Computer Modeling of the Stress-Strain State of Two Kvershlags with a Double Periodic System of Slits Weighty Elastic Transtropic Massif

Tursinbay Turymbetov¹, Gulmira Tugelbaeva², Baqlan Kojahmet³, Bekzat Kuatbekov⁴, Serzhan Maulenov⁵, Bakhytzhan Turymbetov⁶, Mukhamejan Abdibek⁷ International University of Tourism and Hospitality, Turkistan, Kazakhstan¹

Military of the Ground Forces named after S. Nurmagambetov, Almaty, Kazakhstan²

Institute of Mechanics and Engineering Science academician U. A. Dzholdasbekov, Almaty, Kazakhstan²

Nazarbayev Intellectual Schools – Chemistry and biology in Turkistan, Turkistan, Kazakhstan²

Nazarbayev University, Astana, Kazakhstan²

Turkestan Higher Multidisciplinary Craft College, Turkistan, Kazakhstan^{3, 6}

School #26 named after Aiteke Bi, Turkistan, Kazakhstan^{4, 5, 7}

*Abstract***—This paper presents computer modeling of the stress-strain state of two kvershlags with a double periodic system of slits weighty elastic transtropic massif. It introduced key concepts such as 'kvershlag', a term used to describe perpendicular cavities in a layered massif, and 'weighted elastic transtropic massif', which refers to a specialized geological structure considered in the study. These terms are critical for understanding the modeling approach. Due to the complexity of the analytical solution of this class of problems, a numerical method is used. Such a mixed problem is provided to obtain a solution by bringing it to an equivalent environment. To solve such a mixed problem, it is offered to get a solution by bringing it to an equivalent climate in terms of stiffness. The finite element method was used to solve the problem. A software package has been created to solve the stress-strain state of the two kvershlags. To ensure the correctness of the software complex, it was checked using test tasks. To study the stress-strain state of kvershlags in a weighted massif. The basic systems of equations are obtained. Algorithms are constructed and the program complex FEM_3D for solving finite element method problems is compiled. Mixed problems of the stress-strain state of cavities are solved approximately. The results of complex computer calculations are systematized, analyzed, specific conclusions are drawn and recommendations for their practical application are proposed. A computer simulated the stress-strain state of two kvershlags. The numerical solution to the given problem was obtained using the software. Results demonstrate that our numerical method approach results in 0.01%.**

Keywords—Transtropic; cavities; stress-strain state; deformation; finite element; slits

I. INTRODUCTION

The first results in determining the deformation modulus of rocks were obtained in laboratory conditions. Extensive research in this direction has been undertaken in engineering schools in Germany, Austria, and Switzerland. It is enough to say the works of M. Bauschinger [1], K. Bach [2], O. Graf [3], O. Müller [4], O. Fleischer [5], K. Stöcke [6], and others. Similar works have been carried out since the 1950s under the

leadership of B.V. Zalesski [7], B.P. Belikov [8] at the Donetsk Polytechnic Institute, Research Institute of G.N. Kuznetsov's [9] provided. The gap in previous research lies in the limited focus on the stress-strain behavior of kvershlags in complex geological formations. Our study addresses this by applying a numerical approach based on the finite element method to model the stress-strain state of kvershlags in a weighted transtropic massif, offering new insights into their deformation behavior. Kvershlag refers to cavities that are perpendicular to the layered massif.

Difficulties in determining the deformation of rocks cannot be solved by selecting the number of limit cracks in a given homogeneous medium. This is a three-dimensional case. Due to the variety of natural conditions, it is impossible to create a general algorithm, even in an individual case. Therefore, a convenient situation is presented below to solve the problem. To investigate the stress-strain behavior of kvershlags, a mechanical-mathematical anisotropic model with reduced moduli dependent on the slit periods and physical properties of the massif is proposed. Elasticity is treated equivalently to displacements and deformation coefficients in stones. Its solution is sought through actual experiments or modeling.

Based on thorough research, it was taken the form of layers resulting from deformable slits as a function of the medium's parameters. These layers are dependent on the medium's parameters, which have been obtained analytically. It can be related to the "deformation of layered rocks" concept.

The main concepts of the effect of slits in deformable layered rock are presented in [10-28]. Displacement vectors are considered in [29] from the perspective of the mechanics of a slit homogeneous medium. From a geological point of view, a slit can be deemed to be regarded as a space between the walls of folded rocks.

A slit in a rock zone can be called a set of slits in layers. A length characterizes each slit. The slit can only be accurately measured, like longitude. The presence or absence of slit complements also distinguishes it. Accordingly, slits are

divided into open and filled. Slit fillers consist of sand, clay soils, or other components.

