Estimating Probability Values Based on Naïve Bayes for Fuzzy Random Regression Model

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Abstract—**In the process of treating uncertainties of fuzziness and randomness in real regression application, fuzzy random regression was introduced to address the limitation of classical regression which can only fit precise data. However, there is no systematic procedure to identify randomness by means of probability theories. Besides, the existing model mostly concerned in fuzzy equation without considering the discussion on probability equation though random plays a pivotal role in fuzzy random regression model. Hence, this paper proposed a systematic procedure of Naïve Bayes to estimate the probabilities value to overcome randomness. From the result, it shows that the accuracy of Naïve Bayes model can be improved by considering the probability estimation.**

Keywords—Naïve Bayes; fuzziness; randomness; probability estimation

I. Introduction

Fuzziness and randomness are two uncertainties involved in practice of real observation where the statistical data are collected from various measurements. Fuzziness comes from incomplete information while randomness can be related to stochastic variability of all possible outcomes of a situation [1]. In mathematical viewpoint, both uncertainties are merged to formulate a fuzzy random variable by means of assigning probability and fuzzy set theories since possible random outcome have to be described by terms of fuzzy set.

Fuzzy random variable had been studied by many researchers over past few decades. The first introduction of fuzzy random variable concept had been given by Kwarkernaak [2][3]. Since then, different researchers studied fuzzy random variable according to different requirements like by Puri and Ralescu [4] and Liu and Liu [5]. Considering the ability of fuzzy random theories in handling simultaneously fuzzy random uncertainties, this approach can be found in various applications such as in regression analysis. Nather [6] presented fuzzy random variable to deal with regression analysis when the statistical has linguistic data.

In the situation where randomness and fuzziness associated in the regression problems, fuzzy random regression was introduced as a solution for real life regression analysis where the data is not only characterized by imprecision and vagueness but there also exist the formalism of random variables. Fuzzy random regression based on fuzzy random variables with confidence interval was proposed in the framework of real regression analysis where there exist uncertainties [7]. The implementation of this technique as an integral component of regression was successfully in achieving the objective to estimate weight in the production of oil palm [8].Considering the statistical used content fuzzy random information, fuzzy random regression was proposed to estimate coefficient in the model setting [9]. In another application, fuzzy random was introduced to build an improved fuzzy random regression for data preparation by using time series data [10][11].

Various applications presented fuzzy random concept due to its capability in handling factors of fuzziness and randomness. However, the existing studies mostly focused on fuzzy equation regarded as the concept of possibility, without considering the probability equation. Moreover, the models are not adequately discussed on how to estimate probability to reduce randomness [6][7][11]. To date, probability theory is used to model randomness which recorded from dispersion of the measured value [9]. Hence, according to abovementioned reason, this study is to present a systematic procedure to control randomness. This study is concentrated on developing a procedure of probability estimation for the fuzzy random data. This systematic procedure is important to guide the identification of probability estimation in defining the fuzzy random data for developing fuzzy random regression model.

The remainder of this paper is arranged as follow. Some preliminaries of uncertainty fuzzy random is covered in Section II. Next section discusses the procedure of proposed method that is Naïve Bayes to estimate random value for fuzzy random regression model. An empirical study is provided in Section IV to illustrate the proposed method. Finally, Section V discusses the conclusion.

II. Theoretical Background

A. Fuzzy Random

The fuzzy random variable was introduced to present the real situation of uncertainty which comes from vagueness, imprecision, randomness etc. [1]-[5]. The concept of fuzzy random variable has been applied in several papers which combine both fuzzy random uncertainties [18]-[22].

Definition 1. Let Y be the fuzzy variable with possibility distribution μ_Y, the possibility, necessity, and credibility of event $\{Y \leq r\}$ are given in equation as follows.

$$Pos\{Y \leq r\} = sup\mu_Y(t), t \leq r \quad (1)$$

$$Nec\{Y \leq r\} = 1 - sup(t), t \geq r \quad (2)$$

$$Cr\{Y \leq r\} = \frac{1}{2}(1 + sup_{t\leq r}\mu_Y(t) - sup_{t\leq r}\mu_Y(t)) \quad (3)$$

where,

$Cr =$ Credibility measure

Pos= possibility

Nec= necessity measure

Credibility measure is an average of the possibility and the necessity measures, $Cr\{.\} = (Pos\{.\} + Nec\{.\})/2$. Credibility measure is presented to expand a certain measure of possibility and necessity, which is a sound of aggregate of two cases.

