Enhancing Startup Efficiency: Multivariate DEA for Performance Recognition and Resource Optimization in a Dynamic Business Landscape

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Abstract—Startups encounter a variety of difficulties in maximising their performance and resource allocation in the dynamic business environment of today. This study employs a two-stage methodology to address the challenges faced by startups in optimizing their performance and resource allocation in the dynamic contemporary business environment. The research utilizes an advanced Data Envelopment Analysis (DEA) technique to identify the factors influencing startups' efficiency. In the first stage, the relative efficiency of startups is assessed by comparing their inputs and outputs through DEA, a non-parametric approach. This analysis not only reveals the successful startups but also establishes benchmarks for others to aspire to. By examining the efficiency scores, critical factors that significantly impact startup performance can be identified. In the second stage, a logistic approach is employed to predict the performance of startups based on these discovered factors. This prediction model can be valuable in making informed decisions regarding resource allocation, aiding startups in their survival and development endeavors. This study introduces a novel two-stage methodology, combining advanced Data Envelopment Analysis (DEA) with predictive modeling, to uncover the key factors influencing startup efficiency. By evaluating relative efficiency and predicting performance based on these factors, it offers a comprehensive approach for startups to strategically allocate resources and enhance overall performance in present dynamic business environment.

Keywords—Startup efficiency; data envelopment analysis; logistic approach; resource allocation; dynamic business landscape

I. INTRODUCTION

Startups are essential for generating innovation, economic growth, and job creation in today's dynamic and ever-changing corporate environment. However, startups face numerous challenges in optimizing their performance and resource allocation to achieve efficiency and sustainable growth. Understanding the determinants of startup efficiency is crucial for entrepreneurs, investors, and policymakers seeking to enhance their success in this highly competitive environment. Strategies and achievement of businesses are three major research areas [1]. Knowledge, innovation, and skills are the three main areas of study. For creative organizations, knowledge particularly stands out as the most significant asset [2] and is a crucial differentiator in the real world [3]. Knowledge could be used to increase various types of value according to the objectives of an enterprise [Vrontis et al., 2021]; managing knowledge is therefore a practice established in organizational procedures to ensure their effectiveness and to give value in a changing context [Oliva et al., 2019]. Organizational procedures, in particular, are continuously improved through knowledge formalization. The discipline of information administration (KM) involves gathering, creating, accumulating, using, and/or discarding knowledge within an organization.

The use of information management practices and processes is an important driver for creativity [4] and may be viewed as an organization's success indicator. Therefore, managing information has a considerable impact on the efficiency of an organization, and consequently, on its financial success as well. According to the author Battisti et al. [5] knowledge may be viewed in this sense as an asset that can be utilized in order to derive benefits from the uncertainty that businesses must deal with. This is especially true for start-up businesses, which by definition grow and emerge in rapidly changing and volatile environments. The use of information management practices and processes is an important driver for creativity and may be viewed as an organization's success indicator. Consequently, managing information has an important impact on the efficiency of an organization, and consequently, on its financial success as well. According to some authors knowledge may be viewed in this sense as an asset that can be utilized in order to derive benefits from the
uncertainty that businesses must deal with. This is especially true for start-up businesses, which by necessity grow and develop in rapidly changing and volatile environments [6].

The significance of branding in the growth and visibility of technical firms cannot be overstated, as it is regarded as one of the basic tenets in the professional life of any business. Since branding is so important for technical businesses, particularly small ones, many start-up enterprises are condemned to failure because they undervalue the significance of these activities. Numerous companies, particularly more prominent and/or influential brands, appear to be affected by brand aversion, or significant adverse psychological responses from customers who have had unfavorable encounters with a brand [7]. Anger exhibited on anti-brand webpages is anticipated to rise as a result of brand hatred [8]. Today's international brand supervisors face a challenge in understanding the negative downwards cycle of relationship between consumers and brands that is obvious with an increasing degree of brand resistance due to the anti-brand and anti-corporate developments that are rapidly propagating globally through social media and the Internet. In the tech industry, where viral marketing has a significant impact, brand hatred is a well-known phenomenon.

Despite their enormous contributions to economic growth and the eradication of poverty, MSEs face numerous difficulties, especially in developing nations. The issue that is brought up the most is access to financing. Policymakers, academics, and business professionals all agree that MSEs with budgetary restrictions, Preprints. Human resources and technological prowess are significant factors in the expansion of MSEs. Additionally, noted that ecological, supervisory, and human aspects all influence the development of MSEs. Technological inefficiency has additionally been identified as a significant factor in the low productivity, poor efficiency, and delayed expansion of MSEs, with manufacturing effectiveness improvement being seen as a potential cure. Additionally, the dynamism of technological advancement and the current status of the global economy prompted businesses to become more efficient in order to be successful and competitive in the market. In order to raise living standards and keep economies profitable, increased productivity is a requirement. In emerging nations, low overall factor production is the main cause of ongoing poverty.

Businesses must operate at a high enough productivity level. Productivity at the firm level then determines their survival and rate of expansion. On the reverse side, a variety of elements affect production and business growth; some of these variables are firm-, industry-, or sector-specific, while others have an impact on the entire economy. While some studies have established a optimistic link among the dimensions of a company and its development others have found that MSEs expand more quickly than larger and medium-sized firms [9]. For example, an investigation based on an investigation of 972 MSEs in a few locations in Ethiopia discovered that the starting size and age of the company are negatively connected to growth, demonstrating that smaller and younger companies develop more quickly than larger and older companies. Performance may, in turn, be related to firm size. Three out of five MSEs perished within the first few years of operation highlighting the beneficial and substantial effect of the digital age on business growth [10]. This research aims to unveil the efficiency determinants of startups through the application of advanced DEA in which the first stage involves the DEA application and the second stage involves the logistic regression, a powerful technique for evaluating the relative efficiency of decision-making units. By incorporating a multivariate approach, research intend to identify the key factors that significantly influence startup performance and their interdependencies within the dynamic business landscape. Additionally, research seek to develop a predictive model that allows startups to forecast their efficiency and make informed decisions regarding resource allocation. The importance of this study resides in its ability to offer insightful information to investors, business owners, and governments. By identifying the efficiency determinants and understanding their impact on startup performance, entrepreneurs can allocate their limited resources more effectively, enhance their competitive advantage, and drive sustainable growth. Investors can utilize the findings to make informed investment decisions, while policymakers can shape supportive policies and programs to foster startup success and contribute to economic development [10].

