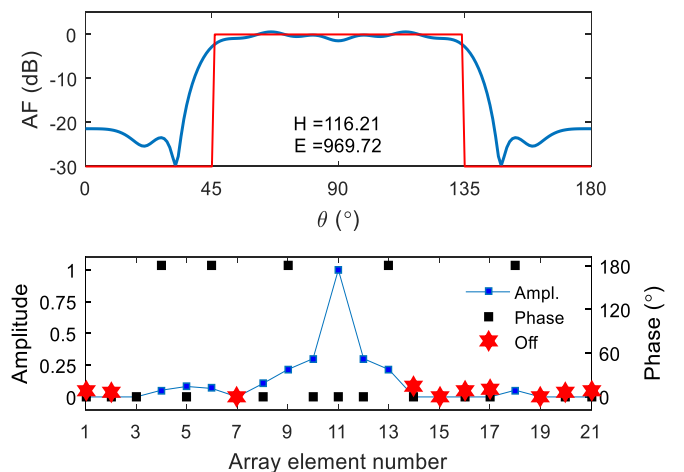


Fig. 7. Array Pattern and Relative Feed Ratio to all 21 Elements with 10 Antenna Elements turned off.

Increasing the number of elements turned off, not necessarily will bring worse results than the case with lower elements turned off. For this purpose, let us compare Fig. 8, Fig. 7 and Fig. 5 (original W-L, no elements turned off). In Fig. 8 there are 11 elements turned off (50% of all antenna elements) and both Euclidean and Hausdorff distances ($E = 760.35$; $H = 107.76$) are better than the case presented in Fig. 7 where both distances presents higher values ($E = 969.72$; $H = 116.21$). This means that we need to carefully choose which elements to turn off to have less distortion in the desired antenna pattern.

Comparing Fig. 9 and Fig. 10 brings a better insight to the above statement. Both configurations have nine antenna elements turned off, but visibly have very different radiation pattern. This can be confirmed also through comparing both metrics used. The solution presented in Fig. 9 has more uniformly distributed powered off elements which is similar to a spare antenna array [17][18].

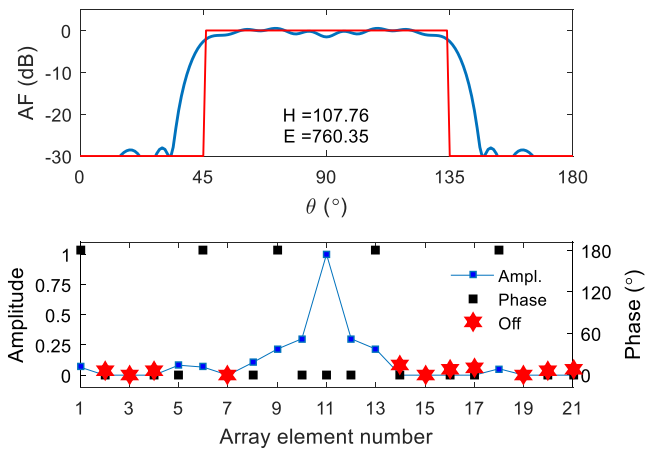


Fig. 8. Array Pattern and Relative Feed Ratio to all 21 Elements with 11 Antenna Elements turned off.

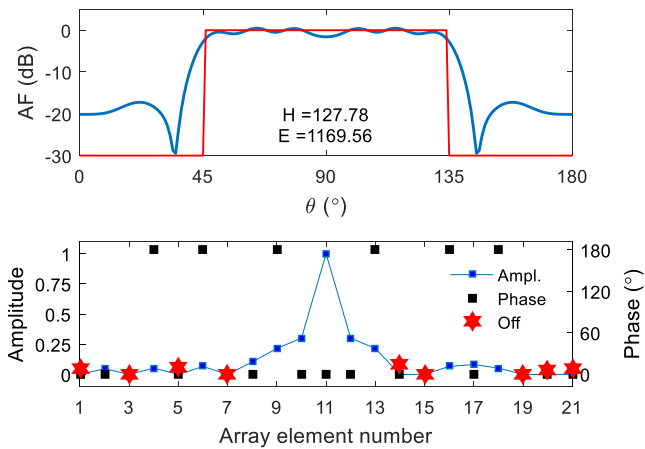


Fig. 9. Array Pattern and Relative Feed Ratio with 9 Antenna Elements turned off.

Referring Euclidean and Hausdorff distances in both figures (Fig. 9 and Fig. 10) compared to the result in Fig. 5 (original W-L configuration), we see an increase of 23% for H and 56% for E in Fig. 9, and 78% for H and 157% for E in Fig. 10.

