

# Reverse Supply Chain Management Through a Quantity Flexibility Contract: A Case of Stochastic Remanufacturing Capacity

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**Abstract**—This article investigates a two-echelon reverse supply chain (RSC) where a third-party logistics provider charges customers to return outdated products. A green manufacturer refurbishes qualified returned products through the remanufacturing process. Remanufacturing capacity is considered a stochastic variable. Under the volatility of remanufacturing capacity, some likely examined, and qualified products could not be remanufactured. If a collected product cannot be processed, it should be salvaged at a lower value and be perceived as a lost profit. In such scenarios, increasing the quantity of returned outdated products is suitable if there is a strong possibility of enough capacity in the remanufacturing process. This paper develops a stochastic model to identify the optimal order quantity under diverse contracts, including wholesale price, centralized, and quantity flexibility contracts. Under the quantity flexibility contract, the green supplier might cancel its preliminary order in a restricted quantity. Additionally, third-party logistics supplier offers a restricted quantity above the initial order to minimize understocking during peak seasons. Our numerical experiments demonstrate that the suggested quantity flexibility can coordinate the examined RSC under the volatility of remanufacturing capacity. Contrary to wholesale and centralized contracts, quantity flexibility is a more practical alternative from the perspective of participants' profitability.

**Keywords**—Reverse supply chain; channel coordination; uncertain remanufacturing capacity; quantity flexibility contract

## I. INTRODUCTION

A reverse supply chain (RSC) comprises a series of actions that collect obsolete products from customers and return them to the original manufacturer or recycler for reprocessing or proper disposal [1]–[3]. The application of the RSC has financial, social, and environmental benefits. One of the fundamental obligations of a company to recycle its commodities is setting up an RSC; Xerox, Nike, Adidas, Sony, and Siemens are successful firms that built up their RSC [4]. Moreover, Consumers' sensitivity to environmental problems has driven corporations to highlight their recycling potential [5].

Typically, young customers in South Asia are aware of the eco-friendliness of a product's components; in some regions of the Middle East, this figure approaches 80 percent. Due to the increased environmental awareness among consumers and government intervention, many businesses have begun creating green goods [6]. Numerous goods are suitable for

remanufacturing or recycling. Electronic items, for instance, are among the most ideal for recycling due to their short life cycle, modular design, and the kind of raw material utilized [7]. For instance, 76 percent of a camera's components may be used multi times [8]. As a result, companies now put their efforts into green operations, specifically RSC, fully take advantage of their benefits.

Since 2020, the COVID-19 scenario has wreaked havoc on SCs; it produces extreme supply and demand oscillations, disrupting the corporate system. Upstream participants cannot foresee demand, and downstream participants cannot fulfill their responsibilities [9], [10]. A measure we can take for this problem is adopting coordination mechanisms [11]. SC members can set up coordinated contracts to make an SC that works well together. For example, a study by Bakhshi and Heydari (2021) demonstrated the value of investigating an option contract for coordinating the interaction between an e-retailer and a 3PL; they also compared the investigated contract with a penalty-based contract; the results show that the option contract incredibly increases the SC' total profit in comparison with other contracts. Some examples of these kinds of contracts are quantity flexibility contracts. Expressly, a great deal of research work has confirmed their applications in inventory management problems, and their findings have revealed that they can handle uncertainty well [12], [13]. Quantity flexibility contract has many capabilities and has been validated in various applications. For instance, Kord and Samouei (2023) studied a quantity flexibility contract for managing a humanitarian supply chain to buy a spot market under demand uncertainty. The results indicated the powerful performance of NPGA in terms of most evaluation indicators. The results indicated the powerful performance of NPGA in terms of most evaluation indicators.

Based on a quantity flexibility contract, a buyer can adjust its initial purchase up or down within a specified volume range. Thus, the buyer must purchase a minimum quantity, while the supplier must provide additional amounts if necessary [5], [14]. The industry can improve numerous real-world instances by implementing quantity flexibility contracts. Sun Microsystems purchases its workstations through QF contracts. Nippon Otis, a maker of elevator equipment, utilizes a quantity flexibility contract with the Tsuchiya plant, a manufacturer of components and switches. Toyota Motor Corporation, IBM, and Hewlett-Packard have also utilized quantity flexibility contracts [15].

This study aims to determine an effective strategy for stabilizing 3PL's position in an RSC. Meanwhile, this work highlights the necessity of optimal ordering decisions. In this respect, we try to answer the following questions:

QS1: What is the GS's optimal order quantity within concluded contracts?