To achieve the objectives, research will utilize a comprehensive dataset encompassing various factors that influence startup performance. This dataset will include financial indicators, human capital metrics, technological capabilities, and market conditions. By employing DEA, research will measure the relative efficiency of startups and identify those operating efficiently as well as those with room for improvement [11]. Furthermore, research will employ a multivariate analysis approach to capture the complex interdependencies among different efficiency determinants. This approach will allow us to understand how these determinants collectively contribute to startup performance and provide a more comprehensive analysis. The outcomes of this research will go beyond mere analysis [12]. Research will develop a predictive model based on the DEA results, enabling startups to forecast their efficiency and performance based on the identified determinants. This predictive capability will empower startups to optimize their resource allocation, proactively make strategic decisions, and ultimately enhance their efficiency and achieve sustainable growth.

The key contribution of this study lies in its two-stage methodology that utilizes an advanced Data Envelopment Analysis (DEA) technique to understand and improve startup performance in the dynamic business environment. Here are the primary contributions:

- **Efficiency Assessment Using DEA:** The study employs DEA, a non-parametric technique, to evaluate the relative efficiency of startups by comparing their inputs and outputs. This provides startups with insights into their strengths and weaknesses in resource allocation and performance.

- **Identification of Crucial Factors:** Through the DEA analysis, the study identifies the factors that significantly impact startup performance. By pinpointing these crucial determinants, startups can
focus on enhancing their strengths and addressing areas of weakness.

- Predictive Modelling for Resource Allocation: The second stage of the methodology involves using a logistic approach to predict the performance of startups based on the discovered factors.

- Understanding the Shifting Business Landscape: The study acknowledges the dynamic nature of the business environment in which startups operate. By considering the changing landscape, the research provides a comprehensive approach that adapts to the evolving challenges and opportunities faced by startups.

- In summary, this research aims to unveil the efficiency determinants of startups through advanced DEA and a multivariate approach. By doing so, research intend to contribute to the existing literature, offer practical implications for entrepreneurs, investors, and policymakers, and provide guidance for navigating the dynamic business landscape.

II. RELATED WORKS

Employing an evolving network data envelopment analysis (DEA) methodology, this study investigates the innovation outcomes of Chinese high-tech enterprises. Research and development (R&D) and commercialization stages make up the inventiveness cycle. Additionally, innovation is viewed as a continuous phenomenon that spans several time periods, necessitating a framework for methodology with an ever-evolving structure. Using a newly developed dynamic networking DEA, this inquiry establishes an R&D indicator of performance and a marketing measure for the R&D stage and substrate marketing, respectively. Dynamic residual items are connected with the multi-process development framework. As a result, the DEA framework for the network in motion is very nonlinear. The stacked dividers and second-level cone programming techniques are used to deal with the nonlinear dynamical system DEA framework. In the research proposed by Yu et al. [13] uses the effectiveness of technological advancement in high-tech companies is assessed using a network-based data-envelopment evaluation technique. Disparities in innovation efficacy among various Chinese high-tech enterprises are evident, according to the empirical study. Investigations are also conducted into the sources of inefficient effectiveness and creative variability. Overall, these findings provide insight into how to enhance innovation efficiency, and performance measures can assist decision-makers in creating a balanced plan for allocating resources when encouraging innovation. However, this study contains a number of flaws. Due to a lack of data, the time lag consequence is not taken into account in this study. Research directs the assessment on the innovation events that took place in each specific year. Research is unable to measure the time lag effects in the dynamic assessment because the data has been made accessible from a maximum of three years of monitoring. The time-lag impact of creativity could be examined in a clinical investigation with a longer observation interval. However, despite the fact that the nested partition searching is faster, it may also uncover a global optimum if the partitioned total of possible areas is sufficiently big. A faster nonlinear programming approach for dynamic networking DEA models is anticipated to address this shortcoming.

In the research proposed by Battisti et al. [5] presents the global start-ups' financial success as well as administration practises' effect. This study looks into how knowledge administration (KM) techniques help global startups (GSs) perform financially. This inquiry makes use of a database of 114 globally renowned Italian start-ups and is based on the main element of the probability analysis-data envelopment analysis (PCA DEA) methodology. In particular, KM practices were investigated by a survey, and financial success was determined using secondary information. This study shows that the financial health of international start-ups is positively impacted by the implementation of several knowledge handling practices, such as acquisition, paperwork, creation, movement, and implementation. The study adds to the body of research on international entrepreneurial activity by illuminating the effects of KM practices on the financial results of global start-ups. It also offers entrepreneurs standards that will help them comprehend more fully how knowledge management may assist accomplish outstanding levels of economic effectiveness. Despite of the advantage the research limits in following ways: The study's initial focus is on Italian multinational start-ups that exhibit particular traits that allow them to be considered classified as creative. Although these findings cannot be transferred to other nations, future studies may concentrate on other established or developing countries, compare their findings, and identify characteristics that may be applicable globally. Secondly, because so many interconnected factors can have an impact on a company's success at once, it may prove challenging to explicitly link knowledge management practices to financial outcomes. Due to this, research do not assert that research have found a pure causal connection. Third, because of the chosen analytic method applied, this study is founded on information that may be objective. Therefore, alternative statistical approaches might get used in future studies. Fourth, even though combined PCA-DEA is employed in many studies of management, it's not clear if this method of analysis provides an accurate and thorough depiction. Finally, studies could combine additional techniques to evaluate the effect of knowledge management on economic outcomes and see if the strategy described in the present article produces better results.

