Enhanced TODIM-TOPSIS Framework for Interior Design Quality Evaluation in Public Spaces Under Hesitant Fuzzy Sets

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***Abstract*—The evaluation of interior landscape design in public spaces involves several aspects, including aesthetics, functionality, sustainability, and user experience. Aesthetic evaluation focuses on the visual appeal and stylistic consistency of the design. Functionality considers the practicality and convenience of the space layout. Sustainability evaluates the environmental friendliness of materials and energy efficiency of the design. Additionally, user experience assessment gathers feedback to gauge comfort and satisfaction. These evaluation criteria help designers optimize spaces to be both attractive and practical while meeting user needs. The interior design quality evaluation in public spaces is multiple-attribute decision-making (MADM) problem. Recently, the TODIM and TOPSIS methods have been applied to address MADM challenges. Hesitant fuzzy sets (HFSs) are used to represent uncertain information in the evaluation of interior landscape design in public spaces. In this study, we developed a hesitant fuzzy TODIM-TOPSIS (HF-TODIM-TOPSIS) approach to tackle Multiple Attribute Decision Making (MADM) issues within the context of HFSs. A numerical case study focused on the interior design quality evaluation in public spaces demonstrates the validity of this approach. The primary contributions of this paper include: (1) Extending the TODIM and TOPSIS approaches to incorporate HFSs; (2) Utilizing information entropy to determine weight values under HFSs; (3) Establishing the HF-TODIM-TOPSIS method for managing MADM in the presence of HFSs; (4) Conducting algorithmic analysis and comparative studies based on a numerical example to assess the practicality and effectiveness of the HF-TODIM-TOPSIS approach.**

***Keywords*—Multiple-attribute decision-making (MADM); hesitant fuzzy sets (HFSs); TODIM; TOPSIS; design quality evaluation**

I. INTRODUCTION

The evaluation of interior landscape design in public spaces involves several key aspects, including aesthetics, functionality, sustainability, and user experience. Aesthetic evaluation focuses on the visual appeal and consistency of style, ensuring overall harmony and beauty. Functionality examines the practicality of layout and convenience of facilities to meet diverse user needs. Sustainability emphasizes the environmental friendliness and energy efficiency of materials, highlighting long-term ecological and economic benefits. Additionally, user experience evaluation gathers feedback to assess comfort and satisfaction. These comprehensive evaluations not only help optimize design but also provide valuable references for future projects, enhancing the overall quality and utility of public spaces. The evaluation of interior landscape design in public spaces has become increasingly important over the past decade as urbanization continues to shape our environments. This literature review synthesizes findings from some significant studies, including some Chinese and some English publications. Li, et al. [1] conducted a pivotal study highlighting the relationship between environmental design and human behavior in public spaces. They advocated for a user-centered approach, emphasizing that aesthetic considerations must align with functional needs to enhance user satisfaction. This work laid the groundwork for understanding how design influences user interactions with public spaces. Kaplan [2] examined the psychological impacts of landscape design, asserting that incorporating natural elements is crucial for fostering emotional well-being in urban environments. This study reinforced the idea that landscapes should do more than just please the eye; they should also promote mental health. Zhao and Chen [3] focused on the aesthetic evaluation of public parks in China. Their research identified key design elements that contribute to user satisfaction, such as visual coherence and cultural relevance. This study was significant in emphasizing the cultural context in landscape design and its effect on user perceptions. Gomez, et al. [4] explored the role of social interaction in public spaces. They proposed that landscape design should facilitate community engagement, suggesting a framework for evaluating designs based on social connectivity and highlighted the social dimension of public spaces, suggesting that design can foster community ties. In the same year, Sun, et al. [5] investigated the effectiveness of biophilic design, concluding that integrating natural elements significantly enhances user satisfaction and well-being in public interiors. Their findings supported the notion that environments rich in nature positively impact users' experiences. Miller [6] further contributed to the discourse by examining sustainable design practices in public spaces. He advocated for eco-friendly materials and practices, providing an evaluation framework for assessing the environmental impact of landscape designs. His work was instrumental in integrating sustainability into the conversation around public space design. In 2017, Zhang and Li [7] discussed the influence of cultural elements on public space design in China. They emphasized the importance of incorporating local heritage into landscape design to enhance community identity, showcasing how cultural context can inform design practices. Haq and Lynch [8] conducted a comparative study on evaluation methods for public space design across different cultures. Their findings underscored the variability in user preferences and highlighted