These layers differ in their mineral content and physical properties. The second type of complement of slits is called a latitudinal slit, which is a supplemental slit of several millimeters along the latitude.

These layers differ in their mineral content and physical properties. The second type of complement of slits is called a latitudinal slit, which is a supplemental slit of several millimeters along the latitude.

In the case of weak connection of slits, the main conclusion about slits is drawn according to the law of asymmetric distribution of the connection along the length and width.

The main contributions of this paper are as follows:

- A computer-modeled transtropic massif with doublyperiodic slit systems as a stiffness-equivalent homogeneous continuous transtropic body.
- Elastic constants are given as functions of geometric and physical parameters of rocks and slits.
- To investigate the elastic state of two kvershlags in a weighted transtropic massif, the representation of the primary systems of solving equations of the finite element method.
- We obtained multivariate calculation results on the distribution patterns of initial stresses near two shallow kvershlags in a rock massif. These patterns depend on various factors, including the parameters of the slits and other initial data.

The rest of this paper is organized as follows:

Section II presents the state of the art in studying the stressstrain state of shallow underground cavities in solid and slotted weakened massifs and discusses methods for analyzing the state of weighted rock strata with two treks—Problem Statements. Section III presents our proposed finite element approach using isoparametric computational elements. A transtropic massif with double periodic slit systems is replaced by a continuous transtropic body equivalent in stiffness to the primary medium by solving the reduction problem. In Section IV we determined the elastic constants of a transtropic body that is equivalent in stiffness to the main massif with slits. The elastic constants depend on the elastic properties of the main massif and the geometry of the slits. In the values of the initial elastic components of stresses and displacements in the transtropic massif around two kvershlags depending on the initial parameters by implementing the developed algorithms, computer program complexes, conducting multivariate calculations and analyzing the numerical results. Section V presents the experimental results, compares the performance of our proposed method with existing state-of-the-art techniques, and discusses the method with existing state-of-the-art techniques and the implications of our findings. Finally, Section VI summarizes the results and suggests directions for future research.

II. RELATED WORKS

In the last century, the works of foreign scientists mainly involved theoretical research on the stress-strain state of underground cavities in the isotropic massif. Using the symmetry of the biharmonic solutions and based on the unique properties of harmonic functions, O.Müller and K. Stocke reviewed the relevant class of problems. G.V. Kolosov, N.I. Muskhelishvili [30], in the solution of plane problems of the theory of elasticity of an isotropic body, has successfully used the method of the Complex Variable Theory. W. Wittke [31] extended the anisotropic jointed rock model (AJRM) and the corresponding analysis methods to a broader spectrum of rock types. His design approach has been applied to many tunneling, dam, and slope design projects.

The analytical function proposed by Appel considers the state of one and many related isotropic bodies with a circular hole. L.A. Filshtinsky considered orthotropic structures with a doubly periodic system of circular holes [32] and a body with elliptical holes A.S. Kosmodamiansky, M.M. Neskorodev [33]. A.S. Kosmodamiansky investigated the stress-strain state of an anisotropic elastic body with three and endless rows of holes, and based on these decisions, Zh.S. Erzhanov, K.K. Kaydarov, M.T. Tusupov [34] studied the effects of the slots on the static stress state of underground workings. Zh.S. Erzhanov, Sh.M. Aytaliev, and Zh.K. Massanov [35] proposed a computational mechanics and mathematical model of the anisotropic elastic deformation of the rock mass with doubly periodic systems slots. They solved the problem by bringing the elastic constants obtained from the transtropic body, the equivalent stiffness central massif with slots, depending on the elastic properties and the geometry of the slots. Based on this model, they studied static initial elastic states, mainly single underground cavities deep foundation of rigorous and approximate methods, and subsequent creeping cavities state-based ∋-Algebra Operators U.N. Rabotnov [36] and the Theory of Creep of Rocks Zh.S. Erzhanov [37].

Different types of immersed tunnels (IMT) as well as their construction methods are discussed by [38-45].

These scientists have made significant contributions to the theory of the Finite Element Method and its application to solving complex problems of statics and dynamics of solid mechanics: L. Segerlind [46], B.Z. Amusin, A.B. Fadeev [47], Zh.S. Erzhanov, T.D. Karimbaev [48], A.D. Omarov, Zh.K. Massanov, N.M.Mahmetova [49], L.B. [Atymtayeva,](https://www.scopus.com/authid/detail.uri?authorId=42061007500) B.E. [Yagaliyeva](https://www.scopus.com/authid/detail.uri?authorId=42062442900) [50] and others [51-53].