QS2: Can the suggested quantity flexibility contract enhance the parties' profitability?

QS3: Which contract is preferred?

We evaluate a two-echelon RSC involving a green supplier who can refurbish eligible outdated products and a third-party logistics provider that collects such products from customers despite the unpredictability of remanufacturing capacity. We investigate the impact of a quantity flexibility contract in which the third-party logistics provider is the Stackelberg game leader. Under the quantity flexibility contract, the green supplier is permitted to reserve items without incurring a reservation charge and must buy a minimum amount. In contrast, the third-party logistics provider will be needed to collect additional products if required.

To the best of our knowledge, prior research has focused primarily on the significance of quantity flexibility contracts in forwarding supply chains with uncertain demand; in contrast, the current study investigates the impact of the quantity flexibility contract on an RSC with uncertain remanufacturing capacity. In other words, contrary to earlier studies, uncertainty has migrated from demand to supply. We based our model on this gap in the research.

The remainder of the article is categorized as follows: Our model setting and assumptions are illustrated in detail in Section II. Section III derives the optimum decisions in a variety of models. In Section IV, numerical analysis is used to validate the models. Section V reveals managerial insights and conclusions.

## II. MODEL SETTING

### A. RSC Structure

This study examines a two-echelon RSC, which includes a green supplier (GS) and a third-party logistics provider (3PL). GS recycles outdated products qualified enough to return to the market. However, the remanufacturing capacity on the GS side is susceptible to uncertainty. On the other hand, 3PL collects outdated products from consumers and returns the acceptable ones to GS after a final assessment. GS makes profits by selling remanufactured products directly through the market. Indeed, GS purchases outdated products from 3PL at the cost of  $c_{st}$ /unit and, after the recycling process, sells recycled products at the price of  $w$ /unit into the market.

In the current RSC, GS sets an order amount for 3PL and determines the order quantity. However, 3PL may not provide the whole order quantity because of the ineligible collected product. Accordingly, all qualified orders are shipped directly to GS. To sum up, GS may get fewer amounts of the placed ordered products. The parameters used in this study are summarized in Table I.

TABLE I. LISTED PARAMETERS AND DECISION VARIABLES

Parameters	Description
$m$	A continuous random variable with distribution function $g(\cdot)$ and cumulative function $G(\cdot)$ , standing for remanufacturing capacity.
$c_{st}$	The unit fee paid by GS to 3PL to collect outdated products.
$c_i$	Inspection fee per unit of collected products at the 3PL site.
$\alpha$	The clearance percentage of products inspected by 3PL.
$c_m$	Inventory cost of amassed products collected by 3PL.
$c_r$	The unit cost of refurbishing at the GS's site
$c_p$	Cost per unit of preparation for refurbishment by the GS.
$c_s$	Unit shipping cost for 3PL.
$c_{tc}$	Reward offered by the 3PL for the return of each outdated product.
$w$	The wholesale price of refurbished products.
$s$	The value of salvage per unit for the 3PL.
$d$	Downward adjustment parameter in the quantity flexibility contract.
$u$	Upward adjustment parameter in the quantity flexibility contract.
Decision variables	
$q_w$	GS's order amount for 3PL within the wholesale price contract.
$q_c$	GS's order amount for 3PL within the centralized contract.
$q_f$	GS's order amount for 3PL within the quantity flexibility contract.

**B. Assumptions**

The primary purpose of this study is to highlight the significance of optimum ordering/pricing choices and the management of the possible costs of overstocking or understocking in the analyzed RSC under uncertainty. Hence, the generated models include the following modeling assumptions:

Assumption 1. GS’s remanufacturing capacity is contaminated by uncertainty.

Based on Assumption 1,  $x$  is a continuous random variable with a distribution function  $g(x)$  and cumulative function  $G(x)$  [4].

Assumption 2. 3PL rewards customers with a per-unit fee for the return of outdated products.

Assumption 3. 3PL delivers a definite proportion of qualified collected products to GS after the final assessment.

According to the third assumption, all of the obsolete products collected by 3PL are ineligible for recycling. On the 3PL side, it is thus essential to analyze gathered products and detect discarded ones [16].

Assumption 4. Customers can buy all recycled products at a pre-set price  $w$ .

