Optimization Performance Analysis for Adaptive Genetic Algorithm with Nonlinear Probabilities

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Abstract—Genetic Algorithm (GA) has been proven to be easy in falling into local optimal value due to its fixed crossover probability and mutation probability, while Adaptive Genetic Algorithm (AGA) has strong global search capability because the two probabilities adjust adaptively. There are two categories of AGA according to the different adjustment methods for crossover and mutation probabilities: probabilistic linear adjustment AGA and probabilistic non-linear adjustment AGA. AGA with linear adjustment of probability values cannot solve the problems of local optimal value and premature convergence. The nonlinear adaptive probability adjustment strategy can avoid premature convergence, poor stability and slow convergence speed. The typical AGA with nonlinear adjustment of probabilities are compared and analyzed through benchmark functions. The optimization performance of typical AGA algorithms is compared and analyzed by 10 benchmark functions. Compared with traditional GA and other AGA algorithms, AGA with crossover and mutation probabilities adjusted nonlinearly at both ends of the average fitness value has higher computational stability and is easy to find the global optimal solution, which provides ideas for the application of adaptive genetic algorithm.

Keywords—Adaptive genetic algorithm; genetic algorithm; nonlinear adjustment; probability

I. INTRODUCTION

Genetic Algorithm (GA) was first put forward by Professor Holland [1]. GA is a highly parallel random search algorithm developed from natural selection and evolution mechanism in biology, providing a solution to complex optimization problems. It has the characteristics of fast search speed, simple process and the flexibility is strong [2]. However, the optimization results easily fall into local optimal values due to its fixed crossover probability and mutation probability [3-4].

For this reason, many researchers put forward an improved method of GA to enhance its global search ability. Generally speaking, there are three categories of these improvements [5]:

The first category is a hybrid optimization algorithm based on GA [6-8]. Combining GA with other optimization algorithms to improve the performance is meaningful for solving different types of optimization problems. However, because different methods are relatively independent, their improvement work can only be carried out in their respective fields.

The second category is to improve the crossover operator [9] or mutation operator [10] in GA, which improves the optimization performance of GA to some extent, but the improvement is not significant [11].

The third category is Adaptive Genetic Algorithm (AGA) [11-14], which adaptively adjusts crossover probability (Pc) and mutation probability (Pm). Because the Pc and Pm adapted to the population are used in each evolutionary generation, instead of the fixed probability value, the adaptive ability of GA can be greatly improved, and the algorithm can converge to the global optimal solution more easily. Moreover, this improved method has wide application range and strong universality [5].

AGA was first proposed by Srinivas et al. [15]. In AGA algorithm proposed by Srinivas et al., when the fitness value of an individual is equal to the maximum fitness value of the individual in this generation, the values of Pc and Pm will be zero, which will lead to premature optimization calculation. For this reason, many scholars have put forward improved methods for AGA. Several different versions of improved AGA have been found. There are two categories of improved AGA methods: linear adjustment and nonlinear adjustment according to the different adjustment methods of crossover and mutation probabilities. This paper focuses on AGA algorithm for nonlinear adjustment of probability.

II. ADAPTIVE GENETIC ALGORITHM

The adaptive ability of genetic algorithm should be reflected in that individuals in the population automatically discover the characteristics and rules of the environment according to the changes of the environment. The most obvious environmental feature is the individual's adaptability to the environment, and the most obvious evolutionary law is the relationship between the individual's fitness value and the average fitness value of all individuals in the population and the minimum fitness value and the maximum fitness value in the population.

In evolution, individuals don't remember which generation they evolved into, only whether they improved their ability to adapt to their environment. If there are improvements, then a better evolutionary pattern will be found, and as much as possible this pattern will be preserved in the algorithm design. Otherwise, the model is likely to become obsolete by nature.
The main difference between GA and AGA is the choice of \( P_c \) and \( P_m \) values. In GA, these two probabilities are randomly determined or based on insufficient prior knowledge reference, while AGA depends on the state of population in the evolution process to select the optimal value. The optimization process of AGA is in Fig. 1.

