General Variable Neighborhood Search for the Quote-Travelling Repairman Problem

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***Abstract*—The Quota-Travelling Repairman Problem (Q-TRP) tries to find a tour that minimizes the waiting time while the profit collected by a repairman is not less than a predefined value. The Q-TRP is an extended variant of the Travelling Repairman Problem (TRP). The problem is NP-hard problem; therefore, metaheuristic is a natural approach to provide near-optimal solutions for large instance sizes in a short time. Currently, several algorithms are proposed to solve the TRP. However, the quote constraint does not include, and these algorithms cannot be adapted to the Q-TRP. Therefore, developing an efficient algorithm for the Q-TRP is necessary. In this paper, we suggest a General Variable Neighborhood Search (GVNS) that combines with the perturbation and Adaptive Memory (AM) techniques to prevent the search from local optima. The algorithm is implemented with a benchmark dataset. The results demonstrate that good solutions, even the optimal solutions for the problem with 100 vertices, can be reached in a short time. Moreover, the algorithm is comparable with the other metaheuristic algorithms in accordance with the solution quality.**

***Keywords*—Q-TRP; GVNS; AM; GRASP**

I. Introduction

The Quota-Travelling Repairman Problem (Q-TRP) in the case of Travelling Repairman Problem (TRP) has been studied in the numerous articles [1], [2], [3], [4], [5], [6], [8], [10], [14], [15]. It is often called the Travelling Repairman Problem (TRP). The TRP takes a customer-oriented approach when the objective function is to minimize the total customers' waiting time. In addition, the TRP has many practical applications in, e.g., disk head scheduling [1], [2], [4], [5], [6]. In the general case, the Q-TRP is an extended version of the TRP case. Informally, the Q-TRP tries to find a tour that minimizes the waiting time while the profit collected by a repairman is not less than a predefined value of P_{min}. The practical application for the problem can be described as follows: A Repairman must sell a quota P_{min} of goods. Repairman knows the distances between cities and how many goods he could sell in his tour. His objective is to travel along a tour while minimizing the waiting time and selling the required quota of goods. We formulate the Q-TRP as followings: Let $K_n = (V, E)$ be a complete graph in which $V = \{1, 2, \dots n\}$, E are a set of vertices and edges, respectively. For each edge $(v_i, v_j) \in E$, which connects the two vertices v_i and v_j, there exist a cost $c(v_i, v_j)$ and the quantity of goods sold $r(v_i, v_j)$. Suppose that $T = (v_1, \dots, v_k, \dots, v_n)$ is a tour. Let $P(v_1, v_k)$ be the path from v_1 to v_k on it and $l(P(v_1, v_k))$ is its length. The waiting time of a vertex $v_k (1 < k \leq n)$ on T is calculated as follows:

$$w(v_k) = \sum_{i=1}^{k-1} c(v_i, v_{i+1}). \quad (1)$$

The cost of the tour T is calculated:

$$L(T) = \sum_{k=1}^{n+1} w(v_k). \quad (2)$$

The quantity of goods sold on the tour is the sum of the amount of goods sold of its edges. The tour must satisfy the following constraint:

$$\sum_{i=1}^{n} r(v_i, v_{i+1}) \geq P_{min}. \quad (3)$$

The term v_{i+1} for *i*=*n* in the preceding formula coincides with v_1, so that the repairman in the very end returns to the vertex from which he started his trip. The solution then is a Hamiltonian Cycle.

Generally speaking, for NP-hard problems, some types of algorithms are applied to solve the Q-TRP. Firstly, exact algorithms find the optimal solution, but they can only solve the problem with small sizes. Secondly, an α-approximation algorithm produces a solution with the cost of no more than α times the optimal solution. Lastly, metaheuristic performs well in practice, and their efficiency is evaluated on a benchmark dataset.

The algorithm is one of the first metaheuristics to solve this problem. In this paper, we propose a General Variable Neighborhood Search (GVNS) [13] that combines with the perturbation and Adaptive Memory (AM) techniques [11] to prevent the search from local optima. Currently, there is no metaheuristic to solve this problem can be found in the literature to compare directly. Therefore, it is difficult to evaluate the efficiency of the A-GVNS exactly. To overcome the issue, several state-of-the-art metaheuristic algorithms for the TRP is chosen to compare to the A-GVNS. The A-GVNS is implemented with benchmark instances. The results demonstrate that good solutions, even the optimal solutions for the problem with 100 vertices, can be reached in a short time. Moreover, the algorithm is comparable with the other metaheuristic algorithms in accordance with the solution quality.

The rest of this paper is demonstrated as follows. Section 2 provides the algorithm. Section 3 presents computational evaluations. Sections 4 and 5 discuss and conclude, respectively.

II. The proposed algorithm

The modified GVNS (notation: A-GVNS) includes the GRASP [7] in the construction phase, and then the GVNS [13], the Adaptive Memory [11], and perturbation technique

[12] in the improvement phase, respectively. The good meta-heuristic needs to maintain the balance between diversification and intensification. Diversification means to generate diverse solutions to explore the unvisited solution space, while intensification means to focus on the search in a current region by exploiting it. In the algorithm, the A-GVNS ensures the intensification while the techniques maintain the diversification. This combination maintains the simplicity spirit of the GVNS while it effectively explores the search space.

An outline of the A-GVNS is shown in Algorithm 3. The A-GVNS starts with iterations of the construction to create an initial solution. At each iteration in the A-GVNS algorithm, a random perturbation is picked to shake the solution, and the local search is used to improve it. The best solution obtained by using a local search is saved in the AM list. The best solution in the AM, in accordance with the formula (5), is used as a starting solution in the next iteration. The algorithm returns the best solution when its computation time reaches t_{max}. We realize that the statements from line 16 to 20 are removed, the A-GVNS becomes the original GVNS.

A. Neighborhood Structures

Several neighborhoods are widely applied in the literature to explore the solution space of this problem [16]. Let us denote $N_k(k = 1, ..., k_{max})$, a finite set of pre-selected neighborhood structures, and with $N_k(T)$ the set of solutions in the $k-th$ neighborhood of *T*. We describe more details about seven neighborhoods as follows:

- **forward** (N_1) pushes a vertex to move forward a position in the tour. Its complexity is $\Theta(n)$.
- **backward** (N_2) pushes a vertex to move backward a position in the tour. Its complexity is $\Theta(n)$.
- **shift** (N_3) locates a vertex to another position in the tour. Its complexity is $\Theta(n)$.
- **adjacent-swap** (N_4) exchanges the positions of each pair of adjacent vertices in the tour. Its complexity is $\Theta(n)$.
- **swap** (N_5) a pair of vertices in the tour are interchanged. Its complexity is $\Theta(n^2)$.
- **2-opt** (N_6) removes each pair of vertices from the tour, and the reconnects them according to the reversing order. Its complexity is $\Theta(n^2)$.
- **Or-opt** (N_7) is located three adjacent vertices to another position of the tour. Its complexity is $\Theta(n^2)$.

In the Q-TRP, the calculation of a neighboring solutions' cost requires exactly $\Theta(n)$ time. It leads to $\Theta(n)$ operations for each move evaluation resulting in $\Theta(n^3)$ operations for a full neighborhood search.

B. Penalty on Infeasible Solution

The search is not constrained to feasible solutions. During the search, we penalize infeasible solutions by incorporating a penalty value. For each solution T, let $L(T)$ denote the total cost and let $V(T)$ denote the violation of the constraint of the quantity of goods sold. The total cost violation $V(T)$ is

Algorithm 1 Construction

Input: v_1, V, α are a starting vertex, the set of vertices in K_n, and $\mid RCL \mid$, respectively.
Output: An initial solution T.

```
T ← T ← v_1; {v_1 is the starting vertex}
while |T| < n do
    {v_e is the last vertex in T}
    Create RCL that includes α nearest vertices to v_e in V;
    Select randomly vertex v ∈ {v_i | v_i ∈ RCL and v_i ∉ T};
    T ← {v_i}
end while
return T;
```

Algorithm 2 VND

Input: T, k_{max} are an initial solution, and the number of neighborhoods, respectively.
Output: the best solution T^*.

```
k = 1;
repeat
    Find the best neighborhood T' of T ∈ N_k(T); {local search}
    {L(T) is the objective function value of T}
    if (L(T') < L(T)) || (L(T') < L(T*)) then
        T = T'; {centre the search around T' and search again in the first neighborhood}
        if (L(T') < L(T*)) then
            T* = T';
        end if
        k = 1;
    else
        k = k + 1; {switch to another neighborhood}
    end if
until k < k_max;
T* = T';
return T*;
```

computed on a tour basis with respect to the value of P_{min}. Specifically, it is equal to

$$\max\{P_{min} - LP, 0\}, \tag{4}$$

where *LP* is the quantity of goods sold in the current solution (feasible or infeasible).

Solutions are then evaluated according to the weighted fitness function $L' = L + PF \times V(T)$, where *PF* is the penalty value. Obviously, $LP \geq P_{min}$ and $L' = L$ when it is a feasible one.

C. The Construction Phase

Algorithm 1 shows the constructive procedure by using GRASP [7]. The algorithm works iteratively until an initial solution is found. At each step, we use a Restricted Candidate List (notation: *RCL*) of all non-visited vertices according to a greedy that evaluate the benefit of adding them in the solution. After that, one random vertex is chosen from the *RCL* to add to the current partial tour. When all vertices are selected, it stops, and we obtain the initial solution. The size of *RCL* controls the balance between greedy and random strategies.

Algorithm 3 A-GVNS

Input: T, k_{max}, t_{max} are an initial solution, the number of neighborhoods, and the maximum time to run, respectively.
Output: the best solution T^*.

repeat
  $k = 1$;
  repeat
    $T^{'}$ = Perturbation(T);
    {implement VND}
    $T^{''} \leftarrow$ VND($T^{'}, k_{max}$);
    if $(L(T^{''}) < L(T)) \parallel (L(T^{''}) < L(T^*))$ **then**
      $T = T^{''}$; {centre the search around $T^{''}$ and search again in the first neighborhood}
      if $(L(T^{''}) < L(T^*))$ **then**
        $T^* = T^{''}$;
      end if
      $k = 1$;
    else
      $k = k + 1$;
    end if
    $AM = AM \cup T^{''}$;
    if $(\mid AM \mid == m)$ **then**
      Clear *AM*;
    end if
    T = Select the best one from *AM* according to (5);
  until $k < k_{max}$
until $time < t_{max}$
$T^* = T^{'}$;
return T^*;

D. The Improvement Phase

The A-GVNS incorporates the Adaptive Memory [11] into the GVNS framework. The Adaptive Memory (AM) is a technique used in the local search provided by I. Mathlouthi et al. [11]. This technique not only allows to diversify the search by exploring solutions but also ensures to intensify the search to a promising region. However, when the technique does not provide enough diversification, the perturbation mechanism is applied. It drives the search to unexplored solution space. The combination helps the balance between diversification and intensification.

