Efficiency and Performance of Optimized Robust Controllers in Hydraulic System

Chong Chee Soon1, Rozaimi Ghazali2, Shin Horng Chong3, Chai Mau Shern4, Yahaya Md. Sam5, Zulfatman Has6
Centre for Robotics and Industrial Automation, Faculty of Electrical Engineering, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia1,2,3,4
Department of Control and Mechatronics Engineering, School of Electrical Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia2
Electrical Engineering Department, University of Muhammadiyah Malang, 65144 Malang, Indonesia6

Abstract—Common applications involved hydraulic system in real-time including heavy machinery, air-craft system, and transportation. Real-time applications, however, are notorious to suffer with nonlinearities due to unavoidable mechanical structures. Thus, control system engaged to manipulate and minimize the effect resulting from the nonlinearities. In this paper, few control approaches are executed in the Electro-Hydraulic Actuator (EHA) system. The control approaches including the generally used Proportional-Integral-Derivative (PID) controller, the reinforced Fractional-Order (FO) PID controller, the Sliding Mode Controller (SMC), and the enhanced hybrid SMC-PID controller. In order to obtain proper parameter of each controller, the Particle Swarm Optimization (PSO) technique is applied. The output data are then analysed based on the performance indices in terms of the consumption of the energy and the error produced. The performance indices including Root Mean Square Error/Voltage (RMSE/V), Integral Square Error/Voltage (ISE/V), Integral Time Square Error/Voltage (ITSE/V), Integral Absolute Error/Voltage (IAE/V), and Integral Time Absolute Error/Voltage (ITAE/V). It is observed in the results, based on the performance indices in terms of error and voltage, the hybrid SMC-PID capable of generating better outcomes with reference to tracking capabilities and energy usage.

Keywords—Robust Control Design; optimization; tracking efficiency analysis; controller effort analysis; electro-hydraulic actuator system

I. INTRODUCTION

The behavior of a system in industrial sector is highly connected with the control system that manage and manipulated the end function of particular machineries. Practical mechanical machineries such as Electro-Hydraulic Actuator (EHA) system, however, commonly undergoes the well-known uncertainties and nonlinearities characteristics [1]. These characteristics generating direct impact to the machine over time and increase the complexities in the controller design. Improper design of the controller system may lead to the common problem such as imprecise and energy efficiency.

In hydraulic, a mathematical model that represent the physical behaviour of this system is difficult to be modelled due to the uncertainties and nonlinearities characteristics [2]. Usually, the compressibility of the oil due to the working temperature, the internal and external friction of the cylinder, the fluid flow characteristic in valve and cylinder are the trait of the nonlinearities. While two primary form of uncertainties are discovered in the past including parametric uncertainties and uncertain nonlinearities [3]. Therefore, control system raised to dealing with these issues.

Over decades, it is famed in the control and industry fields about the Proportional-Integral-Derivative (PID) controller implementation in the EHA system [4-5]. This controller became a topic of interest by researchers and academia on the basis of practical and user-friendly advantage. For years, varieties of approaches have been inserted to this controller. One of the typical approaches that often seen is the modification in terms of the controller’s structure. For example, the fractional order control and the gain scheduling control that have been generally integrated with the PID controller [6-7]. These methods are proven to have more efficient performance compared with the traditional PID controller.

Adaptability and robustness are the major concern by control engineers. When addressing these features, the outstanding Sliding Mode Control (SMC) approach performed significant achievement applied in different machineries [8-9]. Unlike the PID controller, SMC is identified to commonly gain its parameter by means of try and error process [10-12]. Therefore, computer based tuning approaches including Particle Swarm Optimization (PSO), Gravitational Search Algorithm (GSA), and Genetic Algorithm (GA) have been gradually attract the attention of the control field due to their prominent performance in searching for the controller’s optimal parameter [13-15].

It is noticed in the literature, EHA system is mostly executed in positioning control. The accurate positioning control are usually required in the applications such as vehicle [16], robotic [17], construction machinery [18], and aerodynamic [19]. Owing to this matter, this paper aims to develop several control approaches for the purpose of comparison in the manner of positioning performance. Additionally, further analyses in terms of precision and energy usage based on the performance indices, including Root Mean Square Error/Voltage (RMSE/V), Integral Square Error/Voltage (ISE/V), Integral Time Square Error/Voltage (ITSE/V), Integral Absolute Error/Voltage (IAE/V), and
Integral Time Absolute Error/Voltage (ITAE/V) are carried out.