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the need for culturally sensitive evaluation frameworks in landscape design. In 2020, Chen, et al. [9] focused on the integration of technology in evaluating public spaces. Their study introduced a digital evaluation tool aimed at enhancing user feedback, marking a shift toward more interactive and user-driven design evaluations. Fang and Zhao [10] explored the psychological effects of urban green spaces on public health. They proposed a robust evaluation framework that prioritized mental and physical well-being, further emphasizing the health-related aspects of landscape design. The following year, Martin, et al. [11] investigated community participation in the design and evaluation of public spaces. Their findings indicated that involving users in the design process leads to better outcomes and ensures that designs meet community needs. Wang and Liu [12] analyzed climate-responsive design strategies in public spaces, advocating for adaptive landscapes that respond to environmental changes. Their work highlighted the importance of resilience in landscape design, particularly in the face of climate change. In 2023, Zhou, et al. [13] proposed a comprehensive framework that combines qualitative and quantitative methods to assess user experiences in public spaces. Their emphasis on inclusivity and accessibility reflects a growing recognition of diverse user needs in landscape design. Smith and Johnson [14] examined the role of sensory experiences in public space design. They proposed that effective evaluations should consider auditory, visual, and tactile elements, thereby broadening the scope of design evaluations. Lastly, Lu, et al. [15] focused on the integration of smart technologies in public space design evaluation. Their research suggested that smart solutions can enhance user engagement and feedback processes, indicating a future direction for landscape design. The past decade has witnessed significant advancements in the evaluation of interior landscape design in public spaces. Emerging trends emphasize user experience, sustainability, cultural relevance, and the integration of technology. The studies reviewed highlight the necessity of adopting a multidimensional approach to effectively evaluate and enhance public spaces, ultimately contributing to better urban environments.

Multi-attribute decision-making (MADM) is.20000000000000000000000000000000001111111111110 a method used to evaluate and select options when multiple conflicting criteria are involved [16-20]. It's widely applied in fields like engineering, economics, and management [21-25]. The decision-making process typically includes defining objectives and evaluation criteria, assigning weights to each criterion, assessing the performance of each option against these criteria, and ultimately selecting the optimal solution based on a comprehensive score [26-29]. Due to the cognitive limitations of decision-makers and the complexities of the decision-making environment [30-34], the process of MADM is often characterized by significant uncertainties, which preclude the accurate representation of evaluation objects using precise numerical values [35-39]. In response, Zadeh [40] proposed the use of fuzzy numbers to address decision-making challenges. However, in specific contexts, a single numerical value fails to adequately capture the nuances of the evaluation object. Consequently, Torra [41] introduced hesitant fuzzy numbers as a means to enhance the understanding of uncertainty within the decision-making process, thereby yielding more precise outcomes [42-48]. This advancement represents a significant breakthrough in MADM research. Interior design quality evaluation in public spaces exemplifies classical MADM. Recently, the TODIM approach [49-52] and the TOPSIS method [53-57] have been applied to address MADM issues. Hesitant fuzzy sets (HFSs) [41] serve as a tool for characterizing uncertain information in this context. To date, few approaches have integrated information entropy [58] with the TODIM-TOPSIS framework under HFSs [41]. Therefore, an integrated hesitant fuzzy TODIM-TOPSIS (HF-TODIM-TOPSIS) approach has been developed to manage Multi-Attribute Group Decision Making (MAGDM). This study presents a numerical example of interior design quality evaluation in public spaces and conducts a comparative analysis to validate the HF-TODIM-TOPSIS approach. The primary research objectives and motivations outlined in this paper are: (1) the extension of the TODIM and TOPSIS methods to HFSs; (2) the application of information entropy to manage weight values within HFSs; (3) the establishment of HF-TODIM-TOPSIS framework for managing MADM under HFSs; and (4) an algorithmic analysis of interior design quality evaluation in public spaces, supported by numerical example to demonstrate the feasibility and effectiveness of HF-TODIM-TOPSIS approach.

The structure of this paper is outlined as follows. Section II discusses the management of HFSs. Section III presents the HF-TODIM-TOPSIS approach applied to HFSs using the entropy method. Section IV provides an illustrative example of evaluating interior landscape design in public spaces, along with a comparative analysis. Concluding remarks are presented in Section V.

II. PRELIMINARIES

The HFSs is constructed.

Definition 1 [41]. The HFSs is demonstrated:

$$V = \left\{ \left\langle \theta, u_V\left(\theta\right) \right\rangle \middle| \theta \in \Theta \right\} \tag{1}$$

where $\mu_V\left(\theta\right) \subset \left[0,1\right]$ is possible membership of element $\theta \in \Theta$, $\forall\ \theta \in \Theta$. Then, $v\theta = \left(vu\right)$ is demonstrated as HFN.