1) local search: Local search procedures are developed by combining the seven neighborhoods. From an initial solution, they generate their neighborhoods. The final solution should be a local minimum in comparison with all neighborhoods. The order of neighborhoods is deterministic. In a preliminary experiment, we realize that the results of the algorithm relatively depends on the order of exploring neighborhoods. The order of them is determined, from small to large size as follows: swap-adjacent, forward, backward, shift, swap, 2-opt, and or-opt.

2) The adaptive memory: The Adaptive Memory (AM) [11] is a dynamic memory technique. It stores different solutions received by the local search improvement. For each solution in the AM, we count its cost and diversity in a set of solutions that present in the AM as follows:

$$R(T) = \beta \times (|AM| - RF(T) + 1) + (1 - \beta) \times (|AM| - RD(T) + 1), \quad (5)$$

where
$\mid AM \mid$ is the current size of *AM*.
$\beta \in [0, 1]$. The parameter controls the balance between two rank values. $RF(T)$ is the rank of solution *T* based on the objective function.
$RD(T)$ is the rank of solution *T* based on its diversity contribution.

$$\overline{d}(T) = \frac{\sum_{i=1}^{n} d(T, T_i)}{n} \quad (6)$$

$d(T, T_i)$ is the metric distance between T, and T_i, and $\overline{d}(T)$ is the average distance metric of *T* in the AM. In natural way, the distance is defined as the minimum number of transformations from T to T_i, denoted $d(T, T_i)$. Since no polynomial method for computing $d(T, T_i)$ is known, $d(T, T_i)$ to be *n* minus the number of vertices which has the same position in both of *T* and T_i. The larger $\overline{d}(T)$ is, the higher its $RD(T)$ value is. The smaller $L(T)$ is, the higher its $RF(T)$ value is. The solution with the largest $R(T)$ value is selected from the *AM*.

3) The perturbation technique: Perturbation techniques prevent the algorithm from getting trapped into local optimal by driving the search to unexplored solution space. Two perturbations are selected as follows: 1) the double-bridge [12] is applied; 2) we select randomly two vertices and then swap them.

III. EVALUATIONS

We evaluate the A-GVNS in percent as followings: $Gap_1[\%] = \frac{best.sol - LB}{LB} \times 100\%$, and $Gap_1[\%] = \frac{best.sol - UB}{UB} \times 100\%$, where the *OPT*, *LB*, *best.sol*, and *UB* correspond to the optimal solution, lower bound of the optimal solution, best, and the result of the VND, respectively. Note that: The value of *LB* in the Q-TRP is the optimal solution of the TRP (the Q-TRP reduces to the TRP by removing the quantity of goods sold constraint). In the TRP, we can obtain the optimal solutions from the exact algorithm in [2]. However, the exact algorithm can solve the problem with up to 40 vertices. Therefore, for the small instances, the efficiency of the algorithms is evaluated according to the Gap_1 value.

Currently, we do not find any metaheuristic for this problem to compare. Therefore, our solution is compared with the result of the VND. In practice, the VND produces a quite good solution. Moreover, to present the efficiency of the A-GVNS, we implement it on some TRP-instances. The results can compare directly with several state-of-the-art algorithms in [14], [15].

A. Datasets

The A-GVNS is implemented on CPU Core i7, 2.10 GHz, and Ram 8 GB. The parameters in the algorithm are determined through pilot experiments: $t_{max} = 1$h, $\mid AM \mid = 100, \beta = 0.75$, $PF = 10$, and $RCL = 10$. In all tables, we report on the time since the best solution is reached.

The algorithms (VND, GVNS, and A-GVNS) are implemented on the benchmark for the Q-TRP and TRP [14], [17], [18]. These are: 1) A set of the Q-TRP instances is generated randomly as follows: The matrix costs, c_{ij}, are chosen randomly from integers in the range [0, 200]. The quantity of goods sold, p_{ij}, are chosen randomly from integers in the range [0, 200]. The matrices are symmetric that satisfy the triangle inequality. Minimum quantity of goods sold, P_{min}, is created using the following formula:

$$P_{min} = \rho \times \sum_{i \in V} \sum_{j \in V} p_{ij} x^p_{ij} \quad (7)$$

The values x^p_{ij} represent the optimal solution of the problem

$$\sum_{i=1}^{n} r(v_i, v_{i+1}) \rightarrow \max,$$

where the cost matrix is defined by the matrix p_{ij}. The problem with the above objective function has known as the Max-TSP. For the Max-TSP, we choose the Concorde tool [19] to solve exactly the problem. However, the tool solves the TSP problem with a minimization objective function. Since we are interested in the maximization problem, say $\max f$, then an equivalent minimization problem is $\min -f$. That is, minimizing *-f* is the same as maximizing *f*. Therefore, we can adapt the tool to find P_{min}. However, for the instances with 100 vertices, we have customized the way to compute P_{min}. In the formula (7), the approximate solutions are used instead of the optimal solutions. Approximate solutions are computed using the Chained Lin-Kernighan algorithm in Concorde's tool. In the equation, ρ is a parameter to control the tightness of the number of goods sold constraint. In the case of $\rho = 0$, the optimal solution is not affected by the constraint. On the other hand, since $\rho = 1$, it becomes very tight. The algorithm is implemented with the values of ρ =0, 0.5, 0.75, 0.95, and 1. The size of the instances is selected between 30 to 100 vertices. The combination creates four hundred cases. The test instances are available at the web page [18]; 2) A. Salehipour et al. [14]: They provided five sets, where each of them is included 20 instances from 10 to 200 vertices, respectively; 3) Finally, several instances from the TSPLIB are selected by Abeledo et al. [1], [17].

Note that: The VND, GVNS, and A-GVNS are the algorithms in this article while GRASP-VNS, and ILS are the algorithms of Salihepour et al. 's [14], N. Silva et al. 's [15], respectively. Note that: The GVNS is the same as the A-GVNS when statement lines from 16 to 20 in Algorithm 1 are removed.

B. Results

In the tables, *aver.sol*, *best.sol*, and *T* are corresponding to the average, best solution, and average time in seconds of 10 executions, respectively, while *cTime* corresponds to the running time on a Pentium 4, 2 GHz according to the factors of Dongarra in [9]. Tables 1 to 20 show the results of the VND, GVNS, and A-GVNS, while in Tables 22 to 23, our results are compared with the previous algorithms in the TRP case. The average Gap_2 calculated from Table 1 to 20, is illustrated in Table 21.

In Tables 1 to 20, the VND, and GVNS obtain the feasible ones in some cases when the constraint is not tight. Since the constraint is very tight (the ρ value is 1), no feasible solution can be reached by the VND, while several feasible ones can be found by the GVNS. On the other hand, the A-GVNS reaches the feasible solutions in 96 out of 100 tested instances. It is understandable because the larger and larger value of ρ is the tighter and tighter the problem becomes. In Table 21, with the ρ value of 0, 0.5, 0.75, and 0.95, the average Gap_2 is from 2.43% to 25.50%. It is implied that the solutions of the GVNS and A-GVNS are much better than those of the VND.

In Tables 1 to 10 show that in average Gap_1, the solutions found by the GVNS, and A-GVNS are near to the optimal solutions since the difference between our solution and the lower bound is below 5.63%. When the constraint is very tight, the solutions are within 10.02% of the optimum. Moreover, with $\rho = 0$ (the Q-TRP becomes the TRP), the A-GVNS always finds the optimal solutions for the problem with up to 40 vertices. For the instances from 50 to 100 vertices, the exact algorithm in [2] cannot find the optimal solutions for the TRP. Therefore, the lower bounds are not remembered, and there are not the average Gap_1 values in these cases.

The GVNS and A-GVNS obtain better solutions than the VND. It is understandable since VND only ensures the intensification. Conversely, the A-GVNS maintains the diversification better by using the perturbation technique. Therefore, the perturbation technique plays an important role in improving the quality of the solution. In all results, the A-GVNS reaches better solutions than the GVNS does. Even in many cases with the value of ρ of 1, the A-GVNS finds the feasible solutions while the GVNS cannot do. Obviously, the AM technique brings efficiency to the A-GVNS when it can balance the intensification and diversification. More specifically, this technique allows implementing diversification in the search by exploring solutions that ensure the difference from each other. In addition, it intensifies the search to find better local optima in a promising solution space.

The experimental results show that the best-known TRP-solutions (the Q-TRP with $\rho = 0$) can be infeasible solutions for the Q-TRP. Specifically, the best solutions of the TRP in Table 5, 10, 15, and 20 are not feasible for the Q-TRP when $\rho = 1$. Therefore, the good methods for the TRP may not be applied to solve the Q-TRP. Developing an efficient algorithm for the Q-TRP is necessary.

Due to the lack of the works related to the metaheuristic algorithms of the Q-TRP, therefore we compare our solution with the algorithms for the TRP in Tables 22 and 23 [14], [15]. Specifically, in Table 22, our algorithm obtains better solutions than the GRASP–VNS [14] in all cases. In comparison with ILS [15], our solutions are the same for most of the instances. The results are significant because the algorithms in [14], [15] are the state-of-the-art metaheuristic algorithms. Moreover, our algorithm also is implemented on some TSP-instances. The optimal solutions for these instances can be extracted in [1]. The experimental results show that the optimal solutions can be reached for the problems with up to 100 vertices in several seconds [1] in Table 23. Obviously, our algorithm can solve well to the TRP.

The average running time of our algorithm is faster than the GRASP–VNS [14] and comparable with the ILS [15].

IV. DISCUSSIONS

Due to the NP-Hard problem, metaheuristic is a suitable approach for the Q-TRP. The metaheuristic can provide the near-optimal solution faster but without a guarantee of optimality.