A. The Task

Investigated the static stress and strain state of two kvershlags lying in a heavy transtropic massif, depending on the degree of discontinuity, conform to small sloping layers at an angle φ . Let *H* denote the depth of the distance between their centers two $2L$ (Fig. 1).

Fig. 1. Three-dimensional view.

B. The Task Explained

The plane of the cross-sectional areas with anisotropic inplane deformation slits; efforts are at infinity (Fig. 2).

$$
\sigma_x^{(\infty)} = p, \quad \sigma_z^{(\infty)} = q, \quad \tau_{xz}^{(\infty)} = r \tag{1}
$$

Fig. 2. Surface with a periodic system of slits.

Here γ_{00} - main crack; γ_{nm} - circuits and their length; n, m indices, ω_1 , *i* ω_2 - periods of slits in the directions of the axes x and z; circuits are free of external loads. E_j , V_j , G_j ($j = 1,2$) - elastic properties transtropic massif slots. By solving the problem of bringing to an anisotropic body with the boundary conditions (1) given elastic parameters E_i^e , v_i^e , G_2^e , $(i = 1, 2)$, transtropic solid body, equivalent stiffness anisotropic massif with slits are the following formulas:

$$
E_1^{\circ} = E_1, \quad V_1^{\circ} = V_1, \quad V_2^{\circ} = V_2,
$$
\n
$$
E_2^{\circ^{-1}} = E_2^{-1} + 2\omega^{-1} < 2 \operatorname{Re} \sum_{j=1}^2 q_j \Phi_j \left(x + i\beta_j 0.5\omega, q \right) > ,
$$
\n
$$
G_2^{\circ^{-1}} = G_2^{-1} + 2\omega^{-1} < 2 \operatorname{Re} \sum_{j=1}^2 \left[p_j \Phi_j \left(x + i\beta_j 0.5\omega, r \right) + q_j \Phi_j \left(x + i\beta_j 0.5\omega, r \right) \right]
$$
\n
$$
(2)
$$

Here $\langle \rangle$ - symbol averaging values, β_j - anisotropy parameters;

$$
\Phi_j(z_j) = (B_j + iC_j)z_j + \sum_{k=1}^{\infty} a_{2k-1,j} \zeta_j^{-(2k-1)} +
$$

\n
$$
\sum_{k=1}^{\infty} \sum_{k=1}^{\infty} a_{2k-1,j} B_{jik} (\zeta_j^{2k-1} + \zeta_j^{-(2k-1)})
$$

\n
$$
B_1 = 0.5 (p + \beta_2^2 q) (\beta_2^2 - \beta_1^2)^{-1}
$$

\n
$$
B_2 = 0.5 (p + \beta_1^2 q) (\beta_2^2 - \beta_1^2)^{-1}
$$

\n
$$
C_1 = 0, C_2 = 0.5 r \beta_2^{-1};
$$

\nIII. PROPosen METHOD

A. The Solving Problem

Hooke's law of transtropic massif with cavities with generalized plane strain relative to the Cartesian coordinate system $0xyz$ (see Fig. 1):

$$
\{\sigma\} = \left[\overline{D}\right]\!\{\varepsilon\}\,;
$$
\n(4)

where $\{\sigma\} = (\sigma_x, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy})^T$, $[\overline{D}] = [d_{ij}]$, $(i, j = 1, 2, ..., 5)$; - deformation coefficients defined by the formulas [54].

Here $E_k^e, V_k^e, G_2^e (k = 1, 2)$ - effective elastic constants transtropic massif equivalent stiffness transtropic massif with slits, which depends on the elastic constants of the last and the geometry of the slits $a, \omega, i\omega$.

B. The Use of Numerical Methods

The cross-section in plane ABCD kvershlag planes of deformation using units to isoparametric calculation elements (Fig. 3).

Fig. 3. Two-dimensional view.

Constitute the essential resolution of the system of algebraic equations finite element method's three $3n$ -order relative to the projections of moving points, and it can be solved with the following boundary conditions [55]:

Base BD calculation area ABCD non-deformable –

$$
u = v = w = 0;
$$
\n⁽⁵⁾

Sides АВ and СD under the weight of rocks moved only in the vertical direction due to a lack of influence of cavities –

$$
u = v = 0, \quad w = w(z)
$$
\n⁽⁶⁾

The study estimated the area with cavities is automatically split into isoparametric elements using the object-oriented program FEM_3D (Fig. 4).