Definition 2 [59]. Let $v\theta_1 = \left(vu_1\right)$ and $v\theta_2 = \left(vu_2\right)$ be two HFNs, the operation is demonstrated:

$$v\theta_1 \oplus v\theta_2 = \bigcup_{v\gamma_1 \in z\theta_1, v\gamma_2 \in v\theta_2} \left\{ v\gamma_1 + v\gamma_2 - v\gamma_1 v\gamma_2 \right\} \tag{2}$$

$$v\theta_1 \otimes v\theta_2 = \bigcup_{v\gamma_1 \in v\theta_1, v\gamma_2 \in v\theta_2} \left\{ v\gamma_1 \cdot v\gamma_2 \right\} \tag{3}$$

$$\lambda v\theta_1 = \bigcup_{v\gamma_1 \in v\theta_1} \left\{ 1 - \left(1 - v\gamma_1\right)^{\lambda} \right\}, \lambda > 0 \tag{4}$$

$$\lambda v\theta_1 = \bigcup_{v\gamma_1 \in v\theta_1} \left\{ \left(v\gamma_1 \right)^{\lambda} \right\}, \lambda > 0 \tag{5}$$

From Definition 2, the operation laws are built [59].

$$(1) v\theta_1 \oplus v\theta_2 = v\theta_2 \oplus v\theta_1, v\theta_1 \otimes v\theta_2 = v\theta_2 \otimes v\theta_1, \left(\left(v\theta_1 \right)^{\lambda_1} \right)^{\lambda_2} = \left(v\theta_1 \right)^{\lambda_1 \lambda_2};$$

$$(2)\ \lambda \left(v\theta_1 \oplus v\theta_2 \right) = \lambda v\theta_1 \oplus \lambda v\theta_2, \left(v\theta_1 \otimes v\theta_2 \right)^{\lambda} = \left(v\theta_1 \right)^{\lambda} \otimes \left(v\theta_2 \right)^{\lambda};$$

$$(3)\ \lambda_1 v\theta_1 \oplus \lambda_2 v\theta_1 = \left(\lambda_1 + \lambda_2 \right) v\theta_1, \left(v\theta_1 \right)^{\lambda_1} \otimes \left(v\theta_1 \right)^{\lambda_2} = \left(v\theta_1 \right)^{\left(\lambda_1 + \lambda_2 \right)}.$$

Definition 3 [59]. Let $v\theta_1 = \left(vu_1 \right)$ and $v\theta_2 = \left(vu_2 \right)$ be HFNs, the score functions of $v\theta_1$ and $v\theta_2$ is demonstrated:

$$SF\left(v\theta_1 \right) = \frac{1}{\# v\theta_1} \sum_{v\gamma_1 \in v\theta_1} v\gamma_1 \tag{6}$$

$$SF\left(v\theta_2 \right) = \frac{1}{\# v\theta_2} \sum_{v\gamma_2 \in v\theta_2} v\gamma_2 \tag{7}$$

where $\# v\theta_1$ and $\# v\theta_2$ are numbers of the elements in $v\theta_1 = \left(vu_1 \right)$ and $v\theta_2 = \left(vu_2 \right)$.

For $v\theta_1 = \left(vu_1 \right)$ and $v\theta_2 = \left(vu_2 \right)$, then

$$(1)\ if\ SF\left(v\theta_1 \right) < SF\left(v\theta_2 \right),\ v\theta_1 < v\theta_2;$$

$$(2)\ if\ SF\left(v\theta_1 \right) = SF\left(v\theta_2 \right), AF\left(v\theta_1 \right) = AF\left(v\theta_2 \right),\ v\theta_1 = v\theta_2.$$

Definition 4 [60]. Let $v\theta_1 = \left(vu_1 \right)$ and $v\theta_2 = \left(vu_2 \right)$ be HFNs, the HFN Hamming distance (HFNHD) and HFN Euclidean distance(HFNED) are demonstrated:

$$HFED\left(v\theta_1, v\theta_2 \right) = \frac{1}{\# v\theta} \sum_{k=1}^{\# v\theta} \left| v\gamma_1\left(\sigma(k) \right) - v\gamma_2\left(\sigma(k) \right) \right| \tag{8}$$

$$HFED\left(v\theta_1, v\theta_2 \right) = \sqrt{\frac{1}{\# v\theta} \sum_{k=1}^{\# v\theta} \left| v\gamma_1\left(\sigma(k) \right) - v\gamma_2\left(\sigma(k) \right) \right|^2} \tag{9}$$

where $v\gamma_1\left(\sigma(k) \right)$ and $v\gamma_2\left(\sigma(k) \right)$ are kth largest values in $v\theta_1 = \left(vu_1 \right)$ and $v\theta_2 = \left(vu_2 \right)$ and $\# v\theta = \# v\theta_1 = \# v\theta_2$.

The HFWA and HFWG approach is demonstrated [59].

III. HF-TODIM-TOPSIS APPROACH FOR MADM WITH ENTROPY

A. HF-MAGDM Issues

The HF-TODIM-TOPSIS approach (Fig. 1) is demonstrated for MADM. Let $VA = \left\{ VA_1, VA_2, \cdots, VA_m \right\}$ be alternatives, and the attributes set $VG = \left\{ VG_1, VG_2, \cdots, VG_n \right\}$ with weight values wv, where $wv_j \in [0,1]$, $\sum_{j=1}^{n} wv_j = 1$. Then, HF-TODIM-TOPSIS approach is demonstrated for MAGDM.