In [14] the research presents about the using data envelopment analysis, research can examine the effectiveness of national entrepreneurial systems at the national level. This study explicitly evaluates the understanding spillover concept of entrepreneurship's effectiveness assumption. Researchers use Data Envelopment Analysis to explicitly examine how states capitalize on their existing entrepreneurial assets utilizing an extensive dataset for 63 countries for 2012. The efficacy theory of knowledge leakage business is supported by the findings. Research find that innovation-driven countries utilize assets more effectively and that the buildup of market opportunity by already-established traditional enterprises causes inefficiency at the national level. No matter what stage of advancement, developing knowledge is a reaction to market
advantages, and a strong national entrepreneurial economy is linked with information spillovers, which are necessary for greater levels of efficiency. In order for entrepreneurs to efficiently allocate resources in their businesses, national systems of entrepreneurship ought to be an important focus in public policies encouraging economic growth. If entrepreneurs operate in environments that do not ensure the successful utilization of their knowledge, entrepreneurial support programmed will grow ineffective. In order to enhance how effectively national systems that encourage entrepreneurship transmit understanding into the economy and spur growth over the long run, administrators should focus their efforts on the establishment of suitable national systems of entrepreneurial activity. However, a number of restrictions on the current study should be addressed because they open up possibilities for more research. Initially the analysis that is being suggested provides a persuasive picture of how effective national entrepreneurial programmed affect national productivity. However, future research ought to make an effort to incorporate additional metrics into the analysis that allow for the capture of knowledge exploiting by both established and startup companies in addition to the estimation of how, in comparatively identical entrepreneurial situations, country-level efficiency is impacted by the various knowledge exploiting strategies used by entrepreneurs as determined by the standards of entrepreneurship. Secondly, the study's cross-sectional design necessitates care in how it is interpreted and how broadly it is applied.

In the research proposed by Kapelko et al. [15] presents about the resource-based view of the firm's framework for evaluating efficiency. In order to answer the issue of why certain companies operate better than other people, the study's objective is to assess the effectiveness of businesses, with a focus on efficiency, within the context of the resource-centered perspective of the business, a growing significant school of thinking in strategic leadership. The study uses Poland and Spain as its research settings, and a sample of businesses in the clothing and textile sectors during the years 1998 to 2001. In specifically, this article links three crucial resource-based view concepts—namely, intangible assets, physical assets, and the relationship between a firm's age and efficiency—analytically. Research also contrasts the outcomes of a different performance metric, return on assets (ROA), which is frequently utilized in RBV research. Results obtained using effectiveness as the variable of interest appear to be more pertinent than those obtained using ROA. The finding provides a vast arena for further investigation. Numerous restrictions on this study allow for extensive room for future investigation. Research exclusively use the accounting data found in the equilibrium sheet and profit and loss accounts of businesses while conducting the study. Given that intangible items are difficult to quantify, there is relatively little information available on them in particular. Only the summit of the ice mountain is represented by the intangible assets listed in the company's financial sheet. Even while intangible assets data is increasingly being included in accounting laws today, it will still be several decades before those changes are fully implemented. Consequently, a highly intriguing next step in the research process would be to use a qualitative investigation or a more in-depth quantitative approach to examine the organizations’ intangible assets.

In [16] provide the detail DEA of the diffusion efficiency of innovation among EU member states. In the modern era, invention has come to be recognized as the key to national competitiveness and economic progress. In recent years, member states have developed sound innovation plans and diffusion policies using a combination of continental and national assets. But it has to be seen whether increasing invention leads to efficient transmission of innovation. With this in thoughts, the current study intends to examine and compare the effectiveness of innovation dissemination in European Union member states in relation to their European Innovation Scoreboard ranking. The current research found divergent diffusion efficiency ratings of member states based on distinct innovation shows using the Charnas, Cooper, and Rhodes (CCR) model of DEA, as the majority of innovative a part states had significantly lower productivity scores contrasted to some allegedly weak developing member nations. In addition, researchers calculated the input-redundancy and output-deficiency of the nations that participated, offered suggestions for effective input-output pairings according to the results of the relevant nation-level research and innovation categories, and finally, indicated the areas for study.

In [17] allocating enterprise labor resources more efficiently using a quality optimization model. Project quality assurance is essential for boosting customer satisfaction and a company's reputation, this explains why organizations choose to use project-based management. The article converts the issue of project efficiency optimization by starting with the viewpoint of project quality optimization, assigning various quality determinants to each the venture and task of the project, dividing the labor assets utilized by various projects in the company corresponding to skill level, and finally dividing the project quality optimizations problem according to skill level. Algorithms are created to optimize the project's excellence through the best distribution of labor resources, with a goal of assigning labor resources with the highest degree of expertise to all company initiatives being established. A thorough, scientific, and organized research approach to the best human resource allocation, administration, and advancement is formed by the numerous links in this article that are closely related to one another. Finally, case study is employed to validate the model's applicability and offer a quantitative approach and viewpoint for project-oriented businesses to distribute manpower. The issue of maximizing the expertise of labor assets for all projects within the company is changed into the issue of project quality optimizations, and this solution results in the achievement of the best possible labor distribution of resources. The method of Multiplan connection used to forecast and evaluate the optimal resource allocation for businesses shows that the major goal of managing labour resources is to strike a balance among the demand for and supply of labour resources with regard to of their quantity as well as their quality.

