Emergency Decision Model by Combining Preference Relations with Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Priority Weighted Averaging PROMETHEE Approach

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Abstract—**The outbreak of COVID-19 in 2019 has brought greater international attention to emergency decision making and management. Since emergency situations are often uncertain, prevention and control are crucial. For better prevent and control, according to the characteristics of emergency incidents, the paper proposes a new form of linguistic expression trapezoidal Pythagorean fuzzy probabilistic linguistic variables to express decision-making information. Next, the paper develops the operational rules, value index and ambiguity of trapezoidal Pythagorean fuzzy probabilistic linguistic variables. Then, the new trapezoidal Pythagorean fuzzy probabilistic linguistic priority weighted averaging PROMETHEE approach is introduced to aggregate the trapezoidal Pythagorean fuzzy probabilistic linguistic information combining with preference relation. Finally, an emergency decision making case of prevention of infectious diseases analysis illustrate the necessity and effectiveness of this method, the results of comparative and experimental analyses demonstrate that the constructed new trapezoidal Pythagorean fuzzy probabilistic linguistic priority weighted averaging PROMETHEE approach owns better performances in terms of effectiveness and reasonability.**

Keywords—COVID-19; emergency decision model; trapezoidal Pythagorean fuzzy probabilistic linguistic variables; preference relations; PROMETHEE approach

I. Introduction

COVID-19 first appeared in Wuhan, China, on December 30, 2019. Due to its pandemic characteristics, long incubation period and strong transmission capacity, COVID-19 is now expanding globally, with more than 133.14 million people infected worldwide as of April 5, 2021, a lot of researchers have done a lot of research on this [1], [2], [3], [4]. This poses a great threat to people all over the world. After the COVID-19 incident happened, China and the international community have paid more attention to emergency management. Hence, how to choose an effective emergency response plan and organize it quickly to reduce casualties and property losses has been reconsidered by governments, public and scholars all over the world. But in real life, all kinds of sudden events occur frequently, and the evolution of abrupt events is often uncertain. This may lead to multiple emergency scenarios. Considering the urgency of emergencies and the complexity of the matter, the emergency command department often organizes many experts from the relevant departments, and the expert may prefer to express their opinion by linguistic terms, just like [5], [6], [7], [8]. For instance, the linguistic term ”good”, ”bad” or ”very bad” can be used to express the alternative. Due to the ambiguity and complexity of human cognition, one issue of emergency decision making is how to express the experts evaluations or preference information accurately [9], [10], [11].

In several real situations, professional judgments could not be expressed and interpreted as certain qualitative numbers; In other words, data and certain numbers are insufficient for modeling the real-world systems because of ambiguity and uncertainty in the judgment of decision-makers. In previous researches on emergency management, the linguistic term set (LTS) has been introduced into various emergency decision-making processes, like [12], [13], [14]. Whereafter, Xu et al.[9] applied the probabilistic linguistic term set(PLTS) to emergency management. But the trapezoidal Pythagorean fuzzy probabilistic linguistic term set(TrPFPLTS) has not been applied to emergency management events so far, even the application of PTS to emergency management is rare. In fact, TRPFPLTS has greater flexibility and wider applicability than other PLTS or LTS. It not only allows experts to use more than one linguistic term to express their preference, but also reflects the different probability of occurrence of all possible linguistic terms. At the same time, it is more comprehensive in the expression of fuzzy information. In this study, the paper will select the TRPFPLTS to represent the fuzzy decision information and uncertainty probability of the decision makers(DMs) for the emergency event. Take prevention and control of a local outbreak of infectious diseases as an example. When an infectious disease breaks out somewhere and its source is unknown, the emergency command department invites several experts to assess and judge the infectious ways: population density, air pollution index, number of parks and entertainment equipment, density of restaurants, density of cultural and educational centers. In different cases, the proportion of influencing factors is different, for that reason it is very necessary to introduce TRPFPLTS to represent the real situation and the behavior of DMs.

It is unlikely that a single expert will consider all aspects of infectious disease control. Multiple attribute group decision making (MAGDM) is to find a collective solution to a decision-making problem in which a group of experts express their opinions regarding multiple alternatives. However even in a friendly environment, a lot of differences across the expert group are generated inevitably, and greatly discordant opinions are aggregated and this may lead to an intermediate opinion with which no expert totally agrees. In MAGDM, the information provided by the experts has different forms. Because of the nature of infectious diseases, it is difficult for emergency response authorities to provide an accurate assessment and determine the route of transmission among the many influencing factors. Therefore, in order to determine the cause of the outbreak of the infectious disease, we introduced a preference relationship(PR) to make a pairwise comparison of the influencing factors. Because linguistic variables are the natural expressions of DMs, researchers often use linguistic preference relations(LPR) to express DMs' subjective preferences. Herrera et al. [15], Liu et al. [16], Liu. p et al. [17] and Dong[18] respectively apply the LPR to multi-attribute group decision making(MAGDM). Due to the neglect of the probabilistic nature of linguistic terms in DMs's judgment, the linguistic preference relation has some limitations. Hence Zhang et al. [19], [20], Alonso et al. [21], Wang et al. [22] put forward different probabilistic language preference relations respectively and applied them to different fields.

It's important to find proper way to aggregate the PRs. So far, various formats of aggregation approach have been put forward to aggregate decision making preferences. Due to the increasing complexity of social problems, more and more researchers are using PROMETHEE approach to provide solutions for DMs through priority relationships, which is based on pairwise comparisons between alternatives. The PROMETHEE method is extremely useful in complex decision making processes, especially real world MAGDM problems involving human perception and subjective judgment of experts. It is worth noting that the PROMETHEE method has considerable advantages when collaboration among specialists are restricted by their distinct fields of expertise. The PROMETHEE method is a robust decision making approach since it determines the relative merits of alternatives by comparing them in pairs, rather than ranking all of them directly. This approach can avoid the round off error which may occur during data normalization. Lolli et al. [23] defined the elicitation of criteria weights in PROMETHEE based ranking methods. Gul et al. [24] propoesed a fuzzy logic based PROMETHEE method for material selection problems.

From the previous research, with the diversity, complexity of the MAGDM problem, although a lot of researches on the PRs at this stage, there are still many defects:

(1) They are limited in their depiction of each person's point of view, using only a single linguistic term to express an evaluation of an object. It can be seen that due to the limited knowledge and the complexity of reality, people often carry out the evaluation with a certain degree of uncertainty. For example, one may use ”very good”, ”good”, ”somewhat good” to describe the quality of a product, rather than simply "good" or "bad".

(2) They did not fully take into account the poor structural of the information itself, and only considers the fuzziness of the MAGDM problem. Therefore, the paper have made several innovations on the basis of previous studies.

(3) When a simple aggregation operator aggregates group decision information, it retains less information and loses the original information, which seriously leads to the loss of fuzzy information.

Based on the above investigation, we've improved and expanded on previous methods. This paper makes significant contributions on the probabilistic linguistic MAGDM problems:

(1) The paper proposes a new form of linguistic expression trapezoidal Pythagorean fuzzy probabilistic linguistic variables (TrPFPLVs) and trapezoidal Pythagorean fuzzy probabilistic linguistic preference relations (TrPFPLPRs) to express decision-making information.

(2) The paper develop the operational rules, value index and ambiguity index of trapezoidal Pythagorean fuzzy probabilistic linguistic variables (TrPFPLVs).

(3) The paper introduce the new trapezoidal Pythagorean fuzzy probabilistic linguistic priority weight(TrPFPLPW) PROMETHEE approach to aggregate the trapezoidal Pythagorean fuzzy probabilistic linguistic information.

The remainder of this paper is organized as follows. Section 2 reviews the concept of linguistic term sets, probabilistic linguistic term set and probabilistic linguistic preference relationship. Section 3 proposes the TrPFPLVs and TrPFPLPRs to express the MAGDM information, and proposed the operations and operational rules of TrPFPLVs. Section 4 develops TrPFPLPWA-PROMETHEE approach to aggregate the trapezoidal Pythagorean fuzzy probabilistic linguistic information. An emergency decision making model is introduced in Section 5. Section 6 presents a case study: prevention of infectious diseases. The paper ends with some concluding remarks in Section 7.

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II. Preliminaries

This section present some basic definitions to facilitate our presentation.

A. The Linguistic Term Sets

A linguistic value is less precise than a number, however it is closer to human cognitive process. Therefore it is used to solve uncertain problems successfully. The LTSs are used to express the DMs's opinion over the considered objects, which is initial and totally ordered. It can be defined as follows [25]:

Definition 1. *Suppose that $S = \{s_i | i = -\tau, ..., -1, 0, 1, ..., \tau\}$ be a pre-established finite and totally ordered discrete linguistic term set, where s_i denotes the ith linguistic value of S and τ represents the cardinality of S.*

In which $s_i < s_j$ iff $i < j$. Usually, in these cases, it is often required that the linguistic term set satisfies the following additional characteristics [26], [28], [27], [29]:

(1) *There is the negation operator, $Neg(\tilde{s}_i) = s_j$, $j = \tau - i + 1$.*
(2) *The maximum operator:*$\mathrm{Max}(s_i, s_j) = s_i$ *if* $s_i \geq s_j$.
(3) *The minimum operator:*$\mathrm{Min}(s_i, s_j) = s_i$ *if* $s_i \leq s_j$.

Example 1. *The set of nine terms S could be given as follows[30], [31]:*

$S = \{s_{-4} = extremely\ bad(EB), s_{-3} = very\ bad(VB),$
$s_{-2} = bad(B), s_{-1} = little\ bad(LB), s_0 = general(G),$
$s_1 = little\ good(LG), s_2 = good(G), s_3 = extremely$
$good(EG), s_4 = perfect(P)\},$

B. Probabilistic Linguistic Term Sets

In LTSs, each linguistic term value is equally important by default while ignoring the preference information in group decision making. Then Pang et al. [32] defined the PLTS to solve this problem.

