Game Theory Approach for Open Innovation Systems Analysis in Duopolistic Market

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***Abstract*—The approach used in this study involves applying the Cournot model, which is initially based on the analysis of product quantities in the market. Building upon the obtained equilibrium, a second analysis is conducted to examine the impact of the open innovation integration rate, utilizing a dynamic model. The obtained results have demonstrated that multiple equilibria are possible, and under certain conditions, competing firms have a stake in carefully analyzing the integration rate of open innovation.**

Keywords—Duopoly; open innovation; closed innovation; Cournot model

I. Introduction

The closed innovation model is a strategy that prioritizes the use of internal resources to optimize the innovation process, ultimately leading to the development of innovative products and services [1]. Essentially, companies focus on building and nurturing the necessary competencies in-house to become leaders in their respective markets. By keeping innovation activities in-house, companies can have more control over the entire innovation process, from ideation to product launch, and can better protect their intellectual property.

The primary goal of closed innovation is to ensure that the necessary resources are developed and improved to implement the innovation process effectively[2]. This approach facilitates the creation of new products and services while simultaneously minimizing risks and creating barriers to imitation by competitors. Closed innovation also allows companies to build a competitive advantage by cultivating in-house expertise and refining their innovation processes. By relying on internal resources, companies can optimize the innovation process, leading to more efficient product development, better quality products, and higher profits[3].

According to Chesbrough, the closed innovation model was effective for much of the 20th century. However, this approach to innovation faced two significant limitations. The first limitation was succinctly expressed by Bill Joy, the co-founder of Sun Microsystems, who noted that "No matter who you are, most of the smartest people work for someone else". In other words, relying solely on internal resources means missing the valuable expertise and ideas that exist beyond the organization.

However, Open innovation is viewed as a sustainable innovation approach that depends on international collaboration between companies and countries[4]. As companies seek to gain a competitive advantage through innovation, open innovation has become increasingly popular among academics and practitioners. However, the current literature has mostly focused on the benefits of open innovation and overlooked its potential failures [5]. The integration of open innovation (OI) and the circular economy (CE) has the potential to contribute to a more sustainable economy.

However, there is a lack of understanding of how OI can be leveraged to promote the adoption of CE. As an important aspect of the economy, it is crucial to investigate the relationship between OI and CE and identify ways to overcome the barriers to CE adoption[6]. The second limitation is related to the high level of investment required to support the innovation process. Since closed innovation relies on internal resources, there is a higher level of investment needed to supply the innovation process. This investment also comes with a higher risk as developed ideas may not be supported by the organization, resulting in wasted resources and missed opportunities[7].

The concept of open innovation has been widely applied in various fields, particularly in innovation management for firms of different sizes. Its emphasis on sharing and collaboration has made it a popular topic of interest[8]. Numerous studies have highlighted the discovery of an inverted U-shaped relationship among open innovation, knowledge reorganization, and innovation performance. Moreover, it has been observed that knowledge reorganization and reuse play a mitigating role by alleviating the adverse effects of excessive open innovation on innovation performance[9]. Furthermore, the observed correlation among open innovation, generic strategies (cost-leadership and differentiation), and business performance indicates that the influence of open innovation on business performance is mediated by the adoption of cost-leadership and differentiation strategies[10][11].

Open Innovation has gained significant attention in both research and management practices[12]. As radical innovation and new business development often necessitate external technologies and commercialization methods, many companies have transitioned from a Closed to an Open Innovation model[13]. However, firms frequently encounter challenges during the implementation phase, with the focus primarily placed on external ideas, technologies, and identification processes, while cultural obstacles are often overlooked[14]. While the open innovation literature has extensively discussed strategies, processes, and business models, it has largely neglected the importance of the underlying innovation culture[15].

Researchers have conducted multiple studies to examine the differences between Open Innovation and Closed Innovation, with the objective of characterizing each type of innovation. These studies have revealed that Chesbrough's six principles of open innovation rely on a false dichotomy that necessarily opposes closed innovation to open innovation[13]. Within the same context, researchers have conducted studies to explore the implementation of inbound, outbound, and combined open innovation practices. These investigations have examined multiple factors such as organizational context, company structure, collaborative arrangements, the involvement of diverse actors, and the outcomes achieved, providing insights into the role and influence of these factors on the efficacy of open innovation practices in companies[16].

The dilemma between Open Innovation and Closed Innovation lies in the strategic choice that companies face regarding their approach to innovation. Closed Innovation is based on the principle that the company should internally control and develop its innovations, relying on its own resources and capabilities. On the other hand, Open Innovation takes a more open approach, seeking to integrate external ideas, knowledge, and resources through collaborations, partnerships, and leveraging the innovation ecosystem.

The challenge arises when companies are faced with the decision of selecting the optimal approach to embrace. Closed Innovation provides enhanced control and protection of internal knowledge; however, it may limit exposure to new ideas and opportunities[17]. Conversely, Open Innovation offers access to a broad spectrum of external knowledge and resources, fostering innovation, yet it entails risks such as potential intellectual property disclosure and difficulties in coordinating with external partners[18]. Therefore, companies must navigate between adopting a more secure and internally-focused approach or embracing a collaborative and open approach to innovation, carefully considering the benefits and drawbacks of each, as well as their unique organizational context and environment.

