The Multi-Objective Design of Laminated Structure with Non-Dominated Sorting Genetic Algorithm

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***Abstract*—Non-dominated sorting genetic algorithm has shown excellent advantages in solving complicated optimization problems with discrete variables in a variety of domains. In this paper, we implement a multi-objective genetic algorithm to guide the design of the laminated structure with two objectives: minimizing the mass and maximizing the strength of a specified structure simultaneously, classical lamination theory and failure theory are adopted to compute the strength of a laminate. The simulation results have shown that a non-dominated genetic algorithm has great advantages in the design of laminated composite material. Experiment results also suggest that optimal run times are from 16 to 32 for the design of glass-epoxy laminate with non-dominated sorting genetic algorithm. We also observed that two stages involve the optimization process in which the number of individuals in the first frontier first increases, and then decreases. These simulation results are helpful to decide the proper run times of genetic algorithms for glass-epoxy design and reduce computation costs.**

Keywords*—*Non-dominated sorting genetic algorithm; optimization; failure theory; laminated composite material; classical lamination theory

I. Introduction

Non-dominated sorting genetic algorithm (NSGA-II) provides [1], [2], [3] a collection of techniques to maintain multiple solutions in the mating pool and has shown excellent performance in domains [4], [5], [1], [6], [7], [8], [9], [10], [11], [12], [13]. Slowik and Kwasnicka [9] present the family of evolutionary algorithms for real-life application, such as genetic algorithms, genetic programming, differential evolution, and evolution strategies. Lu [10] et al. adopt NSGA for neural architecture search and has demonstrated the ability in finding competitive neural architecture with less computational resources. Kou [11] et al. propose a two-stage multiobjective feature-selection method for bankruptcy prediction of small and medium-sized enterprises.

The design of a laminate in nature is a tricky optimization problem involving several discrete variables and additional constraints. The traditional wisdom [14], [15], [16], [17], [18], [19], [20], [21], [22] suggests using an evolutionary algorithm to solve this problem with a single objective function, appending additional constraints to the objective function as punishment items, in which the coefficient of each punishment is a random number with a range from 0 to 1. Adams [23] et al. use a genetic algorithm approach by locally reducing a thick laminate to generate and evaluate valid globally blended designs for composite panel structure optimization. Cho and Rowlands [24] implement a genetic algorithm to minimize tensile stress concentrations in a perforated laminated structure, obtaining more than one favorable stacking sequence with different fiber orientations. An [25] et al. present the two-objective design of composite laminates: minimizing cost and maximizing fundamental frequency and frequency gaps.

Although the non-dominated sorting genetic algorithm has demonstrated great efficiency in fields, there is rare literature on the application of NSGA-II for the design of a laminated structure. In this work, we implement this algorithm to guide the design of laminated composite material with multiple constraints. The experiment results have shown that NSGA offers great advantages in assisting the design of a laminate, where it provides a set of solutions. As far as we know, this is the first time to adopt a non-dominated sorting genetic algorithm for laminate design.

The rest of this work is organized as follows: Section II reviews the non-dominated genetic algorithm; Section III gives a brief introduction to laminate and covers the strength calculation process; in Section IV, we formulate the objective functions; in Section V, we present the experiment; finally, we analyze the simulation result and give the conclusion.

II. Non-Dominated Sorting Genetic Algorithm and Laminate Representation

Non-dominated sorting genetic algorithm is an evolutionary algorithm that maintains solutions in the mating pool. The solutions are in the same frontier if none of them dominate each other, and NSGA-II can reserve frontiers in the population. NSGA-II outperforms other multiobjective algorithms in three aspects: 1) $O(MN^2)$ computational cost, where M is the number of objectives and N is the population size; 2) without specifying sharing parameters; 3) new selection operator which combines parents and children and selects individuals from the combination. Fig. 2 shows the process of NSGA-II in which the non-dominated sorting technique reduces the sorting time complexity from $O(MN^3)$ to $O(MN^2)$. The crowding distance sorting trick ranks the individual according to the values of the individual's objective function. We propose to adopt these techniques to guide the design of a laminate.