Definition 2. *Let $S = \{s_i | i = -\tau, ..., -1, 0, 1, ..., \tau\}$ be a LTS, the PLTS can be defined as:*

$$L(p) = \{L^{(k)}(P^{(k)}) | L^{(k)} \in S, P^{(k)} \geq 0, k = 1, 2, ..., \#L(p), \quad (1)$$

and

$$\sum_{l=k}^{\#L(p)} P^{(k)} \leq 1\} \quad (2)$$

where $L^{(k)}(P^{(k)})$ denotes the kth linguistic term $L^{(k)}$ with the probability $P^{(k)}$, and $\#L(p)$ is the number of linguistic terms in $L(p)$. For the PLTS $L(p)$, let $p^{N(k)} = p^{(k)} / \sum_{k=1}^{\#L(p)} P^{(k)} (k = 1, 2, ..., \#L(p))$, then get the normalized PLTS (NPLTS), denoted as $L(p)^N = \{L^{N(k)}(p^{N(k)}) | k = 1, 2, ..., \#L(p)\}$. The PLTS is composed of the linguistic terms and the corresponding probabilities rather than the linguistic terms only. To be concise, the elements in the PLTS, i,e., $L^{(k)}(p^{(k)}) (k = 1, 2, ..., \#L(p))$, are called probabilistic linguistic elements (PLEs).

To conduct computation, some operations are defined[32]:

(1) $L(p)_1 \oplus L(p)_2 = \{L_3^{(k_3)} p_3^{(k_3)} | k_3 = 1, ..., \#L(p)_3\}$, where $L_3^{(k_3)} = L_1^{(k_1)} \oplus L_2^{(k_2)}$, $p_3^{(k_3)} = p_1^{(k_1)} p_2^{(k_2)} (k_1 = 1, ..., \#L(p)_1; k_2 = 1, ..., \#L(p)_2)$;

(2) $\lambda L(p)_1 = \{\lambda L_1^{(k_1)} p_1^{(k_1)} | k_1 = 1, ..., \#L(p)_1\}$.

The operational laws related to the linguistic terms in PLTS satisfy[30]:

(1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$, , where $\lambda \in [0, 1]$;

(2) $neg(s_\alpha) = s_{-\alpha}$, especially, $neg(s_0) = s_0$.

(3) $\lambda s_\alpha = s_{\lambda\alpha}$

C. Probabilistic Linguistic Preference Relations

In many MAGDM problems, the experts use the PLTSs to express their preference degrees of one alternative over another. The preference relation with the PLTSs is called PLPR. For a finite set of alternatives $X = x_1, x_2, \ldots, x_n (n \geq 2)$, the PLPR is defined on the linguistic evaluation scale $S = \{s_\alpha | \alpha = -\tau, ..., -1, 0, 1, ..., \tau\}$.

Definition 3. *[33]. Given a PLPR $B = (L(p)_{ij})_{n \times n} \subset X \times X$, $L(p)_{ij}$ indicates the preference degrees of the alternative x_i over x_j and $L(p)_{ij}$ satisfies the following conditions: (1) $p_{ij}^{(k)} = p_{ji}^k$, $L_{ij}^{(k)} = neg(L_{ji}^{(k)})$,$L(p)_{ii} = \{s_0(1)\} = \{s_0\}$, $\#L(p)_{ij} = \#L(p)_{ji}$; (2)$L_{ij}^k p_{ij}^k \neq L_{ij}^{(k+1)} p_{ij}^{(k+1)}$ for $i \neq j$,$L_{ji}^k p_{ji}^k \geq L_{ji}^{(k+1)} p_{ji}^{(k+1)}$ for $i \geq j$.*

III. Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Variables and Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Preference Relations

In this part, TrPFPLVs and PPLPRs are introduced.

A. Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Variables

In order to describe the uncertain , complexity and poor structural probabilistic linguistic information more accurately and completely, the paper propose TrPFPLVs to express making decision information.

Definition 4. *Let $\hat{t_k} = ([s_{\varphi_{\tilde{t}_k}}]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$ for all $k = 1, 2, ..., n$ be a PFPLVs, where $s_{\varphi_{\tilde{t}_k}}$ represents a possible value for a linguistic label, If $s_{\varphi_{\tilde{t}_k}} = s_{(\alpha_k, \beta_k, \gamma_k, \theta_k)}$, that is $\hat{t_k} = ([s_{(\alpha_k, \beta_k, \gamma_k, \theta_k)}]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$ be the TrPFPLV. If $s_{\varphi_{\tilde{t}_k}} = s_{(\alpha_k, \beta_k, \gamma_k, \theta_k)}$ and $\beta_k = \gamma_k$, which is triangle Pythagorean fuzzy probabilistic variables(TPFPVs). $\mu_{\tilde{t}_k}(x_k)$ and $\nu_{\tilde{t}_k}(x_k)$ represent the degrees of membership and nonmembership respectively, and satisfy $0 \leq \mu_{\tilde{t}_k}(x_k) \leq 1$, $0 \leq \nu_{\tilde{t}_k}(x_k) \leq 1$ and $0 \leq \mu^2_{\tilde{t}_k}(x_k) + \nu^2_{\tilde{t}_k}(x_k) \leq 1$. $\pi^2_{\tilde{t}_k}(x_k) = 1 - \mu^2_{\tilde{t}_k}(x_k) - \nu^2_{\tilde{t}_k}(x_k)$ is interpreted as indeterminacy degree or a hesitancy degree. $p_{\tilde{r}_k})$ indicates the degree of certainty of his/her preference for the decision problem.*

Such as his/her preference can be expressed as $([\hat{s}_3]; \mu_{\tilde{r}_k}(x_k), \nu_{\tilde{r}_k}(x_k); 0.5)$, it can be interpreted as he/she is 50% sure that the alternative is bad in comparison with other alternative.

if a decision maker prefers the alternative, he/she will use “positive” linguistic labels, such as “bad” or “little bad”, to describe his/her degree of preference. Different “positive” linguistic labels reflect different preference degrees of the DMs. Inspired by [34], [31], the TrPFPLVs set of nine terms T can be given follows:

$\hat{T} = \{\hat{t}_1 = ([s_0, s_1, s_2, s_3]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= extremely\ bad(EB),$
$\hat{t}_2 = ([s_1, s_2, s_3, s_4]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= very\ bad(VB),$
$\hat{t}_3 = ([s_2, s_3, s_4, s_5]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= bad(B),$
$\hat{t}_4 = ([s_3, s_4, s_5, s_6]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= little\ bad(LB),$
$\hat{t}_5 = ([s_4, s_5, s_6, s_7]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= general(G),$
$\hat{t}_6 = ([s_5, s_6, s_7, s_8]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= little\ good(LG),$
$\hat{t}_7 = ([s_6, s_7, s_8, s_9]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= good(G),$
$\hat{t}_8 = ([s_7, s_7, s_9, s_9]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= extremely\ good(EG),$
$\hat{t}_9 = ([s_8, s_9, s_9, s_9]; \mu_{\tilde{t}_k}(x_k), \nu_{\tilde{t}_k}(x_k); p_{\tilde{t}_k})$
$= perfect(P)\}$

Inspired by Xian's work [35], the paper further propose operational laws for TrPFPLVs to facilitate the calculation.

Definition 5. *Let $\hat{T} = (\hat{t}_1, \hat{t}_2, ..., \hat{t}_n)$ be the set of all TrPFPLVs, and $\hat{t}_1 = ([s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\theta_1}]; \mu_{\hat{t}_1}, \nu_{\hat{t}_1}; p_{\hat{t}_1})$, $\hat{t}_2 = ([s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\theta_2}]; \mu_{\hat{t}_2}, \nu_{\hat{t}_2}; p_{\hat{t}_2})$, $\hat{t} = ([s_\alpha, s_\beta, s_\gamma, s_\theta]; \mu_{\hat{t}}, \nu_{\hat{t}}; p_{\hat{t}}) \in \hat{S}$. $\lambda, \lambda_1, \lambda_2 \in [0,1]$, their operational laws and properties are defined as follow:*

(1) $\hat{t}_1 \oplus \hat{t}_2 = ([s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}, s_{\gamma_1+\gamma_2}, s_{\theta_1+\theta_2}]; \sqrt{\mu_{\hat{t}_1}{}^2 + \mu_{\hat{t}_2}{}^2 - \mu_{\hat{t}_1}{}^2\mu_{\hat{t}_2}{}^2}, \nu_{\hat{t}_1}\nu_{\hat{t}_2}; p_{\hat{t}_1}p_{\hat{t}_2})$;

(2) $\lambda \odot \hat{t} = ([s_{\lambda\alpha}, s_{\lambda\beta}, s_{\lambda\gamma}, s_{\lambda\theta}]; \sqrt{1-(1-\mu_{\hat{t}}{}^2)^\lambda}, \nu_{\hat{t}}^\lambda; p_{\hat{t}})$;

(3) $\hat{t}_1 \oplus \hat{t}_2 = \hat{t}_2 \oplus \hat{t}_1; \hat{t}_1 \otimes \hat{t}_2 = \hat{t}_2 \otimes \hat{t}_1$;

(4) $\lambda \odot (\hat{t}_1 \oplus \hat{t}_2) = \lambda \odot \hat{t}_1 \oplus \lambda \odot \hat{t}_2$;

(5) $(\lambda_1 + \lambda_2) \odot \hat{t} = \lambda_1 \odot \hat{t} \oplus \lambda_2 \odot \hat{t}$.