Fig. 4. A layout of the estimated area for isoparametric elements.

In solving the problem, we considered a generalized plane calculation algorithm for points isoparametric element (Fig. 5).

Fig. 5. Four points isoparametric elements.

Each point acts as the vertical force of the weight:

$$
f_{z_i} = -\frac{\gamma S}{4}; \ (i = 1, 2, 3, 4)
$$

To solve the fundamental system of equations with the Finite Element Method's displacement components with the boundary conditions (5), (6) rigorous methods are complex; therefore, it can be solved in an iterative method of Gauss-Seidel-relaxation factor with a given accuracy [56]:

$$
\{F\} = [K]\{U\} \tag{7}
$$

here $[K] = \sum [K]$ $=$ $=$ *n i* K] = $\sum [k^e]_i$ 1 $[k^e]_i$ – stiffness matrix of the system;

$$
\{U\} = (u_1, ..., u_R, w_1, ..., w_R, v_1, ..., v_R)^T - \text{displacement vector};
$$

$$
\{F\} = \left(F_{x_1}, \dots, F_{x_R}, F_{z_1}, \dots, F_{z_R}, F_{y_1}, \dots, F_{y_R}\right)^T
$$
 – force vector.

An attractive feature of this method is as follows: the stiffness matrix in this method is prepared only once and can be used for iterating both its own elements and the column elements of the matrix. During the first iteration, the value of the matrix is unknown, but in subsequent iterations, the value of the previous iteration is used.

IV. EXPERIMENTAL RESULTS

Thus, the searched $E_1^3, E_2^3, G_2^3, v_1^3, v_2^3$ parameters describe the anisotropic medium weakened by two periodic slits according to the formulas (2) and (3). Elastic and geometrical parameters of slits shape them. It has been shown above that the equivalent homogeneous folding condition for anisotropy is realized by introducing the measure of elastic parameters. Below (Table I) is the elasticity parameter of anisotropic rocks weakened by two periodic slits [48].

Table I gives elastic parameters and their anisotropy parameters for anisotropic siltstone [35], and several cases are given by entering different values of relations of geometrical parameters.

 $\frac{a}{a} = \infty$ $\frac{\omega}{\omega} = \infty$ The given values correspond to the limit values of

the layer. The first value represents the scenario with no slits, while the second value indicates that the connection between the layers is discontinuous.

Table II presents the verification of the correct operation of the developed algorithms and software systems to solve the test problem of the elastic stress state circular cavity in an anisotropic massif with the horizontal plane of isotropy in the plane strain and hydrostatic stress distribution in a pristine environment. Because of the symmetry of the problem, a quarter of the area of the cavity is divided into 342 isoparametric elements with the help of 380 points. The basic system of equations is solved in about 1140 with 1000 iterations. Unlike values of displacements at characteristic points of contour obtained by iterative and strict known methods, it is no more than 1-2%.

$\theta,$	$\cdot\sigma_\theta^{\scriptscriptstyle cont}$ / $\gamma\!H$				
deg	σ_{θ}^{anal} / γH Precise method (test)	$\overline{\sigma}_{\theta}^{FEM}$ / γH FEM	$\left \sigma_{\theta}^{anal} / \gamma H - \sigma_{\theta}^{FEM} / \gamma H\right $	σ_{θ}^{FEM} $\sigma_{\scriptscriptstyle{\theta}}^{\scriptscriptstyle \it{anal}}$ $ \sigma_\theta^{\textit{anal}} $	
Ω	3.079	3.040	0.039	0.01	
30	1.510	1.493	0.017	0.01	
60	1.706	1.694	0.012	0.007	
90	2.692	2.631	0.061	0.022	

TABLE II. COMPARISON OF THE SHEAR IN THE CONTOUR POINTS OBTAINED BY ITERATION AND RIGOROUS METHODS

A. Results

In this section, we demonstrate the results that were obtained. Twelve points were obtained on the boundary of a double kvershlags in transtropic rocks weakened by weighted elastic two-period slits (Fig. 6). As a result of multivariate calculations, the change of tangential stress at the boundaries of the two kvershlags was studied. In this case, two kvershlags are located at depths of H=10M and distances between cavities of L=5M. Our findings align with recent studies (e.g., Turymbetov et al., 2020) on the stress distribution patterns in complex geological structures, but provide additional insights into the behavior of kvershlags when subjected to various stress conditions. A key difference is the extended analysis of the interaction between two closely positioned kvershlags, which has not been previously examined in detail.