Fig. 1 shows individuals in the mating pool with two objective functions, f_1 and f_2. In this figure, the cuboid is to measure the distance of individuals in the same frontier instead

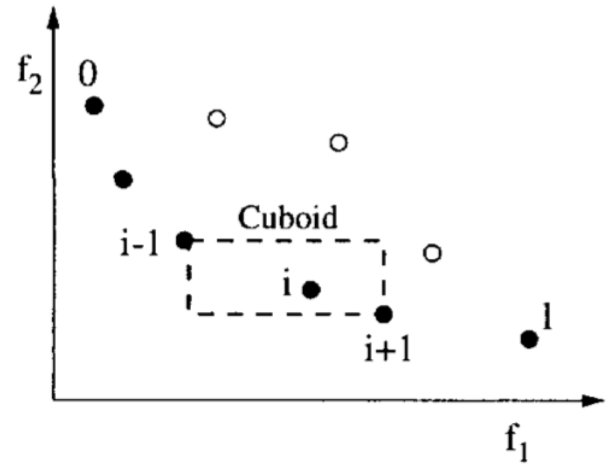


Fig. 1. Frontiers in the Population in which Individuals Marked with the same Color belong to One Frontier [26].

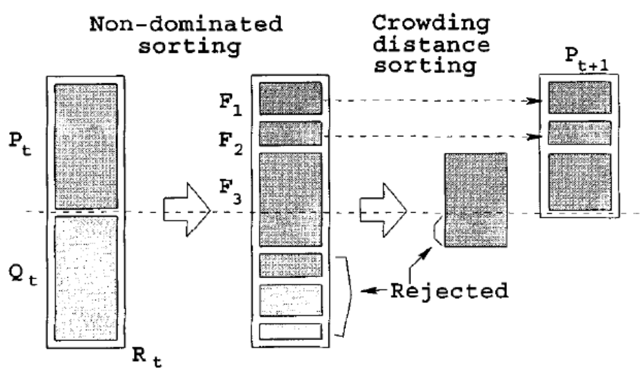


Fig. 2. NSGA-II Procedure[27].

of using sharing function. This technique can also measure the distance of laminates because the distance calculation only requires the value of objective functions. And a sequence of integers is able to represent the structure of a laminate. So we can evaluate the objective function according to the integer representation of a laminate.

III. A Laminate and the Strength Prediction

As shown in Fig. 3, a laminate sequence of lamina binding together along the thickness direction, and lamina is a special composite material whose properties are determined by several variables: ply angle, ply thickness, and material properties. In this paper, to decide the strength of a laminate, it is necessary to know how to compute the strength of a single lamina.

A lamina's strength is highly related to the stress and strain within it. For a lamina under load, it is straightforward to calculate the stress and strain in a lamina by using of three-dimensional stress and strain model, as shown in

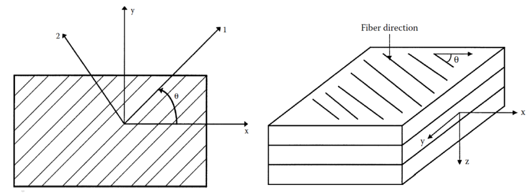


Fig. 3. A Lamina and the Structure of a Laminate.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}. \tag{1}$$

In this equation, $Q_{11}, Q_{12}, Q_{22}, Q_{66}$ are engineering constants, $\sigma_1, \sigma_2, \tau_{12}, \varepsilon_1, \varepsilon_2, \gamma_{12}$ are stress and strain along different directions.

Then failure theories can predict the strength of a lamina according to obtained stress and strain. Various failure theories have been proposed to compute the strength of lamina, and each has its advantage and disadvantages. Here we adopt the two most widely adopted criteria to calculate the strength: Tsai-Wu [28], [29] failure theory and Maximum stress [30], [31] failure theory.

The Tsai-Wu failure theory can compute the strength ratio of a laminate with the following equation. The strength ratio is an indicator of a material's strength under load.

$$\begin{aligned} H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 \\ + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \end{aligned} \tag{2}$$

In this equation, $H_1, H_2, H_6, H_{11}, H_{22}, H_{66}$ are coefficients related to five engineering constants$\sigma_1^T, \sigma_2^T, \sigma_1^C, \sigma_2^C, \tau_{12}$. The relation among them are as follows:

0
90
90
0
90

Fig. 4. Cross-Ply Laminate.