In order to rank alternatives, it is necessary to consider how to compare two TrPFPLVs. Pang et al. [32] defined the comparison between PLTSs, Xian et al. [35] defined the concepts of the compare the TrPFLVs, but there is some set of information can not be compared by the TrPFPVs. Consequently we put forward a method to compare multiple TrPFPVs. In order to do so, in the following, we define the score of TrPFPVs:

Definition 6. *Suppose Let $\hat{t_k} = ([s_{\varphi_{\tilde{r}_k}}]; \mu_{\tilde{r}_k}(x_k), \nu_{\tilde{r}_k}(x_k); p_{\tilde{r}_k})$ for all $k = 1, 2, ..., n$ be a TrPFPLVs, the value index of TrPFPVs are defined as:*

$$L_{\mu_i}(\varphi^+(A_i)) = (m_1\alpha_i + m_2\beta_i + m_3\gamma_i + m_4\theta_i)\mu_i(\varphi^+(A_i)) \tag{3}$$

$$L_{\nu_i}(\varphi^+(A_i)) = (m_1\alpha_i + m_2\beta_i + m_3\gamma_i + m_4\theta_i)\nu_i(\varphi^+(A_i)) \tag{4}$$

$$L_{\mu_i}(\varphi^-(A_i)) = (m_1\alpha_i + m_2\beta_i + m_3\gamma_i + m_4\theta_i)\mu_i(\varphi^-(A_i)) \tag{5}$$

$$L_{\nu_i}(\varphi^-(A_i)) = (m_1\alpha_i + m_2\beta_i + m_3\gamma_i + m_4\theta_i)\nu_i(\varphi^-(A_i)) \tag{6}$$

$$P_{\mu_i}(\varphi^+(A_i)) = (-m_1\alpha_i - m_2\beta_i + m_3\gamma_i + m_4\theta_i)\mu_i(\varphi^+(A_i))p(A_i) \tag{7}$$

$$P_{\nu_i}(\varphi^+(A_i)) = (-m_1\alpha_i - m_2\beta_i + m_3\gamma_i + m_4\theta_i)\nu_i(\varphi^+(A_i))p(A_i) \tag{8}$$

$$P_{\mu_i}(\varphi^-(A_i)) = (-m_1\alpha_i - m_2\beta_i + m_3\gamma_i + m_4\theta_i)\mu_i(\varphi^-(A_i))p(A_i) \tag{9}$$

$$P_{\nu_i}(\varphi^-(A_i)) = (-m_1\alpha_i - m_2\beta_i + m_3\gamma_i + m_4\theta_i)\nu_i(\varphi^-(A_i))p(A_i) \tag{10}$$

where $m_1, m_2, m_3, m_4 \in [0,1]$, $m_1 + m_2 + m_3 + m_4 = 1$. The value of m_1, m_2, m_3, m_4 is different and they depend on the degree of preference of the DMs for MAGDM problems. $\varphi^+(A_i)$ and $\varphi^-(A_i)$ are respectively trapezoidal Pythagorean fuzzy positive dominant flow and trapezoidal Pythagorean fuzzy negative dominant flow, they are introduced in Section 4, which will not be repeated here.

On the basic of the concept of Definition 6, a method compare between two TrPFPLVs is introduced in detail.

Definition 7. *Let $\hat{t_k} = ([s_\alpha, s_\beta, s_\gamma, s_\theta]; \mu_{\tilde{r}}, \nu_{\tilde{r}}; p_{\tilde{r}})$, $\hat{t_1} = ([s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\theta_1}]; \mu_{\tilde{r}_1}, \nu_{\tilde{r}_1}; p_{\tilde{r}_1})$, $\hat{t_2} = ([s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\theta_2}]; \mu_{\tilde{r}_2}, \nu_{\tilde{r}_2}; p_{\tilde{r}_2})$ be any three TrPFPLVs.*

$$E(\varphi^+(A_i)) = \rho_i L_{\mu_i}(\varphi^+(A_i)) + (1-\rho_i)L_{\nu_i}(\varphi^+(A_i)) \tag{11}$$

$$E(\varphi^-(A_i)) = \rho_i L_{\mu_i}(\varphi^-(A_i)) + (1-\rho_i)L_{\nu_i}(\varphi^-(A_i)) \tag{12}$$

$$E(\varphi(A_i)) = \frac{E(\varphi^+(A_i))}{E(\varphi^+(A_i)) + E(\varphi^-(A_i))} \tag{13}$$

$$P(\varphi^+(A_i)) = \rho_i P_{\mu_i}(\varphi^+(A_i)) + (1-\rho_i)P_{\nu_i}(\varphi^+(A_i)) \tag{14}$$

$$P(\varphi^-(A_i)) = \rho_i P_{\mu_i}(\varphi^-(A_i)) + (1-\rho_i)P_{\nu_i}(\varphi^-(A_i)) \tag{15}$$

$$P(\varphi(A_i)) = \frac{P(\varphi^+(A_i))}{P(\varphi^+(A_i)) + P(\varphi^-(A_i))} \tag{16}$$

Where ρ_i is a parameter used to demonstrate the different degrees between two alternatives, $\rho_i \in [0,1]$. Then the paper have

(1) If $E(\varphi(A_1)) < E(\varphi(A_2))$, then A_1 is smaller than A_2, denoted by $A_1 < A_2$.

(2) If $E(\varphi(A_1)) = E(\varphi(A_2))$, then:

(a)If $P(\varphi(A_1)) < P(\varphi(A_2))$, then A_1 is smaller than A_2, denoted by $A_1 < A_2$.

(b) If $P(\varphi(A_1)) = P(\varphi(A_2))$, then A_1 and A_2 represent the same information, denoted by $A_1 \sim A_2$.

Example 2. *If $\varphi^+(A_1) = ([s_{3.3}, s_{3.9}, s_{4.3}, s_{4.75}]; 0.8329, 0.1000; 0.0039)$, $\varphi^-(A_1) = ([s_{1.5}, s_{2.05}, s_{2.65}, s_{3.25}]; 0.8944, 0.1861; 0.0059)$, $\varphi^+(A_2) = ([s_{2.35}, s_{2.85}, s_{3.4}, s_{3.95}]; 0.8456, 0.1414; 0.0020)$, $\varphi^-(A_2) = ([s_{2.45}, s_{3.05}, s_{3.55}, s_{4.1}]; 0.8372, 0.1414; 0.0013)$, be four TrPFPLVs, Though Definition 6 , we can get*

$L_{\mu_1}(\varphi^+(A_1)) = 3.2418$, $L_{\nu_1}(\varphi^+(A_1)) = 0.4108$, $L_{\mu_1}(\varphi^-(A_1)) = 2.1093$, $L_{\nu_1}(\varphi^-(A_1)) = 0.4389$,

$L_{\mu_2}(\varphi^+(A_2)) = 2.6495$, $L_{\nu_2}(\varphi^+(A_2)) = 0.4430$, $L_{\mu_2}(\varphi^-(A_2)) = 2.7557$, $L_{\nu_2}(\varphi^-(A_2)) = 0.4654$,

Then though Definition 7, we can get $E(\varphi^{+}(A_1)) = 0.9770$, $E(\varphi^{-}(A_1)) = 0.7730$

$E(\varphi^{+}(A_2)) = 0.8843$, $E(\varphi^{-}(A_2)) = 0.9235$,

$E(\varphi(A_1)) = 0.5583$ *and* $E(\varphi(A_2)) = 0.4892$, *we can clearly see that* $0.5583 > 0.4892$, *that is to say* $E(\varphi(A_1)) > E(\varphi(A_2))$.

B. Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Preference Relations

In the actual decision-making process, the DMs usually need to express their preference information through pairwise comparison. Because of the ambiguity of information and the incomplete understanding of the preference degree between any pair of alternatives, the DMs can not give the exact membership degree of the preference information. On the basis of Zhang et al. [33], the paper defined the trapezoidal Pythagorean fuzzy probabilistic linguistic preference relations(TrPFPLPRs).

Definition 8. *A Trapezoidal Pythagorean fuzzy probabilistic linguistic preference relations (TrPFPLPRs) on the set* $X = x_1, x_2, \ldots, x_n$ *is represented by a matrix* $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$, *where* $\hat{t_k} = \tilde{r}_{ij} = ([s_{\varphi_{\tilde{r}_{ij}}}]; \mu_{\tilde{r}_{ij}}(x_{ij}), \nu_{\tilde{r}_{ij}}(x_{ij}); p_{\tilde{r}_{ij}})$ *for all* $i, j, k = 1, 2, ..., n$, *where* $\mu_{\tilde{r}_{ij}}(x_{ij})$ *denotes the degree to which the object* x_i *is preferred to the object* x_j, $\nu_{\tilde{r}_{ij}}(x_{ij})$ *indicates the degree to which the object* x_i *is not preferred to the object* x_j, *and* $\pi_{\tilde{r}_{ij}}(x_{ij}) = 1 - \mu^2_{\tilde{r}_{ij}}(x_{ij}) - \nu^2_{\tilde{r}_{ij}}(x_{ij})$ *is interpreted as indeterminacy degree or a hesitancy degree, where* $s_{\varphi_{\tilde{r}_{ij}}}$ *represents a possible value for a linguistic label, if a decision maker prefers A to B, he/she will use "positive" linguistic labels, such as "good" or "little good", to express his/her preference. Different "positive" linguistic labels reflect different preference degrees of the DMs.* $p_{\tilde{r}_{ij}}$ *represents the credibility of* $s_{\varphi_{\tilde{r}_{ij}}}$ *given by the experts when evaluating alternatives and* $s_{\varphi_{\tilde{r}_{ij}}} = s_{(\alpha_i,\beta_i,\gamma_i,\theta_i)}$.