Fig. 6. Given points on the boundary of the cavity.

Tables III-V presents the variation of tangential stresses in the values w/a=∞, w/a=6, and w/a=4, the ratios of geometrical parameters.

TABLE III. THE CHANGE OF TANGENTIAL STRESS AT THE BOUNDARY OF TWO KVERSHLAGS IN THE GEOMETRIC PARAMETER OF SLITS IS W/A=∞

Left cavity					
	$\varphi = 0$	$\varphi = 30^\circ$	$\varphi = 45^\circ$	$\varphi = 60^0$	$φ=900$
1	$-0,2$	-0.408	-0.141	-0.166	-0.37
2	-0.6	-0.579	-0.785	-0.831	-0.79
3	-1.585	-2.366	-2.495	-2.066	-1.876
$\overline{4}$	-2.405	$-3,277$	-3.396	-2.535	-2.093
5	$-2,218$	$-3,15$	-3.121	-2.335	-1.89
6	$-3,33$	$-5,929$	$-4,687$	$-3,811$	$-3,514$
7	0,687	0,712	0,699	0.38	0,193
8	$-3,895$	$-5,382$	$-5,429$	$-4,668$	$-4,066$

9	$-2,082$	$-2,53$	$-2,518$	$-2,093$	$-1,767$
10	$-2,236$	$-2,83$	$-2,869$	$-2,361$	-1.95
11	$-1,727$	$-2,997$	$-2,845$	$-2,142$	$-1,979$
12	$-0,744$	$-2,18$	$-1,305$	-0.712	-0.961
			Right cavity		
1	$-0,216$	-0.54	$-0,644$	-0.465	$-0,372$
2	-0.595	$-0,529$	$-1,301$	$-1,185$	-0.78
3	$-1,583$	$-2,371$	$-2,469$	$-2,003$	$-1,875$
4	$-2,376$	$-3,242$	$-3,056$	$-2,337$	$-2,085$
5	$-2,268$	$-3,263$	$-2,734$	$-2,122$	$-1,955$
6	$-3,925$	$-6,764$	$-6,717$	$-5,199$	$-4,111$
7	0,555	0,557	0,29	0,192	0,17
8	-3.901	$-5,291$	$-5,734$	$-4,929$	$-4,073$
9	$-2,082$	$-2,486$	$-2,447$	$-2,069$	$-1,766$
10	$-2,319$	$-2,899$	$-2,744$	$-2,157$	$-1,992$
11	$-1,727$	$-2,993$	$-2,851$	$-2,144$	$-1,976$
12	$-0,742$	$-2,18$	$-1,534$	$-0,914$	$-0,955$

TABLE IV. THE CHANGE OF TANGENTIAL STRESS AT THE BOUNDARY OF TWO KVERSHLAGS IN THE GEOMETRIC PARAMETER OF SLITS IS $W/A=6$

Left cavity					
	$Q=0$	$\varphi = 30^0$	$\varphi = 45^0$	$\varphi = 60^0$	$\varphi = 90^\circ$
1	$-0,187$	$-0,011$	$-0,09$	$-0,282$	$-0,347$
\overline{c}	$-0,622$	-0.67	$-0,218$	$-0,474$	$-0,775$
3	$-1,575$	$-2,24$	-0.399	$-0,403$	$-1,872$
$\overline{4}$	$-2,406$	$-3,588$	$-1,103$	$-0,7$	$-2,097$
5	$-2,242$	$-3,42$	$-1,24$	-0.688	$-1,895$
6	$-3,372$	$-4,439$	$-1,231$	$-0,429$	$-3,495$
7	1	1,151	0,473	0,233	0,208
8	$-3,975$	$-5,215$	$-1,893$	$-1,683$	$-4,031$
9	$-2,105$	$-2,57$	-0.956	$-0,601$	$-1,773$
10	$-2,235$	$-2,951$	$-1,093$	$-0,586$	$-1,956$
11	$-1,705$	$-2,696$	$-0,772$	-0.415	$-1,973$
12	$-0,762$	$-1,118$	0,172	0,179	$-0,942$
			Right cavity		
1	$-0,209$	$-0,763$	$-0,649$	$-0,599$	$-0,35$
\overline{c}	$-0,619$	$-1,291$	$-1,06$	$-1,102$	$-0,765$
3	$-1,574$	$-2,169$	-0.659	$-0,691$	$-1,871$
$\overline{4}$	$-2,375$	$-3,196$	$-0,826$	$-0,521$	$-2,089$
5	$-2,286$	$-3,017$	$-0,767$	$-0,267$	$-1,961$
6	$-4,013$	$-6,67$	$-2,537$	$-1,61$	$-4,089$
7	0,782	0,53	$-0,123$	$-0,116$	0,185
8	$-3,98$	$-5,582$	$-2,675$	$-2,275$	$-4,038$
9	$-2,105$	$-2,486$	-0.954	$-0,642$	$-1,772$
10	$-2,317$	$-2,799$	$-0,896$	$-0,617$	$-1,997$
11	$-1,705$	$-2,653$	$-0,646$	$-0,474$	$-1,971$
12	$-0,76$	$-1,329$	0,05	0,011	$-0,936$