Various alternative approaches are being used to examine open innovation[19]. These include empirical studies, which involve collecting real-world data and analyzing its impact on firm performance through surveys, interviews, case studies, and quantitative analysis. Network analysis explores the structure and dynamics of innovation networks, investigating collaborations, partnerships, and knowledge flows to identify key actors and understand their influence on innovation outcomes. Qualitative research, such as ethnography and in-depth interviews, delves into the experiences, perspectives, and behaviours of individuals and organizations involved in open innovation. Technological platforms and data analytics leverage advanced technologies to analyze large-scale datasets, uncovering patterns, trends, and correlations relevant to open innovation. Simulation models simulate scenarios to understand the complexities, trade-offs, and uncertainties of open innovation, facilitating the testing of different strategies and policies. Comparative studies compare industries, sectors, or regions to identify variations in open innovation practices, outcomes, and contextual factors, providing insights into industry-specific challenges, best practices, and policy implications for promoting open innovation.

Game theory has proven to be a useful tool for modeling the interactions that take place in Open Innovation ecosystems. Specifically, the Cournot duopoly has emerged as a popular game theory model to simulate the strategic behavior of firms in Open Innovation[20]. For example, an Open Innovation process in a Cournot duopoly is analyzed using a differential game approach that incorporates knowledge spillover. The optimal licensing contract for a patentor with a quality improvement innovation in a Cournot duopoly market is analyzed in this paper. s are endogenously determined via the R&D process[21]. Another study examined the optimization of technology licensing contracts for quality improvement innovation in the context of Cournot competition[22]. A similar study has addressed the problem of patent licensing in a Cournot duopoly, where one of the firms acts as the innovator (patentee) and encounters capacity limitations. The focus of this study revolves around investigating the challenges associated with patent licensing within the context of a Cournot duopoly, where one firm holds the role of the patentee and faces capacity constraints[23]. Another study focused on a differentiated Cournot model and a differentiated Bertrand model, in which one of the firms engages in an R&D process resulting in an endogenous cost-reducing innovation[24]. The role of platform economics in facilitating open innovation while addressing the challenges of information stickiness and product diversification risks was studied in [25].

II. Methodology

A. *Goals and Assumptions Underlying the Study*

The purpose of this paper is to explore the adoption of open innovation versus closed innovation by a firm operating in a duopoly model, specifically using the Cournot model. The study incorporates an integration rate parameter to assess its impact on the firm's innovation strategy. It is important to mention that the analysis does not consider the specific activities of the competing firms. Additionally, this study builds upon and draws inspiration from several related works in the field. In this regard, the study considers the work on the complex dynamics of R&D competition with one-way spillover based on intellectual property protection[25].

This study specifically focuses on a dynamic two-stage model. Indeed, investigations on the two-stage model have increasingly captured the interest of economists. Whereas initially, many scholars were primarily focused on examining the properties of the static model. It is undeniable that the static model has its limitations. One of these limitations is its ability to only analyze the individual supply and demand equilibrium between firms. When the factors of supply and demand undergo changes, the corresponding supply relationship will also shift.

This research employs nonlinear dynamics theory to examine the evolutionary process within firms' games. Various scholars have conducted previous studies on this topic. The local and global dynamic properties of a two-stage oligopoly game model with an adaptive dynamic mechanism, highlighting its complex evolutionary behaviors was studied in [26]. Also, another paper investigated the properties of a dynamic Cournot duopoly game model with a nonlinear demand function[27]. Similar works can be found in [28].

Moreover, the economic dynamical system has shown significant interest in the dynamical two-stage game. For example, a dynamic model of a two-stage remanufacturing closed-loop supply chain was used to investigate how technological innovation, Big Data marketing, and overconfidence influence the decision-making process of supply chain members[29]. The stability of a two-stage duopoly Cournot game model, which incorporates a nonlinear inverse demand function and R&D spillover, is investigated. The results indicate that the final state of the system is influenced by its initial state[30].

B. Mathematical Model

To assess the effects of open innovation versus closed innovation, the Cournot duopoly model is used. The model assumes the presence of two companies, labeled i (i=1,2), in a market, offering identical products. Recognizing the value of innovation as a means of achieving a competitive edge, each firm adopts a strategy that enables it to emerge as the leader in the market.

Our model consists of two game stages that take into account the time required for innovation before introducing products to the market. In the first stage, the innovation parameter is considered, followed by the standard Cournot model where a balance is sought in relation to the quantities of products on the market. Differences in the levels of innovation integration (i.e., open innovation) between firms can result in differences in product quality. The industry is characterized by a linear inverse demand function expressed as:

$$p_i = a - bQ \quad (i = 1,2)$$

Where, $a > 0 \; and \; b \in [0,1]$,

$Q \; is \; the \; total \; quantity \; in \; the \; market \; Q = \sum_{i=1,2} q_i$, q_i is the outputs of the products producing by the firm i.

This work consider that the two firms decide to integrate the innovation in their strategies. In order to model the OI and CI, This paper introduce the parameter $\sigma_i \in [0,1]$ that corresponds to the OI integration rate. The effective marginal cost of firm i is represented as follow:

$$C_i(\sigma_i) = A + c \times (1 - \sigma_i), \; i = 1,2 \qquad (2)$$

According to this cost equation, if a firm i decides to outsource the innovation the marginal cost will be A, since the rate σ_i will be equal to one. However, if firm i decides to internalize, completely, the innovation, the marginal cost will be high since it will be equal to A+c.

