Modified Gradient Algorithm based Noise Subspace Estimation with Full Rank Update for Blind CSI Estimator in OFDM Systems

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***Abstract*—This paper presents a modified Gradient-based method to directly compute the noise subspace iteratively from the received Orthogonal Frequency Division Multiplexing (OFDM) symbols to estimate Channel State Information (CSI). By invoking the matrix inversion lemma which is extensively used in Recursive Least Square (RLS) algorithms, the proposed computationally efficient method enables direct computation of noise subspace using the inverse of the autocorrelation matrix of the received OFDM symbols. In the case of a vector input, the modified Gradient algorithm uses rank one update to calculate noise subspace recursively. For an input in the matrix form, the modified Gradient algorithm uses a full rank update. The validity, efficacy, and accuracy of the proposed modified Gradient algorithm have been substantiated through a relative comparison of the results with the conventional Singular Value Decomposition (SVD) algorithm, which is in wide use in the estimation of the subspaces. The simulation results obtained through the modified Gradient algorithm show a satisfactory correlation with the results of SVD, even though the computational complexity involved in modified Gradient is relatively less. Apart from the results encompassing various power levels of the multipath channel, this paper also discusses the adaptive tracking of CSI and presents a comparative study.**

Keywords—Orthogonal Frequency Division Multiplexing (OFDM); Carrier Frequency Offset (CFO); Channel State Information (CSI); Recursive Least Square (RLS); Singular Value Decomposition (SVD); Channel Impulse Response (CIR); BPSK; QPSK; QAM

I. INTRODUCTION

Radio systems based on Orthogonal Frequency Division Multiplexing (OFDM) are increasingly being adopted by many wireless communication standards [1]. With the ability of OFDM systems to effectively handle impairments of wireless channels, communication engineers and system designers can use standards for broadband digital data communication standards such as IEEE 802.11 [a, b, j, n], IEEE 802.15.3a, etc. OFDM is adopted as the waveform in IEEE 802.16 [d, e], IEEE 802.120, digital video broadcasting, digital audio broadcasting and cellular [3G, 4G]. Additionally, in high-speed wireless data communication using OFDM waveforms, the effects of multipath and the delayed spread of channels have a significant impact on data throughput. Figure 1 describes a typical OFDM transmitter and Receiver operating in a multipath wireless channel. For efficient operation and throughput of the OFDM system, it is important to estimate the multipath wireless channel (Channel State Information) at the receiver end.

In summary, accurate estimation of Carrier Frequency Offset (CFO) and CSI is essential for ensuring the satisfactory performance of the OFDM system. The attainment of time synchronization between OFDM symbols is assumed to be present. The emphasis of this paper is on CSI which deals with the estimation of the multipath wireless channel. Instead of conventional SVD, this paper presents a computationally efficient modified gradient algorithm to estimate the noise subspace directly (Instead of signal subspace first and then through it, the noise subspace). The noise subspace is then utilized for the estimation of CSI.

This paper is structured as follows. Section II presents the review of CSI estimation techniques. Section III presents the OFDM system model. Section IV deals with the second-order statistics and the blind CSI estimation process of the subspace. Section V discusses the proposed modified Gradient based noise subspace. Section VI presents the formulation for the estimation of CSI using the proposed modified gradient algorithm. Section VII analyses the performance of modified Gradient based CSI estimator, to substantiate its efficacy and ability in improving the estimation accuracy of CSI. Section VIII presents the conclusions of the paper.

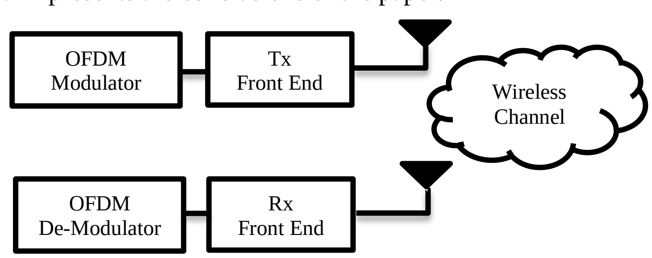


Fig. 1. OFDM System Block Diagram.

II. RELATED WORK

This section presents a comprehensive review of research studies pertaining to OFDM in general and estimation of CSI in particular.

The authors in [2] propose an Integrated OFDM (I-OFDM) system to meet the BER performance of Enhanced Long-Term Assessment (LTE-A). For the pilot-aided CSI estimation technique, pilot arrangement of comb type as well as convolution (Channel) encoder with best-known bounds of Coding Bounds are used. "Maximum A Posteriori (MAP)" decoder for the convolution encoder is at the receiver. The evaluation of BER performance was through convolution codes of Recursive systematic and non-systematic nature. The proposed I-OFDM showed significant improvement in the system performance combining traditional IM and multiple-mode IM in [3]. To achieve the varied diversity order and spectral efficiency required next-generation wireless communication networks, CIM techniques (subcarrier-wise and subblock-wise) are presented.

To achieve higher data bandwidth in such systems, it is of greater importance to estimate the CSI, the transfer function of the wireless channel. Author in [4] addresses the clustering strategy for heterogeneous wireless sensor networks. The clustered heads are chosen based on the CSI, the nodes' residual energy, and the network's average energy. Author proposes periodic transmission of a data packet containing the information about the energy dissipation of the wireless sensor nodes using Bluetooth low energy, ZigBee, and ANT protocols for the appropriate choice for the protocol selection.