Step 1. Implement the HFN-matrix $VR = \left[v\phi_{ij} \right]_{m \times n} = \left(vu_{ij} \right)_{m \times n}$:

$$VR = \left[v\phi_{ij} \right]_{m \times n} = \begin{array}{c} \\ VA_1 \\ VA_2 \\ \vdots \\ VA_m \end{array} \begin{array}{c} \begin{array}{cccc} VG_1 & VG_2 & \cdots & VG_n \end{array} \\ \begin{bmatrix} v\phi_{11} & v\phi_{12} & \cdots & v\phi_{1n} \\ v\phi_{21} & v\phi_{22} & \cdots & v\phi_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ v\phi_{m1} & v\phi_{m2} & \cdots & v\phi_{mn} \end{bmatrix} \end{array} \tag{8}$$

Step 2. Normalize the $VR = \left[v\phi_{ij} \right]_{m \times n}$ into $VN = \left[n\phi_{ij} \right]_{m \times n} = \left(nvu_{ij} \right)_{m \times n}$.

For benefit attributes:

$$n\phi_{ij} = \left(nvu_{ij} \right) = \left(vu_{ij} \right) \tag{10}$$

For cost attributes:

$$n\phi_{ij} = \left(nvu_{ij} \right) = \left(1 - vu_{ij} \right) \tag{11}$$

B. Implement the Attributes Weight through Entropy

Step 3. Implement the attributes weight through entropy.

The information entropy is employed under different environment[61-65]. Entropy [58] is used to derive weight values. The $HFNM_{ij}$ is demonstrated:

$$HFNM_{ij} = \frac{SF\left(nvu_{ij}\right)+1}{\sum_{i=1}^{m}\left(SF\left(nvu_{ij}\right)+1\right)}, \tag{12}$$

The HFN Shannon entropy $HFNSE = \left(HFNSE_1, HFNSE_2, \cdots, HFNSE_n\right)$ is demonstrated:

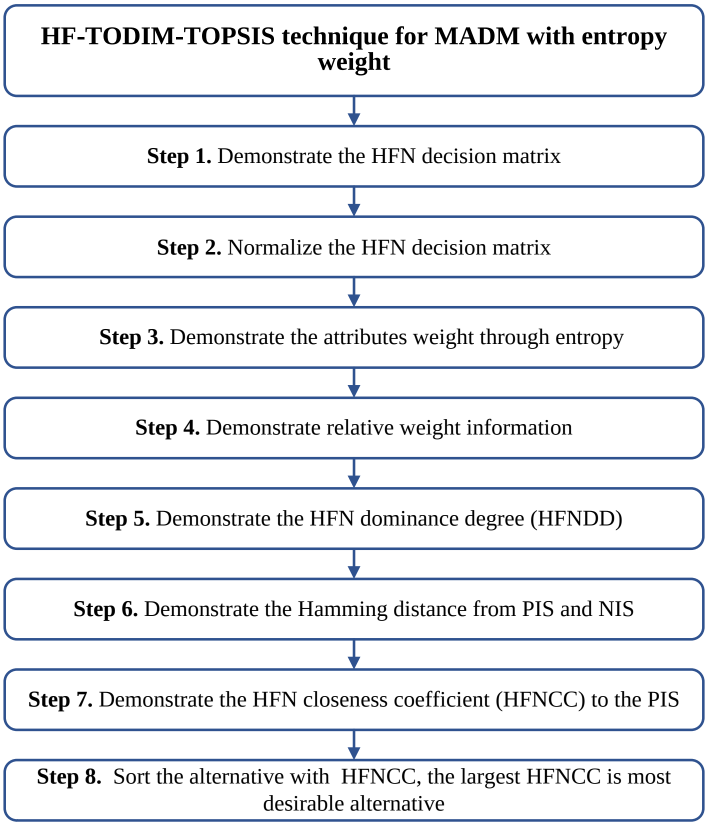

Fig. 1. HF-TODIM-TOPSIS approach for MADM with entropy weight.

$$HFNSE_j = -\frac{1}{\ln m}\sum_{i=1}^{m} HFNM_{ij} \ln HFNM_{ij} \tag{13}$$

and $HFNM_{ij} \ln HFNM_{ij} = 0$ if $HFNM_{ij} = 0$.

The weights $wv = \left(wv_1, wv_2, \cdots, wv_n\right)$ is demonstrated:

$$wv_j = \frac{1-HFNSE_j}{\sum_{j=1}^{n}\left(1-HFNSE_j\right)}, \quad j = 1, 2, \cdots, n. \tag{14}$$

C. HF-TODIM-TOPSIS Approach for MADM

The HF-TODIM-TOPSIS approach is demonstrated for MADM.

Step 4. Implement relative weight of VG_j as:

$$rwv_j = wv_j \Big/ \max_j wv_j, \tag{15}$$

Step 5. Illustrate the HFN dominance degree (HFNDD).