Considering the gains that can be obtained from corporate innovation (CI), such as high-powered incentives, firm-owned property rights, and reuse cost, it can be hypothesized that as long as the firm perceives a decrease in these gains, its rate of Open innovation will diminish. Therefore, it represents losses generated by the massive use of Open innovation, charges for the firm and therefore an additional cost.

Furthermore, based on several works of duopolistic models[31], the expressions of a quadratic cost equation, loss of a firm i can be expressed as:

$$L(\sigma_i) = \gamma {\sigma_i}^2/2 (i = 1,2), \qquad (3)$$

Where γ is a spillover parameter.

C. Profit of each Firm

According to the propositions given above, the profit equations for the two firms

$$\begin{cases} \pi_1(q_1, q_2, \sigma_1, \sigma_2) = [p_1(q_1, q_2) - C_1(\sigma_1, \sigma_2)]q_1 - L(\sigma_1), \\ \pi_2(q_1, q_2, \sigma_1, \sigma_2) = [p_2(q_1, q_2) - C_2(\sigma_1, \sigma_2)]q_2 - L(\sigma_2). \end{cases} \quad (4)$$

Substituting Eq. (1) and (2) into Eq. (4), the expression of profit function for each firm is:

$$\begin{cases} \pi_1(q_1, q_2, \sigma_1, \sigma_2) = -\mathrm{b}{q_1}^2 + \big(a - bq_2 - A - c(1 - \sigma_1)\big)q_1 - \gamma \frac{1}{2}{\sigma_1}^2 \\ \pi_2(q_1, q_2, \sigma_1, \sigma_2) = -\mathrm{b}{q_2}^2 + \big(a - bq_1 - A - c(1 - \sigma_2)\big)q_1 - \gamma \frac{1}{2}{\sigma_2}^2 \end{cases}$$
(5)

Now, the marginal profits of these two firms are:

$$\begin{cases} \frac{\partial \pi_1}{\partial q_1} = -2\mathrm{b}q_1 + \big(a - bq_2 - A - c(1 - \sigma_1)\big) \\ \frac{\partial \pi_2}{\partial q_2} = -2\mathrm{b}q_2 + \big(a - bq_1 - A - c(1 - \sigma_2)\big) \end{cases} \qquad (6)$$

The second-order conditions are met because

$$\frac{\partial^2 \pi_1}{\partial^2 q_1} = \frac{\partial^2 \pi_2}{\partial^2 q_2} = -2 < 0.$$

According to Eq. (6), the reaction function of the firm 1 and firm 2 by setting $\frac{\partial \pi_1}{\partial q_1} = 0, \frac{\partial \pi_2}{\partial q_2} = 0$ is as follow:

$$\begin{cases} R_1\,(q_2, \sigma_1) = q_2^* = \frac{(a - bq_1 - A - c(1-\sigma_1))}{2\mathrm{b}} \\ R_2\,(q_1, \sigma_2) = q_1^* = \frac{(a - bq_2 - A - c(1-\sigma_2))}{2\mathrm{b}} \end{cases} \qquad (7)$$

Let $U = \frac{(a - A - c)}{2\mathrm{b}}$

Cournot equilibrium can be expressed by replacing q_1^* and q_2^* in reaction's function.

$$\begin{cases} q_1^* = \frac{2}{3}\Big(U - \frac{c}{2b}(\sigma_2 - \frac{1}{2}\sigma_1)\Big) \\ q_2^* = \frac{2}{3}\Big(U - \frac{c}{2b}(\sigma_1 - \frac{1}{2}\sigma_2)\Big) \end{cases} \qquad (8)$$

Provided that $\frac{2b}{c}U > \Big(\sigma_2 - \frac{1}{2}\sigma_1\Big)$ and $\frac{2b}{c}U > \Big(\sigma_1 - \frac{1}{2}\sigma_2\Big)$.

Therefore, in the subsequent scenario, it is assumed that a Cournot equilibrium exists.

D. First Stage of Equilibrium Analysis

According to this result, the rate of OI determines the production strategy of the quantities to be produced for each firm. Also, by subtracting q_2^* from q_1^*,

$$\Delta Q = q_1^* - q_2^* = \frac{c}{3b}(\sigma_1 - \sigma_2) \qquad (9)$$

According to $\Delta Q \; value$, the equilibrium quantities depend on the innovation integration rates for each firm. Thus, assuming firm 1 chooses an OI approach (σ_1=1) and firm 2 chooses CI as an opposite approach (σ_2=0), $\Delta Q > 0$. Firm 1 must always deliver quantities greater than those of firm 2, since $\frac{c}{3b}$ is positive.

The profit function about innovation rate σ_i captured by taking Eq. (8) into Eq. (5) in reverse order can be obtained as:

$$\begin{cases} \pi_1(\sigma_1,\sigma_2) = \left(\frac{5}{36}\frac{c}{b} - \frac{1}{2}\gamma\right)\sigma_1{}^2 + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{18}\frac{c}{b}\sigma_2\right)\sigma_1 - \frac{1}{9}U(3-2b)\frac{c}{2b}\sigma_2 + \frac{1}{9}U^2(3-2b) \\ \pi_2(\sigma_1,\sigma_2) = \left(\frac{5}{36}\frac{c}{b} - \frac{1}{2}\gamma\right)\sigma_2{}^2 + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{9}\frac{c}{2b}\sigma_1\right)\sigma_2 - \frac{1}{9}U(3-2b)\frac{c}{2b}\sigma_1 + \frac{1}{9}U^2(3-2b) \end{cases} \quad (10)$$

The equation of the profits of the two firms makes it possible to calculate the maximum local profit according to the rates of integration of the IO.