The authors in [5] propose the OFDM/OFDMA system's improved performance by introducing closed-loop rotate modulation schemes involving BPSK, QPSK, and QAM. The method utilizes the feedback of the complex-valued CSI. The rotate modulation serves the role of channel equalizer and does not need the guard interval insertion. With BPSK and QPSK, the proposed method does not degrade BER performance with feedback delay. However, with QAM, the feedback delay degrades BER performance.

The authors of [6] discuss the potential demands and challenges of the emerging 6G wireless communication network. The authors address system capacity requirements, data rate, latency, enhanced security, and the improved quality of service of 6G relative to 5G. The paper also dwells on the potential applications of mm-wave, terahertz communications, and massive MIMO systems in 6G.

This paper [7] addresses the issue of Pilot contamination in a massive MIMO system. The authors proposed a scheme for reducing pilot contamination by combining time-shifting protocol, the directional pilot scheme, and a greedy algorithm-based pilot allocation scheme for channel estimation. The simulation results show the significantly enhanced performance of a massive MIMO system with the proposed combinatorial scheme compared to the individual constituents of the combination.

The receiver improves the overall bandwidth efficiency of the system by detecting CSI accurately and efficiently. CSI acquisition is generally performed using pilot or non-pilot techniques. To estimate CSI, pilot-based techniques typically use significant bandwidth to train the channel estimator at the receiver and send the training sequence. Paper [7] addresses the issue of pilot contamination in large-scale MIMO systems. The authors proposed a scheme to reduce pilot contamination by combining a time-shift protocol, a directional pilot scheme, and a pilot allocation scheme based on a greedy channel estimation algorithm. The simulation results show that the performance of large MIMO systems using the proposed combination scheme has improved significantly.

For non-pilot assisted methods, CSI estimation requires statistical information about the data received. In addition, non-pilot assistive techniques (often referred to as blind techniques) are more bandwidth efficient because they do not require a unique training sequence.

The authors in [8,9] proposed pilot aided technique for channel estimation. The authors investigated the performance of block type and comb type pilot insertion technique in estimation of the channel. Comb time pilot insertion scheme is found to be more appropriate to track time varying channel. The technique involves the pilot insertion on the estimation. The frequency domain interpolations are carried in case of the block type pilot insertions and time domain interpolation is carried in case of the comb type pilot insertions.

In [10], the authors presented the pilot aided technique for channel estimation in OFDM. The performance of two pilot based estimators performance is evaluated at different SNRs. Bayesian based Minimum Mean Square Estimator (MMSE) performs better at low SNR when compared to the Maximum Likelihood (ML) estimator. Both MMS and Bayesian estimators require a prior information about the channel statistics. The estimator requires more number of pilot tones as compared to the Channel Impulse Response (CIR) length.

Linear Redundancy Precoding (LRP) uses either cyclic prefixes (CPs) or zero pads (ZPs) in OFDM systems [11, 12]. LRP is one of the categories of blind techniques for estimating the CSI of OFDM systems. The estimation accuracy of the LRP technique depends on the CP / ZP length of the OFDM symbol. The second category of blinding techniques relies on the subspace of the secondary statistics of the received sample to estimate the CSI of the OFDM system. The performance of the subspace-based method depends heavily on the accuracy in estimating the autocorrelation matrix of the received sample. This paper focuses on a subspace-based approach with improved stability in CSI numerical estimation and improved channel bandwidth efficiency.

In [12] and [13], the underlying method for estimating CSI is based on the singular vector of the covariance matrix. CSI estimation requires a noise subspace of the autocorrelation matrix. Computing the required noise subspace with traditional algorithms such as SVD requires extensive computation. A zero padded OFDM system is explored in [14] for CSI estimation based on subspace.

Numerical methods for calculating noise subspaces are not as common as signal subspace estimators. In addition, most algorithms for estimation [15] that combine signal and noise subspaces are available with high complexity. The estimation algorithm exists exclusively for the signal subspace, but there seems to be no algorithm dedicated to the noise subspace.

This paper presents a Gradient [16] based method for iteratively calculating noise subspaces from received OFDM symbols. The method proposed in this paper directly calculates the noise subspace required to estimate the CSI. In the proposed method, the calculation of the noise subspace requires the inverse of the autocorrelation matrix of the received OFDM symbols. In addition, this paper employs the matrix inversion lemma, which is very commonly used in recursive least squares algorithms (RLS) [17], to overcome the high computational cost of direct inversion of autocorrelation matrices.

Also, the noise subspace of the autocorrelation matrix is just the signal subspace of the inverse autocorrelation and is calculated using the numerically stable Gradient algorithm. In this paper, the modified Gradient refers to fitting the inverse matrix to compute the inverse of the autocorrelation matrix and using the numerically stable Gradient algorithm to estimate the noise subspace. This paper also shows two schemes of modified gradient based on whether the underlying input to the modified Gradient algorithm is a vector or a matrix. For vector inputs, the modified Gradient recursively calculates the noise subspace using rank one updates. On the other hand, modified Gradient algorithm recursively computes the noise subspace using a full-rank update if the input is in the form of a matrix. Without loss of generality, this paper assumes channel matrix to be a full-rank Hankel matrix.

The emphasis of this paper is on CSI which deals with estimation of multipath wireless channel. Instead of conventional direct SVD, this paper presents a computationally efficient modified gradient algorithm to estimate the noise subspace directly. The noise subspace is then utilized for the estimation of CSI. The application of the new algorithm has been substantiated in the estimation of CFO using MUSIC algorithm [18], which also requires the noise subspace. This paper extends the utility of the modified gradient algorithm for the estimation of CSI. It is envisaged that the modified gradient algorithm proposed by the authors [18] is new way of estimating CFO and CSI, since modified Gradient algorithm is computationally efficient compared to the conventional direct SVD[19].