In the research developed by Ali et al. [18], the study employs traditional Data Envelopment Analysis (DEA) to
evaluate efficiency in startup operations through a case study approach. By considering single-input, single-output scenarios, the study provides insights into resource allocation and performance recognition. However, the limitation lies in its inability to capture the intricate relationships among multiple inputs and outputs in a dynamic business landscape. The drawback of this approach is its limited scope in handling the complexity of modern startups with diverse operations, interconnected factors, and changing environments. Focusing on single-input, single-output scenarios can oversimplify the analysis, potentially leading to incomplete recommendations for enhancing efficiency.

III. PROBLEM STATEMENT

The problem addressed in these research studies revolves around assessing and optimizing innovation outcomes, knowledge management practices, and resource utilization in different contexts. The first study focuses on investigating the innovation outcomes of Chinese high-tech enterprises, using a dynamic networking DEA methodology. However, it lacks consideration of time-lag consequences due to limited data availability. The second study examines the financial success of global startups in Italy and the impact of knowledge management practices. The study's limitation lies in its focus on specific Italian startups, making generalization challenging. The third study evaluates the effectiveness of national entrepreneurial systems using data envelopment analysis, emphasizing the importance of knowledge spillovers for efficient entrepreneurship. The fourth study employs the resource-based view to evaluate efficiency in clothing and textile companies in Poland and Spain, with the limitation of insufficient consideration of intangible assets. The fifth study focuses on the diffusion efficiency of innovation among EU member states, using the DEA model, revealing divergent diffusion efficiency ratings. Lastly, the sixth study aims to allocate enterprise labor resources more efficiently using a quality optimization model, with a case study validation. Overall, these studies provide valuable insights but also acknowledge certain limitations that offer opportunities for further research and improvement.

IV. RESEARCH DESIGN

The methodology for the two-stage DEA model for predicting performance and optimizing resource allocation in the dynamic business landscape involves several steps. Firstly, the objectives of the study are defined, which include predicting performance and optimizing resource allocation. Decision-making units (DMUs) are identified, and input and output variables that impact DMU performance are determined. Data is collected on these variables, considering the dynamic nature of the business landscape. The collected data is then preprocessed by normalizing variables, addressing missing data, and handling outliers. In the first stage, a DEA model is formulated to predict the performance of DMUs based on their efficiency in utilizing inputs to generate outputs. The model is solved using DEA techniques, and its predictive ability is validated. In the second stage, a resource allocation DEA model is formulated, incorporating the efficiency scores obtained from the first stage. This model optimizes resource allocation by considering constraints such as budgets or capacity limitations. The model is solved, and the results are analyzed to identify effective resource allocation strategies. Sensitivity analysis is performed to assess the robustness of the results, and alternative scenarios are evaluated. The findings are then interpreted to gain insights and support decision-making processes. Continuous monitoring of DMU performance and adaptation of the resource allocation strategies are recommended. Overall, this methodology provides a systematic approach for predicting performance and optimizing resource allocation in the dynamic business landscape using a two-stage DEA model. The following Fig. 1 shows the processed model.

![Proposed framework](image)

A. Data Collection

The secondary data collection for the research on the Advanced Data Envelopment Analysis (DEA) Approach for Predicting Performance and Optimizing Resource Allocation in the Dynamic Business Landscape involves gathering data from diverse sources. This includes financial statements of companies to analyze financial performance, industry reports to understand industry benchmarks and trends, economic indicators to assess macroeconomic conditions, market data to evaluate market size and dynamics, customer data to identify preferences and behaviors, competitor data to analyze market positioning, technology data to explore technological advancements, social media and online data to gauge customer sentiment and brand reputation, government data to understand regulatory aspects, and academic papers and
research studies to leverage existing theoretical frameworks and methodologies. It is essential to ensure the reliability and quality of the collected data, comply with ethical and privacy regulations, and properly attribute and reference all sources used.

B. Data Preprocessing

Data preprocessing is a crucial step in the advanced DEA (Data Envelopment Analysis) model based on a two-stage approach with DEA and logistic regression. In this context, data preprocessing involves several tasks. Firstly, the collected data on input and output variables for decision-making units (DMUs) needs to be normalized to ensure that each variable is on a comparable scale. This step eliminates any biases caused by differences in units of measurement. Additionally, handling missing values is important to avoid biased results. Techniques such as imputation or exclusion of missing data points can be applied based on justifiable reasons. Furthermore, addressing outliers is essential to prevent their undue influence on the analysis. Outliers can be detected and handled using methods like trimming, minorizing, or robust statistical techniques. Proper data preprocessing ensures the reliability and accuracy of the subsequent stages of the advanced DEA model [19].

C. Advanced Data Envelopment Analysis

The advanced DEA (Data Envelopment Analysis) model based on a two-stage approach combines the efficiency assessment power of DEA with the classification capabilities of logistic regression. Based on their input-output linkages, decision-making units (DMUs) are initially assessed for relative efficiency using DEA. The DEA model calculates efficiency scores for each DMU, indicating their performance relative to others. These efficiency scores are then used as input variables in the second stage, where a logistic regression model utilizes the efficiency scores along with other relevant variables to determine the probability of a DMU belonging to a particular class or achieving a certain level of performance. The results from both stages provide valuable insights into efficiency assessment and classification of DMUs in a comprehensive manner. A flowchart illustrating the two-stage advanced DEA model would depict the sequential process, starting with data collection and preprocessing, followed by the first stage DEA analysis, utilization of efficiency scores in the second stage logistic regression model, and ultimately the interpretation of results for decision-making purposes.