Currently, several metaheuristic algorithms [1], [14], [15] are proposed to solve the TRP. However, the quote constraint in the works does not include, and their corresponding algorithms cannot be adapted to the Q-TRP. That means that we cannot use the above algorithms to solve the Q-TRP. Therefore, developing an efficient algorithm for the Q-TRP is necessary. There are no previous works in the literature to solve the Q-TRP, neither exact nor heuristically. Our contribution to this article is to propose the efficient algorithms for the problem. These algorithms are the first metaheuristics for the problem. In this work, three algorithms are used to solve the problem, such as the VND, GVNS, and GVNS-AM. Among the algorithms, the VND outputs the worse results. It is understandable because the VND only implements the intensification while the others maintain the diversification by using the perturbation technique. The GVNS-AM outperforms than the GVNS. Obviously, the AM brings the efficiency well since it balances between the diversity and intensification. The GVNS-AM is the most effective algorithm in terms of the quality of solution for the Q-TRP as well as TRP, although it consumes than the others. In the TRP case, it can find the optimal solutions to the problems with up to 100 vertices as well as provide the near-optimal solutions to the larger instances. Though our aim is not to propose metaheuristic for the TRP, the good results for this problem demonstrate the efficiency and broad applicability of our metaheuristics.

V. CONCLUSIONS

In this paper, we propose metaheuristic algorithms for the Q-TRP problem. The experimental results show that our algorithm obtains good solutions for the Q-TRP in a short time. In the TRP case, the optimal solutions can be reached for the instances with 100 vertices in some seconds. Our solutions are compared with the previous algorithms in both of the solution quality as well as running time.

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TABLE I. THE EXPERIMENTAL RESULTS FOR TEST-30-X WITH $\rho = 1.0$

instances	*LB*	VND			GVNS				A-GVNS			
		best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*Time*
test-30-1	6415	-	-	-	6415	6415	0.00	1.88	6415	6415	0.00	1.89
test-30-2	7186	-	-	-	7263	7263	1.07	1.82	7263	7263	1.07	2.07
test-30-3	6286	-	-	-	6961	6961	10.74	1.85	6961	6961	10.74	2.14
test-30-4	6348	-	-	-	7093	7093	11.74	1.83	7093	7093	11.74	1.94
test-30-5	6239	-	-	-	7416	7416	18.87	1.86	7416	7416	18.87	2.11
test-30-6	6612	-	-	-	7004	7004	5.93	1.89	7004	7004	5.93	2.07
test-30-7	7305	-	-	-	7601	7601	4.05	1.91	7601	7601	4.05	1.80
test-30-8	5907	-	-	-	6283	6283	6.37	1.89	6283	6283	6.37	2.04
test-30-9	5988	-	-	-	6379	6379	6.53	1.98	6379	6379	6.53	1.95
test-30-10	7154	-	-	-	7366	7366	2.96	1.90	7366	7366	2.96	2.17
test-30-11	7154	-	-	-	7366	7366	2.96	1.99	7366	7366	2.96	1.80
test-30-12	6173	-	-	-	6655	6655	7.81	1.93	6655	6655	7.81	1.98
test-30-13	6031	-	-	-	6115	6115	1.39	1.99	6115	6115	1.39	1.97
test-30-14	5852	-	-	-	6250	6250	6.80	1.85	6250	6250	6.80	1.98
test-30-15	6059	-	-	-	6457	6457	6.57	1.94	6457	6457	6.57	2.11
test-30-16	6669	-	-	-	6884	6884	3.22	1.86	6884	6884	3.22	1.93
test-30-17	6937	-	-	-	7302	7302	5.26	1.93	7302	7302	5.26	2.11
test-30-18	6862	-	-	-	6967	6967	1.53	1.94	6967	6967	1.53	1.99
test-30-19	7469	-	-	-	7558	7558	1.19	1.81	7558	7558	1.19	1.81
test-30-20	6240	-	-	-	6508	6508	4.29	1.85	6508	6508	4.29	1.87
Aver							5.46				1.99	

LB: Since $\rho = 0$, the Q-TRP becomes to the TRP. The optimal solution of the TRP found by using the exact algorithm in [2] are the lower bound of the optimal solution of the Q-TRP.

TABLE II. THE EXPERIMENTAL RESULTS FOR TEST-30-X WITH $\rho = 0.95$

instances	*LB*	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap*$_1$	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*Gap*$_2$	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*Gap*$_2$	*time*
test-30-1	6415	6914	7230.27	7.78	0.36	6719	7249.73	4.74	-2.82	1.75	6675	7108.27	4.05	-3.46	1.87
test-30-2	7186	7807	8203.91	8.64	0.16	7402	7557.18	3.01	-5.19	1.7	7384	7488.45	2.76	-5.42	1.84
test-30-3	6286	7343	7826.64	16.82	0.3	6404	6570.18	1.88	-12.79	1.72	6347	6508.36	0.97	-13.56	2.07
test-30-4	6348	6645	7020.36	4.68	0.17	6616	6763.36	4.22	-0.44	1.74	6567	6703.91	3.45	-1.17	1.85
test-30-5	6239	6656	7078.27	6.68	0.13	6489	6660.73	4.01	-2.51	1.81	6288	6585.27	0.79	-5.53	1.8
test-30-6	6612	7287	7533.64	10.21	0.27	7122	7233.36	7.71	-2.26	1.74	7113	7190.18	7.58	-2.39	1.81
test-30-7	7305	7658	7756.36	4.83	0.23	7598	7827.64	4.01	-0.78	1.8	7367	7723.64	0.85	-3.80	1.92
test-30-8	5907	6479	6676.64	9.68	0.23	6222	6298.45	5.33	-3.97	1.74	6037	6254.36	2.20	-6.82	1.83
test-30-9	5988	6555	6756.36	9.47	0.29	6454	6593.64	7.78	-1.54	1.82	6408	6530.73	7.01	-2.24	1.87
test-30-10	7154	7899	8265.18	10.41	0.32	7563	7937.91	5.72	-4.25	1.8	7333	7767.09	2.50	-7.17	1.95
test-30-11	7154	7899	8265.18	10.41	0.2	7563	7937.91	5.72	-4.25	1.78	7333	7767.09	2.50	-7.17	1.89
test-30-12	6173	7395	7532.64	19.80	0.29	6355	6576.09	2.95	-14.06	1.75	6336	6501.73	2.64	-14.32	1.83
test-30-13	6031	6962	7323.64	15.44	0.22	6556	6794.18	8.71	-5.83	1.74	6302	6668.27	4.49	-9.48	1.95
test-30-14	5852	6785	7538.82	15.94	0.34	6234	6467.27	6.53	-8.12	1.79	6126	6379.73	4.68	-9.71	1.83
test-30-15	6059	6571	6698.45	8.45	0.33	6174	6405.18	1.90	-6.04	1.72	6158	6333.73	1.63	-6.29	1.88
test-30-16	6669	7226	7403.45	8.35	0.17	6794	6920.45	1.87	-5.98	1.75	6777	6863.09	1.62	-6.21	1.96
test-30-17	6937	7644	7782.45	10.19	0.27	7142	7314.82	2.96	-6.57	1.79	7063	7244.18	1.82	-7.60	1.84
test-30-18	6862	7527	7650.18	9.69	0.26	6996	7329	1.95	-7.05	1.81	6977	7179.73	1.68	-7.31	2.07
test-30-19	7469	8108	8259.27	8.56	0.25	7887	8062.82	5.60	-2.73	1.73	7858	8000.73	5.21	-3.08	1.9
test-30-20	6240	6841	7064.55	9.63	0.34	6566	6666.73	5.22	-4.02	1.86	6506	6621.45	4.26	-4.90	1.8
Aver				10.28	0.26			4.59	-5.06	1.77			3.13	-6.38	1.89

TABLE III. THE EXPERIMENTAL RESULTS FOR TEST-30-X WITH $\rho = 0.75$

instances	*LB*	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap_1*	*time*	*best.sol*	*aver.sol*	*Gap_1*	*Gap_2*	*time*	*best.sol*	*aver.sol*	*Gap_1*	*Gap_2*	*time*
test-30-1	6415	7116	7432.09	10.93	0.36	6415	6481.91	0.00	-9.85	1.75	6415	6460.27	0.00	-9.85	1.87
test-30-2	7186	8297	8503.91	15.46	0.16	7248	7329.91	0.86	-12.64	1.7	7211	7294.36	0.35	-13.09	1.84
test-30-3	6286	7602	7726.55	20.94	0.3	6300	6376.64	0.22	-17.13	1.72	6286	6349.55	0.00	-17.31	2.07
test-30-4	6348	7131	7411	12.33	0.17	6366	6458.82	0.28	-10.73	1.74	6348	6411.55	0.00	-10.98	1.85
test-30-5	6239	6997	7261.91	12.15	0.13	6247	6452.45	0.13	-10.72	1.81	6239	6367.27	0.00	-10.83	1.8
test-30-6	6612	7467	7787.27	12.93	0.27	6652	6805.18	0.60	-10.91	1.74	6612	6742.64	0.00	-11.45	1.81
test-30-7	7305	7918	8270.18	8.39	0.23	7312	7439.36	0.10	-7.65	1.8	7305	7384.09	0.00	-7.74	1.92
test-30-8	5907	6441	6711.09	9.04	0.23	5911	6006.27	0.07	-8.23	1.74	5907	5973	0.00	-8.29	1.83
test-30-9	5988	7343	7654.55	22.63	0.29	6008	6073.55	0.33	-18.18	1.82	5988	6045.18	0.00	-18.45	1.87
test-30-10	7154	8136	8401.45	13.73	0.32	7164	7625.18	0.14	-11.95	1.8	7154	7479.64	0.00	-12.07	1.95
test-30-11	7154	8136	8401.45	13.73	0.2	7164	7625.18	0.14	-11.95	1.78	7154	7479.64	0.00	-12.07	1.89
test-30-12	6173	6453	6839.36	4.54	0.29	6183	6261.09	0.16	-4.18	1.75	6173	6226.91	0.00	-4.34	1.83
test-30-13	6031	6251	6571	3.65	0.22	6049	6180.82	0.30	-3.23	1.74	6031	6127.55	0.00	-3.52	1.95
test-30-14	5852	7460	8127.18	27.48	0.34	5859	5939.91	0.12	-21.46	1.79	5852	5909.27	0.00	-21.55	1.83
test-30-15	6059	6257	6483.55	3.27	0.33	6066	6241.91	0.12	-3.05	1.72	6059	6176.27	0.00	-3.16	1.88
test-30-16	6669	7446	7695.18	11.65	0.17	6728	6831.36	0.88	-9.64	1.75	6714	6799.09	0.67	-9.83	1.96
test-30-17	6937	7163	7253.09	3.26	0.27	6937	7003.45	0.00	-3.16	1.79	6931	6968.91	-0.09	-3.24	1.84
test-30-18	6862	7961	8120.82	16.02	0.26	6888	6982.64	0.38	-13.48	1.81	6862	6940.36	0.00	-13.80	2.07
test-30-19	7469	7934	8336.09	6.23	0.25	7470	7595	0.01	-5.85	1.73	7469	7553.36	0.00	-5.86	1.9
test-30-20	6240	6517	6788.64	4.44	0.34	6253	6426.82	0.21	-4.05	1.86	6240	6357.18	0.00	-4.25	1.8
Aver				11.64	0.26			0.25	-9.90	1.77			0.05	-10.09	1.89