III. OFDM System Model

In this section, the basic OFDM symbol is formed by N carriers. The basic OFDM symbol is followed by L zeros. Where $L \geq L_H$ (length of channel impulse response). In this document, the zero-pad OFDM system eliminates ISI. Equation (1) represents the nth received OFDM symbol Y_n after full time synchronization.

$$Y_n = HF^H x_n + w_n \tag{1}$$

Y_n is n^{th} Received OFDM symbol with L zero padding of size (N+L) x 1

H is channel convolution matrix of size N+L x N.

F^H is the orthonormal IIFT matrix.

x_n is n^{th} i.i.d unit norm data vector of size N x 1.

w_n is i.i.d additive white Gaussian noise of variance σ^2.

$$H = \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ \vdots & h(0) & \ddots & \vdots \\ h(L_H) & \vdots & \ddots & 0 \\ 0 & h(L_H) & \ddots & h(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h(L_H) \end{bmatrix}$$

H is a full-rank non-negative Toeplitz channel convolution matrix. The element of matrix H is the normalized channel impulse response of length L_H. The channel is assumed to be time-invariant and frequency-selective over the symbol period. To overcome the effects of ISI, the length of the selected ZP will be $L \geq L_H$. The CSI estimate assumes carrier frequency offset (CFO) cancellation introduced by the Doppler or local oscillator of the received OFDM symbol Y_n.

IV. Blind CSI Estimation

The blind CSI estimation algorithm in this document uses the noise subspace calculated from the quadratic statistics of the received OFDM symbols [14]. The quadratic statistics (autocorrelation) of the received OFDM symbols contain only the information about the transmitted OFDM symbols and the convoluted channels. However, due to the orthonormal nature of the inverse Fast Fourier Transform (IFFT) and the nature of the independent identical distribution and unit norm of the transmitted data X_n, the autocorrelation matrix can be used to estimate the CSI [14].

$$R_n = E\{Y_n Y_n^H\} \tag{2}$$

$$R_n = HH^H + \sigma_{noise}^2 I_{N+L} \tag{3}$$

Where R_n is the autocorrelation matrix and E is the expected value operator. The matrix HH^H is the Hermitian positive semi-finite matrix, I_{N+L} is the identity matrix, and σ^2_{noise} is the noise power of the channel. The size of the resulting autocorrelation matrix R_n is N+L x N+L. The identity matrix can diagonalize the resulting R_n by applying the spectral theorem. SVD is used to diagonalize R_n.

$$R_n = U \Sigma V^H \tag{4}$$

$$U = [U_{signal} U_{nosie}] \tag{5}$$

$$V^H = [U_{signal} U_{noise}]^H \tag{6}$$

$$\Sigma = \begin{bmatrix} \Delta + \sigma_{noise}^2 I_N & 0 \\ 0 & \sigma_{nosie}^2 I_L \end{bmatrix} \tag{7}$$

The orthonormal matrices U and V^H contain subspace components of signal and noise subspaces. U_{signal} corresponds to the signal subspace of dimension N+L x N of the autocorrelation matrix R_n, and U_{noise} corresponds to the noise subspace of dimension N+L x L. Δ is a diagonal matrix with diagonal elements representing the signal power of each N-subcarrier. Equation (1) shows that the received OFDM symbol Y_n is basically in the signal space across the channel convolution matrix H. A linear combination of the channel convolution matrix H multiplied by $F^H x_n$ yields the received OFDM symbol Y_n. The channel convolution matrix H is a Toeplitz matrix of size N+L x N, so there are N non-identical columns and the rank is N. This indicates that the rank of the main subspace (signal subspace) of this matrix H is N, and the rank of that subspace (noise subspace) is L.

$$U_{noise}^{H} \times U_{siganl} = 0 \quad (8)$$

Equation (8) states that the U_{signal} and U_{noise} are orthogonal subspaces. Each vector in the noise space is orthogonal to the entire signal space U_{signal}. Similarly, each vector in the signal space is orthogonal to the total noise space U_{noise}. This special property is used to estimate the channel impulse response h_n. Since the columns of the channel convolution matrix H span the entire signal space, the received OFDM symbol Y_n is also in the signal space across H . Therefore, this space is also orthogonal to any vector in the noise space Unoise calculated from the SVD on the autocorrelation matrix R_n.

$$U_{noise}^{H} \times H = 0 \quad (9)$$

$$u^{H} = [u_1 \quad u_2 \quad \cdot \quad \cdot \quad u_{N+L}] \quad (10)$$

$$u^{H} \times H = 0 \quad (11)$$

if u^H in Equation (10) is a vector of length N+L in the noise subspace, it is also an orthogonal complement of the channel convolution matrix H by equation (11). From Equation (11) to Equation (12) rewritten by the Toeplitz structure of the channel convolution matrix H. Where V is a Henkel matrix of size L+1 x N formed using the elements of u, which is the noise vector of the noise subspace U_{noise}. The cost function for estimating CSI includes the complex conjugate of equation (12).

$$h^{H} \times V = 0 \quad (12)$$

$$(h^{H} \times V) \times (h^{H} \times V)^{H} \quad (13)$$

$$h^{H} V V^{H} h = 0 \quad (14)$$

By using all the noise vectors in the noise subspace U_{noise}, Equation (14) is rewritten as the cost function of Equation (16) to estimate the CSI.

$$W = \sum_{i=0}^{L} V_i V_i^{H} \quad (15)$$

$$h^{H} W h = 0 \quad (16)$$

V_i is a Hankel matrix formed from the individual noise vector elements of the noise subspace U_{noise}. Equation (16) means that the vector h, which is a small singular vector of W, can minimize the cost function. This small normalized singular vector is an estimate of CSI with phase ambiguity. Each complex element, when scaled, is a small singular vector that is also the solution to the cost function defined in Equation (16).