III. LITERATURE REVIEW ABOUT NONLINEAR ADJUSTMENT AGA

There are two categories of AGA according to the different adjustment methods for \( P_c \) and \( P_m \): probabilistic linear adjustment AGA [16-18] and probabilistic nonlinear adjustment AGA [19-22]. AGA with linear adjustment of probability values cannot solve the problems of local optimal value and premature problem [19-22]. The nonlinear adaptive probability adjustment strategy can avoid premature problem, poor stability and slow convergence speed. According to whether the probability is adjusted at two branches of the average fitness value, the AGA with nonlinear adjustment can be divided into three types: one end is fixed and the other end is nonlinear adjustment (taking the average fitness value as the cutoff point) [18-19]; linear adjustment at one end and nonlinear adjustment at the other [21-22]; nonlinear adjustment at two ends.

A. One Branch is Fixed and the Other is Adjusted Nonlinearly [19-22]

1) Nonlinear adjustment of probability value by exponential function [19]: Equation (1) and equation (2) give the adjustment formulas of \( P_c \) and \( P_m \) through exponential function [19] in solving the minimum optimization problem, respectively. This AGA is labeled as AGA-1.

\[
P_c = \begin{cases} 
\frac{(P_{c_{\max}} - P_{c_{\min}})}{\exp \left( \frac{1}{A} \left( f_{\text{avg}} - f' \right) \right)} + P_{c_{\min}}, & f' \leq f_{\text{avg}} \\
\frac{(P_{c_{\max}} - P_{c_{\min}})}{\exp \left( \frac{-1}{A} \left( f_{\text{avg}} - f' \right) \right)} + P_{c_{\min}}, & f' > f_{\text{avg}}
\end{cases}
\]

\[
P_m = \begin{cases} 
\frac{(P_{m_{\max}} - P_{m_{\min}})}{\exp \left( \frac{1}{A} \left( f_{\text{avg}} - f' \right) \right)} + P_{m_{\min}}, & f' \leq f_{\text{avg}} \\
\frac{(P_{m_{\max}} - P_{m_{\min}})}{\exp \left( \frac{-1}{A} \left( f_{\text{avg}} - f' \right) \right)} + P_{m_{\min}}, & f' > f_{\text{avg}}
\end{cases}
\]

Fig. 2 shows the adaptive curves of \( P_c \) and \( P_m \) for AGA-1 and AGA-2. \( P_c \) and \( P_m \) will adjust nonlinearly according to the fitness of individuals between \( f_{\text{avg}} \) and \( f_{\text{min}} \).

Fig. 2. Adaptive \( P_c \) and \( P_m \) of AGA-1 and AGA-2.
B. One Branch is Linear and the other is Nonlinear (Mark as AGA-3) [22]

Although AGA-1 and AGA-2 can avoid the local optimization and the premature convergence problem to some extent due to the adoption of a nonlinear adjustment strategy on one branch, it may still lead to the algorithm to fall into local optimum solution easily when the individuals’ fitness value are larger than average fitness value and $P_c$ and $P_m$ are both fixed. For this reason, Wang [21] proposed the following improved adjustment strategy as shown in equation (5) and equation (6).

$$
P_c = \begin{cases} 
    P_{c_3} + (P_{c_2} - P_{c_3}) \exp \left( \frac{10(f - f_{avg})}{f_{avg} - f_{min}} \right), & f < f_{avg} \\
    P_{c_2} - \frac{(P_{c_1} - P_{c_2})(f_{avg} - f)}{f_{max} - f_{avg}}, & f > f_{avg}
\end{cases}$$

$$
P_m = \begin{cases} 
    P_{m_3} + (P_{m_2} - P_{m_3}) \exp \left( \frac{10(f - f_{avg})}{f_{avg} - f_{min}} \right), & f < f_{avg} \\
    P_{m_2} - \frac{(P_{m_1} - P_{m_2})(f_{avg} - f)}{f_{max} - f_{avg}}, & f > f_{avg}
\end{cases}$$

Adjustment curves of $P_c$ and $P_m$ for AGA-3 are in Fig. 3.