For this, the derivative of the system of equation (10) give

$$\begin{cases} \frac{\partial \pi_1(\sigma_1,\sigma_2)}{\partial \sigma_1} = \left(\frac{5}{18}\frac{c}{b} - \gamma\right)\sigma_1 + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{18}\frac{c}{b}\sigma_2\right) \\ \frac{\partial \pi_2(\sigma_1,\sigma_2)}{\partial \sigma_2} = \left(\frac{5}{18}\frac{c}{b} - \gamma\right)\sigma_2 + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{9}\frac{c}{2b}\sigma_1\right) \end{cases} \quad (11)$$

Based on this approach, different expectations are assumed from the two firms. Indeed, supposing that firm 1 is rational in a bounded way and firm 2 is a local approximation. The limited rational actor 1 does not have complete knowledge of the market; hence, they try to use local information based on marginal profit.

E. Second Stage of Equilibrium Analysis

In this section, the impact of adjustment mechanisms on the competitive outcomes of enterprises, building on the work of Dixit is discussed. Dixit's research focuses on constructing a competitive model of two companies with an adjustment mechanism and estimating the marginal profit to describe the production evolution[32][33]. In this paper, two different scenarios are given: when two companies co-exist in the same market and when one company takes full control of the market after dislodging the other. The findings indicate that the adjustment mechanism is effective in reducing the output and profit gap between the companies, and in some cases, it can lead to the elimination of this difference and the attainment of Nash equilibrium[34].

Therefore, it is supposed that Firm 1 decides to proceed with the decisions concerning the rate of integration of the IO by either increasing or decreasing it. Thus, the dynamic adjustment mechanism can be modeled as follows:

$$\begin{cases} \sigma_1(t+1) = \sigma_1(t) + \vartheta_1\sigma_1(t)\frac{\partial \pi_1(\sigma_1,\sigma_2)}{\partial \sigma_1} \\ \sigma_2(t+1) = \sigma_2(t) + \vartheta_2\sigma_2(t)\frac{\partial \pi_2(\sigma_1,\sigma_2)}{\partial \sigma_2} \end{cases} \quad (12)$$

Where, ϑ_1, ϑ_2 are positive parameters, which represent respectively the speed of adjustment of firm 1 and firm 2.

By replacing the profit given in equation (11) in the equation, (12) system become:

$$\begin{cases} \sigma_1(t+1) = \sigma_1(t) + \vartheta_1\sigma_1(t)\left(\left(\frac{5}{18}\frac{c}{b} - \gamma\right)\sigma_1(t) + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{18}\frac{c}{b}\sigma_2(t)\right)\right) \\ \sigma_2(t+1) = \sigma_2(t) + \vartheta_2\sigma_2(t)\left(\left(\frac{5}{18}\frac{c}{b} - \gamma\right)\sigma_2(t) + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{18}\frac{c}{b}\sigma_1(t)\right)\right) \end{cases} \quad (13)$$

III. Equilibrium Points and Local Stability

The system of Eq. (13) given is a set of coupled first-order nonlinear difference equations. To analyze the equilibrium and stability of the system, the fixed points of the system by setting σ1 (t+1) = σ1 (t) and σ2 (t+1) = σ2(t).

Setting $\sigma_1(t+1) = \sigma_1(t)$ and $\sigma_2(t+1) = \sigma_2(t)$, the system become:

$$\begin{cases} \vartheta_1\sigma_1(t)\left(\left(\frac{5}{18}\frac{c}{b} - \gamma\right)\sigma_1(t) + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{18}\frac{c}{b}\sigma_2(t)\right)\right) = 0 \\ \vartheta_2\sigma_2(t)\left(\left(\frac{5}{18}\frac{c}{b} - \gamma\right)\sigma_2(t) + \frac{1}{9}\left(8U - \frac{c}{4} - \frac{5}{18}\frac{c}{b}\sigma_1(t)\right)\right) = 0 \end{cases} \quad (14)$$

After performing computational analysis, it was determined that the system given by Eq. (16) has four equilibrium points. These points are:

$$(\sigma_1, \sigma_2) = (0,0)$$

$$(\sigma_1, \sigma_2) = \left(\left(\frac{16U - c}{\frac{5c}{9b} - 2\gamma}\right), 0\right)$$

$$(\sigma_1, \sigma_2) = \left(0, \left(\frac{16U - c}{\frac{5c}{9b} - 2\gamma}\right)\right)$$

$$(\sigma_1, \sigma_2) = \left(\left(\frac{40U - 5c + 9\gamma\frac{c}{b}}{18b\gamma - 25c}\right), \left(\frac{40U - 5c + 9\gamma\frac{c}{b}}{18b\gamma - 25c}\right)\right)$$

To guarantee that all four equilibrium points of the system (10) are non-negative, the following conditions must be satisfied: $5c - 18b\gamma > 0$ and $U < \frac{c}{16}$. Additionally, it is important to note that the analysis assumes positive values for ϑ1 and ϑ2.