V_i is the Henkel matrix formed by using the elements of the individual noise vectors of the noise subspace U_{noise}. Equation (16) implies that the vector h, a minor singular vector of W, can minimize the cost function. This minor singular vector normalized is the estimate of CSI with a phase ambiguity. Any complex element, when scaled, a minor singular vector, is also a solution to the cost function defined by Equation (16). This phase ambiguity is attributed to the above.

Considering the process of CSI estimation from the reception of OFDM symbol Y_n to the solution of equation (16), the calculation of equations (4) and (16) uses a computationally intensive algorithm (SVD) in two steps. Analysis of the steps involved in the CSI estimation shows that noise space is more important in the CSI estimation. In fact, of the results of Equations (4) and (16), the noise vector from each calculation is used in the subsequent process of the CSI estimation method. However, the computational cost of the $O((N+L)^3)$ SVD method makes this estimation method a non-viable option for real-time implementations. In the next section, this paper proposes a new algorithm for efficiently estimating the noise vectors in Equation (4) (using rank-one updates) and equation (16) (using full-rank updates). The proposed algorithm has computational advantages over existing algorithms such as SVD.

V. Modified Gradient Algorithm

Let x(t) be a column vector in complex vector space C^n observed at instant t. In time domain spectral analysis, it is a vector of n consecutive samples of summation of r non coherent complex sinusoids corrupted by additive complex Gaussian noise n(t) with variance σ^2.

$$x(t) = \sum_{k=1}^{r} s_k \, a(w_k) + n(t) \quad (17)$$

$$x(t) = As(t) + n(t) \quad (18)$$

Where $A = [a(w_1)\, a(w_2) \ldots a(w_r)]$ is the deterministic matrix of size n x r. $a(w_p) = [1 \; e^{jw_p} \; e^{j2w_p} \ldots e^{j(n-1)w_p}]^T$ is the frequency vector and $S(t) = [s_1 \, s_2 \ldots s_r]$ is the random source vector. The correlation matrix R is formed from the snapshot vector x(t).

$$R = E[x(t)\, x^{H}(t)] = AC_sA^{H} + I\sigma^2 \quad (19)$$

Where $C_s = E[S(t)S^{H}(t)]$, I denotes the identity matrix and E stands for expectation. Taking Eigen decomposition of Equation (5).

$$R = U\Sigma U^{H} \quad (20)$$

$$U = [U_s U_n] \quad (21)$$

U_s contains the signal space vectors and U_n has the noise space vector. The first r vectors in U (Equation (21)) form the signal subspace vector alias column subspace vectors. The remaining (n-r) vectors form the noise subspace vectors alias left Null subspace vector. Similarly, $\sum$ (Equation (20)) is the diagonal matrix containing the Eigen values corresponding to the signal and noise subspace respectively. It is important to note that both the matrices A and U_s span the same column space alias the signal subspace. The U_s signal subspace vectors and U_n noise subspace vectors are orthogonal and they complement each other. The above-mentioned properties are exploited in high resolution spectral estimation technique using MUSIC [20] and ESPIRIT algorithms [21].

It is clear that the matrices A and U_s span the same column space alias the signal subspace. This enables to build a cost function to estimate the signal subspace. Let y(t) be a vector in the column space of the ${U_s}^{H}$. Then,

$$y(t) = {U_s}^{H} x(t) \quad (22)$$

$$x(t) = U_s \, y(t) \quad (23)$$

$$x(t) = U_s U_s^H x(t) \tag{24}$$

Superscript H denotes the Hermitian transpose. For convenience U_s and U_s^H are rewritten as W and W^H respectively. The scalar cost function can be arrived as

$$J(W) = E\|x(t) - WW^H x(t)\|^2 \tag{25}$$

$J(W)$ has global minimum only at when the columns of W span the signal space of A.

Global minimum of Equation (25) can be found recursively using the Gradient Descent method [16]. As a first step, the gradient of the unconstrained cost function with respect to W is derived.

$$\nabla J = [-2R + RWW^H + WW^H R]W \tag{26}$$

The update on the subspace can be written as

$$W(t) = W(t-1) + [2R(t) - R(t)W(t-1)W^H(t-1) - W(t-1)W^H(t-1)R(t)]W(t-1) \tag{27}$$

The above Equation (27) converges to the signal subspace of the correlation matrix $R(t)$. This implies that the conventional Gradient algorithm facilitates the computation of signal subspace only. One has to compute the noise subspace after computing the signal subspace. The proposed modified Gradient algorithm is aimed for the direct computation of noise subspace instead of first computing the signal subspace and then computing the noise subspace from the knowledge of auto correlation matrix. To obtain the noise subspace, R(t) in Equation (27) is to be replaced with $R_{inv}(t)$, where $R_{inv}(t)$ is the inverse of correlation matrix $R(t)$. With this modification, Equation (27) can be rewritten as shown in Equation (28).

$$W(t) = W(t-1) + [2R_{inv}(t) - R_{inv}(t)W(t-1)W^H(t-1) - W(t-1)W^H(t-1)R_{inv}(t)]W(t-1) \tag{28}$$

Equation (28) is the modified Gradient Method for the noise subspace estimation. The computational complexity of Equation (28) is comparatively less when compared to the batch based SVD or EVD techniques [19] which is of the order of $O(n^3)$ operations.