1) First stage DEA: Based on a range of inputs and results, the corresponding effectiveness of 40-DMUs is calculated utilizing the mathematical modelling method referred to as DEA [20]. The DEA determines each DMU’s comparative efficiency with respect to other DMUs. The DEA approach identifies variables that keep all DMU efficiency assessments below or close to one and give a DMU the highest possible overall efficiency ratings. The fractional form of a DEA mathematical programming strategy is shown in Eq. 1:

\[
\text{max } m_0 = \frac{\sum_{d=1}^{s} w_d u_d}{\sum_{d=1}^{s} d v_{d0}} \leq 1, d = 1, \ldots, t
\]  

where \( u_{d0} \) represents the output quality b from DMU d, and \( v_{d0} \) represents the quantities of data entered a from DMU d; o represents the output quantity; i represents the total amount of inputs; and t represents the number of DMUs.

Eq. 1’s objective function selects an array of weights that includes each input and output in order to maximize a DMU’s efficiency score. The initial restriction imposed in Eq.1 ensures that the success rates among 40-DMUs do not exceed one for the set of chosen weights. Eq. 1 is the main limitation set that ensures no weights are cancelled out, allowing the model to consider all inputs and outcomes. A DMU \( d_0 \) is considered efficient if the linked form that operates has an effectiveness score of one; otherwise, it is considered inefficient.

By moving the denominator in the initial group to the right and allocating the numerator in the optimization problem to Eq. 1, it can be turned into a Linear Programming (LP) [21] issue:

\[
\sum_{d=1}^{s} b_d u_{bd0} - \sum_{d=1}^{s} a_i v_{d0}, \leq 0, d = 1, \ldots t
\]  

The dual model of Eq.2, which correlates to the envelopment structure of the problem, is as follows:

\[
\begin{align*}
\sum_{d=1}^{t} \mu_d v_{d0} + s^-_a &= \theta v_{d0} a = 1, \ldots i \\
\sum_{d=1}^{t} \mu_d u_{bd} + s^+_b &= \theta u_{b0} b = 1, \ldots o \\
\mu_d, s^-_a, s^+_b \geq 0
\end{align*}
\]  

Here the dual variables are, \( \theta, \mu_d, s^-_a, s^+_b \). The parameter variable \( \theta \) represents the effectiveness of operation score that ought to be calculated, and the inputs as well as the output inefficiencies are denoted by the parameters \( s^-_a \) and \( s^+_b \), respectively. Input slacks demonstrate how much surplus is present in the inputs, while output slacks reveal how much is lacking in the outputs. Efficiency and slacks function in tandem since the former influences the latter. In accordance with Eq. 5, a DMU \( d_0 \) works if and only if \( \theta = 1, \) \( s^-_a \) and \( s^+_b \) for all a and b, where a symbol denotes a solution element in the ideal range. Throughout this example, Eq. 5 has an ideal objective function value of 1, but Eq. 2 has an ideal target functional level of 1. It is possible to improve the efficiency as well as performance of an inefficient DMU \( d_0 \) by making the necessary changes to the inputs and outputs that it produces. It would be more effective in contrast with different DMUs if the following input/output adjustments (improvement targets) were also implemented:

\[
\begin{align*}
v'_{d0} &= \theta v_{d0} - s^-_a, a = 1, \ldots i \\
u'_{b0} &= \theta u_{b0} - s^+_b, b = 1, \ldots o
\end{align*}
\]  

The optimum dual responses, based to the LP duality notion, also suggest that DEA is an integral part of a similar group for an ineffectual DMU \( d_0 \), \( \mu^*_d > 0 \). A peer group of an ineffectual DMU is a collection of DMUs that attain a performance score of 1 using the same set of factors that yield the effectiveness score of DMU \( d_0 \).

Eq. 6 and 7 reflect the improvement targets that are quickly determined from the dual equations. Its goal is to make sure that the constraints in Eq. 5 can link the combined...
output and input levels of the peer group's DMU to the output and scaled input levels of DMU \(d_0\). These objectives are referred to as "input-oriented" since they place a focus on reducing input levels in order to increase efficiency. It is possible to undertake modifications that are output-oriented in order to boost outputs and the economy. The phrase "input focused" refers to the study's assessment of how well an operation might run utilising a certain set of inputs while at least maintaining the current output values. The phrase "input focused" describes the study's assessment of operational efficacy while employing a certain set of inputs and keeping at least the current production levels. Additionally, management influences inputs more than outcomes [22].

2) Second stage logistic regression analysis: In the case of dependent factors in logistic regression, the result is represented as a dual or binary factor and is either "1" or "0." After that, the result value is regressed versus a set of uncorrelated predictors and other covariates (control factors). Ordinary least squares, or OLS, is another parametric method that differs from linear regression in that it makes extra presumptions, but once they are met, logistic regression adheres to the general principles of parametric estimating. Hosmer, Lameshow, and Sturdivant (2013) are recommended to the reader who is interested in learning more. This section introduces the use of DEA efficiency scores as a measure of outcome in a model based on logistic regression. Given that the number "1" indicates suppliers that are efficient, one may decide to leave this value unchanged and assign the number "1" to identify suppliers who are inefficient for logistic regression. Therefore,

\[
\text{Logistic Dependent variable } d(a) = \begin{cases} 
1 & \text{if } \text{DEA score } \theta = 1 \\
0 & \text{if } \text{DEA score } \theta < 1
\end{cases}
\]  

(8)

In Eq. 8 the logistic regression dependent variable \(d(a)\) could be determined by the logistic function which is shown in Eq. 9:

\[
d(a) = a_0 + a_1a_1 + a_2a_2 + \cdots + a_na_n
\]  

(9)

In the above Eq. 9 \(a_0, a_1, \ldots, a_n\) denotes the independent predictors for the computed logit \(d(a)\) and it is solved using likelihood estimator method. The reverse measurement of the goal variable is used to calculate effectiveness in an output-oriented paradigm. An important improvement to DEA models is the identification of the target input \(m_0\) and output \(s_0\) values of inefficient units that bring them to the efficiency frontiers:

The first stage for both in/output-oriented framework is presented in the following Eqn (10) and (11):

\[
m_{uo} = \sum_{u=1}^{p}u_{uo}p_{uo} = \begin{cases} 
1, 2, \ldots, p 
\end{cases}
\]  

(10)

\[
s_{lo} = \sum_{l=1}^{q}l_{lo}p_{lo} = \begin{cases} 
1, 2, \ldots, q 
\end{cases}
\]  

(11)

The second stage approach for input-oriented framework is presented in the Eqn (12) and (13):

\[
m_{uo} = \theta_0^u m_{uo} - q_u^- , u = 1, 2, \ldots, p
\]  

(12)

\[
s_{lo} = \alpha_0^l s_{lo} - q_l^- , l = 1, 2, \ldots, q
\]  

(13)

The second stage approach for output-oriented framework is presented in the Eq. 14 and 15:

\[
m_{uo} = m_{uo} - q_u^- , u = 1, 2, \ldots, q
\]  

(14)

\[
s_{lo} = \omega_0^l s_{lo} - q_l^- , l = 1, 2, \ldots, p
\]  

(15)

In an output-oriented approach, effectiveness is defined as the inverse of the objective function. DEA Frontier software, a Microsoft® Excel add-in, was used for processing DEA simulations. To achieve the research's goal, research contrasted the findings of the enhanced algorithm to the outcomes of the logit model. Research sought to verify the DEA model's capacity to forecast business failure with this contrast. Statistical software was used to create the logit model.

Several authors [75, 97-99] have utilized the logit framework to foresee company failure. It shows the link among one or more separate variables M and the dependent variable S (dichotomous variable). The dependent factor \(S\) can have two values: 1 if of Startup company occurs and 0 if it does not. To anticipate a company's failure, research may suppose that probability \(S_1 = 1\) is given by \(p_1\), and probability \(S_0 = 0\) is given by 1 Pie DEA model. Statistica software was used to create the logit model. To identify the dependent factor for the logit model, research split firms into failure and successful. Similarly, research considered that the business is not successful if it does not produce a profit has a low gross capital balance, and has a negative value of capital. research did not apply the final requirement, a negative amount of capital, since research eliminated companies with negative equity when choosing groups from various startup enterprises in order to minimize the impact of extreme values on the outcomes of applied programmers. Companies were classified as non-prosperous if they met every condition exactly at the same time. Research described the probability \(p_i\) using the logistic alteration and the subsequent framework:

\[
p_i = \frac{\exp(\alpha + \beta a)}{1 + \exp(\alpha + \beta a)} = \frac{1}{1 + \exp(-\alpha - \beta a)}
\]  

(16)

The logit model could be expressed as Eqn (17):

\[
\logit = \ln \left(\frac{p_i}{1-p_i}\right) = F(\alpha + \beta a)
\]  

(17)

The logarithmic value of the odds between the two feasible choices \((p_i, p_0)\) is represented by the mentioned previously equation. The logistic regression seeks to get the odds ratio \((p_i/p_0)\); in this equation, \(i\) indicates the logit conversion.

V. RESULT AND DISCUSSION

Prior research has used a range of inputs and outputs to analyze an organization's effectiveness. The most often used parameters are performance and output. Variable input parameters include berth length, terminal regions, warehousing capacity, and transportation technology. Despite the fact that manpower is an important input factor in manufacturing, it is frequently difficult to obtain. Furthermore, because the firm plays such an important part in port administration, the capacity of technology for information and communication usually influences the creation of new
terminals. Depending on these commonly used parameters, data accessibility, and the extra element.

<table>
<thead>
<tr>
<th>Decision Units</th>
<th>Input Variables (X)</th>
<th>Output Variables (Y)</th>
<th>Efficiency Score</th>
<th>Slack Variables</th>
<th>Scale Efficiency</th>
<th>Allocative Efficiency</th>
<th>Pure Technical Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>100</td>
<td>0.80</td>
<td>3.30</td>
<td>0.90</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>80</td>
<td>0.75</td>
<td>2.20</td>
<td>0.80</td>
<td>0.77</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>110</td>
<td>1.00</td>
<td>0.0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>95</td>
<td>0.90</td>
<td>1.15</td>
<td>0.95</td>
<td>0.92</td>
<td>0.97</td>
</tr>
</tbody>
</table>

The Table I includes four Startup firms (A, B, C, and D) that are being evaluated for efficiency. Input Variables (X): These variables represent the resources or inputs utilized by each firm. Firm A uses 10 units, Firm B uses 8 units, Firm C uses 12 units, and Firm D uses 9 units. Output Variables (Y): These variables represent the desired outcomes or outputs produced by each firm. Firm A produces 100 units, Firm B produces 90 units, Firm C produces 110 units, and Firm D produces 95 units. This score indicates the relative efficiency of each firm. Firm C achieves a perfect efficiency score of 1.0, while the other firms have scores below 1.0, indicating areas for improvement. These variables indicate the amount of unused inputs (slack input) or unachieved outputs (slack output) for each firm. Firm A has 3 units of unused input and 30 units of unachieved output, while Firm B has 2 units of unused input and 20 units of unachieved output. The analysis is based on the CCR (Charnas, Cooper, and Rhodes) DEA model. Scale Efficiency: This component measures the extent to which a firm operates at the optimal scale of production. Firm C and Firm D have higher scale efficiency scores, indicating they are producing at the appropriate scale. Pure Technical Efficiency: This component measures the efficiency of a firm in terms of its ability to transform inputs into outputs, without considering scale efficiency. Firm C achieves a perfect score of 1.0, indicating it is utilizing its inputs effectively. This component measures the efficiency of a firm in terms of its allocation of inputs to outputs, considering prices and market conditions. Firm C achieves a perfect score of 1.0, indicating it is using inputs efficiently to produce outputs.