$$J = \begin{bmatrix} 1 + \vartheta_1\left(\left(\frac{5c}{18b} - \gamma\right)\sigma_1 + \frac{1}{9b}\left(8U - \frac{c}{4} - \frac{5c}{18b}\sigma_2\right)\right) & -\frac{1}{9b}\vartheta_1\sigma_1\left(\frac{5c}{18b}\right) \\ -\frac{1}{9b}\vartheta_2\sigma_2\left(\frac{5c}{18b}\right) & 1 + \vartheta_2\left(\left(\frac{5c}{18b} - \gamma\right)\sigma_2 + \frac{1}{9b}\left(8U - \frac{c}{4} - \frac{5c}{18b}\sigma_1\right)\right) \end{bmatrix} \quad (15)$$

After identifying the four equilibrium points of the system given by Eq. (16), the next step is to study their stability. This is an important step as it helps us determine the behavior of the system around these equilibrium points. The stability of an equilibrium point can be classified as either stable, unstable, or semi-stable. A stable equilibrium point is one where any small disturbance from its position will cause the system to return to that point. An unstable equilibrium point, on the other hand, is one where any small disturbance will cause the system to move away from that point. Lastly, a semi-stable equilibrium point has one stable direction and one unstable direction. By analyzing the stability of each equilibrium point, insights can be gained into the behavior of the system and its evolution over time.

A. First Equilibrium Point:

The Jacobian matrix at the equilibrium point $(\sigma_1, \sigma_2) = (0,0)$ is:

$$J(0,0) = \begin{bmatrix} 1 + \vartheta_1\left(\frac{1}{9b}\left(8U - \frac{c}{4}\right)\right) & 0 \\ 0 & 1 + \vartheta_2\left(\frac{1}{9b}\left(8U - \frac{c}{4}\right)\right) \end{bmatrix}$$

The eigenvalues are the solutions to this equation, which are:

$$\lambda_1 = 1 + \vartheta_1 \frac{\left(8U - \frac{c}{4}\right)}{9b}$$

$$\lambda_2 = 1 + \vartheta_2 \frac{\left(8U - \frac{c}{4}\right)}{9b}$$

Since ϑ_1 and ϑ_2 are positive, and that $\frac{c}{4} < 8U$, then both eigenvalues are positive. This means that the equilibrium point (0,0) is unstable node.

In terms of the physical interpretation of the system, this result suggests that the equilibrium point (0,0) is unstable when the gain parameters for the feedback loops, ϑ_1 and ϑ_2, are positive and the net effect of the feedback loops on the system is positive, as represented by the positive value of (8U - c/4).

B. Second Equilibrium Point

The Jacobian matrix at the equilibrium point

$$(\sigma_1, \sigma_2) = \left(\left(\frac{16U-c}{\frac{5c}{9b}-2\gamma}\right), 0\right) \text{ is:}$$

$$J\left(\left(\frac{16U-c}{\frac{5c}{9b}-2\gamma}\right), 0\right) = \begin{bmatrix} 1 + \vartheta_1\left(\left(\frac{40U-9c}{45b-18\gamma}\right)\right) & -\vartheta_1 \frac{4U-c}{45b-18\gamma} \\ 0 & 1 + \vartheta_2\left(\frac{40U-9c}{45b-18\gamma}\right) \end{bmatrix}$$

The Jacobian matrix is a diagonal matrix; thus, its corresponding distinct eigenvalues are:

$$J\left(\left(\frac{16U-c}{\frac{5c}{9b}-2\gamma}\right), 0\right) = \begin{bmatrix} 1 + \vartheta_1\left(\left(\frac{40U-9c}{45b-18\gamma}\right)\right) & -\vartheta_1 \frac{4U-c}{45b-18\gamma} \\ 0 & 1 + \vartheta_2\left(\frac{40U-9c}{45b-18\gamma}\right) \end{bmatrix}$$

If U > 9c/40 and b > 2γ/5, the equilibrium point is an unstable node, meaning that the trajectories of the system move away from this point. This implies that the market will not reach a stable state and will continue to fluctuate. On the other hand, if U > 9c/40 or b > 2γ/5, the equilibrium point is a stable node, meaning that the system will move towards this point as time progresses. In other words, the market will reach a stable state, either with high U or high b values.

The stability analysis of the equilibrium point is crucial in understanding the behavior of the system. The results obtained suggest that the stability of the equilibrium point is influenced by the values of U and b. Therefore, firms can use this information to adjust their strategies and optimize their profits. By maintaining the optimal values of U and b, enterprises can stabilize their position in the market and achieve long-term success.

C. Third Equilibrium Point

The Jacobian matrix at the equilibrium point is:

$$J\left(0, \left(\frac{16U-c}{\frac{5c}{9b}-2\gamma}\right)\right) = \begin{bmatrix} 1 + \vartheta_1\left(\left(\frac{40U-9c}{45b-18\gamma}\right)\right) & 0 \\ -\vartheta_1 \frac{4U-c}{45b-18\gamma} & 1 + \vartheta_2\left(\frac{40U-9c}{45b-18\gamma}\right) \end{bmatrix}$$

The Jacobian matrix is a diagonal matrix; thus, its corresponding distinct eigenvalues are:

$$\lambda_1 = 1 + \frac{\vartheta_1(40U-9c)}{45b-18\gamma}$$

$$\lambda_2 = 1 + \frac{\vartheta_2(40U-9c)}{45b-18\gamma}$$

The types of the second equilibrium point are similar to the boundary equilibrium points. If U > 9c/40 and b > 2γ/5 then is the equilibrium point is unstable node. However if U > 9c/40 or b> 2γ/5 the equilibrium point is stable node.