For convenience, let $\hat{t}_k = \tilde{r}_{ij} =< [s_{\varphi_{\tilde{r}_{ij}}}]; \mu_{ij}, \nu_{ij}; p_{ij} >$ *is TrPFPLVs, with the conditions:*

$$\begin{cases} S = \{s_{\varphi_{\tilde{r}_{ij}}} | \varphi_{\tilde{r}_{ij}} = -\tau, ..., -1, 0, 1, ..., \tau\} \\ \mu_{ij}, \nu_{ij} \in [0,1], \mu_{ji}, \nu_{ji} \in [0,1], \mu_{ij} = \nu_{ji}, \mu_{ii} = \nu_{ii} = \sqrt{0.5}, \\ \pi^2_{ij} = 1 - \mu^2_{ij} - \nu^2_{ij} \\ p_{ij} \in [0,1], p_{ij} = p_{ji} \end{cases} \tag{17}$$

where the for all $i, j = 1, 2, ..., n$. *Specially, the TrPFPLPR, in which each TrPFPLTS is normalized, is called the normalized TrPFPLPR (NTrPFPLPR), denoted as* $\tilde{R}^N = (\tilde{r}^N_{ij})_{n\times n}$.

Remark 1.

(1) If neglect the linguistic $s_{\varphi_{\tilde{r}_{ij}}}$ *and* $p_{\tilde{r}_{ij}}$, *that is* $\tilde{R} = (\tilde{r}_{ij})_{n\times n} = (\hat{t}_k)_{n\times n} = (\mu_{\tilde{r}_{ij}}(x_{ij}), \nu_{\tilde{r}_{ij}}(x_{ij}))_{n\times n}$, *we can get trapezoidal Pythagorean fuzzy preference relations (TrPFPRs).*

(2) If $0 \le \mu_{\tilde{r}_{ij}}(x_{ij}) + \nu_{\tilde{r}_{ij}}(x_{ij}) \le 1$, *we can get trapezoidal fuzzy probabilistic linguistic preference relations (TrFPLPRs).*

(3) If neglect the linguistic $s_{\varphi_{\tilde{r}_{ij}}}$ *and* $p_{\tilde{r}_{ij}}$, $0 \le \mu_{\tilde{r}_{ij}}(x_{ij}) + \nu_{\tilde{r}_{ij}}(x_{ij}) \le 1$, *we can get intuitionistic fuzzy preference relations (IFPRs).*

(4) If neglect the probabilistic linguistic term set $p_{\tilde{r}_{ij}}$, *we can get trapezoidal Pythagorean fuzzy linguistic preference relations (TrPFLPRs).*

IV. Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Priority Weighted Averaging PROMETHEE Approach

In this part, we propose trapezoidal Pythagorean fuzzy probabilistic linguistic priority weighted averaging PROMETHEE (TrPFPLPWA-PROMETHEE) approach. In dealing with trapezoidal Pythagorean fuzzy probabilistic linguistic making decision problems, it is not enough for traditional aggregation approach to consider only the fuzziness of preference, but it is very important to consider the fuzzy weight among attributes for poor structural making decision problems, so that we develop the TrPFPLPWA-PROMETHEE approach.

A. Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Priority Weighted Averaging PROMETHEE

The PROMETHEE method is a classic method to deal with making decision problems. The PROMETHEE method is a sorting method based on levels above relationships. By defining the priority function, it can judge the degree of superiority between alternative according to the difference between the attribute values of each alternative. The value of the priority function is $0 \sim 1$, the smaller the function value is, the smaller the priority degree between the two schemes under the same attribute, and the larger the function value is, the greater the priority degree between the two schemes under the same attribute. It not only considers the fuzzy preference among alternatives, but also considers the weight among attributes. At present, many scholars have done a lot of research on the PROMETHEE, for instance, Le Teno et al.[36]. Therefore in this part, we develop the TrPFPLPWA-PROMETHEE approach.

Definition 9. *Let* $\hat{t_k} = \tilde{r}_{ij} = ([s_{\varphi_{\tilde{r}_{ij}}}]; \mu_{\tilde{r}_{ij}}(x_{ij}), \nu_{\tilde{r}_{ij}}(x_{ij}); p_{\tilde{r}_{ij}})$ *for all* $i, j, k = 1, 2, ..., n$, *then we can get TrPFPLPWA-PROMETHEE approach:*

$$\varphi^{+}(A_i) = \frac{1}{m-1} \oplus^m_{k=1,k\neq i} r(A_i, A_k) = \frac{1}{m-1} \oplus^m_{k=1,k\neq i} r_{ik} \tag{18}$$

$$\varphi^{-}(A_i) = \frac{1}{m-1} \oplus^m_{k=1,k\neq i} r(A_k, A_i) = \frac{1}{m-1} \oplus^m_{k=1,k\neq i} r_{ki} \tag{19}$$

where

$$r(A_k, A_i) = \oplus^n_{j=1}(w_j \oplus r^{(j)}_{ik}) = \Big\{[s_{\sum\limits_{i=1}^{n}\omega_j\alpha_i}, s_{\sum\limits_{i=1}^{n}\omega_j\beta_i}, s_{\sum\limits_{i=1}^{n}\omega_j\gamma_i}, s_{\sum\limits_{i=1}^{n}\omega_j\theta_i}]; \sqrt{1 - \prod_{j=1}^{n}(1-\mu^{2(j)}_{ik})^{w_j}}, \prod_{j=1}^{n}\nu^{w_j(j)}_{ik}; p^{(j)}_i p^{(j)}_k\Big\} \tag{20}$$

and $\varphi^{+}(A_i)$ *is a trapezoidal Pythagorean fuzzy positive dominant flow, the larger* $\varphi^{+}(A_i)$ *is, the higher* A_i *is relative to other alternatives.* $\varphi^{-}(A_i)$ *is a trapezoidal Pythagorean fuzzy negative dominant flow, the smaller* $\varphi^{-}(A_i)$ *is, and the*

TABLE I. PREFERENCE RELATIONS OF THE EXPERT 1

	A_1	A_2	A_3	A_4	A_5
A_1	$-$	$([s_3]; 0.7, 0.6; 0.8)$	$([s_4]; 0.7, 0.2; 0.9)$	$([s_{-1}]; 0.6, 0.3; 0.4)$	$([s_2]; 0.8, 0.3; 0.6)$
A_2	$([s_{-3}]; 0.6, 0.7; 0.8)$	$-$	$([s_3]; 0.6, 0.3; 0.7)$	$([s_1]; 0.5, 0.4; 0.4)$	$([s_{-2}]; 0.5, 0.4; 0.5)$
A_3	$([s_{-4}]; 0.2, 0.7; 0.9)$	$([s_{-3}]; 0.3, 0.6; 0.7)$	$-$	$([s_{-1}]; 0.8, 0.5; 0.6)$	$([s_2]; 0.6, 0.4; 0.8)$
A_4	$([s_1]; 0.3, 0.6; 0.4)$	$([s_{-1}]; 0.4, 0.5; 0.4)$	$([s_1]; 0.5, 0.8; 0.6)$	$-$	$([s_3]; 0.7, 0.2; 0.4)$
A_5	$([s_{-2}]; 0.3, 0.8; 0.6)$	$([s_2]; 0.4, 0.5; 0.5)$	$([s_{-2}]; 0.4, 0.6; 0.8)$	$([s_{-3}]; 0.2, 0.7; 0.4)$	$-$

TABLE II. PREFERENCE RELATIONS OF THE EXPERT 2

	A_1	A_2	A_3	A_4	A_5
A_1	$-$	$([s_1]; 0.8, 0.5; 0.7)$	$([s_2]; 0.4, 0.6; 0.8)$	$([s_0]; 0.1, 0.8; 0.4)$	$([s_4]; 0.6, 0.7; 0.6)$
A_2	$([s_{-1}]; 0.5, 0.8; 0.7)$	$-$	$([s_{-4}]; 0.6, 0.3; 0.7)$	$([s_3]; 0.5, 0.7; 0.3)$	$([s_{-1}]; 0.4, 0.6; 0.6)$
A_3	$([s_{-2}]; 0.6, 0.4; 0.8)$	$([s_4]; 0.3, 0.6; 0.7)$	$-$	$([s_2]; 0.4, 0.7; 0.6)$	$([s_1]; 0.5, 0.4; 0.7)$
A_4	$([s_0]; 0.8, 0.1; 0.4)$	$([s_{-3}]; 0.7, 0.5; 0.3)$	$([s_{-2}]; 0.7, 0.4; 0.6)$	$-$	$([s_{-3}]; 0.2, 0.7; 0.4)$
A_5	$([s_{-4}]; 0.7, 0.6; 0.6)$	$([s_1]; 0.6, 0.4; 0.6)$	$([s_{-1}]; 0.4, 0.5; 0.7)$	$([s_3]; 0.7, 0.2; 0.4)$	$-$

TABLE III. PREFERENCE RELATIONS OF THE EXPERT 3

	A_1	A_2	A_3	A_4	A_5
A_1	$-$	$([s_2]; 0.6, 0.6; 0.6)$	$([s_0]; 0.7, 0.4; 0.7)$	$([s_{-3}]; 0.7, 0.8; 0.5)$	$([s_4]; 0.9, 0.4; 0.8)$
A_2	$([s_{-2}]; 0.6, 0.6; 0.9)$	$-$	$([s_{-1}]; 0.5, 0.3; 0.8)$	$([s_2]; 0.9, 0.9; 0.7)$	$([s_4]; 0.3, 0.8; 0.4)$
A_3	$([s_0]; 0.4, 0.7; 0.7)$	$([s_1]; 0.3, 0.5; 0.8)$	$-$	$([s_3]; 0.6, 0.7; 0.7)$	$([s_{-3}]; 0.4, 0.8; 0.8)$
A_4	$([s_3]; 0.8, 0.7; 0.5)$	$([s_{-2}]; 0.9, 0.9; 0.7)$	$([s_{-3}]; 0.7, 0.6; 0.7)$	$-$	$([s_{-2}]; 0.6, 0.3; 0.7)$
A_5	$([s_{-4}]; 0.4, 0.9; 0.8)$	$([s_{-4}]; 0.8, 0.3; 0.4)$	$([s_3]; 0.8, 0.4; 0.8)$	$([s_2]; 0.3, 0.6; 0.7)$	$-$