This paper is organized as, modelling and control methods are briefly discussed in Section 2. The performance of each controller is presented in Section 3. Lastly, the summary of the outcome is drawn in Section 4.

II. MATHEMATICAL MODELLING

This study utilizes the valve operated transmission system instead of pump operated transmission system due to the efficiency [20]. Refer to Fig. 1, the general structure of the EHA system composed of valve and actuator units, sensing unit, and computer control unit.

![Fig 1. General Component in the EHA System.](image)

The motor responsible to drive the spool in servo-valve. The voltage, \( V_v \), that drive the current, \( I_v \), flow to the coil that connected to the servo-valve generating torque of the motor as expressed in (1).

\[
V_v = \frac{dI_v}{dt} L_v + R_v I_v
\]

where the coil consists of inductance and resistance as denoted in \( L_v \) and \( R_v \), respectively.

The voltage and current that generating torque to the motor concurrently produces dynamic motion to the servo-valve that represented in a second order differential equation as expressed in (2).

\[
\frac{d^2 x_v}{dt^2} + 2 \zeta \omega_n \frac{dx_v}{dt} + \omega_n^2 x_v = I_v \omega_n^2
\]

The spool-valve that controlling the flow rate, \( Q \), in chambers consist of different pressure, \( P \), the position of the spool valve, \( x_v \), and the gain of the servo-valve, \( K_v \), as written in (3).

\[
Q = K_v x_v \sqrt{P_1 - P_2}
\]

The fluid flow characteristic in each chamber by neglecting the effect of internal leakage occurs in servo-valve can be expressed in (4) and (5).

\[
Q_1 = \begin{cases} 
K_v x_v \sqrt{P_1 - P_2} & ; x_v \geq 0,
\end{cases}
\]

\[
Q_2 = \begin{cases} 
-K_v x_v \sqrt{P_2 - P_1} & ; x_v < 0,
\end{cases}
\]

where the servo-valve gain coefficient is assumed for a symmetrical valve as \( K_v = K_{v1} = K_{v2} \).

The pump will generate fluid flow that simultaneously produce supply pressure, \( P_s \), that driving the servo-valve. At present, the EHA system generally equipped along with pressure regulator, which can adjust the maximum operating pressure supported by particular applications. The generated pressure will form the dynamics between the pump and the servo-valve can be expressed in (6).

\[
P_s = \frac{\beta}{V_v} (Q_{pump} - Q_s) dt
\]

A. PID and FOPID

It is widely known that PID controller composed with three parameters which playing vital role in transient response and steady-state response analyses. Commonly, PID controller contains the transfer function as expressed in (7).

\[
G(s) = K_p + \frac{K_i}{s} + \frac{K_d}{T_d s}
\]

where proportional gain represented by \( K_p \), integral gain denoted by \( K_i \) and derivative gain defined by \( K_d \). The performance of the transient response that proportional to the steady-state error response is handled by \( K_p \). The responsibility in reduction or elimination of error in the steady-state through the compensation of low frequency executed by \( K_i \). While the transient response performance is improved by \( K_d \) through the compensation of high frequency [21].

For the FOPID, in 20th century, a researcher named Igor Podlubny has introduced the Fractional Order calculus [22]. This calculus is introduced in control and dynamic system by extend general differential equations into fractional order differential equation [23]. The fractional order calculus has integrated to the PID controller due to its flexibilities that emerged Fractional Order (FO-PID) controller.

Alternatively, two additional gains have been merged in the fractional order controller that yielding five parameters which are \( K_p \), \( K_i \), \( K_d \), \( \lambda \), and \( \mu \) denoted as proportional-integral-derivative-integral order, and derivative order respectively [23-25]. Generally, PID controller composed with the element of the transfer function as expressed in (7). Whereas the integration of the additional parameters from the fractional order calculus to the PID controller leads to the transfer function in (8).
It is clearly perceived that if both $\lambda$ and $\mu$ are presumed to be one, then conventional PID can be obtained. Closed-loop control system commonly composed with the interchangeable of integer or fractional order system and controller.

FO-PID or know as $P^\lambda D^\mu$ controller emerged decades and proven to be capable to elevate the performance of the conventional PID controller. However, in some practical stand point, the additional parameters or the numbers of parameters that required to be achieved somehow affecting the computational burden.