VI. Modified Gradient in CSI Estimation

This section presents a unique method for calculating the noise vector of R_n, the autocorrelation matrix by altering the current Gradient method [16]. The Gradient-based subspace estimation method is a member of the iterative power-based subspace estimation algorithm class. These categories of algorithms estimate and track signal vectors more effectively than the noise vector of the subject matrix. In addition, the iterative power-based approach predicts the greatest vector in the signal space. Typically, signal subspace vectors are computed prior to noise subspace estimation. The noise subspace created by inverting R_n, the autocorrelation matrix eliminates the need to compute the noise subspace following the signal subspace. Taking into account the inversion of the subjected R_n matrix, the same power-based technique will be more effective at directly estimating and tracking the noise space vector or minor vectors.

The inverse of the autocorrelation matrix R_n is calculated in the proposed modified Gradient, X_n, by utilizing the underlying received OFDM symbol Y_n. This is carried out in a recursive fashion. The classic matrix inversion lemma is used in the recursive least square estimation techniques, and it is utilized by the modified Gradient. The underlying matrix that will be employed in tracking the noise vectors in Equations (4) and (16) of interest for estimating the CSI will be the inverted version of X_n, which will have a dimension of N+L x N+L. During the process of estimating the CSI, this modified Gradient method will be used to calculate and keep track of the noise vectors. In terms of the amount of computing power required, a technique of this iterative nature is quite effective. The comparative performance of blind CSI estimation using the proposed modified Gradient algorithm and SVD based approach will be described later in section VI with a random complex channel. The focus of this section is on the performance of blind CSI estimation using modified Gradient method.

This paper describes the procedure for computing the noise subspace vectors specified in Equation (4) and (16) using the modified Gradient approach. A pseudocode for the CSI estimation is also presented. Estimation of CSI begins with the OFDM signals that were successfully received. X_n denotes the n^{th} estimate of the inverse of the autocorrelation matrix, R_n.

A brief explanation of pseudo-code is presented here in order to estimate the noise subspace vectors making use of modified Gradient, with rank one update being performed by the underlying OFDM signal Y_n. As shown in the Equation (31), the rank-one update of the inverse autocorrelation matrix X_n is computed using Y_n, the received OFDM signal.

Initialize

$$V_{nosie}(0) = [I]_{N+L\ X\ L}$$

$$X(0) = [I]_{N+L\ X\ N+L}$$

Where N is the number of carriers and L is the zero-padding length. $V_{nosie}(0)$ is the initial noise subspace matrix of X_n^H , and X(0) is the initial inverse of R_n.

For n = 1,2,3… ,

$$KalGain\ A(n) = X(n-1)Y(n) \tag{29}$$

$$Gamma\ A(n) = \frac{1-\lambda}{\lambda + (1-\lambda)Y^H(n)KalGainA(n)} \tag{30}$$

$$X(n) = \frac{1}{\lambda}(X(n-1) - GammaA(n)KalGainA(n)KalGainA^H(n)) \tag{31}$$

$$V_{nosie}(n) = V_{nosie}(n-1) + \left[2X(n) - X(n)V_{nosie}(n-1)V_{nosie}{}^H(n-1) - V_{nosie}(n-1)V_{nosie}{}^H(n-1)X(n)\right] V_{nosie}(n-1) \tag{32}$$

End

λ is the forgetting factor between 0 and 1.

V_{nosie}, which has the noise vectors corresponding to the noise subspace vector of Equation (10). The L_{noise} vectors from V_{nosie} construct the Henkel matrix V needed to compute W, as shown in Equation (15). The smallest singular vector of W is obtained using the procedure listed below.

A summary of pseudo code for estimating the noise vectors using the modified Gradient method is presented with full-rank updating of W, which is computed from the V_{nosie} as described in the Equation (15).

Initialize

$$Q(0) = \begin{bmatrix} I \\ 0 \end{bmatrix}_{N+L\ X\ L}$$

$$W_{inv}(0) = [I]_{LXL}$$

for n = 1,2,3…

$$KalGainB(n) = W_{inv}(n-1)W(n) \quad (33)$$

$$GammaB(n) = \frac{1-\lambda}{\lambda + (1-\lambda)\det(W^H(n)KalGainB(n))} \quad (34)$$

$$W_{inv}(n) =$$

$$\frac{1}{\lambda}(W_{inv}(n-1) - GammaB(n)KalGainB(n)KalGainB^H(n)) \quad (35)$$

$$Q(n) = Q(n-1) + [2W_{inv}(n) - W_{inv}(n)Q(n-1)Q^H(n-1) - Q(n-1)Q^H(n-1)W_{inv}(n)]Q(n-1) \quad (36)$$

End

Q in Equation (36) contains noise subspace vectors of W_{inv} of Equation (35). The first singular vector is the estimate of the channel's impulse response. The impulse response estimate is normalized to get the estimate of CSI. The estimated normalized impulse response will have a phase ambiguity and is resolved with the help of a single pilot carrier.

The Gradient class of subspace estimation algorithms typically exhibit a complexity of $O((N+L)L)$ with the additional complexity of $O((N+L)^2)$ for inversion of matrix using matrix inversion lemma of Equation (31). In general, computational complexity in the estimation of noise spaces through the proposed modified Gradient scheme is $O((N+L)^2+(N+L)L))$, instead of $O((N+L)^3)$ operations of the conventional SVD based methods. This in turn implies a reduction in computational complexity of modified Gradient method. Table I presents the comparison of computational complexity between the direct SVD and modified Gradient algorithms in computing the noise subspace with rank one update. As shown in Table I, the computational complexity of modified Gradient is comparatively less than the direct SVD based algorithms for various values of N and L.