$$
\begin{aligned}
H_1 &= \frac{1}{\left(\sigma_1^T\right)_{ult}} - \frac{1}{\left(\sigma_1^C\right)_{ult}},\\
H_{11} &= \frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}},\\
H_2 &= \frac{1}{\left(\sigma_2^T\right)_{ult}} - \frac{1}{\left(\sigma_2^C\right)_{ult}},\\
H_{22} &= \frac{1}{\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}},\\
H_{66} &= \frac{1}{\left(\tau_{12}\right)_{ult}^2},\\
H_{12} &= -\frac{1}{2}\sqrt{\frac{1}{\left(\sigma_1^T\right)_{ult}\left(\sigma_1^C\right)_{ult}\left(\sigma_2^T\right)_{ult}\left(\sigma_2^C\right)_{ult}}}.
\end{aligned} \tag{3}
$$

The five engineering constants are as follows: $(\sigma_1^T)_{ult}$ ultimate longitudinal tensile strength(in direction 1); $(\sigma_1^C)_{ult}$ ultimate longitudinal compressive strength; $(\sigma_2^T)_{ult}$ ultimate transverse tensile strength; $(\sigma_2^C)_{ult}$ ultimate transverse compressive strength; and $(\tau_{12})_{ult}$ and ultimate in-plane shear strength.

A laminate consists of laminas with a specified sequence, in which the ply angle, thickness, and composite material of one lamina could be different from another; Therefore, the strength computation of a laminate is more complicate than the strength prediction of a lamina. Classical lamination theory [32] is an analytical tool to compute the stress and strain for every lamina in a laminate. For a laminate, the relation between stress and strain is formulated as

$$
\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}. \tag{4}
$$

In this equation, $\sigma_x\sigma_y, \tau_{xy}$ and $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ are stress and strain in global coordinate. And we can compute $\bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{16}, \bar{Q}_{22}, \bar{Q}_{26}, \bar{Q}_{66}$ with the following equations.

$$
\begin{aligned}
\bar{Q}_{11} = & \; Q_{11}cos^4\theta + 2\left(Q_{12} + 2Q_{66}\right)sin^2\theta cos^2\theta + \\
& Q_{22}sin^4\theta \\
\bar{Q}_{12} = & \left(Q_{11} + Q_{22} - 4Q_{66}\right)sin^2\theta cos^2\theta + \\
& Q_{12}\left(cos^4\theta + sin^2\theta\right) \\
\bar{Q}_{22} = & \; Q_{11}sin^4\theta + 2\left(Q_{12} + 2Q_{66}\right)sin^2\theta cos^2\theta + \\
& Q_{22}cos^4\theta \\
\bar{Q}_{16} = & \left(Q_{11} - Q_{12} - 2Q_{66}\right)cos^3\theta sin\theta - \\
& \left(Q_{22} - Q_{12} - 2Q_{66}\right)sin^3\theta cos\theta \\
\bar{Q}_{26} = & \left(Q_{11} - Q_{12} - 2Q_{66}\right)cos\theta sin^3\theta - \\
& \left(Q_{22} - Q_{12} - 2Q_{66}\right)cos^3\theta sin\theta \\
\bar{Q}_{66} = & \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right) \\
& sin\theta^2 cos\theta^2 + Q_{66}\left(sin\theta^4 + cos\theta^4\right)
\end{aligned} \tag{5}
$$

The mid-plane strains and curvature of laminate global coordinates are obtained with the following equation:

$$
\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{11} & B_{12} & B_{16} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}.
\end{aligned} \tag{6}
$$

We can use Equation 7 to calculate every entry in matrice A, B, and D.

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n}(\overline{Q_{ij}})_k(h_k - h_{k-1})i = 1,2,6, j = 1,2,6 \\
B_{ij} &= \frac{1}{2}\sum_{k=1}^{n}(\overline{Q_{ij}})_k(h_k^2 - h_{k-1}^2)i = 1,2,6, j = 1,2,6 \\
D_{ij} &= \frac{1}{3}\sum_{k=1}^{n}(\overline{Q_{ij}})_k(h_k^3 - h_{k-1}^3)i = 1,2,6, j = 1,2,6
\end{aligned} \tag{7}
$$

In Equation 7, h_k is the local coordinate of every lamina. With these equations, we can obtain the strength ratio of a laminate.

In this work, the experiment material is a laminate with 0 and 90 ply orientation, also known as cross-ply laminate, as shown in Fig. 4.