(1) The dominance degree $HFNDD_j\left(VA_i, VA_t\right)$ of VA_i over VA_t for VG_j is demonstrated:

$$HFNDD_j\left(VA_i,VA_t\right)=\begin{cases}\dfrac{rwv_j\times\left(\dfrac{HFNHD\left(n\phi_{ij},n\phi_{tj}\right)+HFEND\left(n\phi_{ij},n\phi_{tj}\right)}{2}\right)^{\alpha}}{\sum_{j=1}^{n}rwv_j} & \text{if } SF\left(n\phi_{ij}\right)>SF\left(n\phi_{tj}\right)\\ 0 & \text{if } SF\left(n\phi_{ij}\right)=SF\left(n\phi_{tj}\right)\\ -\dfrac{1}{\theta}\dfrac{\sum_{j=1}^{n}rwv_j\times\left(\dfrac{HFNHD\left(n\phi_{ij},n\phi_{tj}\right)+HFEND\left(n\phi_{ij},n\phi_{tj}\right)}{2}\right)^{\beta}}{rwv_j} & \text{if } SF\left(n\phi_{ij}\right)<SF\left(n\phi_{tj}\right)\end{cases} \tag{16}$$

The values of α,β is determined from study [66].

(2) The $HFNDD_j\left(VA_i\right)\left(j=1,2,\cdots,n\right)$ with respect to VG_j is defined:

$$HFNDD_j\left(VA_i\right)=\left[HFNDD_j\left(VA_i,VA_t\right)\right]_{m\times m}$$

$$=\begin{array}{c} \\ VA_1\\ VA_2\\ \vdots\\ VA_m\end{array}\begin{array}{c}\begin{array}{cccc}VA_1 & VA_2 & \cdots & VA_m\end{array}\\ \begin{bmatrix}0 & HFNDD_j\left(VA_1,VA_2\right) & \cdots & HFNDD_j\left(VA_1,VA_m\right)\\ HFNDD_j\left(VA_2,VA_1\right) & 0 & \cdots & HFNDD_j\left(VA_2,VA_m\right)\\ \vdots & \vdots & \cdots & \vdots\\ HFNDD_j\left(VA_m,VA_1\right) & HFNDD_j\left(VA_m,VA_2\right) & \cdots & 0\end{bmatrix}\end{array}$$

(3) Implement the overall HFNDD of alternative VA_i over other alternatives for VG_j:

$$HFNDD_j\left(VA_i\right)=\sum_{t=1}^{m}HFNDD_j\left(VA_i,VA_t\right) \tag{12}$$

The overall HFNDD matrix is defined:

$$HFNDD=\left(HFNDD_{ij}\right)_{m\times n}$$

$$=\begin{bmatrix} & VG_1 & VG_2 & \dots & VG_n\\ VA_1 & \sum_{t=1}^{m}HFNDD_1\left(VA_1,VA_t\right) & \sum_{t=1}^{m}HFNDD_2\left(VA_1,VA_t\right) & \dots & \sum_{t=1}^{m}HFNDD_n\left(VA_1,VA_t\right)\\ VA_2 & \sum_{t=1}^{m}HFNDD_1\left(VA_1,VA_t\right) & \sum_{t=1}^{m}HFNDD_2\left(VA_1,VA_t\right) & \dots & \sum_{t=1}^{m}HFNDD_n\left(VA_1,VA_t\right)\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ VA_m & \sum_{t=1}^{m}HFNDD_1\left(VA_m,VA_t\right) & \sum_{t=1}^{m}HFNDD_2\left(VA_m,VA_t\right) & \dots & \sum_{t=1}^{m}HFNDD_n\left(VA_m,VA_t\right)\end{bmatrix}$$

(4) Implement the positive ideal solution (PIS) and negative ideal solution (NIS):

$$PIS=\left(PIS_1,PIS_1,\cdots,PIS_n\right) \tag{27}$$

$$NIS=\left(NIS_1,NIS_1,\cdots,NIS_n\right) \tag{28}$$

$$PIS_j=\max_{j=1}^{n}HFNDD_{ij},NIS_j=\min_{j=1}^{n}HFNDD_{ij} \tag{29}$$

Step 6. Implement the Euclidean distance from PIS and NIS.

$$ED\left(VA_i,PIS\right)=\sum_{j=1}^{n}\left|HFNDD_{ij}-PIS_j\right| \tag{30}$$

$$ED\left(VA_i,NIS\right)=\sum_{j=1}^{n}\left|HFNDD_{ij}-NIS_j\right| \tag{31}$$

Step 7. Implement the HFN closeness coefficient (HFNCC) from PIS.

$$HFNCC\left(VA_i, PIS\right)=\frac{ED\left(VA_i, NIS\right)}{ED\left(VA_i, PIS\right)+ED\left(VA_i, PIS\right)} \tag{32}$$

Step 8. Sort the alternative in line with the HFNCC, the largest HFNCC is the most desirable alternative.