TABLE V. THE CHANGE OF TANGENTIAL STRESS AT THE BOUNDARY OF TWO KVERSHLAGS IN THE GEOMETRIC PARAMETER OF SLITS IS W/A=4

V. DISCUSSION

Despite certain achievements in studying the state of individual deep cavities using isotropic and anisotropic elastic computational models of rock strata, there has been no systematic study of the stress-strain state of two drifts with arbitrary cross-sectional shapes and depths of location in a large inclined layered massif with a system of slits under conditions of elastic deformation of rocks.

The study of the regularity of distribution of elastic stresses and displacements in the vicinity of cavities of arbitrary depth and cross-sectional shapes depending on the inhomogeneous fractured structure is not only of theoretical interest but also has direct practical significance.

This study systematically investigates the distribution patterns of elastic stresses and displacements near two kvershlags of arbitrary profile shape and depth using the finite

element method. The study is based on a homogeneous transtropic computer-modeled inclined-fine-layered massif with a double periodic system of slits, and is conducted under conditions of generalized plane deformation.

VI. CONCLUSION

Future research could focus on refining the numerical model by incorporating real-time field data, which could improve the accuracy of predictions regarding the stress-strain behavior of kvershlags in various geological settings. Additionally, the extension of this model to analyze other types of underground structures could further broaden its application.

In this paper, we presented a finite-element computational model of anisotropic rocks with a double periodic system of slits and elastic properties in the form of a transtropic body with given moduli. To ensure the generalizability and adaptability of the model, we also performed multivariate calculations.

Using an accepted model of deformation for the weighted massif, we formulated a boundary problem to determine the initial elastic stress-strain state of the kvershlag cavity. We obtained the basic equation system for solving this problem using the finite element method.

A calculation algorithm and a program complex for studying the elastic state of double kvershlags of arbitrary depth have been developed.

We conducted numerical calculations and analyzed the influence of geometric and physical parameters, as well as rock elasticity properties, on the components of stresses and displacements in the vicinity of cavities. Based on these analyses, we arrived at specific conclusions.

Our analysis revealed that as the slits approach each other, the value of displacements increases, and the value of stresses changes. Additionally, we observed that the numerical value of stresses and displacements increases as the cavity location deepens and the distance between them decreases.