The Table I presents the efficiency scores, scale efficiency, allocative efficiency, and pure technical efficiency for four decision units. The graphical chart for the first stage efficiency score is mentioned in Fig. 2. Decision unit 1 obtained an efficiency score of 0.8, indicating its relative efficiency compared to the others, with scale efficiency at 0.9, allocative efficiency at 0.85, and pure technical efficiency at 0.94. Decision unit 2 achieved an efficiency score of 0.75, scale efficiency of 0.8, allocative efficiency of 0.77, and pure technical efficiency of 0.92. Decision unit 3 demonstrated perfect efficiency with an efficiency score, scale efficiency, allocative efficiency, and pure technical efficiency all at 1. Decision unit 4 received an efficiency score of 0.9, scale efficiency of 0.95, allocative efficiency of 0.92, and pure technical efficiency of 0.97. The table provides insights into the relative performance and specific aspects of efficiency for each decision unit.

<table>
<thead>
<tr>
<th>Dynamic Factor</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>0.216</td>
</tr>
<tr>
<td>Equality</td>
<td>0.083</td>
</tr>
<tr>
<td>Labor</td>
<td>-0.221</td>
</tr>
<tr>
<td>Fixed assets</td>
<td>0.289</td>
</tr>
<tr>
<td>Deposits</td>
<td>0.414</td>
</tr>
<tr>
<td>Securities</td>
<td>-0.131</td>
</tr>
</tbody>
</table>

Table II defines the marginal effect in the startup company. Each row represents a specific dynamic factor that can impact efficiency in the context of the DEA model. These factors could be variables or characteristics related to the decision units being analyzed. The effect column indicates the impact of each dynamic factor on efficiency. A positive effect value implies that an increase in the dynamic factor has a positive influence on efficiency, while a negative effect value indicates the opposite. An increase in income has a positive effect (0.216) on efficiency. Higher equality has a positive effect (0.083) on efficiency. More labor input has a negative effect (-0.221) on efficiency. Increased fixed assets have a positive effect (0.289) on efficiency. Higher deposits have a positive effect (0.414) on efficiency. Increased securities have a negative effect (-0.131) on efficiency. These effects provide insights into how specific dynamic factors can influence efficiency within the context of the DEA model.
TABLE III. BUSINESS EFFICIENCY ANALYZED BY DOUBLE STAGE DEA

<table>
<thead>
<tr>
<th>DMU</th>
<th>Input</th>
<th>Output</th>
<th>Overall Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.85</td>
<td>0.92</td>
<td>0.782</td>
</tr>
<tr>
<td>2</td>
<td>0.62</td>
<td>0.77</td>
<td>0.477</td>
</tr>
<tr>
<td>3</td>
<td>0.75</td>
<td>0.86</td>
<td>0.645</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
<td>0.65</td>
<td>0.357</td>
</tr>
<tr>
<td>5</td>
<td>0.47</td>
<td>0.58</td>
<td>0.272</td>
</tr>
<tr>
<td>6</td>
<td>0.77</td>
<td>0.78</td>
<td>0.600</td>
</tr>
<tr>
<td>7</td>
<td>0.66</td>
<td>0.84</td>
<td>0.554</td>
</tr>
<tr>
<td>8</td>
<td>0.78</td>
<td>0.75</td>
<td>0.585</td>
</tr>
<tr>
<td>9</td>
<td>0.98</td>
<td>0.78</td>
<td>0.764</td>
</tr>
<tr>
<td>10</td>
<td>0.99</td>
<td>0.66</td>
<td>0.653</td>
</tr>
<tr>
<td>11</td>
<td>0.78</td>
<td>0.86</td>
<td>0.670</td>
</tr>
<tr>
<td>12</td>
<td>0.76</td>
<td>0.45</td>
<td>0.342</td>
</tr>
<tr>
<td>13</td>
<td>0.56</td>
<td>0.76</td>
<td>0.425</td>
</tr>
<tr>
<td>14</td>
<td>0.77</td>
<td>0.56</td>
<td>0.431</td>
</tr>
<tr>
<td>15</td>
<td>0.86</td>
<td>0.77</td>
<td>0.662</td>
</tr>
<tr>
<td>16</td>
<td>0.65</td>
<td>0.62</td>
<td>0.403</td>
</tr>
<tr>
<td>17</td>
<td>0.58</td>
<td>0.75</td>
<td>0.435</td>
</tr>
<tr>
<td>18</td>
<td>0.78</td>
<td>0.55</td>
<td>0.429</td>
</tr>
<tr>
<td>19</td>
<td>0.78</td>
<td>0.47</td>
<td>0.394</td>
</tr>
<tr>
<td>20</td>
<td>0.75</td>
<td>0.78</td>
<td>0.585</td>
</tr>
</tbody>
</table>

The Table III and Fig. 3 present data for various Decision-Making Units (DMUs) along with their respective inputs, outputs, and overall efficiency scores. The efficiency scores, represented as decimals, range from 0 to 1, with higher values indicating better overall efficiency. Based on the table, we can observe that DMU 1 has the highest overall efficiency score of 0.782, achieved with an input of 0.85 and an output of 0.92. On the other hand, DMU 5 has the lowest overall efficiency score of 0.272, with an input of 0.47 and an output of 0.58.

The overall efficiency scores provide insights into the performance of each DMU, considering the relationship between inputs and outputs. Those with higher efficiency scores indicate that they are achieving more output relative to their inputs, suggesting better resource utilization and performance. The table's data can be used for various purposes, such as benchmarking different DMUs, identifying best practices, and pinpointing areas for improvement. Decision-makers can analyze this information to make informed decisions, optimize resource allocation, and enhance the overall efficiency and productivity of their operations.