IV. NUMERICAL EXAMPLE AND COMPARATIVE ANALYSIS

A. *Numerical Example for Interior Design Quality Evaluation in Public Spaces*

Evaluating interior landscape design in public spaces is a multifaceted process aimed at enhancing overall quality and user experience. The primary goal is to ensure the design is both aesthetically pleasing and functional while meeting sustainability requirements. Firstly, aesthetic evaluation is crucial. The visual appeal of a design directly affects users' first impressions and long-term feelings. Color coordination, material selection, and spatial layout need to be harmonious to create a pleasant environment. The design should not only align with current aesthetic trends but also possess innovation to inspire interest and curiosity. Secondly, functionality focuses on practicality and convenience. Public spaces must meet various needs, such as socializing, resting, and activities. Therefore, design should consider traffic flow, seating arrangements, and accessibility of facilities. A well-functioning design improves space efficiency, allowing users to engage in activities with ease. Sustainability is an essential element in modern design. Evaluating the environmental friendliness of materials and energy-saving features can reduce negative environmental impacts. Choosing renewable materials and energy-efficient equipment not only lowers long-term operational costs but also enhances ecological value. Designers need to consider lifecycle costs to balance economic and environmental benefits. User experience evaluation is another key aspect. By collecting and analyzing user feedback, designers can gain insights into their true feelings and needs within the space. This user-centered approach helps in making targeted improvements, increasing satisfaction and comfort. Whether through surveys, interviews, or observing behavior, this data provides crucial support for design adjustments. Finally, comprehensive evaluation results offer valuable references for future projects. By summarizing successful experiences and identifying shortcomings, designers can continuously optimize strategies and improve overall project quality. This process of ongoing improvement not only drives design innovation but also enhances the utility of public spaces.

In summary, evaluating interior landscape design in public spaces is a complex yet essential task that involves technical considerations as well as artistic and humanistic care. Through scientific evaluation methods, it is possible to create more attractive, functional, and sustainable public spaces that meet both social and environmental needs. The interior design quality evaluation in public spaces is a MADM issue. Therefore, the interior design quality evaluation in public spaces is presented to demonstrate the approach developed in this essay. Five potential interior landscape design schemes $VA_i\left(i=1,2,3,4,5\right)$ are assessed with four attributes (see Table I):

TABLE I. FOUR ATTRIBUTES FOR INTERIOR DESIGN QUALITY EVALUATION IN PUBLIC SPACES

Attribute	Description
Aesthetic Value-VG$_1$	Focuses on the visual appeal of the design, ensuring consistency in color, materials, and style to create a harmonious overall effect.
Functionality-VG$_2$	Evaluates whether the spatial layout is reasonable and supports various activity needs, as well as the convenience and accessibility of facilities.
Sustainability-VG$_3$	Examines the environmental characteristics of materials, use of renewable resources, and assesses the energy efficiency and long-term cost benefits.
User Experience-VG$_4$	Collects and analyzes user feedback to evaluate comfort, satisfaction, and whether the design meets user needs and expectations.

All attributes are beneficial. The five possible interior landscape design schemes $VA_i\left(i=1,2,3,4,5\right)$ are evaluated with HFNs. The HF-TODIM-TOPSIS approach is employed to solve the interior design quality evaluation in public spaces.

Step 1. Illustrate the HFN matrix $VR=\left[v\phi_{ij}\right]_{5\times4}$ (see Table II).

TABLE II. THE $VR = \left[v\phi_{ij} \right]_{5\times4}$

	VG1	VG2
VA_1	{0.1342, 0.5621, 0.8173}	{0.2451, 0.6894, 0.9238}
VA_2	{0.2275, 0.6953, 0.0182}	{0.3184, 0.7439, 0.1347}
VA_3	{0.3526, 0.7195, 0.2834}	{0.4731, 0.8274, 0.3142}
VA_4	{0.4173, 0.8021, 0.3652}	{0.5294, 0.8903, 0.4781}
VA_5	{0.5789, 0.9134, 0.4508}	{0.6891, 0.0257, 0.5723}
	VG3	VG4
VA1	{0.3785, 0.7512, 0.8426}	{0.4923, 0.8034, 0.0159}
VA2	{0.4658, 0.8125, 0.2964}	{0.5709, 0.9142, 0.3471}
VA3	{0.5839, 0.9056, 0.4203}	{0.6912, 0.0328, 0.5784}
VA4	{0.6417, 0.0135, 0.5236}	{0.7542, 0.1267, 0.6895}
VA5	{0.7904, 0.1382, 0.6458}	{0.8916, 0.2493, 0.7341}

Step 2. Normalize the $VR = \left[v\phi_{ij} \right]_{5\times4}$ into $VN = \left[n\phi_{ij} \right]_{5\times4}$ (see able III).