REFERENCES

- [1] Bauschinger M. (1884). Versuche über Elastizität und Festigkeit. München (Vol. 10, pp. 50-68).
- [2] Bach K., Baumann R. (1924). Elastizität und Festikgiet. Berlin, 687 p. Springer.
- [3] Graf O. (1926). Versuche über die Druckelastizität von Basalt, Gneis, Muschelkaik, Quarzit, Granit, Bundsandstein, sowie Hochofenstuckshlake. Beton und Eisen. (Vol. 22, pp. 399-406).
- [4] Müller O. (1930). Untersuchungen an Karbongesteinen zur Klärung von Gebirgsdruckfragen. Glückauf (Vol. 47, pp. 1601-1612).
- [5] Fleischer O. (1934). Bergmännische Beobachtungen und Versuche über Gebingsbewegungen im obershlesischen beim. Glückauf (Vol. 31, pp. 716-721).
- [6] Stöcke K. (1937). Für das Gebingsdruckproblem wichtige Begriffe aus der technischen Mechanik. Zeitschrift für des Berg-Hütten und Salinenwesen. Bd. 84. (Vol. 11, pp. 465-467).
- [7] Zalesskiy B.V., Florenskiy K.V. (1952). About some basic prerequisites for testing stone building materials for frost resistance. Proceedings of the Institute of Geological Sciences (pp. 39-50).
- [8] Belikov B.P. (1952). Elasticity moduli of different types of rocks of the USSR. Proceedings of the Institute of Geological Sciences (pp. 3-38).
- [9] Kuznetsov G.N. (1947). Mechanical properties of rocks. M: Ugeltekhizdat, 180 p.
- [10] Krieger N.I. (1951). Cracking and methods of its study at hydrogeological survey. Materials on engineering geology, (Vol. 2, pp. 7-139).
- [11] Kuvaev N.N. (1960). On the persistence of crack orientation at different depths. Works of VNIMI, (Vol. XXXIV, pp. 305-307).
- [12] Protodyakonov M.M., Chirkov S.E. (1964). Cracking and strength of rocks in the massif. M: Nauka, 67 p.
- [13] Laufer H. (1958). Gebirgsklassifizierung für den Stollenbau. Geologie und Bauwesen (Vol. 1, pp. 46-51).
- [14] Belousov V.V. (1952). Tectonic ruptures, their types and mechanism of formation. Proceedings of the Geophysical Institute of the USSR Academy of Sciences (pp. 1-145).
- [15] Mikhailov A.E. (1956). Field methods of studying cracks in rocks. M: Gosgeoltekhizdat, 132 p.
- [16] Permyakov E.N. (1949). Tectonic fracturing of the Russian platform. Materials to the knowledge of the geological structure of the USSR. M: 210 p.
- [17] Chernyshev V.F. (1965). About regularities in the position of spalling and fracturing cracks. Proceedings of the Institute of Geological Sciences of the Academy of Sciences of the USSR, (Vol. 162, pp. 146-150).
- [18] Kushnarev I.P., Lukin L.I. (1960). On the study of fracture tectonics. Problems of Tectonophysics. М: Gosgeoltechizdat, (pp. 99-110).
- [19] Neishtadt L.I. (1957). Methods of geological study of rock fracturing in engineering-geological research. M: Gosenergoizdat, 103 p.
- [20] Sokolov D.S. (1962). Basic conditions of karst development. M: Gosgeoltekhizdat, 322 p.
- [21] Rats M.V., Chernyshev S.N. (1970). Cracking and properties of fractured rocks. M: Nedra, 159 p.
- [22] Nevskiy V.A. (1964). Genetic classification of rock cracks. Investigations of universities, Geology and Exploration, (pp. 3-13).
- [23] Belousov V.V. (1962). Basic questions of geotectonics. M: Gosgeoltekhizdat, 608 p.
- [24] Shultz S.S. (1964). On the study of planetary fracturing. International Geological Congress, XXII session. Problems 4. Reports of Soviet geologists to the XXII session of the IGC. M: Nauka, (pp. 147-152).
- [25] Savarensky F.P. (1939). Engineering Geology. M: State Scientific and Technical Publishing House, 488 p.
- [26] Kolomensky N.V. (1956). Engineering Geology. M: Gosgeoltekhizdat, 320 p.
- [27] Kirillova I.V. (1945). Experience in analyzing fracturing in layered strata of the eastern dip of the Trialet folded region. DAN USSR. pp. 389-892.
- [28] Skaryatin V.D. (1965). Classification of fractures and application of aerial photography and some other methods in the study of rock fracturing. Proceedings of the 2nd All-Union Meeting on fractured oil and gas reservoirs. M: Nedra, (pp. 66-80).
- [29] Barenblatt G.I. (1961). Mathematical theory of equilibrium cracks formed at brittle fracture. Journal of Applied Mechanics and Technical Physics. M: (pp. 3-56).
- [30] Muskhelishvili N.I. (1966). Some basic problems of the mathematical theory of elasticity. M: Science, 707 p.
- [31] Wittke W. (2014). Rock Mechanics Based on an Anisotropic Jointed Rock Model (AJRM). Germany, 891 p.
- [32] Filshtinsky L.A. (1967). Stress in the correct doubly periodic grids (lattices). Engineering journal. Mechanics of Solids (Vol. 1, pp. 20-28).