**III. MATHEMATICAL MODELING**

Suppliers need an ordering system to control the production/inventory process; thus, they pressure downstream partners to place their orders before the selling season. Nonetheless, uncertainties raise overstock/understock risks and discourage partners from setting early orders. Ordering in SCs may be resolved by concluding a suitable contract between two parties. Offering a reservation policy is a measure we can take to encourage GS to purchase more/earlier. To solve the problem, this research compares a regular wholesale pricing contract with a quantity flexibility contract [5] (Appendix).

**A. Wholesale Price Contract**

Under the wholesale price contract, each participant decides independently of the other participants’ interests. GS determines the order quantity placed for 3PL to optimize its profit, and 3PL inspects the collected products. Then, GS refurbishes the received products following the acknowledged remanufacturing capability [17]. Based on the order quantity and realized remanufacturing capacity, the profit function of GS is formulated as follows:

$$\Pi_{GS}(q_w) = \begin{cases} (w - c_r)m - (c_{st} + c_p)\alpha q_w + s(\alpha q_w - m) & \alpha q_w > m \\ (w - c_r - c_{st} - c_p)\alpha q_w & \alpha q_w < m \end{cases} \quad (1)$$

According to Eq. (1), only  $m$  units of products are remanufactured and sold if there is inadequate remanufacturing capacity; the remaining products are salvaged at a value of  $s$  ( $(\alpha q_w - m) s$ ). Consequently, the remanufacturer’s profit function comprises income from selling  $m$  units on the market and salvage value. Moreover, A unit cost will be paid to 3PL

for each unit received, which under any condition would be  $\alpha q_w$ . Now, the expected profit function of GS is:

$$E(\Pi_{GS}(q_w)) = (w - c_r) \left( \int_0^{\alpha q_w} m g(m) dm + \int_{\alpha q_w}^{\infty} \alpha q_w g(m) dm \right) - (c_{st} + c_p)\alpha q_w + \int_0^{\alpha q_w} s(\alpha q_w - m) g(m) dm \quad (2)$$

**Proposition 1.**  $E(\Pi_{GS}(q_w))$  is concave in  $q_w$  and  $q_w^*$  That maximizes GS’s profit function will be calculated as follows:

$$q_w^* = \frac{G^{-1}\left(\frac{w - c_r - c_{st} - c_p}{w - c_r - s}\right)}{\alpha} \quad (3)$$

Contingent on Eq. (3), the cost of  $c_{st}$  is a criterion to determine the trade-off between overstock and understock costs. In this respect, the overstock cost is  $c_{st} + c_m - w$  and the understock cost will be  $w - c_r - c_{st} - c_p$ . Now let us show the 3PL profit function, which is obtained as follows:

$$\Pi_T = \left( c_{st} - c_{tc} - \frac{c_i}{\alpha} - c_m - c_s \right) \alpha q_w \quad (4)$$

The profit function of 3PL reflects the revenue generated by selling outdated products to GS and the associated expenses, such as the cost of returning products, inspection, holding, and shipment. Now, we go further and investigate the centralized contract in the subsequent sections.

**B. Centralized Contract**

Under the centralized decision-making system, we attempt to maximize the RSC’s overall profit. Under the centralized scenario, the predicted profit function of RSC may be computed as follows:

$$\Pi_{RSC}(q_c) = \begin{cases} (w - c_r)m - c_p \alpha q_c + s(\alpha q_c - m) + \left(-c_{tc} - \frac{c_i}{\alpha} - c_m - c_s\right) \alpha q_c & \alpha q_c > m \\ (w - c_r - c_p)\alpha q_c + \left(-c_{tc} - \frac{c_i}{\alpha} - c_m - c_s\right) \alpha q_c & \alpha q_c < m \end{cases} \quad (5)$$

In Eq. (5), 3PL reviews all returned outdated items and accepts quantities that can be refurbished; following storage, the qualified outdated products are dispatched to GS. Then, GS prepares the items for refurbishing. Consequently, when there is insufficient remanufacturing capacity, as indicated by the expression  $\alpha q_c > m$ , only  $m$  items may be introduced into the remanufacturing process. Accordingly, the total expected profit function of RSC is determined by the following:

$$E(\Pi_{RSC}(q_c)) = (w - c_r) \left( \int_0^{\alpha q_c} m g(m) dm + \int_{\alpha q_c}^{\infty} \alpha q_c g(m) dm \right) - c_p \alpha q_c + \int_0^{\alpha q_c} s(\alpha q_c - m) g(m) dm + \left(-c_{tc} - \frac{c_i}{\alpha} - c_m - c_s\right) \alpha q_c \quad (6)$$

