Enhancing Multiple-Attribute Decision-Making with Interval-Valued Neutrosophic Sets: Diverse Applications in Evaluating English Teaching Quality

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***Abstract*—The evaluation of college English teaching quality is a key method for systematically analyzing and providing feedback on the teaching process and outcomes. It aims to comprehensively assess the effectiveness of teaching, student learning outcomes, and the appropriateness of the course design. The evaluation typically covers aspects such as teaching methods, classroom atmosphere, student engagement, use of teaching resources, and learning achievements. By collecting data from student feedback, teaching supervision, and exam results, the evaluation helps to improve teaching strategies, enhance students' English proficiency, and ultimately achieve continuous optimization and improvement of teaching quality. The teaching quality evaluation of college English is viewed as the multiple-attribute decision-making (MADM). In this paper, some Aczel-Alsina operators are produced under interval-valued neutrosophic sets (IVNSs). Then, interval-valued neutrosophic number (IVNN) Aczel-Alsina weighted averaging (IVNNAAWA) operator is employed to cope with MADM problem. Finally, the numerical decision example for teaching quality evaluation of college English is employed to illustrate the produced method.**

Keywords*—Multiple-attribute decision-making; interval-valued neutrosophic sets (IVNSs); Aczel-Alsina operations; teaching quality evaluation

I. Introduction

Evaluation of college English teaching quality is an essential tool for systematically analyzing and assessing the teaching process and outcomes. Its aim is to enhance teaching quality through a scientifically sound evaluation system, meeting students' learning needs and the societal demands for English proficiency. The evaluation typically covers various aspects, including teaching methods, curriculum design, utilization of teaching resources, student learning outcomes, and classroom participation. Diverse evaluation methods, such as student feedback, classroom observation, and test performance analysis, are widely employed to ensure comprehensiveness and objectivity. Through continuous quality evaluation, universities can optimize teaching strategies, improve course design, and promote the development of students' comprehensive English skills, laying a foundation for cultivating internationally competitive talents in line with contemporary demands. Starting in 2013, Lu and Huang [1] explored the construction of English teaching quality evaluation systems and the applicability of cultivating students' comprehensive English abilities. They suggested reforming the existing college English teaching evaluation methods by referencing the Canadian language benchmarks to comprehensively enhance teaching quality and students' comprehensive English proficiency. In the same year, Ma and Li [2] attempted to construct a multidimensional ecological evaluation system under network environments, based on constructivism and educational ecology theories, to promote the ecologicalization and quality improvement of college English teaching. In 2015, Wu and Tang [3] analyzed the current state of classroom teaching quality evaluation systems in universities, identifying existing problems and designing various evaluation forms to enhance the scientific and rational nature of evaluations. Yu [4] proposed a diversified evaluation model, combining modern quality management theory, constructivism, and multiple intelligences theory, emphasizing the use of various methods for comprehensive teaching quality evaluation to improve college English course quality. In 2016, Yuan [5] analyzed teaching evaluation strategies for subsequent college English courses, highlighting the need for differentiated strategies in evaluation subjects, content, and methods to enhance teaching quality across language culture, skills, and language application courses. In 2017, Deng [6] studied the application of participatory teaching models in college English, proposing related teaching quality evaluation methods. The study emphasized that participatory teaching methods should focus not only on students' exam scores but also on their practical application abilities. In 2018, Yang [7] developed a teaching quality evaluation index system for flipped classrooms in vocational college English, emphasizing the importance of developmental and process evaluations. The evaluation indices were refined through expert consultation and surveys to align with the characteristics of flipped classroom teaching. In 2019, Li [8] explored strategies for constructing college English teaching quality evaluation systems under the applied talent training system, proposing the establishment of a scientific and reasonable evaluation system to improve English teaching quality and meet the societal demand for applied talents. In 2020, Xu [9] discussed the application of stratified teaching methods in college English, analyzing its role in reducing student learning differences, enhancing learning interest, and ensuring classroom teaching quality, while proposing specific application strategies. In 2021, Yang and Li [10] constructed a classroom teaching quality evaluation model based on set pair analysis in the context of the digital era to meet the requirements of "golden course" construction. The study emphasized the importance of integrating modern information technology with college English courses. In 2022, Zhang [11] studied college English teaching quality evaluation through value speculation guidance, constructing an evaluation model

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based on the SERVQUAL model. This aimed to identify gaps and deficiencies in teaching by comparing students' "expectations" and "actual perceptions," and proposed improvement measures. In 2023, Gong and Peng [12] proposed a college English teaching quality evaluation model based on ISSA-DRNN, utilizing an improved deep recurrent neural network and optimized sparrow search algorithm to enhance the model's evaluation performance and precision. In 2024, Zhang [13] constructed an English teaching quality evaluation system based on the CIPP model, proposing a comprehensive framework that includes context, input, process, and product evaluations. This provided a scientific and operational evaluation standard for college English teaching.