TABLE IV. THE EXPERIMENTAL RESULTS FOR TEST-30-X WITH $\rho = 0.5$

instances	*LB*	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap_1*	*time*	*best.sol*	*aver.sol*	*Gap_1*	*Gap_2*	*time*	*best.sol*	*aver.sol*	*Gap_1*	*Gap_2*	*time*
test-30-1	6415	7526	7680.55	17.32	0.17	6415	6454.91	0.00	-14.76	1.7	6415	6435.55	0.00	-14.76	2.03
test-30-2	7186	7865	8003	9.45	0.18	7258	7454.09	1.00	-7.72	1.77	7248	7392.18	0.86	-7.84	2.1
test-30-3	6286	6797	7152.18	8.13	0.13	6312	6423.45	0.41	-7.14	1.83	6286	6377.64	0.00	-7.52	1.93
test-30-4	6348	6929	7180.64	9.15	0.34	6366	6573.64	0.28	-8.13	1.78	6348	6503.45	0.00	-8.39	1.99
test-30-5	6239	6966	7349.36	11.65	0.27	6247	6377.82	0.13	-10.32	1.74	6239	6319.91	0.00	-10.44	2.07
test-30-6	6612	7003	7570.55	5.91	0.22	6681	6804.36	1.04	-4.60	1.63	6612	6741.64	0.00	-5.58	1.78
test-30-7	7305	7695	7891.82	5.34	0.27	7324	7451.64	0.26	-4.82	1.77	7305	7413.91	0.00	-5.07	1.94
test-30-8	5907	6486	6938.73	9.80	0.24	5911	6054.09	0.07	-8.87	1.62	5907	5993.82	0.00	-8.93	1.92
test-30-9	5988	6474	6635.73	8.12	0.27	6041	6192.91	0.89	-6.69	1.65	5988	6131.55	0.00	-7.51	2.07
test-30-10	7154	8257	8407.09	15.42	0.24	7184	7531	0.42	-13.00	1.63	7154	7433.27	0.00	-13.36	1.99
test-30-11	7154	8257	8407.09	15.42	0.29	7184	7531	0.42	-13.00	1.75	7154	7433.27	0.00	-13.36	2.03
test-30-12	6173	6728	7550.73	8.99	0.24	6209	6346.73	0.58	-7.71	1.64	6198	6299.82	0.40	-7.88	1.83
test-30-13	6031	7031	7228.91	16.58	0.36	6057	6237.27	0.43	-13.85	1.69	6031	6172.91	0.00	-14.22	1.82
test-30-14	5852	7889	8429.27	34.81	0.16	5922	6013.82	1.20	-24.93	1.8	5852	5971.36	0.00	-25.82	1.81
test-30-15	6059	6523	6753.91	7.66	0.13	6129	6259.27	1.16	-6.04	1.82	6059	6195.36	0.00	-7.11	1.83
test-30-16	6669	8255	8474.27	23.78	0.13	6674	6814.73	0.07	-19.15	1.79	6669	6759.09	0.00	-19.21	2.02
test-30-17	6937	7339	7558.45	5.80	0.12	6958	7075.91	0.30	-5.19	1.71	6957	7024.55	0.29	-5.21	2.06
test-30-18	6862	7476	7656.82	8.95	0.21	6890	7008.45	0.41	-7.84	1.83	6862	6963.91	0.00	-8.21	1.95
test-30-19	7469	8065	8405.36	7.98	0.22	7470	7732.82	0.01	-7.38	1.72	7469	7644.27	0.00	-7.39	1.82
test-30-20	6240	6322	6505.36	1.31	0.2	6280	6466.82	0.64	-0.66	1.64	6240	6407.73	0.00	-1.30	1.81
Aver				11.58	0.22			0.49	-9.59	1.73			0.08	-9.96	1.94

TABLE V. THE EXPERIMENTAL RESULTS FOR TEST-30-X WITH $\rho = 0$

instances	*LB*	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap_1*	*time*	*best.sol*	*aver.sol*	*Gap_1*	*Gap_2*	*time*	*best.sol*	*aver.sol*	*Gap_1*	*Gap_2*	*time*
test-30-1	6415	6505	6830.5	1.40	0.28	6415	6415	0.00	-1.38	0.17	6415	6415	0.00	-1.38	2.04
test-30-2	7186	7379	7561.9	2.69	0.24	7193	7193	0.10	-2.52	0.18	7186	7186	0.00	-2.62	2.06
test-30-3	6286	6286	6514.4	0.00	0.28	6286	6286	0.00	0.00	0.18	6286	6286	0.00	0.00	1.85
test-30-4	6348	6872	7262.6	8.25	0.31	6348	6348	0.00	-7.63	0.19	6348	6348	0.00	-7.63	2
test-30-5	6239	6489	6822.2	4.01	0.32	6239	6239	0.00	-3.85	0.19	6239	6239	0.00	-3.85	2.09
test-30-6	6612	7017	7277.8	6.13	0.14	6612	6612	0.00	-5.77	0.19	6612	6612	0.00	-5.77	2.05
test-30-7	7305	7871	8270.8	7.75	0.13	7323	7323	0.25	-6.96	0.18	7305	7305	0.00	-7.19	1.86
test-30-8	5907	6213	6483.3	5.18	0.26	5907	5907	0.00	-4.93	0.19	5907	5907	0.00	-4.93	1.78
test-30-9	5988	6386	6894.9	6.65	0.12	5988	5988	0.00	-6.23	0.17	5988	5988	0.00	-6.23	2
test-30-10	7154	7689	8049.8	7.48	0.22	7154	7154	0.00	-6.96	0.17	7154	7154	0.00	-6.96	1.94
test-30-11	7154	7689	8049.8	7.48	0.22	7154	7154	0.00	-6.96	0.17	7154	7154	0.00	-6.96	1.87
test-30-12	6173	6435	6726.6	4.24	0.3	6173	6173	0.00	-4.07	0.19	6173	6173	0.00	-4.07	1.81
test-30-13	6031	6885	7197.2	14.16	0.21	6031	6031	0.00	-12.40	0.18	6031	6031	0.00	-12.40	2.08
test-30-14	5852	6029	6463.2	3.02	0.19	5858	5858	0.10	-2.84	0.18	5852	5852	0.00	-2.94	1.99
test-30-15	6059	6878	7092.6	13.52	0.25	6059	6059	0.00	-11.91	0.18	6059	6059	0.00	-11.91	1.83
test-30-16	6669	7682	7966.2	15.19	0.27	6674	6674	0.07	-13.12	0.17	6669	6669	0.00	-13.19	1.91
test-30-17	6937	7275	7664.2	4.87	0.22	6937	6937	0.00	-4.65	0.17	6937	6937	0.00	-4.65	1.86
test-30-18	6862	7260	7892	5.80	0.18	6937	6937	1.09	-4.45	0.2	6862	6862	0.00	-5.48	1.79
test-30-19	7469	7790	8268.9	4.30	0.13	7469	7469	0.00	-4.12	0.19	7469	7469	0.00	-4.12	2
test-30-20	6240	7183	7310.3	15.11	0.23	6240	6240	0.00	-13.13	0.2	6240	6240	0.00	-13.13	2.09
Aver				6.86	0.23			0.08	-6.19	0.18			0.00	-6.27	1.95

TABLE VI. THE EXPERIMENTAL RESULTS FOR TEST-40-X WITH $\rho = 1$

instances	LB	VND			GVNS				A-GVNS			
		best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap₁*	*time*	*best.sol*	*aver.sol*	*Gap₁*	*time*
test-40-1	10651	-	-	-	-	-	-	-	11730	11730	10.13	8.44
test-40-2	9541	-	-	-	10758	10758	12.76	7.52	10758	10758	12.76	8.66
test-40-3	9366	-	-	-	-	-	-	-	12818	12818	36.86	8.06
test-40-4	10210	-	-	-	10858	10858	6.35	7.56	10858	10858	6.35	8.29
test-40-5	9378	-	-	-	9788	9788	4.37	7.92	9788	9788	4.37	8.19
test-40-6	9305	-	-	-	-	-	-	-	11336	11336	21.83	8.20
test-40-7	9241	-	-	-	9674	9674	4.69	8.37	9674	9674	4.69	8.84
test-40-8	9011	-	-	-	10343	10343	14.78	7.91	10343	10343	14.78	8.64
test-40-9	10412	-	-	-	10869	10869	4.39	7.96	10869	10869	4.39	8.67
test-40-10	9516	-	-	-	9606	9606	0.95	7.77	9606	9606	0.95	8.77
test-40-11	9516	-	-	-	9606	9606	0.95	7.50	9606	9606	0.95	8.80
test-40-12	9419	-	-	-	-	-	-	-	9456	9456	0.39	8.99
test-40-13	9995	-	-	-	10785	10785	7.90	7.63	10785	10785	7.90	8.16
test-40-14	9975	-	-	-	10348	10348	3.74	8.22	10348	10348	3.74	8.02
test-40-15	9485	-	-	-	9559	9559	0.78	7.91	9559	9559	0.78	8.65
test-40-16	10312	-	-	-	10877	10877	5.48	8.42	10873	10873	5.44	8.52
test-40-17	9844	-	-	-	-	-	-	-	12191	12191	23.84	8.60
test-40-18	9790	-	-	-	-	-	-	-	12404	12404	26.70	8.40
test-40-19	9229	-	-	-	-	-	-	-	9824	9824	6.45	8.75
test-40-20	8668	-	-	-	9280	9280	7.06	7.94	9280	9280	7.06	8.37
Aver							-				10.02	