### B. Conventional and PSO Tuning Methods

Several conventional tuning approaches for the PID controller including Ziegler-Nichols, Tyreus-Luyben, Damped Oscillation, Chien, Hrones and Reswicht, Cohen and Coon, Fertik, and Ciancone-Marline tuning methods [26]. This article, however, only covers Ziegler-Nichols tuning method due to it prominent performance. Generally, the gains of $K_i$ and $K_d$ are reduced to zero in the procedure of the Ziegler-Nichols tuning method.

In the output result where $K_i$ and $K_d$ are in zero condition, the $K_p$ will be increased to the ultimate gain, $K_u$. In this state, the sustain oscillation occurred. Then, the period of the sustain oscillation, $T_u$ is achieved through the calculation of a full wave cycle as depicted in Fig. 2. The PID controller gains $K_p$, $K_i$, and $K_d$ are then obtained through the formula as tabulated in Table I [27].

\[ G(s) = \frac{U(s)}{E(s)} = K_p \left( 1 + \frac{1}{T_D s^\mu} + T_I s^\nu \right) \]  
\[ (8) \]

For the Particle Swarm Optimization (PSO), the establishment of this algorithm is summarized as depicted in Fig. 3. Generally speaking, the establishment operation begins with random parameter allocation of particle’s velocity and position. These particles are then allocated to the region that existed with problem space, local and global borders, and started to engaging the problem space for the execution. The best solution or defined as fitness for a particle is came up after the execution in the local border and classified as local best value, $l_{best}$. The process repeated for each particle in seeking for their best fitness that consist of position and velocity value. These fitness value will then preserve in arrays and classified as personal best value, $p_{best}$. The operation repeated to the maximum criterion. When the maximum criteria are met, the excellent fitness value among these particles is finally named as global best value, $g_{best}$.

![Ziegler-Nichols Tuning Method](image)

**Fig 2.** Sustain Oscillation to Obtain Ultimate Gain and Period.

<table>
<thead>
<tr>
<th>Gains Type</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.5 $K_u$</td>
<td>Inf</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>0.45 $K_u$</td>
<td>$T_u/1.2$</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>0.6 $K_u$</td>
<td>$T_u/2$</td>
<td>$T_u/8$</td>
</tr>
</tbody>
</table>

**TABLE I.** GAINS FORMULATION IN ZIEGLER-NICHOLS TUNING METHOD

![General Procedures in the Establishment of the PSO Algorithm](image)

**Fig 3.** General Procedures in the Establishment of the PSO Algorithm.
C. Conventional SMC and SMC with PID Sliding Surface

The outstanding work by Russia in early 1960’s had introduced Sliding Mode Control (SMC) in a form of continuous time. Basically, the concept of this controller is to manipulate the control state heading the designed sliding surface. The control state will remain on the surface until the desired condition as depicted in Fig. 4.

\[ s(t) = l + \frac{d}{dt} \left( e(t) + 2 \lambda \dot{e}(t) + \lambda^2 e(t) \right) \]  

(9)

In the SMC design, third order usually acquired for the EHA system. In the conventional SMC design, the sliding surface, \( s(t) \) is proportional to the error, \( e \) and the control gain, \( \lambda \) as expressed in (10).

\[ s(t) = \dot{e}(t) + 2 \lambda \ddot{e}(t) + \lambda^2 e(t) \]  

(10)

By merging the PID controller in the conventional sliding surface design, following expression is obtained with the gains of PID controller \( K_p, K_i, \) and \( K_d \).

\[ s(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \ddot{e}(t) \]  

(11)

In the conditions where sliding surface is in reaching phase or \( s(t) \neq 0 \), the switching control, \( u_{sw} \) undertake the role in carrying the control state to the sliding phase. When the control state reached the sliding phase or \( s(t) = 0 \), take over in guiding the control state to the desired point. Commonly, the design of the SMC composed the elements as expressed in (12).

\[ u_{sw}(t) = u_{eq}(t) + u_{sw}(t) \]  

(12)

In the ordinary SMC and the SMC-PID sliding surface design, first and second derivatives of the sliding surface can be obtained as expressed in (13) and (14) respectively.

\[ \dot{s}(t) = \ddot{e}(t) + 2 \lambda \dddot{e}(t) + \lambda^2 \dddot{e}(t) \]  