IV. Problem Formulation

Our problem is to design cross-ply laminate whose strength ratio should be greater than two. So the ply orientation is 0 and 90. So the search problem can be reformulated as follows:

(1) design variable: $\{\theta_k, n\}$ $\theta_k \in \{0, 90\}$;

(2) objective: maximization of strength ratio and minimization of mass

(3) constraint: strength ratio should be greater than two.

V. Simulation Results and Discussion

This section presents the experiment. Glass/epoxy is the experiment material and its properties are shown in Table I. The dimension of a lamina is $1000 \times 1000 \times 0.165mm^3$, and the load applied to the laminate is 2MPa. There are two objectives in this experiment: maximization of the strength ratio and minimization of the mass.

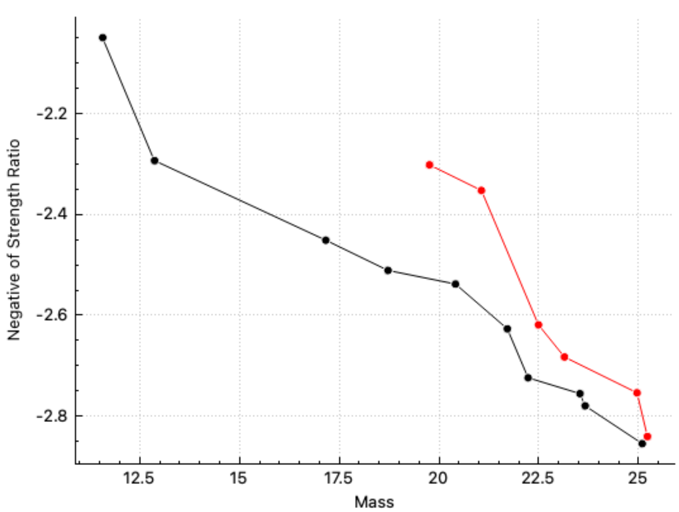
The Fig. 5 displays individuals in the mating pool during the NSGA-II process. In this figure, the x-axis is the mass, and the y-axis is the negative strength ratio because NSGA-II can only deal with minimization problems. Each individual corresponds to one feasible solution which represents the sequence of a laminate. In this figure, individuals are marked with a different color if they belong to a different frontier, then connect individuals in the same frontier, and there are two frontiers: the black and red frontiers. In the same frontier, no individual dominates the rest. For our problem, no individual is better than the other both in mass and strength ratio. This figure demonstrated that NSGA-II could maintain multiple solutions in the mating pool with one-time simulation.

TABLE I. GLASS/EPOXY PROPERTIES

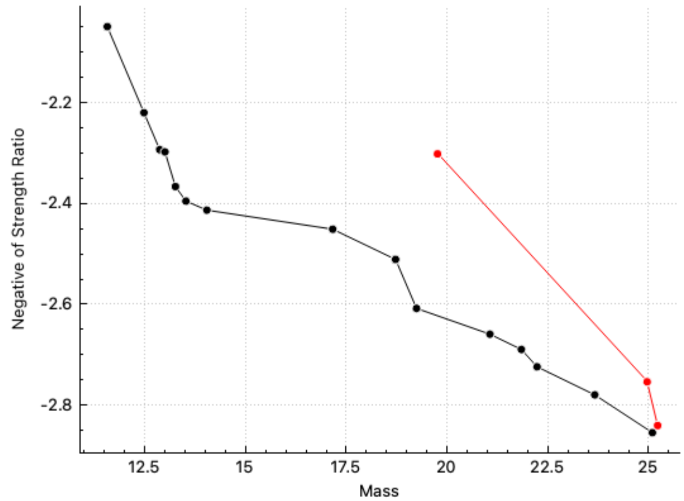
Property	Symbol	Unit	Glass/Epoxy
Longitudinal elastic modulus	E_1	GPa	38.6
Traverse elastic modulus	E_2	GPa	8.27
Major Poisson's ratio	v_{12}		0.26
Shear modulus	G_{12}	GPa	4.14
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MP	1062
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MP	610
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	31
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	118
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	72
Density	ρ	g/cm^3	1.903

It also clearly shows that the whole NSGA-II process is consist of two stages: 1) in the first stage, the number of individuals in the first frontier keeps increasing. At the beginning of this process, there are only 10 individuals in the first frontier, and the number of individuals comes to a peak when the generation is 24th. 2) In the second stage, the number of individuals in the first frontier begins to decrease. As shown in Fig. 5(e), (f), (g), and (h), the number of individuals in the first frontier is less than the number in the previous figure.