TABLE I. COMPARISON OF COMPUTATIONAL COMPLEXITY OF DIRECT SVD AND MODIFIED GRADIENT IN ESTIMATION NOISE SUBSPACE WITH RANK ONE UPDATE

SI. No	N	L	SVD	Modified Gradient
1	128	8	*O*(136^3)	*O*(26.955^3)
2	256	16	*O*(272^3)	*O*(42.788^3)
3	512	32	*O*(544^3)	*O*(67.921^3)

VII. PERFORMANCE ANALYSIS

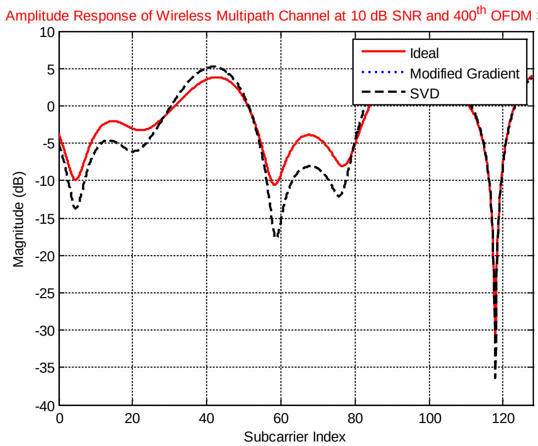
This section presents the results of the simulation to establish the ability modified Gradient based CSI estimation algorithm using the directly computed noise subspace. The simulation model assumes the reception of the OFDM symbol with 128 carriers and zero-padded to the extent of 1/4th of the OFDM symbol. Out of 128 carriers, one carrier is a reference carrier to resolve the issue of phase ambiguity. The simulation of the Rayleigh channel model mimicking the outdoor channel is through 16 tap FIR filter. The subcarriers of the OFDM use the modulation scheme of the QPSK constellation. The simulation is with 1500 OFDM symbols. The changes in the Rayleigh channel model are induced to occur at the time instances of 500th and 1000th OFDM symbols. The Channel Impulse Response, which is nothing but CSI, is estimated and is tracked using a subspace-based technique at various power levels. The estimate of CSI is compared with a response of modelled ideal Rayleigh channel. Direct SVD and proposed modified Gradient techniques are adopted to estimate the noise subspace. Noise subspace, in turn, is used to estimate the CSI. Fig. 2 to Fig. 4 present the comparative performance and analysis of CSI estimates obtained through modified Gradient, Direct SVD, and the ideal channel response.

Fig. 2 to Fig. 4 illustrate the comparative estimation performance of the modified Gradient and direct SVD-based noise subspace estimators in the estimation of CSI of the OFDM system. In the simulation studies presented in this paper, 400 OFDM symbols are utilized to reconstruct the autocorrelation matrix which is required for noise subspace estimation. The results of Fig. 2 correspond to the power level (SNR) of the wireless channel at 10 dB. The ideal Rayleigh channel shown in Fig. 2, is the reference for comparison and has been modeled through 16 tap FIR filter. Fig. 2 depicts the estimation of the Rayleigh channel state (at SNR of 10 dB) at the 400th OFDM symbol. While Fig. 2(a) depicts the amplitude response of the CSI estimation, the corresponding phase response is shown in Fig. 2(b). There is an excellent agreement between the results obtained through the modified Gradient and direct SVD-based blind CSI estimators. The results of Fig. 2(b) reveal a fixed offset between the results of modified Gradient and Direct SVD relative to the ideal Rayleigh channel. The referred fixed offset in the phase of estimated CSI is attributed to the phase ambiguity (which is already explained while discussing Equation (16)). This phase ambiguity is resolved or negated using a single reference subcarrier.

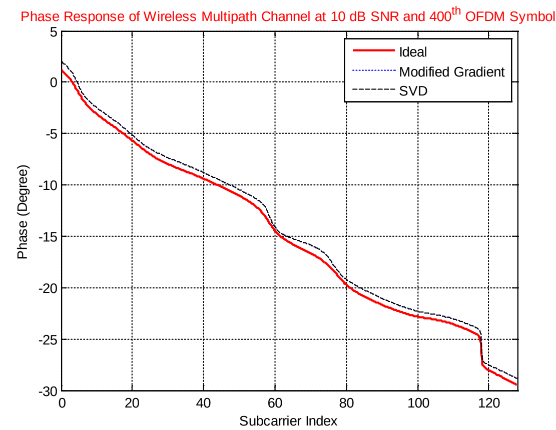
Similarly, Fig. 3(a) and Fig. 3(b) depict the amplitude and the phase of the CSI estimation (SNR of 20 dB) at the 400th OFDM symbol. At increased power level of the channel (SNR), the correlation between the CSI estimation by the blind CSI estimators and the ideal channel improves.

The analogous results at SNR of 30 dB are shown in Figures 4a and 4b. The results on amplitude response of CSI estimation shown in Figure 4a show perfect agreement among the blind CSI estimators and ideal channel. It is pertinent to note that modified Gradient requires relatively lesser computational effort compared to direct SVD.

Fig. 5 depicts the channel tracking performance of the proposed modified Gradient algorithm with direct SVD at SNR of 10 dB. It also illustrates the Relative Error Norm performance of the proposed modified Gradient and direct SVD-based blind CSI estimators. Whenever there is a change in the state of the Rayleigh channel, the blind CSI estimator takes around 100 OFDM symbols for attaining the steady state. Post 100 symbols, the Relative Error Norm falls to around -6 dB at 1000th and 1500th OFDM symbols.