IV. ANALYZE ALGORITHM PERFORMANCE BY BENCHMARK FUNCTION

The performance of the above four AGAs for minimum optimization problems is examined in this section. In order to analyze the results, basic GA is used as a benchmark for comparison. Ten standard benchmark functions commonly
used in literatures are selected for numerical simulation analysis.

A. Parameters Setting of Algorithm

GA and the above four AGAs have the same parameters except \( P_c \) and \( P_m \). All parameters are exhibited in Table I.

| TABLE I. PARAMETERS OF GA AND AGAS |
|-----------------|-----------------|-----------------|
| Parameters      | Value           | Parameters      | Value           |
| Population(P)   | 50              | \( P_{\text{max}} \) for AGA-1 and AGA-2 | 0.75 |
| Generation(G)   | 500             | \( P_{\text{min}} \) for AGA-1 and AGA-2 | 0.3  |
| \( P_c \) for GA | 0.85            | \( P_{\text{max}} \) for AGA-3 and AGA-4 | 0.1  |
| \( P_m \) for GA | 0.01            | \( P_{\text{min}} \) for AGA-3 and AGA-4 | 0.05 |
| \( P_{\text{c1}} \) for AGA-3 and AGA-4 | 0.9 | \( P_{\text{c2}} \) for AGA-3 and AGA-4 | 0.01 |
| \( P_{\text{c3}} \) for AGA-3 and AGA-4 | 0.6 | \( P_{\text{c4}} \) for AGA-1 and AGA-2 | 0.075 |
| \( P_{\text{m}} \) for AGA-3 and AGA-4 | 0.3 | \( P_{\text{m}} \) for AGA-1 and AGA-2 | 0.01 |

B. Benchmark Function

Ten benchmark functions [23] are used here to evaluate the performance of AGA. Their expressions, search ranges and global optimum values are listed in Table II. \( f_1 \)-\( f_5 \) are multimodal functions, and \( f_6 \)-\( f_{10} \) are single modal functions. GOV stands for global optimum value, and D stands for dimensions in Table II.

| TABLE II. BENCHMARK FUNCTION |
|---------------------------|-------------------|-------------------|
| Objective function        | Search range      | GOV\(^{1}\)       | D\(^{2}\)       |
| \( f_1(x) = \sum_{j=1}^{D} \frac{x_j^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \) | [-600,600] | 0 | 10 |
| \( f_2(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{D} \sum_{j=1}^{D} x_j^2} \right) - \exp\left(\frac{1}{D} \sum_{i=1}^{D} \cos(2\pi x_i)\right) + 20 + e \) | [-32.768, 32.768] | 0 | 10 |
| \( f_3(x) = 418.9929D - \sum_{i=1}^{D} x_i \sin(\sqrt{|x_i|}) \) | [-500,500] | 0 | 10 |
| \( f_4(x) = \left[ \sum_{j=1}^{D} i \cos\left((i+1)x_j + i\right) \right] \left[ \sum_{j=1}^{D} i \cos\left((i+1)x_j + i\right) \right] \) | [-10,10] | -186.731 | 2 |
| \( f_5(x) = -0.0001 \left( \sin(x_1) \sin(x_2) \exp\left(\frac{100 - \sqrt{x_1^2 + x_2^2}}{\pi}\right) + 1 \right)^{0.1} \) | [-10,10] | -2.0626 | 2 |
| \( f_6(x) = \sum_{i=1}^{D} \left[ 100(x_{i+1} - x_i)^2 + (x_i - 1)^2 \right] \) | [-2.048, 2.048] | 0 | 10 |
| \( f_7(x) = (x_1 - 1)^2 + \sum_{i=2}^{D} (2x_i^2 - x_{i-1})^2 \) | [-10,10] | 0 | 10 |
| \( f_8(x) = \sum_{i=1}^{D} x_i^2 \) | [-10,10] | 0 | 10 |
| \( f_9(x) = \sum_{i=1}^{D} |x_i| \) | [-1,1] | 0 | 10 |
| \( f_{10}(x) = \sum_{i=1}^{D} x_i \) | [-5.12, 5.12] | 0 | 10 |