TABLE III. THE $VN = \left[n\phi_{ij} \right]_{5\times4}$

	VG1	VG2
VA_1	{0.1342, 0.5621, 0.8173}	{0.2451, 0.6894, 0.9238}
VA_2	{0.0182, 0.2275, 0.6953}	{0.1347, 0.3184, 0.7439}
VA_3	{0.2834, 0.3526, 0.7195}	{0.3142, 0.4731, 0.8274}
VA_4	{0.3652, 0.4173, 0.8021}	{0.4781, 0.5294, 0.8903}
VA_5	{0.4508, 0.5789, 0.9134}	{0.0257, 0.5723, 0.6891}
	VG3	VG4
VA_1	{0.3785, 0.7512, 0.8426}	{0.0159, 0.4923, 0.8034}
VA_2	{0.2964, 0.4658, 0.8125}	{0.3471, 0.5709, 0.9142}
VA_3	{0.4203, 0.5839, 0.9056}	{0.0328, 0.5784, 0.6912}
VA_4	{0.0135, 0.5236, 0.6417}	{0.1267, 0.6895, 0.7542}
VA_5	{0.1382, 0.6458, 0.7904}	{0.2493, 0.7341, 0.8916}

Step 3. Implement the weight:
$wv_1 = 0.2764, wv_2 = 0.1937$

$wv_3 = 0.3429, wv_4 = 0.1870$.

Step 4. Implement the relative weight: $rwv = \{0.8059, 5651, 1.0000, 0.5456\}$

Step 5. Implement the $HFNDD = \left(HFNDD_{ij} \right)_{5\times4}$ (see Table IV):

TABLE IV. THE $HFNDD = \left(HFNDD_{ij} \right)_{5\times4}$

	VG_1	VG_2	VG_3	VG_4
VA_1	0.7143	-0.7965	0.3553	-0.3080
VA_2	-0.2413	0.8392	-1.2242	0.4401
VA_3	-0.3098	0.0711	-0.0800	0.8909
VA_4	0.7102	-0.2481	0.3035	0.3751
VA_5	-0.6798	-0.9131	-0.7609	-0.5322

Step 6. Implement the PIS and NIS (see Table V).

TABLE V. THE PIS AND NIS

	VG$_1$	VG$_2$	VG$_3$	VG$_4$
PIS	0.7143	0.8392	0.3553	0.8909
NIS	-0.6798	-0.9131	-1.2242	-0.5322

Step 7. Implement the $\left|HFNDD_{ij} - PIS_j\right|$ and $\left|HFNDD_{ij} - NIS_j\right|$ (see Table VI to Table VII).

TABLE VI. THE $\left|HFNDD_{ij} - PIS_j\right|$

	VG$_1$	VG$_2$	VG$_3$	VG$_4$
VA$_1$	0.0000	1.6357	0.0000	1.1989
VA$_2$	0.9556	0.0000	1.5795	0.4508
VA$_3$	1.0241	0.7681	0.4353	0.0000
VA$_4$	0.0041	1.0873	0.0518	0.5158
VA$_5$	1.3941	1.7523	1.1162	1.4231

TABLE VII. THE $\left|HFNDD_{ij} - NIS_j\right|$

	VG$_1$	VG$_2$	VG$_3$	VG$_4$
VA$_1$	1.3941	0.1166	1.5795	0.2242
VA$_2$	0.4385	1.7523	0.0000	0.9723
VA$_3$	0.3700	0.9842	1.1442	1.4231
VA$_4$	1.3900	0.6650	1.5277	0.9073
VA$_5$	0.0000	0.0000	0.4633	0.0000

Step 8. Implement the $HD(VA_i, PIS)$, $HD(VA_i, NIS)$ and $CC(VA_i, PIS)$ (see Table VIII to Table IX).

TABLE VIII. THE $HD(VA_i, PIS)$, $HD(VA_i, NIS)$

	$HD(VA_i, PIS)$	$HD(VA_i, NIS)$
VA$_1$	2.8346	3.3144
VA$_2$	2.9859	3.1631
VA$_3$	2.2275	3.9215
VA$_4$	1.6590	4.4900
VA$_5$	5.6857	0.4633

TABLE IX. THE $HFNCC(VA_i, PIS)$ AND ORDER

	$HFNCC(VA_i, PIS)$	Order
VA_1	0.5390	3
VA_2	0.5144	4
VA_3	0.6377	2
VA_4	0.7302	1
VA_5	0.0753	5

Thus, the best interior landscape design scheme is VA_1.

B. Comparative Analysis

Then, the HF-TODIM-TOPSIS approach is compared with HFWA approach[59], HFWG approach [59], HF-MABAC approach [67], HF-CODAS approach [68], HF-EDAS approach [69] and HF-TODIM approach [70]. The comparative results are demonstrated in Table X and Fig. 2.