TABLE IV. DECISION MATRIX OF THE ATTRIBUTES (1)

	A_1	A_2
A_1	$-$	$([s_{3.6}, s_{4.2}, s_{4.6}, s_{5.2}]; 0.8072, 0.1800; 0.3360)$
A_2	$([s_{1.2}, s_{1.8}, s_{2.4}, s_3]; 0.8887, 0.3360; 0.5040)$	$-$
A_3	$([s_{1.2}, s_{1.8}, s_{2.4}, s_3]; 0.9360, 0.1960; 0.5040)$	$([s_{2.8}, s_{3.4}, s_{3.8}, s_{4.2}]; 0.9721, 0.1800; 0.3920)$
A_4	$([s_{3.2}, s_{3.6}, s_{4.2}, s_{4.8}]; 0.8075, 0.0420; 0.0800)$	$([s_{1.2}, s_{1.8}, s_{2.4}, s_3]; 0.7756, 0.2250; 0.0840)$
A_5	$([s_{0.4}, s_1, s_{1.6}, s_{2.2}]; 0.9101, 0.4320; 0.2880)$	$([s_{2.2}, s_{2.8}, s_{3.4}, s_4]; 0.8485, 0.0600; 0.1200)$

smaller the possibility that other alternatives are better than A_i is. Thus, $\varphi^+(A_i)$ and $\varphi^-(A_i)$ can be used to determine the level between alternatives, p_i and p_j can be interpreted as he/she is p_i or p_j sure that the alternative is bad in comparison with other alternative.

Theorem 1. *Let $\hat{t_k} = \tilde{r}_{ij} = ([s_{\varphi_{\tilde{r}_{ij}}}]; \mu_{\tilde{r}_{ij}}(x_{ij}), \nu_{\tilde{r}_{ij}}(x_{ij}); p_{\tilde{r}_{ij}}) \in \hat{T}(i, j, k = 1, 2, ..., n)$ be a collection of TrPFPLVs , the trapezoidal Pythagorean fuzzy positive dominant flow and trapezoidal Pythagorean fuzzy negative dominant flow are also TrPFPLV by Definition 9.*

$$\varphi^+(A_i) = \frac{1}{m-1} \oplus_{k=1, k\neq i}^{m} r(A_i, A_k) \tag{21}$$

$$\varphi^-(A_i) = \frac{1}{m-1} \oplus_{k=1, k\neq i}^{m} r(A_k, A_i) \tag{22}$$

where $r(A_i, A_k) = \Big\{[s_{\sum_{i=1}^{n} \omega_j \alpha_i}, s_{\sum_{i=1}^{n} \omega_j \beta_i}, s_{\sum_{i=1}^{n} \omega_j \gamma_i}, s_{\sum_{i=1}^{n} \omega_j \theta_i}];$

$$\sqrt{1 - \prod_{j=1}^{n} (1 - \mu_{ik}^{2(j)})^{w_j}}, \prod_{j=1}^{n} \nu_{ik}^{w_j(j)}; p_i^{(j)} p_k^{(j)} \Big\} \tag{23}$$

$r(A_k, A_i) = \Big\{[s_{\sum_{i=1}^{n} \omega_j \alpha_k}, s_{\sum_{i=1}^{n} \omega_j \beta_k}, s_{\sum_{i=1}^{n} \omega_j \gamma_k}, s_{\sum_{i=1}^{n} \omega_j \theta_k}];$

$$\sqrt{1 - \prod_{j=1}^{n} (1 - \mu_{ki}^{2(j)})^{w_j}}, \prod_{j=1}^{n} \nu_{ki}^{w_j(j)}; p_k^{(j)} p_i^{(j)} \Big\} \tag{24}$$

Proof 1. *According to Definition 5, we can get first result. In the next, we only prove the above formula by using mathematical induction on n.*

For $n = 2$, since $\hat{t}_1 = ([s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\theta_1}]; \mu_{\hat{t}_1}, \nu_{\hat{t}_1}), \hat{t}_2 = ([s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\theta_2}]; \mu_{\hat{t}_2}, \nu_{\hat{t}_2}),$

then

$$\oplus_{k=1, k\neq i}^{m} r(A_1, A_2) = \omega_1 \odot \hat{s}_1 + \omega_2 \odot \hat{s}_2 = ([s_{\omega_1\alpha_1+\omega_2\alpha_2}, s_{\omega_1\beta_1+\omega_2\beta_2}, s_{\omega_1\gamma_1+\omega_2\gamma_2}, s_{\omega_1\theta_1+\varpi_2\theta_2}]; \sqrt{1 - (1-\mu_1{}^2)^{\omega_1}(1-\mu_2{}^2)^{\omega_2}}, \nu_1{}^{\omega_1}\nu_2{}^{\omega_2})$$

Suppose that, if Eq 20-23 holds for $n = k, k \in N$, that is

$$\oplus_{i=1}^{k} \omega_i \odot \hat{t}_i = ([s_{\sum_{i=1}^{k} \omega_i \alpha_i}, s_{\sum_{i=1}^{k} \omega_i \beta_i}, s_{\sum_{i=1}^{k} \omega_i \gamma_i}, s_{\sum_{i=1}^{k} \omega_i \theta_i}]; \sqrt{1 - \prod_{i=1}^{k} (1 - \mu_{\hat{t}_i}{}^2)^{\omega_i}}, \prod_{i=1}^{k} \nu_{\hat{t}_i}{}^{\omega_i}).$$

Then, according to the operational laws of Definition 5, when $n = k + 1$, we have

TABLE V. DECISION MATRIX OF THE ATTRIBUTES (2)

	A_3	A_4
A_1	$([s_{3.6}, s_{4.2}, s_{4.6}, s_5]; 0.8589, 0.0480; 0.5040)$	$([s_{1.6}, s_{2.2}, s_{2.8}, s_{3.4}]; 0.8932, 0.1920; 0.0800)$
A_2	$([s_2, s_{2.4}, s_3, s_{3.6}]; 0.8888, 0.0270; 0.3920)$	$([s_{3.6}, s_4, s_{4.6}, s_{5.2}]; 0.7996, 0.2520; 0.0840)$
A_3	$-$	$([s_{3.2}, s_{3.6}, s_{4.2}, s_{4.8}]; 0.8485, 0.0245; 0.2520)$
A_4	$([s_{1.6}, s_{2.2}, s_{2.8}, s_{3.4}]; 0.8492, 0.1920; 0.2520)$	$-$
A_5	$([s_{2.4}, s_{2.8}, s_{3.4}, s_4]; 0.8719, 0.1200; 0.4480)$	$([s_{2.8}, s_{3.2}, s_{3.8}, s_{4.4}]; 0.9223, 0.0840; 0.1960)$

TABLE VI. DECISION MATRIX OF THE ATTRIBUTES (3)

	A_5
A_1	$([s_{4.4}, s_5, s_{5.2}, s_{5.4}]; 0.7314, 0.0840; 0.2880)$
A_2	$([s_{2.6}, s_{3.2}, s_{3.6}, s_4]; 0.9459, 0.1920; 0.1200)$
A_3	$([s_{2.4}, s_3, s_{3.6}, s_{4.2}]; 0.9132, 0.1280; 0.4480)$
A_4	$([s_2, s_{2.4}, s_3, s_{3.6}]; 0.8904, 0.0420; 0.1120)$
A_5	$-$

TABLE VII. TRAPEZOIDAL PYTHAGOREAN FUZZY DOMINANT FLOW

	$\varphi^+(A_i)$	$\varphi^-(A_i)$
A_1	$([s_{3.3}, s_{3.9}, s_{4.3}, s_{4.75}]; 0.8329, 0.1000; 0.0039)$	$([s_{1.5}, s_{2.05}, s_{2.65}, s_{3.25}]; 0.8944, 0.1861; 0.0059)$
A_2	$([s_{2.35}, s_{2.85}, s_{3.4}, s_{3.95}]; 0.8456, 0.1414; 0.0020)$	$([s_{2.45}, s_{3.05}, s_{3.55}, s_{4.1}], 0.8372, 0.1414; 0.0013)$
A_3	$([s_{2.4}, s_{2.95}, s_{3.5}, s_{4.05}]; 0.8981, 0.1000; 0.0223)$	$([s_{2.4}, s_{2.9}, s_{3.45}, s_4], 0.6424, 0.1000; 0.0002)$
A_4	$([s_2, s_{2.5}, s_{3.1}, s_{3.86}]; 0.6424, 0.1000; 0.0002)$	$([s_{2.8}, s_{3.25}, s_{3.85}, s_{4.45}], 0.8196, 0.1000; 0.0003)$
A_5	$([s_{1.95}, s_{2.45}, s_{3.05}, s_{3.65}]; 0.8456, 0.1189; 0.0030)$	$([s_{2.8}, s_{3.25}, s_{3.85}, s_{4.45}], 0.8456, 0.1000; 0.0003)$