Definition 2: Let Y be a fuzzy variable. Under the assumption that the two integrals are finite, the expected value of Y is defined as follows in (7):

$$E[Y] = \int_0^x Cr\{Y \geq r\}dr - \int_{-\infty}^0 Cr\{Y \leq r\}dr \quad (4)$$

Following from Equation (4), the expected value of Y is defined as

$$E[Y] = \frac{a^l + 2c + a^r}{4} \quad (5)$$

where $Y = (c, a^l, a^r)_T$ is a triangular fuzzy number and c is a center value.

The expected value of the fuzzy variable $X(\omega)$ is denoted by $E[X(\omega)]$ for any fuzzy random variable X on Ω[5]. Thus, the expected value of the fuzzy random variable X is defined as the mathematical expectation of the random variable $E[X(\omega)]$.

Definition 3: Let X be a fuzzy random variable defined on a probability space $(\Omega, \sum, Pr)$ with expected value e. The expected value of X is defined in Equation (6) as follows.

$$E[\xi = \int \Omega \, [\int_0^{\infty} Cr\{\xi(\omega) > r\}dr - \int_{\infty}^0 Cr\{\xi(\omega) \leq r\}dr]\Pr(d\omega) \quad (6)$$

The variance of X [7] is defined as Equation (7), respectively.

$$Var[X] = E[(X - e)^2] \quad (7)$$

where $e = E[X]$.

B. Naïve Bayes

The Naïve Bayes is a simplification of Bayes Theorem which is used as a classification algorithm with an assumption of independence among predictors [8]. It is known as 'Naïve' because it assumes that the presence of input features is independent of each other. As the feature of the data points is unrelated to any other, therefore, changing of one input feature may not affect others [9].

The general equation for Bayes [10][11] is given as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A).P(A)}{P(B)} \quad (8)$$

where,

$P(A)$ = the probability of A occurring

$P(B)$ = the probability of B occurring

$P(A|B)$ = the probability of A given B

$P(B|A)$ = the probability of B given A

$P(A \cap B)$ = the probability of both A and B occurring

Given A as Hypothesis and B as evidence, the Bayes rules derive the probability of a hypothesis given the evidence. The rule stated the relationship incorporating $P(A)$ distribution in order to generate $P(A|B)$. $P(A)$ is the probability of an event before getting the evidence. The probability of the event based on the current knowledge before an experiment is performed. $P(A|B)$ is called the posterior probability which is calculated by updating the prior probability after taking into consideration new information. Meaning that, the posterior probability is the probability of event A occurring given that even B has occurred.

In this study, the Naïve Bayes theorem was proposed to be used in estimating probability values in constructing confidence intervals for fuzzy random regression model. This probability value is representative of randomness in the fuzzy random regression model. However, most studies [12] – [17] do not clearly describe how to obtain these random values to develop fuzzy random regression models. But these random values are very necessary to manage data that have random and fuzzy uncertainties to create forecasting models. The following section describes the proposed procedure for estimating probabilities for random values using the Naïve Bayes method that will be used to develop a fuzzy random regression prediction model.

III. Estimating Probabilities for Fuzzy Random Regression Procedure

This section describes the standard procedure to estimate probability value in developing confidence interval for Fuzzy Random Regression model. The procedure uses Naïve Bayes to characterize the random uncertainty.

The procedure for implementing the proposed method can be written in the following to determine the probabilities by using Naïve Bayes.

Step 1 : Suppose x_n are the fuzzy data. Transform crisp data x_n into fuzzy random data (FRD), c_n, a_n^l, a_n^r

Step 2 : Estimate probabilities for each FRD by using Naïve Bayes approach based on Equation (8). The fuzzy random data with probabilities can be arranged in the format as in Table I.