(13)

\[ \ddot{s}(t) = K_p \dddot{e}(t) + K_i \dddot{e}(t) + K_d \dddot{e}(t) \]  

(14)

In the simulation environment, the lumped uncertainties, \( L \) can be usually neglected in the design of the equivalent control, \( u_{eq} \). Thus, the \( u_{eq} \) for the convention SMC and SMC-PID controllers can be defined in (15) and (16) accordingly.

\[ u_{eq}(t) = \frac{1}{C} \left( \dddot{e}(t) + A_p \dddot{e}(t) + B \dddot{e}(t) + 2 \lambda \dddot{e}(t) + \lambda^2 \dddot{e}(t) \right) \]  

(15)

\[ u_{eq}(t) = \left( K_p C \right)^{-1} \left( K_p \dddot{e}(t) + K_i \dddot{e}(t) + K_d \dddot{e}(t) + A_p \dddot{e}(t) + B \dddot{e}(t) \right) \]  

(16)

In the \( u_{sw} \) design, signum function, \( sign(s) \) that has a boundary border will be merged to the sliding surface. The \( u_{sw} \) and the boundary function is derived in (17) and (18) accordingly.

\[ u_{sw}(t) = k \cdot sign(s) \]  

(17)

\[ \begin{cases} 1 & ; s(t) > 0 \\ 0 & ; s(t) = 0 \\ -1 & ; s(t) < 0 \end{cases} \]  

(18)

In SMC-PID switching control, \( u_{sw} \), design, the \( sign(s) \) that has a boundary border will also be merged to the sliding surface. The \( u_{sw} \) and the boundary function is derived in (19) and (20) accordingly.

\[ u_{sw}(t) = \lambda s(t) + k \cdot sign(\dot{s}(t)) \]  

(19)

\[ \begin{cases} 1 & ; \dot{s}(t) > 0 \\ 0 & ; \dot{s}(t) = 0 \\ -1 & ; \dot{s}(t) < 0 \end{cases} \]  

(20)

However, signum function is known to has discontinues state that lead to a chattering effect. Thus, authors in [28-30] have introduced a hyperbolic tangent function that can minimize the chattering effect. Then, the \( u_{sw} \) for both conventional SMC and SMC-PID controller can be obtained in (21) and (22) respectively.

\[ u_{sw}(t) = k \cdot \tanh \left( \frac{\dot{s}(t)}{\phi} \right) \]  

(21)

\[ u_{sw}(t) = \lambda s(t) + k \cdot \tanh \left( \frac{\dot{s}(t)}{\phi} \right) \]  

(22)

D. RMSE, ISE, ITSE, IAE and ITAE Analyses

In control system, few types of analyses are commonly used. These analyses including transient response analysis, steady-state error analysis, Root Mean Square Error (RMSE), Integral Square Error (ISE), Integral Time Square Error (ITSE), Integral Absolute Error (IAE), and Integral Time Absolute Error (ITAE).

Depending on the purposes of the article. In this paper, energy usage or efficiency and precision are the major concern which lead to the usage of RMSE, ISE/V, ITSE/V, IAE/V, and ITAE/V. Transient response and steady-state errors analyses are still can be used if necessary. Generally, the RMSE, ISE/V, ITSE/V, IAE/V, and ITAE/V can be expressed as below.

Fig 4. The Common Route to Obtain Desired Condition in SMC.
\[ RMSE = \sqrt{\frac{\sum_{i=1}^{n}(r_i - y_i)^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n}(e_i)^2}{n}} \quad (23) \]

\[ ISE = \int_{0}^{\infty} e^2(t) \, dt \quad (24) \]

\[ ITSE = \int_{0}^{\infty} e^2(t) \, dt \quad (25) \]

\[ IAE = \int_{0}^{\infty} |e(t)| \, dt \quad (26) \]

\[ ITAE = \int_{0}^{\infty} |e(t)| \, dt \quad (27) \]

### III. TRACKING EFFICIENCY AND CONTROLLER EFFORT ANALYSES

In this article, step input response has been applied in the performance examination of the designed controllers. Simulation works are implemented using MATLAB/Simulink 2018. By mimicking the real time environment that addressing fast response, all controller parameters are tuned only one time. It is inferred that, a well-designed control system not simply compensates the existing deficiency, but also able to mitigating an actual endeavor in operating a machine.