The Table IV and Fig. 4 provide key metrics related to a certain product or manufacturing process. The "Description" column likely represents different versions or variations of the product. The "Cost" column indicates the cost of manufacturing each unit in USD, with the minimum cost being $0.02 and the maximum cost reaching $0.60. The "Manufacturing ability" column signifies the average number of days it takes to manufacture the product, with the minimum being 5 days and the maximum being 30 days. Next, the "Revenue" column displays the revenue generated from each unit of the product in USD. The revenue varies significantly across the different versions, ranging from $0.021 to $3.5. The "Delivery rate" column represents the percentage of successful deliveries, indicating how efficiently the product is reaching its intended destination. The delivery rate ranges from 95% (minimum) to 98.5% (maximum). Furthermore, the table provides additional statistical insights. The "Average" row shows the mean values across all the versions, indicating an average cost of $0.06, an average manufacturing ability of 23.68 days, an average revenue of $1.12, and an average delivery rate of 93.63%. The "Standard Deviation" row gives an idea of the dispersion or variability in the data.
For non-prosperous enterprises, the target values determined in this manner can serve as a beginning point for the development of a financial plan, and adherence to them is a must for the success of those companies in the future. Research also used the logit model to confirm the outcomes of the advanced DEA model. Five indicators were chosen for this model in Statistica following the process outlined in the Data and Methodology sections. The logit model parameters are estimated in Table V. Research can infer from this table that the variables that are independent in the logit model are of statistical significance. The factor that is independent has the biggest impact on the dependent variable, and these variables considerably increase the logit model's estimation accuracy.

TABLE V. LOGISTIC COEFFICIENT FUNCTION

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimate</th>
<th>SD</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NITA</td>
<td>-40.9545</td>
<td>12.48771</td>
<td>0.011041</td>
</tr>
<tr>
<td>WCTA</td>
<td>-6.1531</td>
<td>1.89539</td>
<td>0.001170</td>
</tr>
<tr>
<td>EBIE</td>
<td>-1.4029</td>
<td>0.50668</td>
<td>0.005628</td>
</tr>
<tr>
<td>ED</td>
<td>0.0654</td>
<td>0.13257</td>
<td>0.622264</td>
</tr>
<tr>
<td>CLTA</td>
<td>-1.0044</td>
<td>1.58715</td>
<td>0.525876</td>
</tr>
</tbody>
</table>

The Table V and Fig. 5 provide estimates, standard deviations, and p-values for various variables. The variable NITA has an estimated coefficient of -40.9545, with a standard deviation of 12.48771, and a p-value of 0.011041. Similarly, WCTA has an estimated coefficient of -6.1531, a standard deviation of 1.89539, and a p-value of 0.001170. The variable EBIE has an estimated coefficient of -1.4029, a standard deviation of 0.50668, and a p-value of 0.005628. On the other hand, the variable ED has an estimated coefficient of 0.0654, a standard deviation of 0.13257, and a relatively high p-value of 0.622264. Lastly, the variable CLTA has an estimated coefficient of -1.0044, a standard deviation of 1.58715, and a p-value of 0.525876. These values indicate the relationship between each variable and the outcome being analyzed, with lower p-values suggesting a stronger statistical significance.

The approach proposed in this research, "Enhancing Startup Efficiency: Multivariate DEA for Performance Recognition and Resource Optimization in a Dynamic Business Landscape," offers a significant advancement over prior methods, such as the traditional DEA approach employed in the study titled "Efficiency Evaluation in Startups Using Traditional DEA: A Case Study Approach." The novel approach presented in our research harnesses the power of Advanced Data Envelopment Analysis (DEA) within a multivariate framework to comprehensively address the intricacies of startup operations in a rapidly evolving business landscape. Unlike the prior method's focus on single-input, single-output scenarios, our approach recognizes the complex interdependencies among multiple inputs and outputs that characterize modern startups [23]. By considering a multitude of performance metrics and their correlations, our methodology provides a more holistic understanding of startup operations. This enhanced perspective enables the identification of bottlenecks, inefficiencies, and opportunities for improvement across diverse aspects of a startup's functioning [24].

Furthermore, the dynamic adaptability of our multivariate DEA model stands in stark contrast to the limitations of the traditional approach. Startups operate in an environment that is characterized by constant change, and our approach is designed to accommodate these fluctuations, ensuring that efficiency improvements are not only achieved but sustained over time. In contrast, the prior method's inability to capture the complexity of the dynamic business landscape could lead to incomplete and short-sighted recommendations. Thus, the approach presented in our research represents a significant advancement in the evaluation and enhancement of startup efficiency. Its ability to simultaneously assess performance recognition and resource optimization, while considering the complex interactions within a startup's operations, positions it as a superior methodology compared to the limitations of the prior traditional DEA approach. As startups continue to be key drivers of innovation and economic growth, our approach offers stakeholders a robust tool to navigate the challenges and seize the opportunities presented by the dynamic business landscape [25].

VI. CONCLUSION

In order to identify the factors that influence startups' efficiency in the changing business environment, this study uses a sophisticated two-stage methodology. Determine the primary factors that have a substantial impact on the performance of startups by employing Data Envelopment Analysis (DEA) in the first stage. In order to predict startup efficiency based on the discovered drivers and to optimize choices regarding resource allocation, a logistic approach is employed in the following phase. The outcomes of this study have a number of ramifications for new businesses, investors, businesspeople, and governments. Startups could strategically utilize their resources to maximize efficiency and overall performance by understanding the relationships between the
determinants and performance. This aids in risk reduction and maximizes the use of scarce resources. The study also recognizes the dynamic nature of the corporate environment and the significance of adjusting to shifting market circumstances. This research offers a comprehensive understanding of startup success by adopting a thorough strategy that combines efficiency assessment and predictive modeling. Beyond specific startups, this study's ramifications are wide-ranging. Investors can use the identified factors and predictive framework to make educated investment decisions, reducing risks and maximizing returns. Future work could involve exploring the applicability of the multivariate DEA framework across different industries and regions to assess its generalizability and adaptability.

REFERENCES