TABLE VII. THE EXPERIMENTAL RESULTS FOR TEST-40-X WITH $\rho = 0.95$

instances	LB	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap₁*	*time*	*best.sol*	*aver.sol*	*Gap₁*	*Gap₂*	*time*	*best.sol*	*aver.sol*	*Gap₁*	*Gap₂*	*time*
test-40-1	10651	11322	11665.36	6.30	0.31	11474	11573.27	7.73	1.34	7.31	11177	11493.36	4.94	-1.28	8.47
test-40-2	9541	9763	10015.45	2.33	0.34	9728	9824.18	1.96	-0.36	7.59	9657	9777.73	1.22	-1.09	7.74
test-40-3	9366	9751	9824.64	4.11	0.33	10350	10584.36	10.51	6.14	7.42	10284	10496.55	9.80	5.47	7.92
test-40-4	10210	10990	11267.55	7.64	0.32	10984	11137.55	7.58	-0.05	7.61	10940	11073.64	7.15	-0.45	8.16
test-40-5	9378	10151	10378.91	8.24	0.31	10261	10347.91	9.42	1.08	7.97	10175	10299.09	8.50	0.24	8.61
test-40-6	9305	9746	9900.36	4.74	0.32	9729	9858.18	4.56	-0.17	7.95	9693	9788.18	4.17	-0.54	7.92
test-40-7	9241	10067	10266.27	8.94	0.33	9599	9679.64	3.87	-4.65	7.39	9533	9637.27	3.16	-5.30	8.12
test-40-8	9011	9776	10182.82	8.49	0.29	9346	9534.64	3.72	-4.40	8.28	9320	9469.27	3.43	-4.66	8.01
test-40-9	10412	11280	11451.09	8.34	0.3	10916	11367.64	4.84	-3.23	7.59	10825	11230	3.97	-4.03	8.04
test-40-10	9516	10976	11336.82	15.34	0.3	9924	10102.82	4.29	-9.58	7.8	9846	9998.36	3.47	-10.30	7.89
test-40-11	9516	10976	11336.82	15.34	0.34	9924	10102.82	4.29	-9.58	7.97	9846	9998.36	3.47	-10.30	8.41
test-40-12	9419	10526	11093.91	11.75	0.3	9678	10286.18	2.75	-8.06	7.83	9599	10087.36	1.91	-8.81	8.5
test-40-13	9995	10360	10830.27	3.65	0.32	10369	10866.82	3.74	0.09	7.59	10324	10704.18	3.29	-0.35	8.32
test-40-14	9975	11714	12020.45	17.43	0.29	10470	10640.27	4.96	-10.62	8.13	10443	10579.55	4.69	-10.85	7.76
test-40-15	9485	9877	10185.64	4.13	0.33	9986	10099.18	5.28	1.10	7.9	9817	10035.27	3.50	-0.61	8.36
test-40-16	10312	11355	11518.18	10.11	0.3	10537	10929.18	2.18	-7.20	7.53	10353	10789.18	0.40	-8.82	8.19
test-40-17	9844	10676	10991.91	8.45	0.32	10398	11102.45	5.63	-2.60	8.21	10241	10886.27	4.03	-4.07	8.15
test-40-18	9790	10274	10641.09	4.94	0.33	11113	11331.45	13.51	8.17	8.29	10824	11211.64	10.56	5.35	8.58
test-40-19	9229	9837	9999.45	6.59	0.32	9547	9989.73	3.45	-2.95	8.01	9500	9847.09	2.94	-3.43	8.32
test-40-20	8668	9680	9911.82	11.68	0.32	9384	9615.01	8.26	-3.06	7.55	9147	9510.36	5.53	-5.51	8.43
Aver				8.43	0.32			5.63	-2.43	7.80			4.51	-3.47	8.20

TABLE VIII. THE EXPERIMENTAL RESULTS FOR TEST-40-X WITH $\rho = 0.75$

instances	LB	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap₁*	*time*	*best.sol*	*aver.sol*	*Gap₁*	*Gap₂*	*time*	*best.sol*	*aver.sol*	*Gap₁*	*Gap₂*	*time*
test-40-1	10651	11135	11321.64	4.54	0.31	10653	10788.3	0.02	-4.33	7.31	10651	10751.4	0.00	-4.35	8.47
test-40-2	9541	10020	10239.09	5.02	0.34	9541	9618.82	0.00	-4.78	7.59	9541	9587.27	0.00	-4.78	7.74
test-40-3	9366	10602	11474.27	13.20	0.33	9366	9442.82	0.00	-11.66	7.42	9366	9389.09	0.00	-11.66	7.92
test-40-4	10210	10726	11076.73	5.05	0.32	10228	10350.7	0.18	-4.64	7.61	10210	10310.5	0.00	-4.81	8.16
test-40-5	9378	10734	11242.36	14.46	0.31	9410	9497.82	0.34	-12.33	7.97	9378	9455.09	0.00	-12.63	8.61
test-40-6	9305	11591	12135.45	24.57	0.32	9339	9412	0.37	-19.43	7.95	9339	9379.09	0.37	-19.43	7.92
test-40-7	9241	11348	12219.09	22.80	0.33	9281	9414.09	0.43	-18.21	7.39	9266	9367.82	0.27	-18.35	8.12
test-40-8	9011	14288	14621	58.56	0.29	9011	9163.18	0.00	-36.93	8.28	9011	9100	0.00	-36.93	8.01
test-40-9	10412	11780	12115.55	13.14	0.3	10440	10602.4	0.27	-11.38	7.59	10418	10546.5	0.06	-11.56	8.04
test-40-10	9516	10093	10882.82	6.06	0.3	9577	9618.64	0.64	-5.11	7.8	9516	9591.36	0.00	-5.72	7.89
test-40-11	9516	10093	10882.82	6.06	0.34	9577	9618.64	0.64	-5.11	7.97	9516	9591.36	0.00	-5.72	8.41
test-40-12	9419	11733	12735.91	24.57	0.3	9443	9565.55	0.25	-19.52	7.83	9440	9517.91	0.22	-19.54	8.5
test-40-13	9995	11522	11949.18	15.28	0.32	10025	10241.7	0.30	-12.99	7.59	9995	10162.6	0.00	-13.25	8.32
test-40-14	9975	15264	15755	53.02	0.29	9984	10114	0.09	-34.59	8.13	9975	10082.6	0.00	-34.65	7.76
test-40-15	9485	10178	10555.64	7.31	0.33	9485	9569.18	0.00	-6.81	7.9	9485	9542.55	0.00	-6.81	8.36
test-40-16	10312	11777	12135.55	14.21	0.3	10465	10569.4	1.48	-11.14	7.53	10441	10501.7	1.25	-11.34	8.19
test-40-17	9844	10705	11068.27	8.75	0.32	9852	9954.27	0.08	-7.97	8.21	9844	9911.55	0.00	-8.04	8.15
test-40-18	9790	13440	13697	37.28	0.33	9806	9847	0.16	-27.04	8.29	9790	9826.55	0.00	-27.16	8.58
test-40-19	9229	13897	14302.45	50.58	0.32	9362	9442.09	1.44	-32.63	8.01	9267	9402.36	0.41	-33.32	8.32
test-40-20	8668	10040	10607.45	15.83	0.32	8668	8736.45	0.00	-13.67	7.55	8668	8704.55	0.00	-13.67	8.43
Aver				20.01	0.32			0.34	-15.01	7.80			0.13	-15.19	8.20

TABLE IX. THE EXPERIMENTAL RESULTS FOR TEST-40-X WITH $\rho = 0.5$

instances	*LB*	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap*$_1$	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*Gap*$_2$	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*Gap*$_2$	*time*
test-40-1	10651	11143	11489.45	4.62	0.32	10683	10810	0.30	-4.13	7.93	10662	10756.2	0.10	-4.32	8.06
test-40-2	9541	9950	10074.64	4.29	0.3	9541	9633.27	0.00	-4.11	7.87	9541	9601.55	0.00	-4.11	7.6
test-40-3	9366	10143	10256.91	8.30	0.31	9454	9601.64	0.94	-6.79	7.95	9397	9534.82	0.33	-7.35	7.62
test-40-4	10210	10624	10907.09	4.05	0.32	10258	10324.3	0.47	-3.45	7.11	10210	10287.4	0.00	-3.90	8.16
test-40-5	9378	10503	10671.09	12.00	0.31	9430	9560.27	0.55	-10.22	7.38	9410	9511.36	0.34	-10.41	7.88
test-40-6	9305	9624	9827.36	3.43	0.31	9325	9471.73	0.21	-3.11	7.42	9305	9413	0.00	-3.31	7.51
test-40-7	9241	11198	11710.36	21.18	0.29	9288	9376	0.51	-17.06	7.5	9281	9342.55	0.43	-17.12	8.03
test-40-8	9011	10072	10303.18	11.77	0.29	9011	8971.73	0.00	-10.53	7.29	9011	8926.45	0.00	-10.53	7.48
test-40-9	10412	15036	15523.91	44.41	0.32	10431	10532.6	0.18	-30.63	7.33	10412	10500.1	0.00	-30.75	7.56
test-40-10	9516	11149	11603.64	17.16	0.31	9516	9598.73	0.00	-14.65	7.92	9516	9568	0.00	-14.65	8.06
test-40-11	9516	11149	11603.64	17.16	0.32	9516	9598.73	0.00	-14.65	8.09	9516	9568	0.00	-14.65	7.89
test-40-12	9419	10127	10548.36	7.52	0.31	9440	9526.27	0.22	-6.78	7.15	9419	9485.45	0.00	-6.99	8.09
test-40-13	9995	13433	14001.64	34.40	0.31	10000	10273.1	0.05	-25.56	8.09	9995	10185.7	0.00	-25.59	7.93
test-40-14	9975	10147	10339.36	1.72	0.3	9984	10060.8	0.09	-1.61	7.57	9975	10029.4	0.00	-1.70	7.61
test-40-15	9485	10692	11404.45	12.73	0.31	9575	9634.82	0.95	-10.45	7.9	9539	9607.82	0.57	-10.78	7.84
test-40-16	10312	14210	14443.73	37.80	0.3	10344	10542.3	0.31	-27.21	7.59	10312	10472.6	0.00	-27.43	7.41
test-40-17	9844	10866	11218.36	10.38	0.3	9907	10058.9	0.64	-8.83	8.06	9844	10000.5	0.00	-9.41	7.64
test-40-18	9790	11050	11544.55	12.87	0.31	9957	10039.6	1.71	-9.89	7.25	9845	9992	0.56	-10.90	7.91
test-40-19	9229	10569	10865.82	14.52	0.29	9291	9346.36	0.67	-12.09	7.79	9229	9316.45	0.00	-12.68	8.14
test-40-20	8668	9981	10138.64	15.15	0.31	8668	8785.55	0.00	-13.15	8.04	8668	8745.82	0.00	-13.15	7.53
Aver				14.77	0.31			0.39	-11.74	7.66			0.12	-11.99	7.80