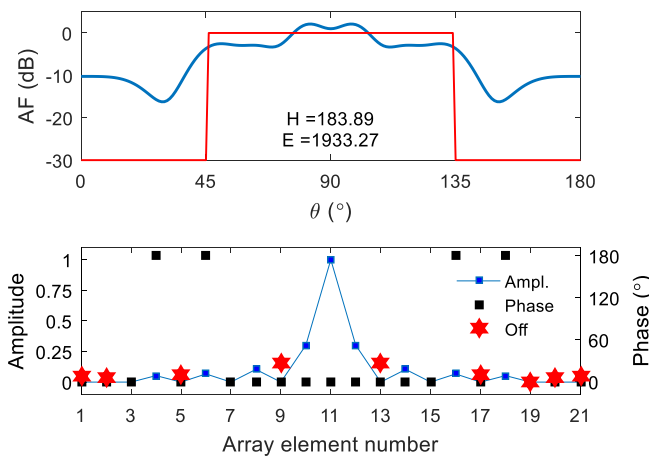


Fig. 10. Array Pattern and Relative Feed Ratio with 9 Antenna Elements turned off especially Element 9 and 13 have Higher Influence to the Pattern.

The last case, Fig. 11, presents the radiation pattern and relative elements powering for the same antenna array with 15 elements powered off. In this case there are around 71% of antenna elements powered off. It's observed an increase in side lobe level, but is maintained a contained variation to the main mask beam.

Both distance metrics can be efficiently used to evaluate how close is the modified antenna pattern to the required radiation mask. For this purpose, all presented structures, are compared in Table I. offering to interested readers all necessary information to recreate obtained results. In Table I is specified also which elements are turned on/off, the relative Euclidean and Hausdorff distances to the mask, and their variation to the original W-L ratio.

The ratio $\Delta E/E_0$ and $\Delta H/H_0$ are defined in (6) as relative E and H distances to the W-L respective distances to the desired mask.

$$\begin{cases} \frac{\Delta E}{E_0} = \frac{E - E_{WL}}{E_{WL}} \times 100\% \\ \frac{\Delta H}{H_0} = \frac{H - H_{WL}}{H_{WL}} \times 100\% \end{cases} \quad (6)$$

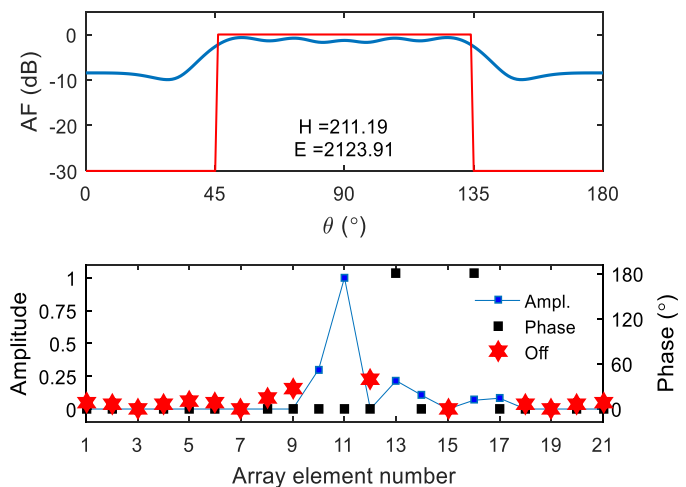


Fig. 11. Array Pattern and Relative Current at each Element with 15 Elements turned off.

TABLE I. EUCLIDEAN AND HAUSDORFF DISTANCES

Fig. No.	Off No.	Off. Elements ^a	E	H	$\Delta E/E_0$ (%)	$\Delta H/H_0$ (%)
5	0	00000000000000000000	750	103	0	0
6	6	01000000000010100111	995	117	32	14
7	10	1100001000000111101111	969	116	29	13
8	11	0111001000000111101111	760	107	1	4
9	9	1010101000000110001111	1169	127	56	23
10	9	1100100010001000101111	1933	183	157	78
11	15	1111111110010010011111	2123	211	183	104

^a Binary representation of powering on/off antenna array elements. 1: element off; 0: element on.

Referred to Table I, both metrics presents an increase in their distances due to the major modification of the radiation pattern. Hausdorff distances presents smaller variations respect to Euclidean distances as per the last row on Table I, where the modified pattern presents 183% variation on E metric and only 104% on H metric. In this way, Hausdorff distance brings a compressed scale for evaluating pattern variations.

V. CONCLUSION AND FUTURE WORK

In this material is presented the analysis of an antenna array by turning off a part of antenna elements, in order to decrease power consumption without major modifications in radiation pattern.

The presented analysis is focused on binary format of turning on/off antenna elements which is easier and does not require higher processing capabilities for beam forming evaluating techniques. Modifying antenna array behavior is faster with just powering on/off elements on demand rather than changing power and phase distribution network or beam forming network.

In this case, the solution of the elements that can be turned off can be stored on look-up table, for faster reference similar to that presented in Table I.

Powering off up to 50% of array elements made possible to have less heat, to save energy without worsening the radiation pattern and increasing the lifecycle of antenna array.

Using Euclidean distance as a metric, brings to a spread of solutions, Hausdorff evaluates the same closer solutions. The other point is that the usage of Hausdorff distance against the traditional one the Euclidean, is more appropriate to choose the deterioration of the radiation models and/or the differences among them.

This work will be the base which will lead us in future works like how to choose in an intelligent way the elements to be powered on/off. Also, the procedure will be extended to a more complex antenna array design as planar and conformal array, and multi beam array systems.

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