TABLE VIII. VALUE INDEX OF A_i

	$L_{\mu_i}(\varphi^+(A_i))$	$L_{\nu_i}(\varphi^+(A_i))$	$L_{\mu_i}(\varphi^-(A_i))$	$L_{\nu_i}(\varphi^-(A_i))$
A_1	3.2418	0.4108	2.1093	0.4389
A_2	2.6495	0.4430	2.7557	0.4654
A_3	2.9864	0.3225	2.0450	0.3183
A_4	1.8266	0.2843	2.9301	0.3575
A_5	2.3394	0.3290	3.0230	0.3575

$$\begin{aligned}
&\omega_1 \odot \hat{t}_1 \oplus \omega_2 \odot \hat{t}_2 \oplus ... \oplus \omega_k \odot \hat{t}_k \oplus \omega_{k+1} \odot \hat{t}_{k+1} \\
&= ([s_{\sum_{i=1}^{k} \omega_i \alpha_i}, s_{\sum_{i=1}^{k} \omega_i \beta_i}, s_{\sum_{i=1}^{k} \omega_i \gamma_i}, s_{\sum_{i=1}^{k} \omega_i \theta_i}]; \\
&\sqrt{1 - \prod_{i=1}^{k} (1 - \mu_{\hat{t}_i}{}^2)^{\omega_i}}, \prod_{i=1}^{k} \nu_{\hat{t}_i}{}^{\omega_i}) \\
&\quad \oplus ([s_{\omega_{k+1}\alpha_{k+1}}, s_{\omega_{k+1}\beta_{k+1}}, s_{\omega_{k+1}\gamma_{k+1}}, s_{\omega_{k+1}\theta_{k+1}}], \\
&\sqrt{1 - (1 - \mu_{k+1}^2)^{\omega_{k+1}}}, \nu_{k+1}{}^{\omega_{k+1}}) \\
&= ([s_{\sum_{i=1}^{k+1} \omega_i \alpha_i}, s_{\sum_{i=1}^{k+1} \omega_i \beta_i}, s_{\sum_{i=1}^{k+1} \omega_i \gamma_i}, s_{\sum_{i=1}^{k+1} \omega_i \theta_i}]; \\
&\sqrt{1 - \prod_{i=1}^{k+1} (1 - \mu_{\hat{t}_i}{}^2)^{\omega_i}}, \prod_{i=1}^{k+1} \nu_{\hat{t}_i}{}^{\omega_i}).
\end{aligned}$$

We can see $\oplus_{k=1,k\neq i}^{m} r(A_i, A_k)$ *is still a TrPFPLV, because*

$$\varphi^+(A_i) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r(A_i, A_k)$$

$$\varphi^-(A_i) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r(A_k, A_i)$$

Then the aggregating value by TrPFPLPWA-PROMETHEE approach $\varphi^+(A_i)$ *and* $\varphi^-(A_i)$ *is still a TrPFPLV.*

Theorem 2. *(Commutativity)* $(\hat{t}_1^*, \hat{t}_2^*, ..., \hat{t}_n^*)$ *is any permutation of the TrPFPLVs vector* $(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n)$*, then* $TrPFPLPWA\text{-}PROMETHEE(\hat{t}_1^*, \hat{t}_2^*, ..., \hat{t}_n^*) =$

$$TrPFPLPWA - PROMETHEE(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n) \quad (25)$$

Proof 2. *Let*

$$TrPFPLPWA\text{-}PROMETHEE(\hat{t}_1^*, \hat{t}_2^*, ..., \hat{t}_n^*) = \frac{1}{m-1} \oplus_{i=1}^{n} \omega_i \odot \hat{t}_i^*$$

$$TrPFPLPWA\text{-}PROMETHEE(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n) = \frac{1}{m-1} \oplus_{i=1}^{n} \omega_i \odot \hat{t}_i$$

Since $(\hat{t}_1^*, \hat{t}_2^*, ..., \hat{t}_n^*)$ *is any permutation of* $(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n)$*, then we have* $\hat{t}_i^* = \hat{t}_i$ *for all* $i(i = 1, 2, ..., n)$*. Consequently*

$$\begin{aligned}
&TrPFPLPWA - PROMETHEE(\hat{t}_1^*, \hat{t}_2^*, ..., \hat{t}_n^*) \\
&= TrPFPLPWA - PROMETHEE(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n)
\end{aligned}$$

Theorem 3. *(Idempotency) If* $\hat{t}_i, \hat{t} \in \hat{T}$*, and* $\hat{t}_i = \hat{t}$ *for all* $i(i = 1, 2, ..., n)$*, where* $\hat{t} = ([s_\alpha, s_\beta, s_\gamma, s_\theta]; \sqrt{1 - (1 - \mu_{\hat{t}}{}^2)}, \nu_{\hat{t}})$*, then*

$$TrPFPLPWA\text{-}PROMETHEE(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n) = \frac{1}{m-1} \odot \hat{t}$$

Proof 3. *Since* $\hat{t}_i = \hat{t}$ *for all* $i(i = 1, 2, ..., n)$*, let*

$$\begin{aligned}
&TrPFPLPWA - PROMETHEE(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n) = \\
&\tfrac{1}{m-1} \odot \omega_1 \odot \hat{t}_1 \otimes \omega_2 \odot \hat{t}_2 \otimes ... \otimes \omega_n \odot \hat{t}_n \\
&= \tfrac{1}{m-1} \odot \omega_1 \odot \hat{t} \otimes \omega_2 \odot \hat{t} \otimes ... \otimes \omega_n \odot \hat{t} \\
&= \tfrac{1}{m-1} \odot (\omega_1 + \omega_2 + ... + \omega_n) \odot \hat{t}.
\end{aligned}$$

According to and $\sum_{i=1}^{n} \omega_i = 1$*, we have* $(\omega_1+\omega_2+...+\omega_n) \odot \hat{t}$

$$\begin{aligned}
&= ([s_{\sum_{i=1}^{n} \omega_i \alpha}, s_{\sum_{i=1}^{n} \omega_i \beta}, s_{\sum_{i=1}^{n} \omega_i \gamma}, s_{\sum_{i=1}^{n} \omega_i \theta}]; \sqrt{1 - \prod_{i=1}^{n}(1 - \nu_{\hat{t}})^{\omega_i}}, \prod_{i=1}^{n} \nu_{\hat{t}}^{\omega_i}) \\
&= ([s_\alpha, s_\beta, s_\gamma, s_\theta]; \sqrt{1 - (1 - \mu_{\hat{t}})^{\sum_{i=1}^{n} \omega_i}}, \nu_{\hat{t}}^{\sum_{i=1}^{n} \omega_i}) \\
&= ([s_\alpha, s_\beta, s_\gamma, s_\theta]; \sqrt{1 - (1 - \mu_{\hat{t}}{}^2)}, \nu_{\hat{t}}) \\
&= \hat{t}.
\end{aligned}$$

Hence, $TrPFPLPWA - PROMETHEE(\mu_1, s_1, ..., \mu_n, s) = \frac{1}{m-1} \odot \hat{t}$.

TABLE IX. VALUE INDEX OF A_i

	$E(\varphi^+(A_1))$	$E(\varphi^-(A_1))$
A_1	0.9770	0.7730
A_2	0.8843	0.9235
A_3	0.8552	0.6636
A_4	0.5928	0.8720
A_5	0.7311	0.8906

TABLE X. RANKING RESULTS OF A_i

	$E(\varphi(A_1))$
A_1	0.5583
A_2	0.4892
A_3	0.5631
A_4	0.4047
A_5	0.4508

TABLE XI. RANKING RESULTS BY AGGREGATE METHOD

	$TrPFLECOWA$	$TrPFLECOWMD$
A_1	2.6756	0.0005
A_2	2.2082	0.0003
A_3	2.4536	0.0043
A_4	1.5181	0.0001
A_5	1.9373	0.0002

Theorem 4. *(Monotonicity) Let $(\hat{t}_1^*, ..., \hat{t}_n^*)$ and $(\hat{t}_1, ..., \hat{t}_n)$ are two TrPFPLVs vector, if $\hat{t}_i < \hat{t}_i^*$ for all $i(i = 1, 2, ..., n)$, then*

$$TrPFPLPWA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) < TrPFPLPWA - PROMETHEE(\hat{t}_1^*, ..., \hat{t}_n^*) \quad (26)$$

Proof 4. *Let* $TrPFPLPWA - PROMETHEE(\hat{t}_1^*, ..., \hat{t}_n^*) = \frac{1}{m-1} \oplus_{i=1}^n \omega_i \odot \hat{t}_i^*$ $= \frac{1}{m-1} \odot \hat{t}_a^*$ $TrPFPLPWA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) = \frac{1}{m-1} \oplus_{i=1}^n \omega_i \odot \hat{t}_i = \frac{1}{m-1} \odot \hat{t}_a$ *Since* $\hat{t}_i < \hat{t}_i^*$ *for all* $i(i = 1, 2, ..., n)$, *it follows that* $\hat{t}_a < \hat{t}_a^*, i = 1, 2, ..., n$. *Then* $TrPFPLPWA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) < TrPFPLPWA - PROMETHEE(\hat{t}_1^*, ..., \hat{t}_n^*)$.

Theorem 5. *(Boundedness) Let $\hat{t}_m = \min_i(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n), \hat{t}_M = \max_i(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n)$, then*

$$\frac{1}{m-1} \odot \hat{t}_m \leq TrPFPLPW - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) \leq \frac{1}{m-1} \odot \hat{t}_M \quad (27)$$

Proof 5. *Since $\hat{t}_m \leq \hat{t}_i \leq \hat{t}_M$ for all $i = (i = 1, 2, ..., n)$ and $\sum_{i=1}^n \omega_i = 1$, according to Theorem 1-4, we have*

$TrPFPLPWA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) = \frac{1}{m-1} \odot \oplus_{i=1}^n \omega_i \odot \hat{t}_i$
$\geq \frac{1}{m-1} \odot \omega_1 \odot \hat{t}_m \oplus \omega_2 \odot \hat{t}_m \oplus ... \oplus \omega_n \odot \hat{t}_m = \frac{1}{m-1} \odot (\sum_{i=1}^n \omega_i) \odot \hat{t}_m$
$= \frac{1}{m-1} \odot (\omega_1 + \omega_2 + ... + \omega_n)\hat{t}_m$
$= \frac{1}{m-1} \odot \hat{t}_m$.