TABLE X. ORDER FOR DIFFERENT APPROACHES

	Order
HFWA approach [59]	$VA_4 > VA_3 > VA_1 > VA_2 > VA_5$
HFWG approach [59]	$VA_4 > VA_3 > VA_2 > VA_1 > VA_5$
HF-MABAC approach [67]	$VA_4 > VA_3 > VA_1 > VA_2 > VA_5$
HF-CODAS approach [68]	$VA_4 > VA_3 > VA_1 > VA_2 > VA_5$
HF-EDAS approach [69]	$VA_4 > VA_3 > VA_1 > VA_2 > VA_5$
HF-TODIM approach [70]	$VA_4 > VA_3 > VA_1 > VA_2 > VA_5$
HF-TODIM-TOPSIS approach	$VA_4 > VA_3 > VA_1 > VA_2 > VA_5$

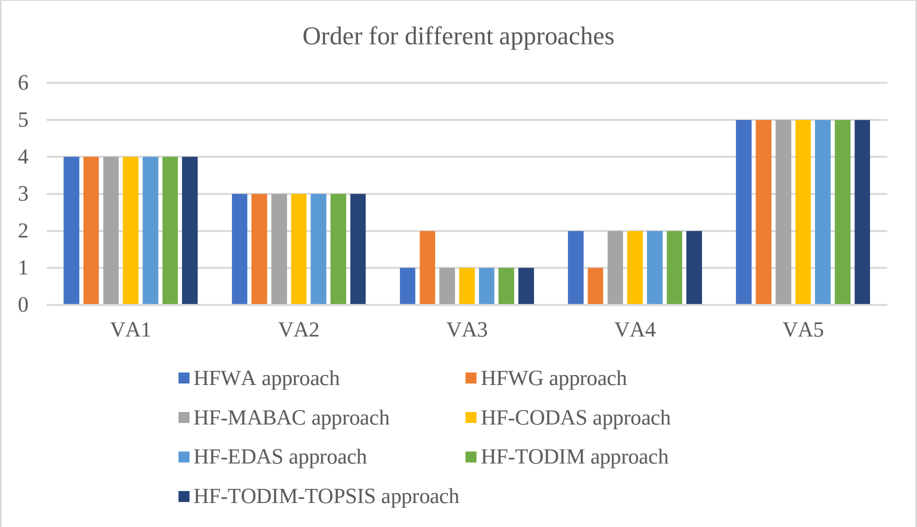

Fig. 2. Order for different approaches.

Through the above analysis, the HF-TODIM-TOPSIS method demonstrates its effectiveness and reliability for multi-attribute decision-making (MADM). The primary advantages of this approach are as follows: (1) the HF-TODIM-TOPSIS method adeptly addresses the uncertainties inherent in real-world MADM scenarios and captures the psychological behaviors of decision-makers during the evaluation of interior landscape design in public spaces; (2) it also explores the dynamics of the TODIM and TOPSIS techniques when integrated into a hybrid model specifically designed for assessing interior landscape design in public spaces. However, a significant limitation of the HF-TODIM-TOPSIS approach is its failure to address issues related to group consensus.

V. CONCLUSION

The evaluation of interior landscape design in public spaces is highly significant. Firstly, it ensures the beauty and functionality of the space, enhancing the overall user experience. By assessing the visual appeal and practicality of the design, it creates environments that are both pleasant and efficient. Secondly, the evaluation process promotes sustainability by emphasizing the use of eco-friendly materials and energy-efficient designs, reducing environmental impact. Additionally, it focuses on user needs and feedback, ensuring the design meets expectations and improves satisfaction and comfort. This comprehensive evaluation not only provides direction for designers to improve but also offers valuable references for future projects, helping to enhance design quality and innovation, ultimately creating better public spaces for everyone. The interior design quality evaluation in public spaces is MADM. Recently, the TODIM and TOPSIS methods have been employed to address challenges in MADM. HFSs are utilized to represent uncertain information in the evaluation of interior landscape design within public spaces. This study introduces the hesitant fuzzy TODIM-TOPSIS (HF-TODIM-TOPSIS) approach to resolve MADM issues in the context of HFSs. A numerical case study focused on the evaluation of interior landscape design in public spaces demonstrates the validity of this method.

The main conclusions and findings of this study can be summarized as follows:(1) Effectiveness of the HF-TODIM-TOPSIS method: The research demonstrated the effectiveness of the HF-TODIM-TOPSIS method in addressing multi-attribute decision-making problems, such as indoor landscape design in public spaces, through numerical case analysis. This method effectively handles uncertainty in the evaluation process and provides more comprehensive and objective evaluation results. (2) Application value of HFSs: The study indicates that the introduction of HFSs can better capture the hesitation and subjectivity of decision-makers during the evaluation process, making the evaluation results more aligned with actual conditions. (3) Weight determination using information entropy: The study utilized information entropy to determine the weights of different evaluation indicators. Compared to traditional subjective weighting methods, this approach is more objective and reduces interference from human factors. (4) Extension of TODIM and TOPSIS methods: The research extends the traditional TODIM and TOPSIS methods to the HFSs framework, providing new ideas and methods for solving more complex multi-attribute decision-making problems.

In summary, this study proposes a new and more effective multi-attribute decision-making method, HF-TODIM-TOPSIS, and validates its practical value in the evaluation of indoor landscape design in public spaces through case analysis.

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