\(^{1}\) Global optimum value.
\(^{2}\) Dimension.
TABLE III. AVERAGE OPTIMUM VALUE

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<tr>
<th></th>
<th>(f_1)</th>
<th>(f_2)</th>
<th>(f_3)</th>
<th>(f_4)</th>
<th>(f_5)</th>
<th>(f_6)</th>
<th>(f_7)</th>
<th>(f_8)</th>
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<td>0</td>
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<tr>
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<td>-2.0621</td>
<td>9.122</td>
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<td>0.122</td>
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<td>AGA-3</td>
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<td>5.670</td>
<td>1.638</td>
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D. Discussion

In order to improve the comparative analysis of different AGAs, the probability adjustment curves of different AGAs are drawn in Fig. 5. Fig. 5 exhibit that the curves of \(P_c\) and \(P_m\) in AGA-3 become steeper when \(f_{\text{avg}}\) and \(f_{\text{min}}\) are closer to each other. As a result, the probabilities of these individuals are greatly different each other even if there is little difference in individual fitness values. Thus, most individuals only have lower \(P_c\) and \(P_m\), causing the evolution to stagnate. In order to avoid the occurrence of the above situation, firstly, the adaptive adjustment curves of \(P_c\) and \(P_m\) should be changed slowly at \(f_{\text{avg}}\) so as to greatly enhance \(P_c\) and \(P_m\) of individuals with fitness closer to \(f_{\text{avg}}\). Secondly, to ensure that better individuals in contemporary population still have a certain crossover and mutation probabilities, the adaptive adjustment curve at \(f_{\text{min}}\) should be smoothed. Although AGA-1’s adaptive adjustment curve is relatively smooth in the near area \(f_{\text{avg}}\), it retains a large probability value in the near area \(f_{\text{min}}\), which is not conducive to the retention of dominant individuals in the late stage of evolution. However, in AGA-2, the \(P_c\) and \(P_m\) vary slowly in the near area \(f_{\text{avg}}\), and \(P_c\) and \(P_m\) values close to \(f_{\text{avg}}\) are significantly improved. At the same time, since the patterns of nearby \(f_{\text{min}}\) individuals are preserved as much as possible, their values are low but greater than zero, which explains why the algorithm tries to get out of local convergence.

On the other hand, the results of AGA-2 are obviously better than AGA-1 and AGA-3, which indicates that a good nonlinear adjustment curve can greatly improve the performance of AGA. However, to some extent, it is easy for AGA to fall into the local optimal solution because AGA-2 adopts a fixed probability value between \(f_{\text{avg}}\) and \(f_{\text{min}}\). AGA-4 makes the \(P_c\) and \(P_m\) curves change nonlinear slowly through nonlinear adjustment, and makes the curves smooth at any point. When the individual population is in a comparable state, AGA-4 pulls apart most individuals near \(f_{\text{avg}}\), thus promoting the process of evolution. It is of positive significance to eliminate local convergence and prevent the algorithm from falling into stagnation.

Fig. 5. Adaptive \(P_c\) and \(P_m\) of AGA-1, AGA-2, AGA-3 and AGA-4.

V. Conclusion

AGA algorithms with nonlinear adjustment of \(P_c\) and \(P_m\) are systematically summarized, and the optimization performance of typical AGA algorithms is compared and
analyzed by 10 benchmark functions. The optimization calculation results of benchmark functions show the superiority of AGA-4 which has crossover probability and mutation probability that are nonlinear adjusted at both ends of $f_{avg}$. Compared with traditional GA and other AGA algorithms, AGA-4 has higher computational stability and it's easy to find the global optimal solution.

In the future, we will study the crossover operator and mutation operator of AGA in order to further improve the performance of the optimization algorithm.

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REFERENCES