$TrPFPLPWA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) = \frac{1}{m-1} \odot \oplus_{i=1}^n \omega_i \odot \hat{t}_i$
$\leq \frac{1}{m-1} \odot \omega_1 \odot \hat{t}_M \oplus \omega_2 \odot \hat{t}_M \oplus ... \oplus \omega_n \odot \hat{t}_M = \frac{1}{m-1} \odot (\sum_{i=1}^n \omega_i) \odot \hat{t}_M$
$= \frac{1}{m-1} \odot (\omega_1 + \omega_2 + ... + \omega_n)\hat{t}_m$
$= \frac{1}{m-1} \odot \hat{t}_M$.

Consequently $\frac{1}{m-1} \odot \hat{t}_m \leq TrPFPLPWA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) \leq \frac{1}{m-1} \odot \hat{t}_M$.

Remark 2. *Let $(\hat{t}_1, \hat{t}_2, ..., \hat{t}_n)$ be a collection of the TrPFPLVs and $W = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weighting vector of the TrPFPLPWA-PROMETHEE with $\omega_i \in [0, 1]$, $\sum_{i=1}^n \omega_i = 1$. Thus,*

(1) If $W = (\omega_1, \omega_2, ..., \omega_n)^T = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})^T$, then we get the TrPFPLPA-PROMETHEE approach as follows: $TrPFPLPA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) = \frac{1}{m-1} \oplus_{i=1}^n \frac{1}{n} \odot \hat{t}_i$
$= \frac{1}{m-1} \odot ([s_{\frac{1}{n}\sum_{i=1}^n \alpha_i}, s_{\frac{1}{n}\sum_{i=1}^n \beta_i}, s_{\frac{1}{n}\sum_{i=1}^n \gamma_i}, s_{\frac{1}{n}\sum_{i=1}^n \theta_i}],$
$\sqrt{1 - [\prod_{i=1}^n (1 - \mu_{\hat{t}_i}{}^2)]^{\frac{1}{n}}}, (\sum_{i=1}^n \nu_{\hat{t}_i})^{\frac{1}{n}})$.

(2) If $W = (\omega_1, \omega_2, ..., \omega_n) = (1, 0, ..., 0)^T$, then $TrPFPLPA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) = \frac{1}{m-1} \odot \max_i \hat{t}_i$.

(3) If $W = (\omega_1, \omega_2, ..., \omega_n) = (0, 0, ..., 1)^T$, then $TrPFPLPA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) = \frac{1}{m-1} \odot \min_i \hat{t}_i$.

(4) If $\omega_i = 1, \omega_j = 0, i \neq j$, then $TrPFPLPA - PROMETHEE(\hat{t}_1, ..., \hat{t}_n) = \frac{1}{m-1} \odot \hat{t}_{a_i}$ *, where $\hat{t}_{a_i}$ is the ith largest of $\mu_{\hat{t}_i}$.*

Remark 3. *Let $\hat{t_k} = \tilde{r}_{ij} = ([s_{\varphi_{\tilde{r}_{ij}}}]; \mu_{\tilde{t}_{ij}}(x_{ij}), \nu_{\tilde{t}_{ij}}(x_{ij}); p_{\tilde{t}_{ij}})$ be TrPFPLSs, if $0 \leq \mu_{\hat{t}}(x_i) + \nu_{\hat{t}}(x_i) \leq 1$, we can get TrIFPLSs.*

Remark 4. *Let $\hat{s} = ([s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}, s_{\theta_i}]; \mu_{\hat{t}_i}, \nu_{\hat{t}_i})$ be the TrPFLV, if $s_{\beta_i} = s_{\gamma_i}$, we have triangle Pythagorean fuzzy probabilistic linguistic variables (TPFPLVs).*

Remark 5. *If $\hat{t_k} = \tilde{r}_{ij} = ([s_{\varphi_{\tilde{t}_{ij}}}]; \mu_{\tilde{t}_{ij}}(x_{ij}), \nu_{\tilde{t}_{ij}}(x_{ij}))$ be*

TrPFLSs, we have trapezoidal Pythagorean fuzzy linguistic variables (TrPFLVs).

Remark 6. *If* $\hat{t_k} = \tilde{r}_{ij} = (\mu_{\tilde{r}_{ij}}(x_{ij}), \nu_{\tilde{r}_{ij}}(x_{ij}); p_{\tilde{r}_{ij}})$ *be PFPLSs, we have Pythagorean fuzzy probabilistic linguistic variables(PFPLVs).*

Remark 7. *If* $\hat{t_k} = \tilde{r}_{ij} = ([s_{\varphi_{\tilde{r}_{ij}}}]; p_{\tilde{r}_{ij}})$ *is a linguistic sets (LSs), if* $s_{\varphi_{\tilde{r}_{ij}}} = s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}, s_{\theta_i}$*; we have trapezoidal probabilistic linguistic sets (TrPLSs).*

V. An Emergency Decision making Model in the Poor Structural's Trapezoidal Pythagorean Fuzzy Probabilistic Linguistic Environment

Step 1. In an emergency decision making problem , there are l alternatives $X = x_1, x_2, ..., x_l$ and q attributes $A = A_1, A_2, ..., A_q$, whose weight vector is $W = (\omega_1, \omega_2, ..., \omega_q)$ be the set of attributes, where the $\sum_{i=1}^{q} \omega_i = 1$. Let $E = e_1, e_2, ..., e_m$ be the set of decision makers.

$$R^{(z)} = \begin{bmatrix} \hat{r}_{11}^{(z)} & \hat{r}_{12}^{(z)} & ...... & \hat{r}_{1q}^{(z)} \\ \hat{r}_{21}^{(z)} & \hat{r}_{22}^{(z)} & ...... & \hat{r}_{2q}^{(z)} \\ . & . & ...... & . \\ . & . & ...... & . \\ . & . & ...... & . \\ \hat{r}_{l1}^{(z)} & \hat{r}_{l2}^{(z)} & ...... & \hat{r}_{lq}^{(z)} \end{bmatrix}$$

where $\hat{r}_{ij}^{(z)} = ([s_\alpha.s_\beta, s_\gamma, s_\theta]; \mu_{\hat{t}_{ij}}, \nu_{\hat{t}_{ij}}; p_{ij})$ be a TrPFPLV, and $\mu_{\tilde{r}_{ij}}, \nu_{\tilde{s}_{ij}}, p_{ij} \in [0,1], 0 \leq (\mu_{\tilde{t}_{ij}})^2 + (\nu_{\tilde{t}_{ij}})^2 \leq 1$. $\hat{r}_{ij}^{(z)} = ([s_\alpha.s_\beta, s_\gamma, s_\theta]; \mu_{\hat{r}_{ij}}, \nu_{\hat{r}_{ij}}; p_{ij})$ represents expert z's preference for attribute x_i in attribute x_j.

Step 2. By formula (19): $r(A_k, A_i) = \oplus_{j=1}^{n}(w_j \oplus r_{ik}^{(j)}) = \left\{[s_{\sum_{i=1}^{n}\omega_j\alpha_i}, s_{\sum_{i=1}^{n}\omega_j\beta_i}, s_{\sum_{i=1}^{n}\omega_j\gamma_i}, s_{\sum_{i=1}^{n}\omega_j\theta_i}]; \sqrt{1 - \prod_{j=1}^{n}(1-\mu_{ik}^{2(j)})^{w_j}}, \prod_{j=1}^{n}\nu_{ik}^{w_j(j)}; p_i^{(j)}p_k^{(j)}\right\}$ to obtain the collective overall preference TrPFPLV and construct the TrPFPLPR matrices of the attributes $T^{(z)} = (\hat{t}_{ij}^{(z)})_{l\times l}$:

$$T^{(z)} = \begin{bmatrix} \hat{t}_{11}^{(z)} & \hat{t}_{12}^{(z)} & ...... & \hat{t}_{1l}^{(z)} \\ \hat{t}_{21}^{(z)} & \hat{t}_{22}^{(z)} & ...... & \hat{t}_{2l}^{(z)} \\ . & . & ...... & . \\ . & . & ...... & . \\ . & . & ...... & . \\ \hat{t}_{l1}^{(z)} & \hat{t}_{l2}^{(z)} & ...... & \hat{t}_{ll}^{(z)} \end{bmatrix}$$

where $\hat{t}_{pn}^{(z)} = ([s_\alpha.s_\beta, s_\gamma, s_\theta]; \mu_{\hat{r}_{pn}}, \nu_{\hat{r}_{pn}}; p_{pn})$ be a TrPFPLV, and $\hat{t}_{pn}^{(z)}$ represents the degree to which attribute A_p is superior to attribute A_n

Step 3. By Definition 9, further calculate trapezoidal Pythagorean fuzzy positive dominant flow and trapezoidal Pythagorean fuzzy negative dominant flow.

$$\varphi^+(A_i) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r(A_i, A_k) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r_{ik}$$

$$\varphi^-(A_i) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r(A_k, A_i) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r_{ki}$$

Step 4. Calculate $L_{\mu_i}(\varphi^+(A_i))$, $L_{\nu_i}(\varphi^+(A_i))$, $L_{\mu_i}(\varphi^-(A_i))$, $L_{\nu_i}(\varphi^-(A_i))$ by Definition 6.

Step 5. Calculate$E(\varphi^+(A_i))$ and $E(\varphi^-(A_i))$ by Definition 7.