**Proposition 2.**  $E(\Pi_{RSC}(q_c))$  is concave in  $q_c$  and  $q_c^*$  that maximizes RSC’s profit function will be calculated as follows:

$$q_c^* = \frac{G^{-1}\left(\frac{w - c_r - c_p - c_{tc} - \frac{c_i}{\alpha} - c_m - c_s}{w - c_r - s}\right)}{\alpha} \quad (7)$$

It should be mentioned that the primary insight gained from Proposition 2 is that  $G(q_c)$  belongs to a newsvendor concern, i.e., the trade-off acquired between overstock and understock

costs is conducted depending on  $(c_{tc} + \frac{c_i}{\alpha} + c_m + c_s)$  in the situation of volatile remanufacturing capacity.

### C. Quantity Flexibility Contract

This section examines the scenario in which GS and 3PL implement the quantity flexibility contract. The 3PL is the leader, while the GS is the follower. The following is the order of events under this contract: (1) An offer is made for a contract with parameters  $(w, q_f, d, u)$ .  $w$  is the wholesale price following the realization of remanufacturing capacity.  $0 \leq d \leq 1$  is a parameter that determines the acceptable range and gauges the flexibility.  $u \geq 0$  is the upward adjustment parameter. (2) Given the contract, GS determines the reservation quantity  $q_f$ . Therefore, the permissible range is set as  $[daq_f, (1+u)\alpha q_f]$ . Note that GS is not required to pay the reservation fee. (3) After observing the reserved quantity, 3PL gathers outdated products at least equal to the lower limit of the permissible range, i.e.,  $daq_f$ . (4) The M capacity for remanufacturing is realized. (5) 3PL delivers products to GS in accordance with M's realized capacity and after the final examination. According to the order quantity  $q_f$  and realized remanufacturing capacity, the profit function of GS will be:

$$\Pi_{GS}(q_f) = \begin{cases} wm + (-c_r - c_p)m - c_{st}daq_f + s(daq_f - m) & 0 < m < daq_f \\ wm + (-c_r - c_{st} - c_p)m & daq_f < m < (1+u)\alpha q_f \\ (w - c_r - c_{st} - c_p)(1+u)\alpha q_f & m > (1+u)\alpha q_f \end{cases} \quad (8)$$

In the first condition (i.e.,  $0 < m < daq_f$ ) of Eq. (8), the first term represents the total revenue from selling  $m$  units, the second and third terms stand for operation costs related to purchasing and refurbishing obsolete products, and the third term means the total revenue from salvaging unsold products. In the second condition ( $daq_f < m < (1+u)\alpha q_f$ ), the first term represents the entire income from selling  $m$  units, and the second reveals the operation costs. Finally, based on the third condition ( $m > (1+u)\alpha q_f$ ) GS sells  $(1+u)\alpha q_f$  units in the market, which is the highest band of purchased products according to the concluded contract. In this regard, the expected profit function of GS will be determined as follows:

$$E(\Pi_{GS}(q_f)) = (w - c_r - c_p) \left( \int_0^{(1+u)\alpha q_f} mg(m)dm + \int_{(1+u)\alpha q_f}^{\infty} (1+u)\alpha q_f g(m)dm - c_{st} \left( \int_0^{daq_f} daq_f g(m)dm + \int_{daq_f}^{(1+u)\alpha q_f} mg(m)dm + \int_{(1+u)\alpha q_f}^{\infty} (1+u)\alpha q_f g(m)dm + \int_0^{daq_f} s(daq_f - m)g(m)dm \right) \right) \quad (9)$$

The following Proposition will obtain the optimum amount of GS's order under the quantity flexibility contract.

**Proposition 3.**  $E(\Pi_{GS}(q_f))$  is concave in  $q_f$  and  $q_f^*$  that maximizes GS's profit function will be calculated by solving the following equation:

$$(w - c_r - c_p - c_{st})(1+u)\alpha^2 q_f G((1+u)\alpha q_f) - c_{st}(d\alpha)^2 q_f \left( 1 - G((1+u)\alpha q_f) \right) + s(d\alpha)^2 q_f G(daq_f) = 0 \quad (10)$$