TABLE X. THE EXPERIMENTAL RESULTS FOR TEST-40-X WITH $\rho = 0$

instances	*LB*	VND				GVNS					A-GVNS				
		best.sol	*aver.sol*	*Gap*$_1$	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*Gap*$_2$	*time*	*best.sol*	*aver.sol*	*Gap*$_1$	*Gap*$_2$	*time*
test-40-1	10651	11085	11196.1	4.07	0.3	10651	10702.3	0.00	-3.92	7.93	10651	10651	0.00	-3.92	7.87
test-40-2	9541	10367	10519.9	8.66	0.31	9541	9593.8	0.00	-7.97	7.08	9541	9541	0.00	-7.97	8.02
test-40-3	9366	9925	10118.7	5.97	0.29	9366	9405.1	0.00	-5.63	7.79	9366	9366	0.00	-5.63	7.68
test-40-4	10210	11851	13162.3	16.07	0.31	10210	10278.6	0.00	-13.85	7.19	10210	10210	0.00	-13.85	7.71
test-40-5	9378	9612	10137.6	2.50	0.3	9378	9467.5	0.00	-2.43	7.33	9378	9378	0.00	-2.43	7.21
test-40-6	9305	9609	9774	3.27	0.31	9305	9338.9	0.00	-3.16	7.32	9305	9305	0.00	-3.16	7.34
test-40-7	9241	11010	13378.8	19.14	0.31	9241	9282.6	0.00	-16.07	6.87	9241	9241	0.00	-16.07	7.83
test-40-8	9011	9454	10026.1	4.92	0.3	9011	9039.5	0.00	-4.69	7.77	9011	9011	0.00	-4.69	7.14
test-40-9	10412	11239	11509.1	7.94	0.3	10412	10457.2	0.00	-7.36	7.33	10412	10412	0.00	-7.36	7.41
test-40-10	9516	12434	15975.6	30.66	0.31	9516	9573.6	0.00	-23.47	6.99	9516	9516	0.00	-23.47	7.55
test-40-11	9516	12434	15975.6	30.66	0.31	9516	9573.6	0.00	-23.47	6.85	9516	9516	0.00	-23.47	7.38
test-40-12	9419	10572	11247.5	12.24	0.3	9419	9445.8	0.00	-10.91	7.5	9419	9419	0.00	-10.91	7.16
test-40-13	9995	11163	12082.4	11.69	0.3	9995	10036.5	0.00	-10.46	7.03	9995	9995	0.00	-10.46	7.55
test-40-14	9975	10321	10931.6	3.47	0.31	9975	9996.6	0.00	-3.35	7.06	9975	9975	0.00	-3.35	7.4
test-40-15	9485	9748	10076.3	2.77	0.29	9485	9540.7	0.00	-2.70	7.17	9485	9485	0.00	-2.70	8.00
test-40-16	10312	11058	11907	7.23	0.31	10312	10413.5	0.00	-6.75	7.18	10312	10312	0.00	-6.75	7.66
test-40-17	9844	12191	13205.3	23.84	0.29	9844	9913.7	0.00	-19.25	7.63	9844	9844	0.00	-19.25	7.52
test-40-18	9790	10913	11290.7	11.47	0.31	9790	9838.8	0.00	-10.29	7.24	9790	9790	0.00	-10.29	7.12
test-40-19	9229	10327	10734.1	11.90	0.29	9229	9322.3	0.00	-10.63	6.83	9229	9229	0.00	-10.63	7.44
test-40-20	8668	8731	9343.4	0.73	0.31	8668	8722.8	0.00	-0.72	6.91	8668	8668	0.00	-0.72	7.81
Aver				10.96	0.30			0.00	-9.35	7.37			0.00	-9.35	7.54

LB: Since $\rho = 0$, the Q-TRP becomes to the TRP. The optimal solution of the TRP found by using the exact algorithm in [2] are the lower bound of the optimal solution of the Q-TRP.

TABLE XI. THE EXPERIMENTAL RESULTS FOR TEST-50-X WITH $\rho = 1$

instances	VND			GVNS			A-GVNS		
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Time*
test-50-1	-	-	-	-	-	-	14158	14158	28.6
test-50-2	-	-	-	14633	14633	26.3	14633	14633	28.7
test-50-3	-	-	-	-	-	-	14632	14632	27.8
test-50-4	-	-	-	-	-	-	16600	16600	27.6
test-50-5	-	-	-	-	-	-	16113	16113	27.0
test-50-6	-	-	-	-	-	-	15897	15897	27.2
test-50-7	-	-	-	15572	15572	24.9	14907	14907	27.3
test-50-8	-	-	-	-	-	-	16796	16796	26.0
test-50-9	-	-	-	15354	15354	25.4	15354	15354	27.2
test-50-10	-	-	-	-	-	-	16861	16861	28.0
test-50-11	-	-	-	-	-	-	14583	14583	28.1
test-50-12	-	-	-	-	-	-	15612	15612	26.4
test-50-13	-	-	-	-	-	-	14249	14249	26.3
test-50-14	-	-	-	-	-	-	15731	15731	26.0
test-50-15	-	-	-	-	-	-	16884	16884	27.1
test-50-16	-	-	-	15827	15827	24.9	13874	13874	27.7
test-50-17	-	-	-	14457	14457	26.4	13155	13155	28.1
test-50-18	-	-	-	14459	14459	24.9	13182	13182	26.0
test-50-19	-	-	-	15662	15662	24.4	15662	15662	28.9
test-50-20	-	-	-	-	-	-	14407	14407	27.6

TABLE XII. THE EXPERIMENTAL RESULTS FOR TEST-50-X WITH $\rho = 0.95$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*
test-50-1	15097	15660.45	1.92	13524	13790	-10.42	23.9	13256	13673.27	-12.19	24.7
test-50-2	17535	17936.36	2.13	12426	12491.73	-29.14	23.3	12305	12452.18	-29.83	24.1
test-50-3	14109	14708.82	2.32	13497	13636.55	-4.34	23.3	13352	13568	-5.37	25.2
test-50-4	17491	17921.64	2.51	14422	14727	-17.55	24.6	14325	14590.09	-18.10	23.3
test-50-5	14419	14531.09	2.4	14348	14871.36	-0.49	24	14194	14663.45	-1.56	24.2
test-50-6	15166	15590.09	2.41	15058	15433.09	-0.71	23.1	14784	15303.82	-2.52	24.4
test-50-7	15192	15469.55	2.02	15418	15825.55	1.49	23.3	15249	15686.18	0.38	23.8
test-50-8	14988	15801.27	2.55	14898	15102.91	-0.60	22.1	14783	15015.09	-1.37	24.2
test-50-9	17273	18060.91	2.5	14977	15218.45	-13.29	23.7	14956	15093.73	-13.41	24.1
test-50-10	14518	15021.91	2.03	13718	13989.09	-5.51	23.8	13625	13885.91	-6.15	24.5
test-50-11	14518	15021.91	2.46	13718	13989.09	-5.51	24	13625	13885.91	-6.15	25
test-50-12	15687	16114.45	2.22	14476	14932.73	-7.72	23.9	14441	14796.73	-7.94	25.3
test-50-13	13079	13326.91	2.52	12496	12716.73	-4.46	22.7	12452	12636.09	-4.79	24.7
test-50-14	14205	15124.73	2.17	13621	13716.91	-4.11	22.3	13515	13653.27	-4.86	24.9
test-50-15	13610	14132.73	2.09	14070	14323.55	3.38	22.2	13915	14204	2.24	25.7
test-50-16	16585	16970.73	2.57	14127	15262.27	-14.82	23.3	13940	14896.18	-15.95	23
test-50-17	13643	13933.91	2.6	13138	13361.82	-3.70	24.2	13095	13284.64	-4.02	23.7
test-50-18	14329	14582.55	2.26	13748	14026.09	-4.05	23.9	13553	13915.73	-5.42	25.8
test-50-19	14959	15452.36	2.06	14781	14992.91	-1.19	23.5	14721	14910.45	-1.59	25.4
test-50-20	15046	15258.73	2.34	14527	14623.82	-3.45	22.3	14370	14569.09	-4.49	24.7
Aver			2.30			-6.31	7.37			-7.15	24.54

TABLE XIII. THE EXPERIMENTAL RESULTS FOR TEST-50-X WITH $\rho = 0.75$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*
test-50-1	13841	14083.18	2.47	12801	13013.45	-7.51	23.1	12695	12921.73	-8.28	27.9
test-50-2	12783	13119.09	2.27	11953	12071.82	-6.49	25.7	11925	12029.73	-6.71	27.9
test-50-3	14700	15256.09	2.6	12682	12763.18	-13.73	23.8	12630	12716.36	-14.08	26.3
test-50-4	15920	16322.91	2.26	14004	14089.36	-12.04	24.1	13979	14055.82	-12.19	27
test-50-5	19703	19915.27	2.12	13909	14042.64	-29.41	24.7	13881	13990	-29.55	26.5
test-50-6	14804	15217	2.07	13974	14056.36	-5.61	24.4	13885	14009.64	-6.21	25.9
test-50-7	19768	21799	2.39	13739	13900.45	-30.50	25.5	13712	13834.55	-30.64	25.9
test-50-8	14973	15601.73	2.1	14291	14354.09	-4.55	25.1	14290	14329.36	-4.56	25.9
test-50-9	15242	15390.27	2.63	14754	14864.64	-3.20	25.1	14716	14822.55	-3.45	25
test-50-10	13829	14166.18	2.33	13381	13472.18	-3.24	24.1	13354	13436.82	-3.43	26.6
test-50-11	13829	14166.18	2.21	13381	13472.18	-3.24	24.6	13354	13436.82	-3.43	25.9
test-50-12	18512	18871.36	2.25	13755	13861.82	-25.70	24.9	13730	13815.27	-25.83	26.5
test-50-13	18263	18502.82	2.52	12308	12469.09	-32.61	24.7	12306	12420.82	-32.62	25.9
test-50-14	16590	16820.18	2.68	13131	13270.45	-20.85	23.5	13078	13228.82	-21.17	25.6
test-50-15	19048	19585.82	2.23	13127	13227.55	-31.08	25.9	13101	13187.18	-31.22	27.7
test-50-16	22257	22602.36	2.02	13667	13704.82	-38.59	25.7	13651	13688.18	-38.67	27.5
test-50-17	13087	13283.82	2.69	12847	12985.27	-1.83	24.2	12842	12925.45	-1.87	25
test-50-18	21169	21831	2.48	12698	12823.91	-40.02	25.3	12684	12765.27	-40.08	27.7
test-50-19	16734	17059	2.34	14443	14646.18	-13.69	25.4	14404	14558.18	-13.92	27.1
test-50-20	15016	15460.55	2.42	13623	13740.91	-9.28	24.2	13606	13695	-9.39	26.7
Aver			2.35			-16.66	24.7			-16.87	26.53