TABLE I. Data Format for Fuzzy Random Data

Sample	Input	FRD	Pr
1	X_1	c_1, a_1^l, a_1^r	Pr_1
2	X_2	c_2, a_2^l, a_2^r	Pr_2
…	…	…	…
n	X_n	c_n, a_n^l, a_n^r	Pr_n

Step 3 : Calculate the expected value, $E(x)$ using the center of triangular fuzzy variable with probability which $Pr + Pr = 1$. The formulation to calculate the expected value is shown in Equation (9) as follows:

$$E(x) = Pr_{v1}.E[x(V_1)] + Pr_{v2}.E[x(V_2)] \quad (9)$$

Step 4 : Calculate variance, $Var(x)$. Define the variance of x by using Equation (7). The $E[(x(v_1) - E[x]^2]$ should be calculated to obtain the $Var(x)$. The calculation to obtain the variance is based on equation (10) respectively.

$$Var\ (x) = E[(x(v_1) - E[x]^2].Pr_{v1} + E[(x(v_2) - E[x]^2].Pr_{v2} \quad (10)$$

Step 5 : Determine the confidence interval, CI of FRD using Equation (11) as follows:

$$CI = [(E(x) - Var(x)), (E(x) + Var(x))] \quad (11)$$

Step6 : Estimate coefficient based on confidence interval in Step 5. The coefficient can be obtained using the following linear programming.

$$\min J(\tilde{A}) = \sum_{k=1}^{K} (\tilde{A}_k^r - \tilde{A}_k^l)$$

$$subject\ to$$

$$\tilde{A}_k^r \geq \tilde{A}_k^l \quad (12)$$

$$\tilde{Y}_i = \sum_{k=1}^{K} \tilde{A}_k 1[ex_{ik,} \sigma x_{i1}] \supseteq_h I[eY_i, \sigma Y_i]$$

$$i = 1, \ldots\ldots, n; 1, \ldots\ldots, K,$$

The fuzzy random regression [7] prediction model was introduced with the advantage of handling data that had dual uncertainties namely fuzziness and randomness. Although this model is good for handling uncertainties, there are constraints for the industry to apply this model in the real world if there is no complete method specially to transform normal data into an acceptable form of data by this model. Then the standard method that has been tested has been introduced [18] – [20]. However, some of them focus on methods of managing fuzzy data only. Thus, this study specializes in the development of standard procedures for determining random value for the development of fuzzy random models.

IV. Numerical Experiment

In this section, a numerical experiment has demonstrated to visualize how the probabilities are estimated using proposed procedure in order to handle randomness. The fuzzy random input and output data are taken from [7] in Table II and Table III, respectively.

Table II shows the fuzzy random input data with two attributes (X_1, X_2). Each attribute has four samples which divided into center, left and right (c, a^l, a^r). Table III shows the fuzzy random output data with four samples of attribute Y and divided into (c, a^l, a^r).

TABLE II. Fuzzy Random Input Data

Sample	X	FRD1			FRD2		
		c	a^l	a^r	c	a^l	a^r
1	$X1_1$	3	2	4	4	3	5
2	$X1_2$	6	4	8	8	6	10
3	$X1_3$	12	10	14	14	12	16
4	$X1_4$	14	12	16	16	14	18
Sample	X	FRD1			FRD2		
		c	a^l	a^r	c	a^l	a^r
1	$X2_1$	2	1	3	4	3	5
2	$X2_2$	3	2	4	4	3	5
3	$X2_3$	12	10	16	14	12	16
4	$X2_4$	18	16	20	21	20	22

TABLE III. Fuzzy Random Output Data

Sample	X	FRD1			FRD2		
		c	a^l	a^r	c	a^l	a^r
1	Y_1	14	10	16	18	16	20
2	Y_2	17	16	18	20	18	22
3	Y_3	22	20	24	26	24	28
4	Y_4	32	30	34	36	32	40

By using the values in fuzzy random data, the probabilities for each value can be determined. Using calculation in Equation (8), the probabilities for each fuzzy random data are estimated.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