In economic terms, the boundary equilibrium points signify a scenario where one of the two firms has exited the market. Equilibrium points $(\sigma_1, \sigma_2) = \left(\left(\frac{16U-c}{\frac{5c}{9b}-2\gamma}\right), 0\right)$ and $(\sigma_1, \sigma_2) = \left(0, \left(\frac{16U-c}{\frac{5c}{9b}-2\gamma}\right)\right)$, on the other hand, it indicates that one of the firms has taken the lead in the oligopoly market, resulting in a monopoly market. The local stability of equilibrium points reflects the short-term stability of the economic market. However, neither of these scenarios is desirable. It is only when both companies restrict each other that the market and the country can achieve stable development. This state is known as "Nash equilibrium," which is reached when both firms maximize their own profits while also ensuring the stable development of the market.

D. Fourth Equilibrium Point

The Jacobian matrix at the equilibrium point is:

$$J = \begin{bmatrix} 1 + \frac{c}{9b}\vartheta_1\left(\frac{-65(8U-c+9\gamma\frac{c}{5b})}{(18b\gamma-25c)} + \frac{8U}{c}\right) & -\vartheta_1 \frac{c}{9b}\frac{(8U-c+9\gamma\frac{c}{5b})}{(18b\gamma-25c)} \\ -\vartheta_2 \frac{c}{9b}\frac{(8U-c+9\gamma\frac{c}{5b})}{(18b\gamma-25c)} & 1 + \frac{c}{9b}\vartheta_2\left(\frac{-65(8U-c+9\gamma\frac{c}{5b})}{(18b\gamma-25c)} + \frac{8U}{c}\right) \end{bmatrix}$$

The trace of the given Jacobian matrix J is:

$$Tr(J) = 2 + \frac{c}{9b}(\vartheta_1 + \vartheta_2)\left(\frac{-65\left(8U-c+9\gamma\frac{c}{5b}\right)}{(18b\gamma-25c)} + \frac{16U}{c}\right)$$

Therefore, the determinant of the given Jacobian matrix J is:

$$Det(J) = 1 + \frac{c}{9b}(\vartheta_1 + \vartheta_2)\left(\frac{-65\left(8U-c+9\gamma\frac{c}{5b}\right)}{(18b\gamma-25c)} + \frac{8U}{c}\right) + \vartheta_1\vartheta_2\left(\frac{c}{9b}\right)^2 \frac{\left(8U-c+9\gamma\frac{c}{5b}\right)^2}{(18b\gamma-25c)^2}$$

The characteristic polynomial of matrix:

$$P(\lambda) = \lambda^2 - Tr(J) + Det(J) = 0$$

The discriminant of the characteristic polynomial is given by:

$$\Delta = Tr(J)^2 - 4Det(J)$$

Substituting the expressions for Tr(J) and Det(J),

$$\Delta = [2+\frac{c}{9b}(\vartheta_1 + \vartheta_2)(\frac{-65\left(8U-c+9\gamma\frac{c}{5b}\right)}{(18b\gamma-25c)} + \frac{16U}{c})]^2[1+\frac{c}{9b}(\vartheta_1 + \vartheta_2)\left(\frac{-65\left(8U-c+9\gamma\frac{c}{5b}\right)}{(18b\gamma-25c)} + \frac{8U}{c}\right) + \vartheta_1\vartheta_2(\frac{c}{9b})^2\frac{\left(8U-c+9\gamma\frac{c}{5b}\right)^2}{(18b\gamma-25c)^2}]$$

Simplifying this expression may lead to a long and complicated expression, but it represents the discriminant of the characteristic polynomial which determines the stability of the equilibrium point.

The given equilibrium is a stable node if the following conditions are satisfied:

- Tr(J) < 0 and Det(J) > 0
- Δ > 0 and Tr(J) < 0

Therefore, these conditions should be satisfied

$$(40U - 5c + 9\gamma c/b)^2 < \frac{(18b\gamma - 25c)\,(64\vartheta_1\vartheta_2 U - 9c\,(\vartheta_1+\vartheta_2)+25\gamma c\vartheta_1\vartheta_2)}{18b}$$

$$\frac{(-65(8U - c + 9\gamma c/5b))}{(18b\gamma - 25c)} < 0$$

$$8U - c + 9\gamma c/5b > 0$$

$$\frac{(40U - 5c + 9\gamma c/b)}{(18b\gamma - 25c)} > 0$$

If these conditions hold, then the equilibrium point $(\sigma_1, \sigma_2) = \left(\left(\frac{40U-5c+9\gamma\frac{c}{b}}{18b\gamma-25c}\right), \left(\frac{40U-5c+9\gamma\frac{c}{b}}{18b\gamma-25c}\right)\right)$ is a stable node.

IV. Numerical Analysis

This section of the article focuses on performing a numerical analysis to investigate stability studied in the previous section. The model used in this analysis assesses the effects of open innovation (OI) versus closed innovation (CI) using the Cournot duopoly model. The model assumes the presence of two companies in a market offering identical products, with the industry characterized by a linear inverse demand function. Each firm adopts a strategy that enables it to emerge as the leader in the market. Three equilibrium points will be studied, although the first equilibrium point will be omitted due to its non-stability. The aim is to demonstrate the impact of open innovation (OI) integration in the context of a Cournot duopoly.