TABLE XIV. THE EXPERIMENTAL RESULTS FOR TEST-50-X WITH $\rho = 0.5$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*
test-50-1	14306	14751.09	2.47	12869	13002.73	-10.04	23.1	12746	12924.45	-10.90	27.9
test-50-2	12581	13114.73	2.27	11939	12143.55	-5.10	25.7	11925	12060.91	-5.21	27.9
test-50-3	18448	18812.55	2.6	12753	12958.73	-30.87	23.8	12610	12887.45	-31.65	26.3
test-50-4	22125	22448.64	2.26	13993	14056.36	-36.75	24.1	13949	14025.45	-36.95	27
test-50-5	15066	15412.64	2.12	14052	14189.27	-6.73	24.7	14037	14141.64	-6.83	26.5
test-50-6	15900	16725	2.07	13918	14009	-12.47	24.4	13905	13972.09	-12.55	25.9
test-50-7	26742	27488.27	2.39	13739	13821	-48.62	25.5	13739	13795.82	-48.62	25.9
test-50-8	14801	15540.91	2.1	14292	14433.27	-3.44	25.1	14290	14379.18	-3.45	25.9
test-50-9	23731	24268.18	2.63	14587	14729.27	-38.53	25.1	14479	14669.36	-38.99	25
test-50-10	13982	14402.82	2.33	13381	13493	-4.30	24.1	13354	13447.91	-4.49	26.6
test-50-11	13982	14402.82	2.21	13381	13493	-4.30	24.6	13354	13447.91	-4.49	25.9
test-50-12	15235	15639.82	2.25	13793	13992.91	-9.47	24.9	13792	13932.73	-9.47	26.5
test-50-13	18515	19144.82	2.52	12308	12433.18	-33.52	24.7	12306	12389.73	-33.53	25.9
test-50-14	16811	16983.73	2.68	13068	13164.55	-22.27	23.5	13065	13124.18	-22.28	25.6
test-50-15	14097	14781.82	2.23	13209	13274.73	-6.30	25.9	13155	13245.45	-6.68	27.7
test-50-16	19852	20921.55	2.02	13667	13748.09	-31.16	25.7	13651	13719.91	-31.24	27.5
test-50-17	15464	16994.64	2.69	12881	13003.09	-16.70	24.2	12872	12929.45	-16.76	25
test-50-18	14661	14864.55	2.48	12684	12795.82	-13.48	25.3	12680	12757.45	-13.51	27.7
test-50-19	18940	19361.91	2.34	14145	14259.27	-25.32	25.4	14124	14205.91	-25.43	27.1
test-50-20	15138	15735.55	2.42	13743	13875.45	-9.22	24.2	13606	13807.64	-10.12	26.7
Aver			2.35			-18.43	24.7			-18.66	26.53

TABLE XV. THE EXPERIMENTAL RESULTS FOR TEST-50-X WITH $\rho = 0$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*	*best.sol*	*aver.sol*	*Gap*$_2$	*time*
test-50-1	14049	15118.1	1.82	12695	12879.27	-9.64	22.3	12695	12801.2	-9.64	24.7
test-50-2	15881	18102.7	2.36	11925	12007.09	-24.91	21.4	11925	11966.2	-24.91	24.1
test-50-3	14976	17840.7	1.95	12740	12808.27	-14.93	21.6	12686	12762.5	-15.29	25.2
test-50-4	17086	18990.3	2.15	13973	14088.45	-18.22	21.6	13961	14028.1	-18.29	23.3
test-50-5	14097	14510.9	1.81	13957	14099.18	-0.99	23.5	13924	14014.4	-1.23	24.2
test-50-6	14683	14915.7	1.98	13858	14014.27	-5.62	21.2	13841	13959.2	-5.73	24.4
test-50-7	17594	21650.4	2.11	13798	13914.18	-21.58	23.5	13739	13840.5	-21.91	23.8
test-50-8	14850	15151.5	2.5	14290	14356.55	-3.77	23.2	14290	14316.7	-3.77	24.2
test-50-9	17706	21430	2.39	14822	14919.18	-16.29	23.3	14762	14853.3	-16.63	24.1
test-50-10	13844	14141.3	1.97	13381	13544.91	-3.34	22.5	13354	13443.1	-3.54	24.5
test-50-11	13844	14141.3	1.93	13381	13544.91	-3.34	21.2	13354	13443.1	-3.54	25
test-50-12	16831	18960.4	2.23	13729	13821.18	-18.43	22	13721	13776.5	-18.48	25.3
test-50-13	13392	13789	2.47	12306	12411.73	-8.11	23.4	12306	12364.1	-8.11	24.7
test-50-14	14580	15316.8	2.35	13065	13104	-10.39	21.4	13065	13080.3	-10.39	24.9
test-50-15	13919	14105.4	2.33	13371	13453.73	-3.94	21.8	13332	13402.7	-4.22	25.7
test-50-16	15056	16270.8	2.41	13651	13734.55	-9.33	23.3	13651	13700.8	-9.33	23
test-50-17	13658	14176.7	2.4	12990	13051.27	-4.89	21.3	12842	12989.4	-5.97	23.7
test-50-18	12967	13232.2	2.25	12688	12778.09	-2.15	21.3	12688	12728.2	-2.15	25.8
test-50-19	15102	15399.8	2.07	14291	14413	-5.37	21.3	14260	14356.4	-5.58	25.4
test-50-20	14299	15254.2	2.26	13766	13869.91	-3.73	21.9	13743	13822.7	-3.89	24.7
Aver			2.19			-9.45	22.15			-9.63	24.54

TABLE XVI. THE EXPERIMENTAL RESULTS FOR TEST-100-X WITH $\rho = 1$

instances	VND			GVNS			A-GVNS		
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Time*
test-100-1	-	-	-	-	-	-	-	-	345.1
test-100-2	-	-	-	-	-	-	39767	39767	346.7
test-100-3	-	-	-	-	-	-	37818	37818	347.8
test-100-4	-	-	-	40216	40216	341.2	36414	36414	346.3
test-100-5	-	-	-	-	-	-	39177	39177	349.1
test-100-6	-	-	-	-	-	-	42625	42625	347.5
test-100-7	-	-	-	44074	44074	342.2	37165	37165	348.4
test-100-8	-	-	-	-	-	-	41231	41231	345.2
test-100-9	-	-	-	-	-	-	40136	40136	347.3
test-100-10	-	-	-	-	-	-	-	-	348.4
test-100-11	-	-	-	-	-	-	43907	43907	348.4
test-100-12	-	-	-	40920	40920	341.3	36345	36345	347.9
test-100-13	-	-	-	44416	44416	340.6	39096	39096	346.1
test-100-14	-	-	-	-	-	-	39892	39892	349.5
test-100-15	-	-	-	-	-	-	-	-	349.6
test-100-16	-	-	-	-	-	344.2	39742	39742	349.8
test-100-17	-	-	-	-	-	342.2	43267	43267	348.7
test-100-18	-	-	-	-	-	340.1	41546	41546	348.4
test-100-19	-	-	-	-	-	341.6	39192	39192	350.0
test-100-20	-	-	-	-	-	-	-	-	346.2

TABLE XVII. THE EXPERIMENTAL RESULTS FOR TEST-100-X WITH $\rho = 0.95$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap_2*	*time*	*best.sol*	*aver.sol*	*Gap_2*	*time*
test-100-1	39552	39845	7.53	37717	38112.36	-4.64	339.2	37532	37927.09	-5.11	345.4
test-100-2	52136	52386.91	6.98	38225	38770	-26.68	341.2	38008	38580.64	-27.10	342.5
test-100-3	51184	51799.36	7.2	38428	38747.45	-24.92	340.1	38362	38632.64	-25.05	342.6
test-100-4	63715	63986.64	7.75	38372	38860.45	-39.78	340.3	38178	38615.91	-40.08	347.8
test-100-5	43053	43476.64	6.95	38571	38815.27	-10.41	339	38525	38712.09	-10.52	344.4
test-100-6	50311	51950.18	7.35	34873	35530.64	-30.69	342.4	34626	35194.82	-31.18	343.1
test-100-7	39333	39792.36	7.29	36698	37335.18	-6.70	338.1	36368	37041.82	-7.54	345.2
test-100-8	51052	51674	7.53	39335	40114.91	-22.95	342.3	39219	39872.09	-23.18	345.3
test-100-9	39003	39445.45	7.2	36018	36508	-7.65	340.6	35990	36335.55	-7.73	342.2
test-100-10	49218	49567.55	7.78	37574	37974.36	-23.66	338.5	37492	37811.55	-23.82	345.6
test-100-11	49218	49567.55	7.72	37574	37974.36	-23.66	341.4	37492	37811.55	-23.82	343.9
test-100-12	50654	51481.55	7.5	37364	37574.91	-26.24	339.1	36887	37466.45	-27.18	343.5
test-100-13	41845	43332.55	7.26	39213	39589.27	-6.29	341.7	39020	39383.73	-6.75	347
test-100-14	57861	58539.27	7.09	40433	40929.73	-30.12	340.2	40329	40725.73	-30.30	347
test-100-15	53683	54339	7.38	37200	37472.36	-30.70	339.3	37137	37346.55	-30.82	342.2
test-100-16	51737	52904.73	7	39283	39627	-24.07	339.3	39134	39502.36	-24.36	345.9
test-100-17	54215	54825.82	7.41	37473	38204.27	-30.88	341.5	36987	37820.27	-31.78	342.8
test-100-18	39935	40116.91	7.52	36157	36522.73	-9.46	338.6	36107	36391.45	-9.59	343.5
test-100-19	53557	53704.36	7.78	37558	37915.82	-29.87	339.8	37386	37680.09	-30.19	342.5
test-100-20	54155	55492.36	6.96	36810	37000.55	-32.03	338.6	36534	36903.36	-32.54	344.6
Aver			7.36			-22.07	340.06			-22.43	344.35