Step 6. Finally, we can calculate $E(\varphi(A_i))$ to obtain the ranking results by Definition 7.

VI. Case Study: Application to Prevention of Infectious Diseases

In this section , we apply the above method to an application in emergency decision support for the prevention of infectious diseases to illustrate the proposed method, and supplies some discussions about the results. In late December 2019, the outbreak of COVID-19 in Wuhan, China, brought global attention to infectious diseases. An epidemic of infectious diseases appears somewhere and is difficult to control. To address the problem, the local health organization invited five experts(e_1, e_2, e_3, e_4) to make decisions on population density(A_1), air pollution index(A_2), number of parks and entertainment equipment(A_3), density of restaurants(A_4), density of cultural and educational centers(A_5) in a certain area to prevent and control the spread the disease.

Step 1. The experts made a decision based on the factors that might affect the infectious disease and established a decision matrix, as shown in Table I to Table III:

Step 2. Constructing the matrix by formula (7), and supposed $\omega_1 = (0.2)$, $\omega_2 = (0.2)$, $\omega_3 = (0.2)$, $\omega_4 = (0.2)$, $\omega_5 = (0.2)$, the decision matrix of the attributes which is shown in Tables IV, V and VI:

Step 3. Calculate trapezoidal Pythagorean fuzzy positive dominant flow and trapezoidal Pythagorean fuzzy negative dominant flow:

$\varphi^+(A_1) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r(A_1, A_k) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r_{1k}$
$= \frac{1}{5-1} * \{([s_{3.6}, s_{4.2}, s_{4.6}, s_{5.2}]; 0.8072, 0.1800; 0.3360) \oplus ([s_{3.6}, s_{4.2}, s_{4.6}, s_5]; 0.8589, 0.0480; 0.0.5040)$
$\oplus ([s_{1.6}, s_{2.2}, s_{2.8}, s_{3.4}]; 0.8932, 0.1920; 0.0800) \oplus ([s_{4.4}, s_5, s_{5.2}, s_{5.4}]; 0.7314, 0.0840; 0.2880)\}$
$= ([s_{3.3}, s_{3.9}, s_{4.3}, s_{4.75}]; 0.8329, 0.1000; 0.0039)$

$\varphi^-(A_1) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r(A_k, A_1) = \frac{1}{m-1} \oplus_{k=1,k\neq i}^{m} r_{k1}$
$= \frac{1}{5-1} * \{([s_{1.2}, s_{1.8}, s_{2.4}, s_3]; 0.8887, 0.3360; 0.5040) \oplus ([s_{1.2}, s_{1.8}, s_{2.4}, s_3]; 0.9360, 0.1960; 0.5040)$
$\oplus ([s_{3.2}, s_{3.6}, s_{4.2}, s_{4.8}]; 0.8075, 0.0420; 0.0800) \oplus ([s_{0.4}, s_1, s_{1.6}, s_{2.2}]; 0.9101, 0.4320; 0.2880)\}$
$= ([s_{1.5}, s_{2.05}, s_{2.65}, s_{3.25}]; 0.8944, 0.1861; 0.0059)$

Similarly, we can get

$\varphi^+(A_2) == ([s_{2.35}, s_{2.85}, s_{3.4}, s_{3.95}]; 0.8456, 0.1414; 0.0020)$

$\varphi^-(A_2) == ([s_{2.45}, s_{3.05}, s_{3.55}, s_{4.1}], 0.8372, 0.1414; 0.0013)$

$\varphi^+(A_3) = ([s_{2.4}, s_{2.95}, s_{3.5}, s_{4.05}]; 0.8981, 0.1000; 0.0223)$

$\varphi^-(A_3) = ([s_{2.4}, s_{2.9}, s_{3.45}, s_4], 0.8099, 0.1000; 0.0223)$

$\varphi^+(A_4) = ([s_2, s_{2.5}, s_{3.1}, s_{3.86}]; 0.6424, 0.1000; 0.0002)$

$\varphi^-(A_4) = ([s_{2.8}, s_{3.25}, s_{3.85}, s_{4.45}], 0.8196, 0.1000; 0.0003)$

$\varphi^+(A_5) = ([s_{1.95}, s_{2.45}, s_{3.05}, s_{3.65}]; 0.8456, 0.1189; 0.0030)$

$\varphi^-(A_5) = ([s_{2.8}, s_{3.25}, s_{3.85}, s_{4.45}], 0.8456, 0.1000; 0.0003)$

Hence, the trapezoidal Pythagorean fuzzy dominant flow can be shown in Table VII.

Step 4. Calculate $L_{\mu_i}(\varphi^+(A_i))$, $L_{\nu_i}(\varphi^+(A_i))$, $L_{\mu_i}(\varphi^-(A_i))$, $L_{\nu_i}(\varphi^-(A_i))$ by Definition 6, which is shown in Table VIII.

Step 5. Calculate$E(\varphi^+(A_i))$ and $E(\varphi^-(A_i))$ by Definition 9.

Therefore, Value index of A_i is shown in Table IX.

Step 6. Finally, we can calculate $E(\varphi(A_i))$ to obtain the ranking results by Definition 9.

Obviously, from Table X, the ranking order is shown as

$$A_3 > A_1 > A_2 > A_5 > A_4$$

We can see that the third influencing -number of parks and entertainment equipment-is the biggest for infectious diseases. the second is the effect of population density. Thus it can be seen that the prevention and control of infectious diseases should be prioritized from the park recreation facilities. Under certain control, then start from the population mobility, because of the high population density.

To illustrate the rationality of our proposed approach in dealing with emergent decision problems, some comparative analysis is performed for the above example if we use the aggregate method shown in Table XI.

The results calculated by different methods are shown in the Fig. 1.

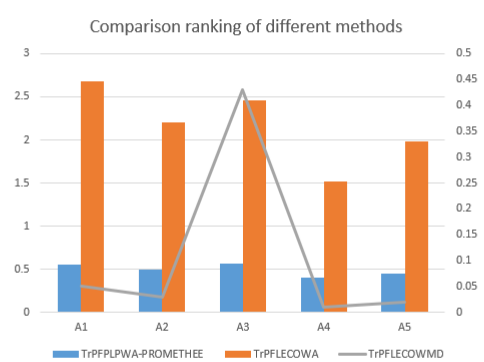

Fig. 1. Ranking Results

We can obtain $A_1 > A_3 > A_2 > A_5 > A_4$ by TrPFLECOWA and $A_3 > A_1 > A_2 > A_5 > A_4$ by TrPFLECOWMD. It can be seen from the results that the influencing factors A_1, A_3 of infectious diseases have different order. The reason is that some information is lost in the fuzzy linguistic environment due to indeterminacy, the relationship between influencing factors is ignored, and the occurrence probability of each influencing factor is not considered in the emergency situation. It is difficult for experts to make reasonable evaluation when making uncertain decisions in emergency situations, hence it is very important to introduce probability language in the poor structural's emergency decisions. Compared with other decision making methods, this paper proposed method has the following advantages:

(1) The TrPFPLVs can describe the fuzzy probability language and its probability, and it is reasonable to make decision in the uncertain environment of emergency.

(2) The TrPFPLPWA-PROMETHEE approach mainly considers the deviation measure between alternatives or attributes, hence it can deal with the decision information with incomplete structure. It is more malleable and applicable in the poor structural's emergency decisions.

VII. Conclusion

The existing methods are limited and defective in terms of tackling MAGDM problems with TrPFPLVs. In addition, the researches on the aggregate operator for TrPFPLV are blank. The aim of this paper is to solve these problems. The case analysis show that the new decision-making approach can not only derive the ideal alternative efficiently, but conquer demerits of the existing methods. The contributions of this paper are summarized in the following:

(1) Compared with trapezoidal Pythagorean fuzzy linguistic variables in [37], [38], [39], [40], [41], a finer variable has been introduced to deal with complex decision-making environment, a new form of linguistic expression trapezoidal Pythagorean fuzzy probabilistic linguistic variables (TrPFPLVs) have been presented to express decision-making information.

(2) Compared with fuzzy linguistic weight averaging operator in [30], [31], [37], [35] , a more complete operator has been introduced to aggregate the trapezoidal Pythagorean fuzzy probabilistic linguistic information, which is the new trapezoidal Pythagorean fuzzy probabilistic linguistic priority weight averaging (TrPFPLPWA) PROMETHEE approach.

(3) Relying on trapezoidal Pythagorean fuzzy probabilistic linguistic variables, this paper develop the operational rules, value index and ambiguity index of trapezoidal Pythagorean fuzzy probabilistic linguistic variables (TrPFPLVs).

In future studies, the TrPFPLPWA-PROMETHEE approach shall further combine with different decision-making models and extend the applications of the proposed method to other domains.

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Ethical Statement

(1) This paper is an original research achievement independently obtained by the authors. The content of the paper

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(3) The authors declare that they have no conflict of interest.

(4) The authors are aware of and consent to the publication of this manuscript in the International Journal of Advanced Computer Science and Applications(IJACSA).

AUTHORSHIP CONTRIBUTIONS

(1) This paper propose a new form of linguistic expression trapezoidal Pythagorean fuzzy probabilistic linguistic variables (TrPFPLVs) and trapezoidal Pythagorean fuzzy probabilistic linguistic preference relations (TrPFPLPRs) to express decision-making information.

(2) This paper develop the operational rules,value index and ambiguity index of trapezoidal Pythagorean fuzzy probabilistic linguistic variables (TrPFPLVs).

(3) This paper introduce the new trapezoidal Pythagorean fuzzy probabilistic linguistic priority weight(TrPFPLPW) PROMETHEE approach to aggregate the trapezoidal Pythagorean fuzzy probabilistic linguistic information.

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