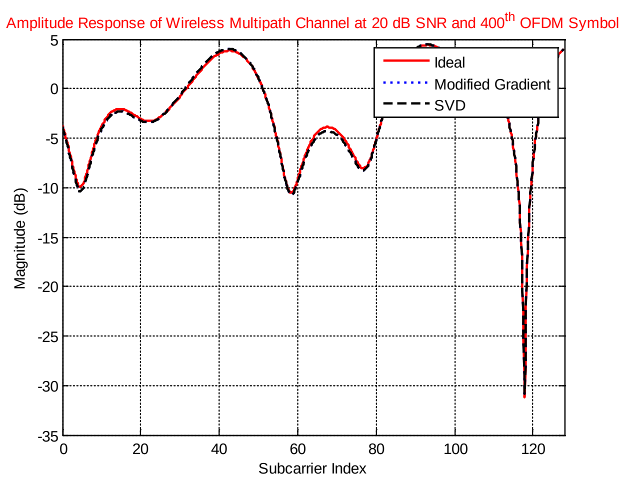


(a)

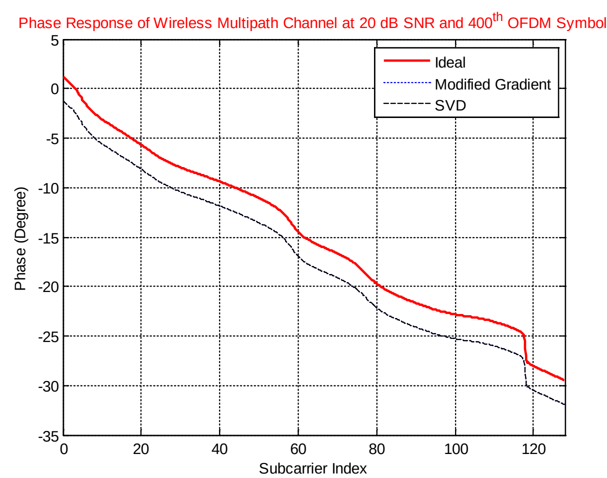


(b)

Fig. 2. (a) Amplitude Response: CSI Estimation for Rayleigh Channel at 400th OFDM Symbol (SNR of 10 dB), (b) Phase Response: CSI Estimation for Rayleigh Channel at 400th OFDM Symbol (SNR of 10 dB).

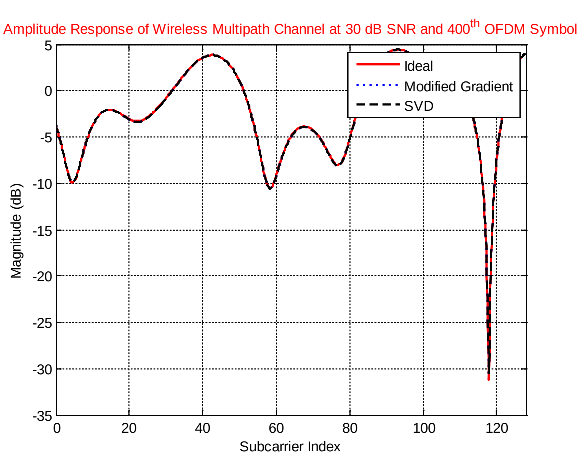


(a)

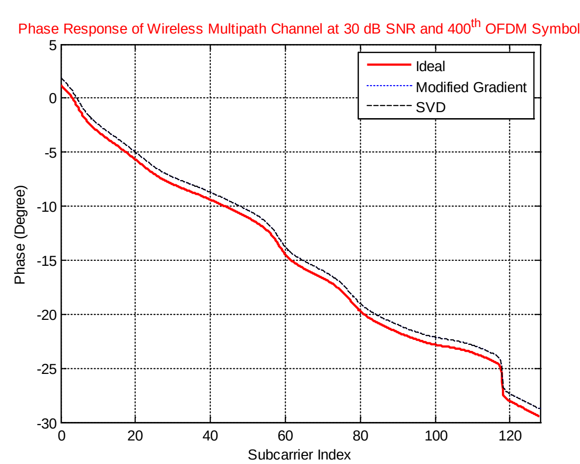


(b)

Fig. 3. (a) Amplitude Response: CSI Estimation for Rayleigh Channel at 400th OFDM Symbol (SNR of 20 dB), (b) Phase Response: CSI Estimation for Rayleigh Channel at 400th OFDM Symbol (SNR of 20 dB)



(a)



(b)

Fig. 4. (a) Amplitude Response: CSI Estimation for Rayleigh Channel at 400th OFDM Symbol (SNR of 30 dB), (b) Phase Response: CSI Estimation for Rayleigh Channel at 400th OFDM Symbol (SNR of 30 dB)

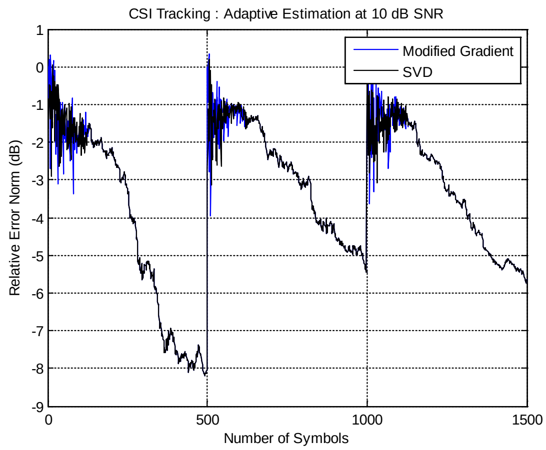


Fig. 5. CSI Tracking of Rayleigh Channel at SNR of 10 dB.

Similarly at the higher SNR value of 20 dB (Fig. 6), the Relative Error Norm shows decreasing trend (as low as -16 dB).