Assuming each feature variable is independent of the rest, calculate the probability of each separate feature given of each class. First step is finding the prior probability of each class in $X1$. Given sample $X1$ has four variables $(X1_1, X1_2, X1_3, X1_4)$. Each variable has one event occur. In mathematical, the probability can be represented as $P(X1_1) = \frac{1}{4} = 0.25$ which mean the occurring event happening is likely one time by considering the total of potential outcome. Thus, the prior probability of each class in $X1$ is as follows.

$$P(X1_1) = \frac{1}{4} = 0.25$$

$$P(X1_2) = \frac{1}{4} = 0.25$$

$$P(X1_2) = \frac{1}{4} = 0.25 \quad (13)$$

$$P(X1_2) = \frac{1}{4} = 0.25$$

Following the calculation in Equation (13), find probabilities for input $X2$ and output Y. The result for each probability is tabulated in Table IV and Table V, respectively.

Table IV and Table V show the probabilities for fuzzy random input and output data. The probabilities for FRD1, Pr_1 are determined using calculation in (13), with value 0.25. Note that, probability is counted by $Pr_1 + Pr_2 = 1$. Therefore, the probability value for FRD2, Pr_2 is 0.75. These probabilities are used to calculate the expected value $E(x)$ using the center of triangular fuzzy variable as in Equation (9). Based on these values of $E(x)$, variance $Var(x)$ can be calculated using Equation (10). The results for $E(x)$ and $Var(x)$ are tabulated in Table VI respectively.

TABLE IV. FUZZY RANDOM DATA WITH PROBABILITIES FOR INPUT DATA

Sample	X1	FRD1				FRD 2			
		c	a^l	a^r	Pr_1	c	a^l	a^r	Pr_2
1	$X1_1$	3	2	4	0.25	4	3	5	0.75
2	$X1_2$	6	4	8	0.25	8	6	10	0.75
3	$X1_3$	12	10	14	0.25	14	12	16	0.75
4	$X1_4$	14	12	16	0.25	16	14	18	0.75
Sample	**X2**	**FRD1**				**FRD 2**			
		c	a^l	a^r	Pr_1	c	a^l	a^r	Pr_2
1	$X2_1$	2	1	3	0.25	4	3	5	0.75
2	$X2_2$	3	2	4	0.25	4	3	5	0.75
3	$X2_3$	12	10	16	0.25	14	12	16	0.75
4	$X2_4$	18	16	20	0.25	21	20	22	0.75

TABLE V. FUZZY RANDOM DATA WITH PROBABILITIES FOR OUTPUT DATA

Sample	Y	FRD1				FRD 2			
		c	a^l	a^r	Pr_1	c	a^l	a^r	Pr_2
1	Y_1	14	10	16	0.25	18	16	20	0.75
2	Y_2	17	16	18	0.25	20	18	22	0.75
3	Y_3	22	20	24	0.25	26	24	28	0.75
4	Y_4	32	30	34	0.25	36	32	40	0.75

Table VI shows the value of expectation and variance for the input output fuzzy random data. The expectation and variance values are used to find confidence interval by using Equation (11). The results are tabulated in Table VII.

Table VII shows the confidence interval result for fuzzy random input and output data. In this study, the confidence interval was considered as one-sigma confidence $(1 \times \sigma)$ interval of each fuzzy random variable. The combination of expectation and variance of fuzzy random variable was induced to define the confidence-interval-based-inclusion [7]. Based on this confidence interval, a fuzzy random regression model can be formulated using mathematical linear programming as in Equation (12) in order to define coefficient.

TABLE VI. EXPECTATION AND VARIANCE OF THE DATA

i	Ex_1, V_{x1}		Ex_2, V_{x2}		E_y, V_y	
1	3.75	0.5729	3.5	1.2031	16.2	10.6688
2	7.5	2.2917	3.75	0.5729	17.6	1.8113
3	13.5	2.2917	13.63	3.467	24.8	4.8125
4	15.5	2.2917	20.35	3.7138	34.4	4.8125