TABLE XVIII. THE EXPERIMENTAL RESULTS FOR TEST-100-X WITH $\rho = 0.75$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap_2*	*time*	*best.sol*	*aver.sol*	*Gap_2*	*time*
test-100-1	39782	40086.18	7.53	35701	35835.73	-10.26	339.2	35696	35778.18	-10.27	345.4
test-100-2	40850	41751	6.98	36254	36380.64	-11.25	341.2	36178	36326.64	-11.44	342.5
test-100-3	49744	50145.18	7.2	36966	37039.36	-25.69	340.1	36168	36921.45	-27.29	342.6
test-100-4	37826	38207.36	7.75	36481	36806.55	-3.56	340.3	36466	36713	-3.60	347.8
test-100-5	55970	57384.36	6.95	37052	37185.73	-33.80	339	36930	37119.64	-34.02	344.4
test-100-6	66094	67482.09	7.35	34806	34950.82	-47.34	342.4	34677	34900.18	-47.53	343.1
test-100-7	51980	52418	7.29	35900	36041.09	-30.93	338.1	35711	35954.91	-31.30	345.2
test-100-8	47856	48351.45	7.53	36787	36942.82	-23.13	342.3	36645	36869	-23.43	345.3
test-100-9	56256	57560	7.2	34731	34890.91	-38.26	340.6	34713	34827.64	-38.29	342.2
test-100-10	39514	39720.09	7.78	36031	36235.36	-8.81	338.5	35849	36143.27	-9.28	345.6
test-100-11	39514	39720.09	7.72	36031	36235.36	-8.81	341.4	35849	36143.27	-9.28	343.9
test-100-12	36985	37368.64	7.5	35015	35428.82	-5.33	339.1	34983	35313.36	-5.41	343.5
test-100-13	40200	40584.73	7.26	37546	37609.82	-6.60	341.7	37422	37570.64	-6.91	347
test-100-14	40118	40654.27	7.09	37626	37681.64	-6.21	340.2	37588	37657.91	-6.31	347
test-100-15	39845	40294.36	7.38	36564	36705.09	-8.23	339.3	36400	36642	-8.65	342.2
test-100-16	46970	47708.45	7	35927	36200.73	-23.51	339.3	35866	36104.91	-23.64	345.9
test-100-17	60588	60978.18	7.41	36142	36294.55	-40.35	341.5	36114	36239.18	-40.39	342.8
test-100-18	38746	39396.64	7.52	35692	35763.64	-7.88	338.6	35647	35732.45	-8.00	343.5
test-100-19	52204	52881.45	7.78	36304	36678.27	-30.46	339.8	36158	36544.64	-30.74	342.5
test-100-20	38485	38936.91	6.96	35364	35583.91	-8.11	338.6	35351	35508.09	-8.14	344.6
Aver			7.36			-18.93	340.06			-19.20	344.35

TABLE XIX. THE EXPERIMENTAL RESULTS FOR TEST-100-X WITH $\rho = 0.5$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap_2*	*time*	*best.sol*	*aver.sol*	*Gap_2*	*time*
test-100-1	58004	58231.27	7.09	36013	36341.27	-37.91	336	35915	36216.82	-38.08	340.8
test-100-2	38230	39685.36	7.56	36423	36873.91	-4.73	337	36279	36727.18	-5.10	341.2
test-100-3	43252	44277.82	7.29	36307	36463.36	-16.06	339.4	36292	36399.27	-16.09	340.6
test-100-4	38100	38574.55	7.38	35931	36153.18	-5.69	336.4	35882	36075.27	-5.82	342.5
test-100-5	38561	39019.45	6.85	36999	37210.91	-4.05	336.1	36988	37147.36	-4.08	345.4
test-100-6	64673	65026.91	7.27	34494	34570.27	-46.66	339.7	34428	34534.36	-46.77	345.7
test-100-7	38083	38466.27	6.8	35987	36059.82	-5.50	338.3	35946	36022.91	-5.61	344.8
test-100-8	58805	59305.45	7.57	36448	36658.09	-38.02	336.8	36322	36575.45	-38.23	344.8
test-100-9	53032	53568.27	6.98	34716	34806.27	-34.54	339.1	34704	34771.27	-34.56	344.2
test-100-10	58146	58615.55	7.45	35942	36054.55	-38.19	338.2	35934	36019.18	-38.20	343.7
test-100-11	58146	58615.55	6.6	35942	36054.55	-38.19	340.7	35934	36019.18	-38.20	344.5
test-100-12	49400	50094.55	7.53	35354	35540.09	-28.43	336.4	35235	35450.36	-28.67	342.4
test-100-13	62370	62851.27	7.37	37268	37371.18	-40.25	338.6	37075	37284.73	-40.56	344.2
test-100-14	41140	42124.82	7.12	37325	37543.27	-9.27	336.1	37088	37457.45	-9.85	340.9
test-100-15	55569	57123.09	7.23	36220	36407.36	-34.82	339.9	36072	36333.45	-35.09	343.3
test-100-16	42710	43159.55	6.98	36000	36169.01	-15.71	339.8	35945	36103.09	-15.84	345.5
test-100-17	42307	43157.18	6.99	36372	36436.11	-14.03	339.4	36287	36393.18	-14.23	340.5
test-100-18	51547	52389.09	6.71	35742	35879.45	-30.66	338.9	35718	35828.45	-30.71	341.5
test-100-19	49862	50832.36	7.57	36196	36382.64	-27.41	339.5	36084	36296.73	-27.63	345.8
test-100-20	55788	56354.27	6.66	35434	35627.14	-36.48	337.5	35299	35541.09	-36.73	340.8
Aver			7.15			-25.33	338.19			-25.50	343.16

TABLE XX. THE EXPERIMENTAL RESULTS FOR TEST-100-X WITH $\rho = 0$

instances	VND			GVNS				A-GVNS			
	best.sol	*aver.sol*	*time*	*best.sol*	*aver.sol*	*Gap₂*	*time*	*best.sol*	*aver.sol*	*Gap₂*	*time*
test-100-1	40252	41601.7	7.11	36082	36239.18	-10.36	337.6	36055	36181.7	-10.43	341.5
test-100-2	46017	55796.8	7.28	36536	36635.27	-20.60	337.7	36497	36561.9	-20.69	340.4
test-100-3	47134	66605.8	6.89	36027	36198.55	-23.56	335.3	35990	36097.9	-23.64	342.5
test-100-4	43528	48294.5	6.62	36221	36597.45	-16.79	337.1	36005	36409.2	-17.28	338.8
test-100-5	46202	49304	6.59	37033	37110.82	-19.85	338.3	37030	37057.9	-19.85	343.1
test-100-6	45522	63332.4	6.42	34866	35041.09	-23.41	334.9	34858	34967.6	-23.43	342.6
test-100-7	39590	39915.6	6.8	35618	35751.09	-10.03	339.2	35608	35695.4	-10.06	343.4
test-100-8	45330	59064.5	6.98	37013	37219.36	-18.35	338.4	36996	37119.5	-18.39	339.4
test-100-9	42800	57751.5	6.54	34712	35015.55	-18.90	339.5	34649	34867.4	-19.04	343.5
test-100-10	46831	55515.6	6.71	35889	36046.09	-23.36	336.4	35825	35952.3	-23.50	338.9
test-100-11	46831	55515.6	6.58	35889	36046.09	-23.36	337.6	35825	35952.3	-23.50	343.9
test-100-12	36743	36942.5	6.77	34987	35247.36	-4.78	336.2	34947	35130.2	-4.89	339.1
test-100-13	37999	38647.8	6.71	36978	37245.73	-2.69	338.3	36942	37129.6	-2.78	342.1
test-100-14	39466	40796	6.6	36819	37283.18	-6.71	334.2	36774	37100.3	-6.82	338.9
test-100-15	42777	52823.5	7.29	36138	36353.45	-15.52	337.3	36111	36262.2	-15.58	341.7
test-100-16	38495	39097.6	7.01	35577	35714.64	-7.58	339.7	35532	35629.8	-7.70	338.8
test-100-17	42166	57323.8	6.51	36337	36479.45	-13.82	336.5	36333	36421	-13.83	340.6
test-100-18	39192	40863.8	6.95	35484	35647.36	-9.46	338.9	35372	35537	-9.75	343
test-100-19	39607	40387.8	6.53	36031	36245.64	-9.03	337.3	35901	36149	-9.36	340.6
test-100-20	38396	39399.2	7.15	35535	35669.91	-7.45	338.8	35534	35615.9	-7.45	340.3
Aver			6.80			-14.28	337.46			-14.40	341.16

TABLE XXI. $The Average Gap_2$

instances	ρ=0		ρ=0.5		ρ=0.75		ρ=0.95		ρ=1	
	GVNS	A-GVNS	GVNS	A-GVNS	GVNS	A-GVNS	GVNS	A-GVNS	GVNS	A-GVNS
30	-6.19	-6.27	-9.59	-9.96	-9.90	-10.09	-5.06	-6.38	-	-
40	-9.35	-9.35	-11.74	-11.99	-15.01	-15.19	-2.43	-3.47	-	-
50	-9.45	-9.63	-18.43	-18.66	-16.66	-16.87	-6.31	-7.15	-	-
100	-14.28	-14.40	-25.33	-25.50	-18.93	-19.20	-22.07	-22.43	-	-

Since ρ=1, there is no the *Gap₂* value because the VNS does not provide any feasible sol utions in these cases.

TABLE XXII. THE EXPERIMENTAL RESULTS FOR A. SALEHIPOUR ET AL.'S DATASET

Instances	GRASP–VNS		ILS		A-GVNS	
	gap₁]%[	*T*	*gap₁*]%[	*cTime*	*gap₁*]%[	*cTime*
10	33.04	0.00	33.04	0.00	**33.04**	0.00
20	40.34	0.04	39.29	0.05	**39.29**	0.18
50	47.20	3.54	43.97	1.36	**43.97**	9.38
100	44.28	103.92	40.82	18.94	**40.82**	25.82
200	38.77	3995.0	38.14	178.72	**38.14**	198.2

TABLE XXIII. THE EXPERIMENTAL RESULTS FOR THE INSTANCES IN TSPLIB

Instances	*OPT*	*best.sol*	*aver.sol*	*T*
dantzig42	12528	**12528**	12528	5.16
att48	209320	**209320**	209320	12.4
eil51	10178	**10178**	10178	17.7
berlin52	143721	**143721**	143721	17.2
st70	20557	**20557**	20557	41.2
KroA100	983128	**983128**	983128	67.5
KroB100	983128	**983128**	983128	64.4
KroC100	961324	**961324**	961324	64.4
KroD100	976965	**976965**	976965	62.3