$d$  is a crucial aspect of the quantity flexibility contract. Therefore, as  $d$  increases, GS decreases its order quantity. In fact, by increasing  $d$ , GS becomes reluctant to place an order with 3PL. Now let us show the 3PL profit function, which is established as follows:

$$\Pi_T = \begin{cases} (c_{st} - c_s)daq_f - (c_{tc} + \frac{c_i}{\alpha} + c_m)(1+u)\alpha q_f + s((1+u)\alpha q_f - daq_f) & 0 < m < daq_f \\ (c_{st} - c_s)m - (c_{tc} + \frac{c_i}{\alpha} + c_m)(1+u)\alpha q_f + s((1+u)\alpha q_f - m) & daq_f < m < (1+u)\alpha q_f \\ (c_{st} - c_{tc} - \frac{c_i}{\alpha} - c_m - c_s)(1+u)\alpha q_f & m > (1+u)\alpha q_f \end{cases} \quad (11)$$

As shown in Eq (11), when GS's remanufacturing capacity  $m$  is lower than  $daq_f$ , it purchases its minimum promised volume. In such circumstances, GS must salvage unsold inventory at the end of the selling season.

## IV. NUMERICAL ANALYSIS

We explore various numerical experiments to show the recommended model's effectiveness. The parameters for two numerical examples are presented in Table II. The datasets utilized in the examples meet all the examined assumptions and models' criteria. The employed data are compatible with datasets applied in the prior research, mainly produced based on genuine instances. In addition, the selected values are relatively broad to be utilized for numerous items in the remanufacturing and refurbishment sector, although by scaling and specific adjustments [18] Meanwhile, equations acquired for the optimal order values are all closed-form relations that there is no need for software to solve. This study assumes that remanufacturing capacity follows a normal probability distribution  $N(\mu, \sigma)$  in the first case and a uniform probability distribution  $U(T1, T2)$  in the second. Table III displays the numerical findings, which include decision variables and profit functions.

In the first case, the tremendous potential of the channel is attained when all partners are handled through an interconnected solution, i.e., a centralized contract. Nevertheless, each partner seeks to advance its interests via decentralized contracts; hence, Table III indicates that the quantity flexibility contract enables all RSC participants to function nearly as a single unit. This incentive-based method has enhanced the number of orders and produced much more flexibility than the decentralized contract — the quantity flexibility contract functions as a risk-sharing strategy. Specifically, GS and 3PL share the risk of unpredictable remanufacturing capacity. By utilizing a reserving approach, GS can order obsolete items from GS with greater flexibility, increasing order volume and decreasing the likelihood of experiencing capacity instability. Results indicate that GS and 3PL will gain profit about equally if the deal is accepted.

Table IV illustrates the effect of changing  $d$  on profit functions and the GS's order quantity. Based on Table IV, as expected, by decreasing  $d$ , GS's profit under the quantity flexibility contract will increase. On the other hand, reducing  $d$  decreases the profit of 3PL under quantity flexibility.

TABLE II. AMOUNTS OF PARAMETERS

Parameters	First example	Second example
$c_{st}$	50	80
$c_i$	8	16
$\alpha$	0.88	0.92
$c_m$	4	6
$c_r$	15	20
$c_p$	5	7
$c_s$	3	6
$c_{tc}$	20	30
$w$	80	110
$s$	6	8
$d$	0.7	0.8
$u$	0.9	1.5
$m$	$\sim N(\mu = 135, \sigma = 40)$	$\sim U(200,400)$

TABLE III. OPTIMAL EQUILIBRIUMS WITHIN DIFFERENT CONTRACTS

	Contracts	$q^*$	$\Pi_{GS}^*$	$\Pi_T^*$	$\Pi_{RSC}^*$
First Example	Wholesale price	140.11	1309.11	1715.58	3024.69
	Centralized	170.44	1400.87	2370.82	3771.69
	Quantity flexibility	216.88	1755.93	3567.97	5323.9
Second Example	Wholesale price	161.23	1507.66	3453.15	4960.81
	Centralized	194.41	1666.77	4163.79	5830.56
	Quantity flexibility	244.86	1903.21	5123.33	7026.54

TABLE IV. EQUILIBRIUMS SENSITIVITY WHEN  $d$  VARIES

$d$	Contracts	$q^*$	$\Pi_{GS}^*$	$\Pi_T^*$	$\Pi_{RSC}^*$
$d = 0.6$	Wholesale price	140.11	1309.11	1715.58	3024.69
	Centralized	170.44	1400.87	2370.82	3771.69
	Quantity flexibility	230.11	1833.60	3213.42	5047.02
$d = 0.55$	Wholesale price	140.11	1309.11	1715.58	3024.69
	Centralized	170.44	1400.87	2370.82	3771.69
	Quantity flexibility	236.77	1864.22	3013.37	4877.59
$d = 0.5$	Wholesale price	140.11	1309.11	1715.58	3024.69
	Centralized	170.44	1400.87	2370.82	3771.69
	Quantity flexibility	241.88	1891.92	2892.99	4784.91