TABLE VII. CONFIDENCE INTERVAL FOR FUZZY RANDOM INPUT OUTPUT DATA

i	$X1$	$X2$	Y
1	[3.177, 4.323]	[2.297, 4.703]	[5.531, 26.869]
2	[5.208, 9.792]	[3.177, 4.323]	[15.789, 19.411]
3	[11.208, 15.792]	[10.158, 17.092]	[19.988, 29.613]
4	[13.208, 17.792]	[16.536, 23.964]	[29.588, 39.213]

$$\min J(\tilde{A}) = \sum_{k=1}^{K} (\tilde{A}_k^r - \tilde{A}_k^l)$$

$$subject\ to$$

$$\tilde{A}_k^r \geq \tilde{A}_k^l$$

$$\tilde{Y}_i = \sum_{k=1}^{K} \tilde{A}_k 1[ex_{ik,} \sigma x_{i1}] \supseteq_h I[eY_i, \sigma Y_i]$$

$$i = 1, \ldots\ldots, n; 1, \ldots\ldots, K,$$

$$min = (a_1^r - a_1^l) + (a_2^r - a_2^l);$$

$$a_1^l <= a_1^r;$$

$$3*a_1^l+2*a_2^l<=3.177075;$$

$$6*a_1^l+4*a_2^l<=5.208325;$$

$$12*a_1^l+10*a_2^l<=11.208333;$$

$$14*a_1^l+12*a_2^l<=13.208333;$$

$$3*a_1^r+5*a_2^r>=4.322925;$$

$$6*a_1^r+10*a_2^r>=9.791675;$$

$$12*a_1^r+16*a_2^r>=15.791667;$$

$$14*a_1^r+18*a_2^r>=17.791667;$$

$$a_1^l>=0; a_1^r>=0;$$

$$a_2^l>=0; a_2^r>=0; \tag{14}$$

The linear programming of the fuzzy random regression was applied to the dataset as shown in Equation (14). This linear programming is performed to generate the coefficient value as tabulated in Table VIII respectively.

TABLE VIII. COEFFICIENT OF THE FUZZY RANDOM INPUT OUTPUT DATA

Item	Coefficient		Width
	A1	A2	
Y	0.00	0.943	0.79
$X1$	0.00	1.044	0.113
$X2$	0.00	1.052	0.073

The coefficient result for the fuzzy random input output data as tabulated in Table VIII shows the values estimated from fuzzy random regression. The attribute which has larger coefficient value is more significant to the total evaluation. In this result, it shows that the evaluation of attributes $A1$ and $A2$ indicate the $A2$ is significant to the total evaluation due to its higher coefficient. The model had a wider coefficient width because of the consideration of the confidence interval in its evaluation. The width in this evaluation plays an important role, as it reflects natural human judgment.

A greater breadth denotes the evaluation's ability to capture more data while using fuzzy judgments. Mean Square Error (MSE) can be defined using the estimated coefficient obtained from the fuzzy random regression and the model in Equation (14). In Table IX, the mean squared error (MSE) is calculated to compare the outcomes of the existing approach and the suggested method.

TABLE IX. MSE RESULT

Watada [7]	Naïve Bayes
196.6845	193.8861

Table IX shows the MSE result using Naïve Bayes approach as compared with current method by Watada *et al.*, [7]. In comparison study, the testing data derived from proposed method have a close majority of the expectation and variance result when compare to [7]. As the majority of the expectation and variance have been captured, therefore, both confidence intervals from testing and current model are quite similar. The evaluation of MSE was considered using current model and testing. From the result shown in Table IX, MSE of the proposed model is smaller than the other. This MSE implies that the prediction error can be reduced significantly.

The outcomes of the experiment demonstrate that the suggested approach is highly accurate at estimating the expectation, variance, and confidence interval of the data. Additionally, it is more accurate than the present technique and has a lower MSE, proving its superiority. These findings imply that the suggested approach can estimate probability and circumvent data unpredictability.

V. CONCLUSION

In this paper, a procedure based on Naïve Bayes is proposed to treat data which contain uncertainty known as fuzzy random data. The uncertainty data of randomness was handled by implementing the Naïve Bayes method to estimate probability. As to demonstrate the potential application of proposed method for accessing estimation, an experimental study using fuzzy random data is illustrated and the results are compared with the result of current method. The result shows that the proposed method has majority close of the expectation, variance and confidence interval. Further, it also has better MSE result than the current method. The result demonstrated that the proposed model is capable to estimate probability and overcome randomness of the data.

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