The model consists of two game stages that take into account the time required for innovation before introducing products to the market. In the first stage, the innovation parameter, followed by the standard Cournot model, where seeking a balance in relation to the quantities of products on the market. The goal of the numerical analysis is to demonstrate the impact of OI integration in the context of Cournot duopoly. Through this analysis, the objective is to gain insights into the stability of the equilibrium point under diverse scenarios and conditions. This endeavor aims to provide valuable information for decision-making and strategic planning. One important aspect of studying these systems is to determine their stability, which refers to how they behave over time under small perturbations.

The stability of this system is investigated by analyzing the behavior of its trajectories for different parameter values. In particular, the values of ϑ1 and ϑ2 affect the stability of the system.

U = 5,c = 1, γ = 2,b = 4, ϑ1 = 3, ϑ2 = 2

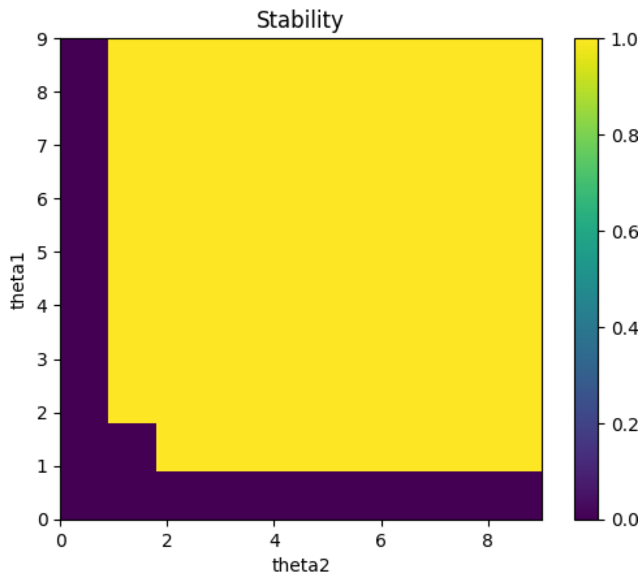

Fig. 1. Stability analysis for the last equilibrium point.

Based on the values of the parameters provided, the graph (Fig. 1) shows the stability of the system as a function of the variables ϑ1 and ϑ2.

The stable region is represented by the purple shaded area, and it corresponds to the values of ϑ1 and ϑ2 for which the system is stable. The unstable region is represented by the yellow shaded area, and it corresponds to the values of ϑ1 and ϑ2 for which the system is unstable.

The stability boundary is represented by the black curve, and it separates the stable region from the unstable region. Points on this curve correspond to values of ϑ1 and ϑ2 for which the system is marginally stable, meaning that small perturbations can cause the system to become unstable.

Overall, this graph provides a visual representation of the stability of the system as a function of the variables ϑ1 and ϑ2, which can be useful for understanding the behavior of the system and for making design decisions.

For the Second equilibrium given the following values, U = 5, c = 1, γ=2, b=4, $\vartheta_1 = 3$, $\vartheta_2 = 4$

This graph (Fig. 2) shows the behavior of the system at the equilibrium point (2, 2) as the parameters U, c, γ, b, ϑ1, and ϑ2 are varied. The color of each point on the plot represents the type of stability of the equilibrium point at that parameter combination. The blue points represent a stable node, the red points represent an unstable node, and the white points represent a saddle point.

As the values of U, c, γ, b, ϑ1, and ϑ2 are varied, the shapes of the stability regions change. In general, as U increases, the stability regions expand, and as c or γ increase, the stability regions contract. The positions of the stability regions depend on the values of b, ϑ1, and ϑ2.

Overall, this graph provides insight into the behavior of the system at the equilibrium point (2, 2) and how it changes as the parameters of the system are varied.

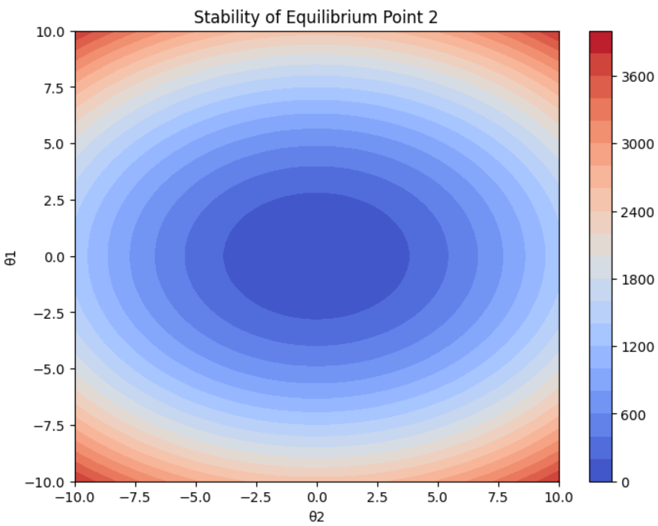

Fig. 2. Stability analysis for the second equilibrium point.

The graph shows the stability of the equilibrium point in a Cournot duopoly model with open innovation, where ϑ1 and ϑ2 represent the speed of adjustment for each firm, and σ_1 and σ_2 represent the ratio of open innovation for each firm.

The red line represents the stability boundary, where any points above the line correspond to a stable equilibrium, while points below the line correspond to an unstable equilibrium. The stability boundary is determined by the Jacobian matrix at the equilibrium point, which in this case is a diagonal matrix with distinct eigenvalues λ1 and λ2.