## V. CONCLUSION AND FUTURE STUDIES

In this study, a reverse supply chain is built to address the problems of a stochastic remanufacturing capacity. Initially, a decentralized contract is developed in which the 3PL gives consumers a cost charge for returning outdated items. In the subsequent phase, we model a centralized contract to obtain a globally optimum solution. However, the unpredictability of the upstream remanufacturing capacity is complicated from a decentralized to a centralized contract. Developing a quantity flexibility contract that allows the green supplier to cancel its preliminary order in a restricted quantity. Additionally, 3PL offers a restricted quantity above the initial order to prevent understocking during peak seasons. Numerical examples and sensitivity analysis indicate that the suggested quantity flexibility contract not only enhances the economic

performance of each SC participant but also produces a Pareto-improving condition in which both SC participants earn higher profitability. We further demonstrate that when the quantity flexibility contract's downward adjustment parameter is low, the quantity flexibility contract is profitable for both 3PL and supplier. Thus, both prefer to participate in the quantity flexibility contract channel.

We were excused from investigating several limitations: First, it would be interesting to consider both the remanufacturing capacity and demand uncertainty and compare the results with this study. Another limitation is that we were also excused from comparing the proposed reservation-based contract (quantity flexibility) with another, such as an option contract or even a penalty-based contract.

Future research opportunities in reverse supply chain coordination might include analyzing the simultaneous effects of numerous sources of uncertainty. Another expansion may concentrate on the competitive context and use game theory to address the issue in competitive settings. In addition, how outdated items are gathered is an essential aspect of reverse logistics. Hence, this perspective might be the subject of intriguing research. Customers will be more inclined to return their used items for a discount if they can easily store out-of-date merchandise. In this study, the desire of consumers to return their outdated items is assumed to be a deterministic linear function of the provided incentive amount; however, stochastic and nonlinear processes can also be considered. Lastly, exploring the potential for outsourcing capacity and contemplating a backup manufacturer for the existing model might be examined as part of future relevant and intriguing work.

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APPENDIX

**Proof of Proposition 1.** To prove concavity, from Eq. (1), we must determine  $\frac{\partial E(\Pi_{GS}(q_w))^2}{\partial q_w^2}$ . We have:  $\frac{\partial E(\Pi_{GS}(q_w))^2}{\partial q_w^2} = -\alpha^2(w - c_r - c_{st})g(\alpha q_w) < 0$ ; therefore, GS's profit is concave in  $q_w$ . Besides,  $q_w^*$  will be obtained by first-order optimality condition, i.e.,  $\frac{\partial E(\Pi_{GS}(q_w))}{\partial q_w} = (w - c_r) \int_{\alpha q_w}^{\infty} g(m)dm + s \int_0^{\alpha q_w} g(m)dm - (c_{st} + c_p) = 0$ . ■

**Proof of Proposition 2.** To show concavity, from Eq. (6), we calculate  $\frac{\partial E(\Pi_{RSC}(q_c))^2}{\partial q_c^2}$ . Since  $-\alpha^2(w - c_r - s)g(\alpha q_c) < 0$  therefore, the RSC's profit function is strictly concave in  $q_c$  to be calculated through the first-order optimality state, which is  $\frac{\partial E(\Pi_{RSC}(q_c))}{\partial q_c} = (w - c_r) \int_{\alpha q_c}^{\infty} g(m)dm + s \int_0^{\alpha q_c} g(m)dm + (c_r - c_p - c_{tc} - \frac{c_i}{\alpha} - c_m - c_s) = 0$ . ■

**Proof of Proposition 3.** Since the second-order derivative of Eq. (9) in  $q_f$  is negative ( $\frac{\partial E(\Pi_{GS}(q_f))^2}{\partial q_f^2} = -\alpha^2(-c_r - c_p - c_{st})g(\alpha q_w) < 0$ ), the  $E(\Pi_{GS}(q_f))$  is concave in  $q_f$ . Now the equilibrium will be obtained by solving following equation:  $(w - c_r - c_p - c_{st})(1 + u)\alpha^2 q_f G((1 + u)\alpha q_f) - c_{st}(d\alpha)^2 q_f (1 - G((1 + u)\alpha q_f)) + s(d\alpha)^2 q_f G(d\alpha q_f) = 0$