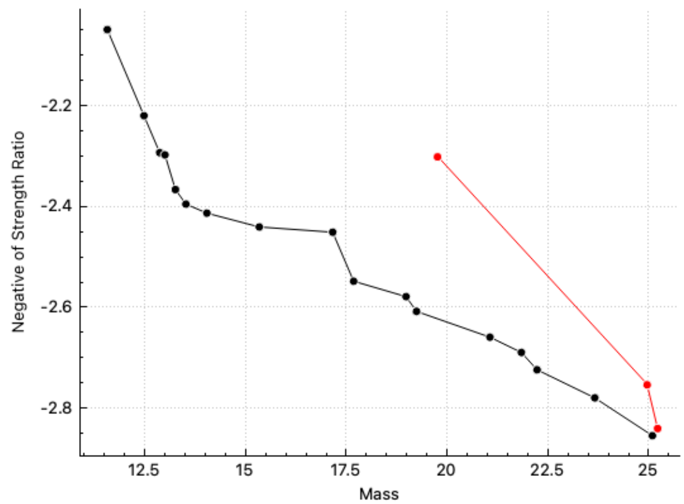
In this experiment, a set of feasible solutions is obtained using NSGA-II, which satisfies different strength requirements. In Javidrad [33] et al. work, only one feasible solution is found after one simulation with a hybrid PSO-SA algorithm, and therefore it is necessary to run this algorithm many times according to different constraints. Using non-dominated sorting genetic algorithms to optimize the design of laminates could reduce simulations times and improve efficiency.



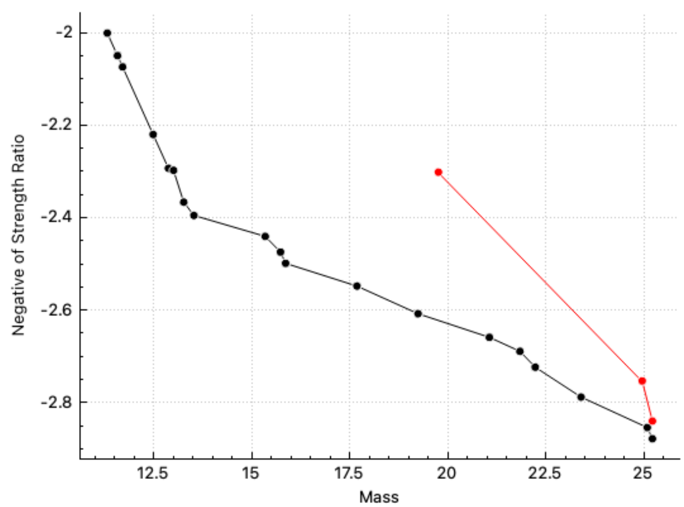
(a) The First Generation.