The eigenvalues can be used to determine the stability of the equilibrium point. If both eigenvalues are negative, then the equilibrium point is stable; if both eigenvalues are positive, then the equilibrium point is unstable; if one eigenvalue is negative and one is positive, then the stability of the equilibrium point depends on the slope of the null clines.

In this case, the stability boundary is curved, which indicates that the stability of the equilibrium point depends on the values of ϑ1 and ϑ2. When ϑ1 is small and ϑ2 is large, the equilibrium point is stable for a wide range of values. However, as ϑ1 increases and ϑ2 decreases, the stability region becomes smaller and shifts to the right.

The fact that the stability boundary is curved indicates that the duopoly model with open innovation is highly nonlinear, and small changes in the values of the parameters can have significant effects on the stability of the equilibrium point. This suggests that firms should be careful in their strategic decision-making, and should take into account the potential effects of their actions on the stability of the market.

The graph (Fig. 3) shows the stability of the third equilibrium point in a Cournot duopoly model with open innovation. The model has four parameters: U = 5.0, c = 2.0, b = 1.5 and gamma = 0.5. The equilibrium point is represented in the graph as a black dot at the origin (0, 0).

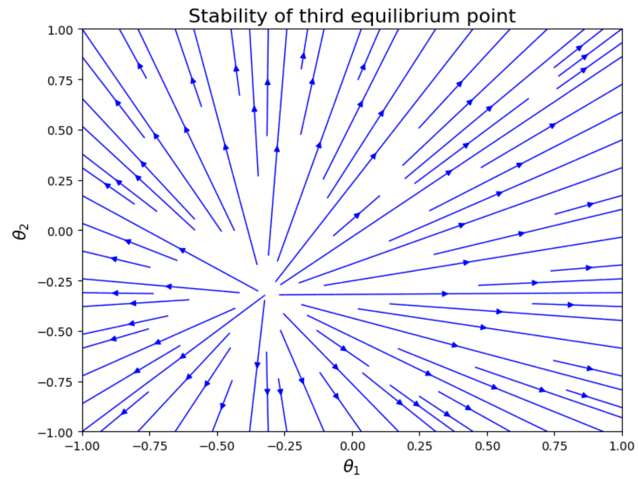

Fig. 3. Stability analysis for the third equilibrium point.

The arrows in the graph in Fig. 3 represent the direction of the trajectories of two firms in the duopoly as they adjust their speed of innovation, with one arrow representing the trajectory of the first firm (θ1) and the other arrow representing the trajectory of the second firm (θ2). The color of the arrows represents the magnitude of the eigenvalues of the Jacobian matrix at each point in the plane, with warmer colors (such as red and orange) indicating more positive eigenvalues and cooler colors (such as blue and purple) indicating more negative eigenvalues.

The graph shows that the equilibrium point at the origin is a saddle point, with one stable direction along the θ2 axis and one unstable direction along the θ1 axis. This means that the equilibrium is locally stable in the direction of the second firm's speed of innovation, but unstable in the direction of the first firm's speed of innovation.

Overall, the graph provides insight into the dynamics of the Cournot duopoly model with open innovation and shows how the stability of the equilibrium point depends on the firms' speed of innovation.

In economic terms, the third equilibrium point represents a scenario where both firms choose not to engage in open innovation, resulting in a monopolistic market. The stability of this equilibrium point reflects the short-term stability of the market. However, this scenario is not desirable in the long run, as it hinders innovation and can lead to market inefficiencies.

In economic terms, the third equilibrium point represents a scenario where one of the firms dominates the market with a monopoly position. This may be due to several factors, such as technological advantages or economies of scale. However, this scenario is not desirable, as it leads to inefficiencies in the market and reduced consumer surplus. It is only when both firms compete and restrict each other's market power that the market can achieve stable development. This state is known as "Nash equilibrium," where both firms maximize their own profits while also ensuring the stable development of the market.

Overall, the graph provides a visual representation of the stability of the third equilibrium point in a duopoly Cournot model, highlighting the importance of competition and market regulation for achieving optimal market outcomes.

V. Conclusion

This study explored the use of the Cournot duopoly model to evaluate the impact of open innovation on competitive advantage. By incorporating the parameter of innovation into our model, the first stage of the game is studied, followed by the standard Cournot model to find a balance in the quantities of products on the market. Our analysis showed that the use of open innovation could lead to higher profits and market share for both firms compared to closed innovation. In addition, the speed of adjustment parameter plays a crucial role in the stability of the system, with a smaller value indicating greater stability. Overall, our study highlights the importance of considering open innovation strategies in a competitive market environment.

Our mathematical model demonstrates that under certain conditions, open innovation can lead to greater market share and profits for both firms, compared to a closed innovation approach. These findings have implications for firms operating in industries with high levels of technological change and innovation, and suggest that collaboration can be a powerful tool for achieving competitive advantage.

However, further research is needed to fully understand the dynamics of open innovation systems, and to explore the impact of different market structures, levels of OI investment, and intellectual property rights on firm performance and market outcomes. By further exploring these significant inquiries, a more nuanced comprehension of the intricate interplay among innovation, collaboration, and competition can be acquired. This, in turn, enables the development of strategies that optimize the advantages of open innovation for firms, consumers, and society.

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