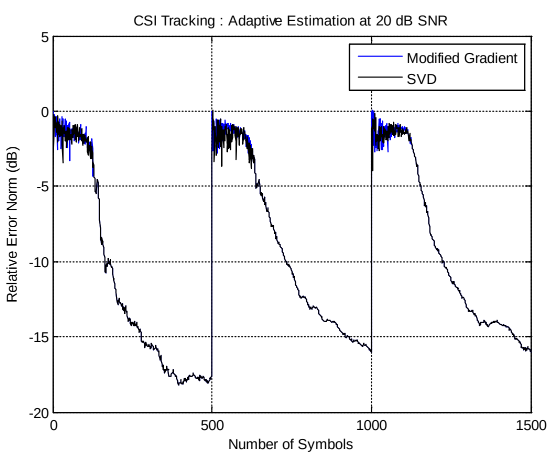


Fig. 6. CSI Tracking of Rayleigh Channel at SNR of 20 dB.

The results of Fig. 7 reveal that for a channel power level (SNR of 30 dB), the Relative Error Norm is as low as -22 dB. As the power level increases, the difference in the Relative Error Norm continuously decreases leading to very good agreement between the results of direct SVD and the modified Gradient method. This substantiates the ability of modified Gradient algorithms to accurately estimate the CSI of the wireless multipath channel at a lower computational cost.

Fig. 8 presents the comparison of the Mean Square Estimate (MSE) of CSI estimation performed through Modified Gradient algorithm and with that of Cramer Rao Lower Bound (CRLB). The subspace-based techniques which are blind in nature are formulated to perform the estimation of CSI without the knowledge of complete information. The proposed modified Gradient algorithm compares favorably with CRLB at high SNR. At SNR of about 15 dB and above, the modified Gradient algorithm requires about 2 to 3 dB additional power gain to attain the limits of CRLB. Indeed, it is exhibiting a good performance in spite of complete blind operation.

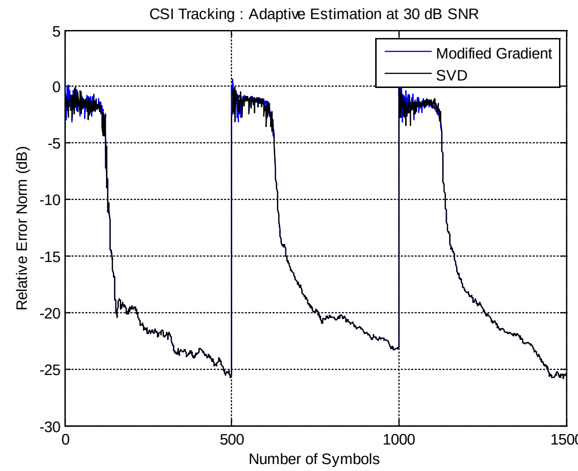


Fig. 7. CSI Tracking of Rayleigh Channel at SNR of 30 dB.

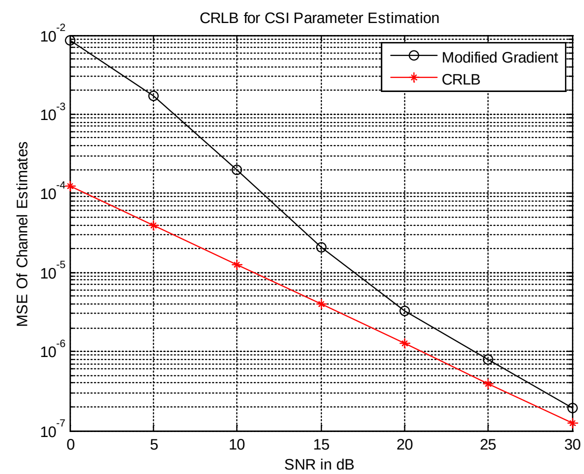


Figure 8: CRLB Comparison.

VIII. CONCLUSION

This paper presents a modified Gradient based method to directly compute the noise subspace iteratively from the received OFDM symbols to estimate CSI. The proposed method enables direct computation of noise subspace using the inverse of the autocorrelation matrix of the received OFDM symbols. This paper adopts a matrix inversion lemma to overcome the heavy computational efforts in the direct inversion of an autocorrelation matrix. This paper also introduced two schemes of modified Gradient based on whether the underlying input to the algorithm is a vector or matrix. In the case of a vector input, the modified Gradient algorithm uses rank one update to calculate noise subspace recursively. For the matrix input, modified Gradient algorithm uses full rank update. The validity, efficacy and the accuracy of the proposed modified Gradient algorithm have been substantiated through a relative comparison of the results with the conventional SVD algorithm, which is in wide use in estimation of the subspaces. The results of performance analysis obtained through modified Gradient algorithm show satisfactory correlation with the results of SVD, even though the computational complexity involved in modified Gradient method is relatively less. This enables to adopt the noise subspace-based CSI estimation in realistic scenario of OFDM system. The reduced computation complexity in the estimation of the noise subspace estimation by the proposed modified Gradient can be of potential utility for the use of a subspace-based technique in the estimation of CSI for coherent demodulation in OFDM systems. Through simulation studies, this paper also has illustrated the ability of the blind CSI estimator in tracking the changes in the wireless channel state at various time instants.

The focus of this paper is on CSI estimation assuming the presence of perfect carrier frequency synchronization. However, the joint CSI and CFO estimation without the assumption of perfect carrier frequency synchronization will be of practical relevance from the performance perspective of the OFDM system. The joint CFO-CSI estimators based on the noise subspace technique is a topic of research interest of the authors.

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