- [33] Kosmodamianski A.S., Neskorodev M.M. (1970). Doubly periodic problem for an anisotropic structure weakened elliptical holes. "Reports of the Academy of Sciences URSR Ser. A (Vol. 7, pp.31-42).
- [34] Erzhanov Zh.S, Kaidarov K.K., Tusupov M.T. (1969). Mountain range with a discontinuous layer coupling (plane problem)."Mechanical processes in the rock mass. Alma-Ata, Nauka, 189 p.
- [35] Erzhanov Zh.S, Aytaliev Sh.M., Massanov Zh.K. (1971). Stability of horizontal caves in the pan-layered massif. Alma-Ata, Science, 160 p.
- [36] Rabotnov U.N. (1948). Some problems in the theory of creep. Vestnik MGU, (Vol. 10, pp. 145-152).
- [37] Erzhanov Zh.S. (1964). The theory of creep of rocks and its application. Alma-Ata, Nauka, 175 p.
- [38] Glerum A. (1992). Options for tunneling: a personal story. Tunn. Undergr. Space Technol. 7 (4), 313–315.
- [39] Rasmussen N.S. (1997). Concrete immersed tunnels Forty years of experience. Tunn. Undergr. Space Technol. 12 (1), 33–46.
- [40] Bucher C., 2006. Applications of random field models in stochastic structural mechanics. In: Advances in Engineering Structures, Mechanics & Construction, vol. 140, pp. 471–484.
- [41] Soubra A.-H., Youssef Abdel Massih, D., Kalfa, M. (2008). Bearing capacity of foundations resting on a spatially random soil. GeoCongress, pp. 66–73.
- [42] Li W., Fang Y., Mo H., Gu R., Chen J., Wang Y., Feng D. (2014). Model test of immersed tube tunnel foundation treated by sand-flow method. Tunn. Undergr. Space Technol. 40, 102–108.
- [43] Hicks M.A., Li Y. (2018). Influence of length effect on embankment slope reliability in 3D. Int. J. Numer. Anal. Methods Geomech. 42 (7), 891– 915.
- [44] Müller S., Schüler L., Zech A., Heße F., 2022. GSTools v1.3: a toolbox for geostatistical modelling in python. Geosci. Model Dev. 15 (7), 3161– 3182.
- [45] Cornelis M.H., Oswaldo M.N., Bas J. (2024). The influence of spatial variation on the design of foundations of immersed tunneles: Advanced probabilistic analysis. Tunnelling and underground space technology, 147, 3-35.
- [46] Segerlind L. (1979). Application of the Finite Element Method. Moscow, Mir, 392p.
- [47] Amusin B.Z., Fadeev A.V. (1975). The Finite Element Method for solving mining mechanics. Moscow, Nedra, 142 p.
- [48] Erzhanov Zh.S., Karimbayev T.D. (1975). The Finite Element Method in problems of rock mechanics. Alma-Ata, Science, 238 p.
- [49] Omarov A.D., Massanov Zh.K, Makhmetova N.M. (2002). Static and seismic stress state transport of underground structures in anisotropic nonlinear geometric massif. Almaty, Bastau, 244 p.
- [50] [Atymtayeva](https://www.scopus.com/authid/detail.uri?authorId=42061007500) L.B., [Masanov](https://www.scopus.com/authid/detail.uri?authorId=42061688400) Z.K., [Yagaliyeva](https://www.scopus.com/authid/detail.uri?authorId=42062442900) B.E. (2013). Analysis of stress-strain state of diagonal mines in multilinked anisotropic massif under the actions of elastic waves. Lecture Notes in Engineering and Computer Science, 134–138
- [51] Marchandise E., Remacle J-F. (2006). A stabilized finite element method using a discontinuous level set approach for solving two phase incompressible flows. J comput phys 219 (2), 780-800.
- [52] Zhao L., Mao J., Bai X., Li T., Williams JJR. Finite element implementation of an improved conservative level set method for twophase flow. Comput & Fluids, 100, 38-54.
- [53] Kardani V., Nazem M., Carter JP., Abbo AJ. (2015). Efficiency of highorder elements in largedeformation problems of geomechanics. Int. J. Geomech. 15 (6), 401-411.
- [54] Turymbetov T., Azhikhanov N., Zhunisov N., Aimeshov Zh. (2015). Stress-strain state of two diagonal cavities weighty inclining layered massif system with slots in terms of elastic-creep deformations. World conference on technology, innovation and entrepreneurship. Procedia-Social and Behavioral Sciences. Istanbul, Turkey, (pp. 2263-2271).
- [55] Turymbetov T., Kuatbekov B., Maulenov S., Aimeshov Z. (2019). The Stress-Strain State of the Two Shtreks a Weighty Obliquely Layered Massif System With Slits in Terms of Elastic Deformation of Rocks. 3rd World Conference on Technology, Innovation and Entrepreneurship (WOCTINE), Procedia Computer Science, Istanbul, Turkey, (Vol.158, pp. 355-360).
- [56] Azhikhanov N.T., Zhumagulov B.T., Turymbetov T.A., Bekbolatov A.B. (2020). Stressed-deformed state of two drifts in a tiltly layered cracked array in the conditions of elastic deformations of rocks. Al-Farabi Kazakh National University. Journal of Mathematics, Mechanics and Computer Science. Almaty, Kazakhstan, 1 (105), 120-128.