This article examines the tracking capability and actual effort in actuating the Electro-Hydraulic Actuator (EHA) system as aforementioned. The feedback of the designed controllers including PID, FOPID, SMC, and SMC-PID controller is portrays in Fig. 5. In the figure, the SMC-PID is apparently outstanding with fastest response or rise time. This effect might be due to the increment of the numbers of gains in the SMC-PID controller. Referring to this, however, the FOPID that contained numbers of gains that not differ much than the SMC-PID controller showing poorest outcome even compared with the PID controller. This situation might be due to the defect of the tuning algorithm. Therefore, further analysis will be carried out on the tuning method and will implement in experiment environment to be more practical.

![Fig 5. The Tracking Performance of the Designed PID, FO-PID, SMC and SMC-PID Controllers.](image)

![Fig 6. The Effort Required by Each Controller in Actuating the EHA System.](image)

By taking the sample between the controller and the system, or particularly the output of the controller, and the input of the system, the control effort can be analyzed. Fig. 6 depicts the effort of the designed controllers in actuating the EHA system.

Referring to the numerical data shown in the y axis, the FOPID controller produced worst performance in terms of control effort, followed by PID, SMC-PID, and SMC controllers. All the feedbacks are executed according to the parameters obtained using PSO algorithm listed in Table II. To comprehensively analyse the outcome, numerical data with respect to Root Mean Square Error (RMSE) implemented to the outcome in Fig. 6 are filed in Table III. The data clearly indicate the FOPID obtained worst performance which is approaching unstable condition. Even the performance indices of ISV, ITSV, IAV, and ITAV tabulated in Table IV clearly indicate the achievement of the FOPID controller.

### TABLE II. PARAMETERS GENERATED BY PSO ALGORITHM

<table>
<thead>
<tr>
<th>Controller</th>
<th>Parameter</th>
<th>(K_p)</th>
<th>(K_i)</th>
<th>(K_d)</th>
<th>(\lambda)</th>
<th>(\delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td></td>
<td>10.0910</td>
<td>0.0013</td>
<td>-4.6985</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FO-PID</td>
<td></td>
<td>34.8991</td>
<td>0.7052</td>
<td>8.5401</td>
<td>2.0296</td>
<td>8.1205</td>
</tr>
<tr>
<td>SMC</td>
<td></td>
<td></td>
<td>87.6240</td>
<td>395.7009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMC-PID</td>
<td></td>
<td>1118.2151</td>
<td>0.0000073</td>
<td>4.0390734</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

### TABLE III. ROOT MEAN SQUARE ERROR WITH RESPECT TO THE CONTROLLER EFFORT

<table>
<thead>
<tr>
<th>Controllers</th>
<th>Root Mean Square Error (Voltage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>3.1558</td>
</tr>
<tr>
<td>FO-PID</td>
<td>6.9289×10^{-3}</td>
</tr>
<tr>
<td>SMC</td>
<td>0.4893</td>
</tr>
<tr>
<td>SMC-PID</td>
<td>5</td>
</tr>
</tbody>
</table>
To investigate the tracking capability of the designed controllers, the performance indices with reference to the error generated by each controller is listed in Table V. It is roughly seen that the SMC-PID controller is able to providing greatest tracking efficiency compared to others even the conventional SMC controller. However, with reference to the controller effort as tabulated in Table IV, conventional SMC outperform the SMC-PID controller. Logically thinking, due to the numbers of gains, and the performance demonstrated by SMC-PID controller, highest precision required highest effort. Simply to said that, the energy usage and the precision can be achieved only either one. Therefore, further enhancement will be focused on this area, which is to designing the best energy usage and precise controller.

IV. Conclusions

In this article, the energy usage that interconnected with the controller effort, and the positioning tracking efficiency that interconnected with precision have been assessed. The modelling of the EHA system also addressed in the study. Then, the discussion in terms of the controller design and the tuning algorithm are carried out. It is notorious the existence of the nonlinear and uncertain characteristics in the EHA system that concurrently increase the complexities in the controller design. In the results, compared to the PID and the FO-PID controllers, the SMC-PID is able to achieving smallest error, while the smallest effort is required by the conventional SMC. It is however, in a practical point of view, only one of the criteria between precision and the controller effort can be achieved which has been proven in the results. Further examination in the experiment environment is necessary to support the aforementioned statement.

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