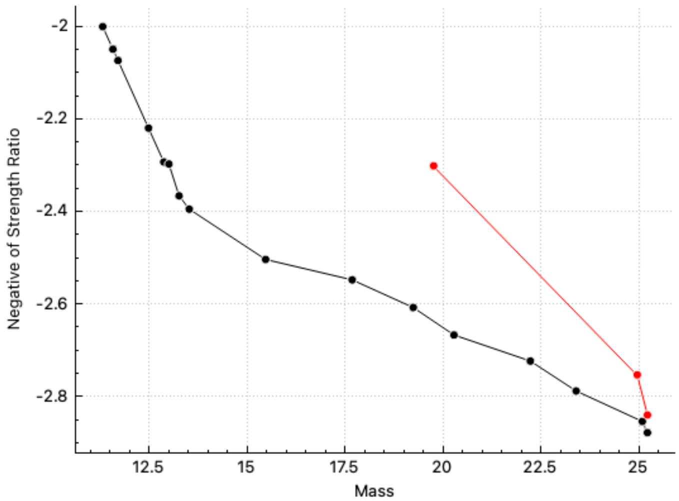
(b) The 8th Generation



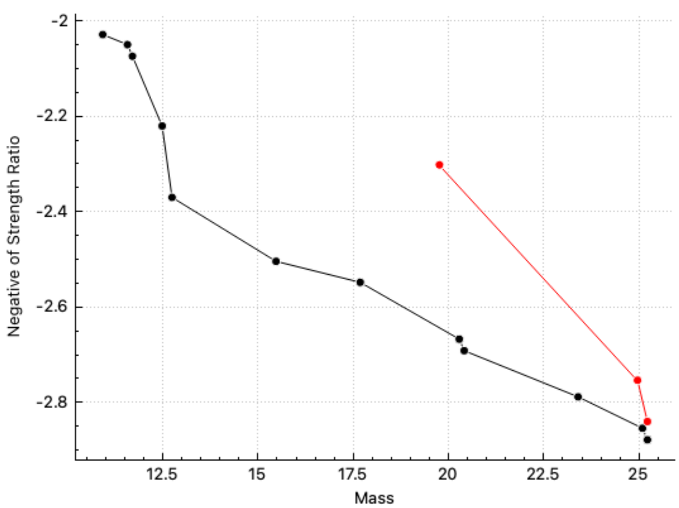
(c) The 16th Generation



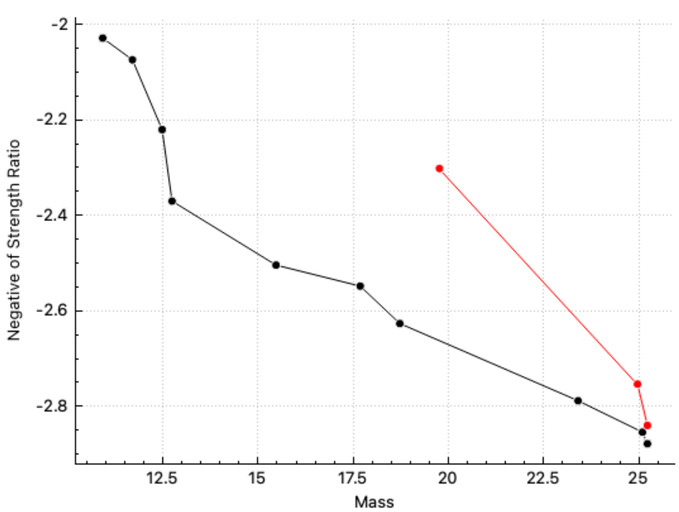
(d) The 24th Generation



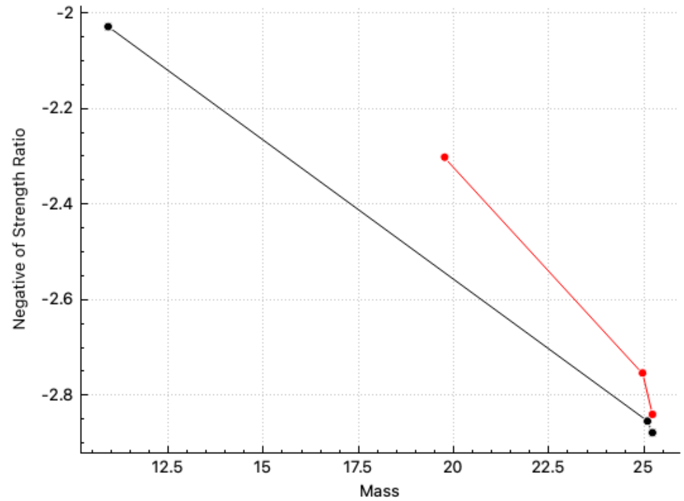
(e) The 32th Generation



(f) The 40th Generation



(g) The 47th Generation



(h) The 50th Generation

Fig. 5. The Variation of Individuals' Number in each Frontier in the Population as the NSGA-II Proceeds.

VI. Conclusion

In this work, we implement the non-dominated sorting genetic algorithm to guide the design of the laminated composite structure with two objectives: minimizing the mass and maximizing the strength ratio simultaneously. The experiment results have demonstrated that NSGA-II is an efficient algorithm to obtain multiple solutions in the first frontier. In our experiment, this algorithm obtains 19 individuals in the mating pool where each individual represent a feasible solution for solving the design problem. No solution dominates others in these individuals.

This simulation also demonstrated that the optimal run times for NSGA-II are from 16 to 32 for the design of glass/epoxy laminates. We also observe that the NSGA-II optimization process is consist of two stages: in the first stage, the number of individuals keeps increasing; however, during the second stage, the number in the first frontier keeps decreasing. The number of individuals in the first frontier comes to a peak if the run times of NSGA-II are from 16 to 32, which would significantly reduce computation cost and obtain an optimal result.

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