Multi-Attribute Decision Making (MADM) is a decision-making method used to address selection problems involving multiple evaluation criteria [14-17]. This type of decision-making approach is widely applied in fields such as management science, engineering, and economics, aiding decision-makers in making rational choices in complex situations [18-21]. In multi-attribute decision making, the decision problem typically involves multiple competing attributes or criteria, which may have different levels of importance [22-25]. Therefore, decision-makers need to weight each attribute to reflect its relative importance. Common weighting methods include expert scoring, Analytic Hierarchy Process (AHP), and entropy method. The basic steps of multi-attribute decision making include: first, clarifying the decision objectives and available alternatives; second, determining the evaluation attributes and assigning weights to them [26-29]; then, collecting and organizing data on the performance of each alternative across different attributes[30-33]; next, applying appropriate decision-making methods (such as TOPSIS, VIKOR, ELECTRE) to comprehensively evaluate the alternatives; finally, selecting the optimal alternative or ranking the alternatives based on the evaluation results. The advantage of this method is that it systematically considers multiple factors, making the decision process more comprehensive and objective. However, the reliability of the decision results largely depends on the accuracy of the attribute weights and the completeness of the data [34-38]. Therefore, decision-makers need to carefully assess the rationality of each step and the accuracy of the data when using multi-attribute decision-making methods. The problems of teaching quality evaluation of college English is MADM [39-45]. Aczél and Alsina [46] structured some new operations named as Aczel-Alsina t-norm and t-conorm operations. Yong et al.[29] and Ashraf et al. [47] structured the Aczel-Alsina decision operations to SVNNs and produced the Aczel-Alsina fused operators of SVNNs for MADM. The main aim of this defined paper is to expand the Aczel-Alsina operations [46] to cope with MADM under IVNSs. The main study motivations are listed: (1) the Aczel-Alsina operations are extended to IVNSs; (2) some Aczel-Alsina aggregating operators are produced under IVNSs; (3) the IVNNAAWA method is designed for MADM; (4) a case study about translation quality decision evaluation of college English is given to show the IVNNAAWA method; (5) some comparative models are used to proof the IVNNAAWA method. The remainder sections of this paper are set out. Section II lists the IVNSs. In Section III, the some Aczel-Alsina aggregating operators are produced under IVNSs. In Section IV, the IVNNAAWA operator is built for MADM. In Section V, a case study for translation quality decision evaluation of college English is listed and some comparative decision methods are done. The defined decision study ends in Section VI.

II. PRELIMINARIES

Wang et al. [48] produced the IVNSs

Definition 1 [49]. The IVNSs A in X is:

$$\tilde{A}=\left\{\left(x,TT_{\tilde{A}}\left(x\right),II_{\tilde{A}}\left(x\right),FF_{\tilde{A}}\left(x\right)\right)\middle|x\in X\right\} \tag{1}$$

with truth-membership $TT_{\tilde{A}}(x)$, indeterminacy-membership $II_{\tilde{A}}(x)$ and falsity-membership $FF_{\tilde{A}}(x)$, $TT_{\tilde{A}}(x),II_{\tilde{A}}(x),FF_{\tilde{A}}(x)\in[0,1]$, $0\le\sup TT_{\tilde{A}}(x)+\sup II_{\tilde{A}}(x)+\sup FF_{\tilde{A}}(x)\le 3$.

The IVNN is expressed as $\tilde{A}=\left(TT_{\tilde{A}},II_{\tilde{A}},FF_{\tilde{A}}\right)=\left(\left[TL_{\tilde{A}},TR_{\tilde{A}}\right],\left[IL_{\tilde{A}},IR_{\tilde{A}}\right],\left[FL_{\tilde{A}},FR_{\tilde{A}}\right]\right)$, where $TT_{\tilde{A}}\subseteq[0,1],II_{\tilde{A}}\subseteq[0,1],FF_{\tilde{A}}\subseteq[0,1]$, $0\le TR_{\tilde{A}}+IR_{\tilde{A}}+FR_{\tilde{A}}\le 3$.

Definition 2[50]. Let $\tilde{A}=\left(\left[TL_{\tilde{A}},TR_{\tilde{A}}\right],\left[IL_{\tilde{A}},IR_{\tilde{A}}\right],\left[FL_{\tilde{A}},FR_{\tilde{A}}\right]\right)$, the score value is expressed:

$$SV\left(\tilde{A}\right)=\frac{\left(2+TL_{\tilde{A}}-IL_{\tilde{A}}-FL_{\tilde{A}}\right)+\left(2+TR_{\tilde{A}}-IR_{\tilde{A}}-FR_{\tilde{A}}\right)}{6},$$

$$SV\left(\tilde{A}\right)\in[0,1]. \tag{2}$$

Definition 3[50]. Let $\tilde{A}=\left(\left[TL_{\tilde{A}},TR_{\tilde{A}}\right],\left[IL_{\tilde{A}},IR_{\tilde{A}}\right],\left[FL_{\tilde{A}},FR_{\tilde{A}}\right]\right)$, the accuracy value is expressed:

$$AV\left(\tilde{A}\right)=\frac{\left(TL_{\tilde{A}}+TR_{\tilde{A}}\right)-\left(FL_{\tilde{A}}+FR_{\tilde{A}}\right)}{2},AV\left(\tilde{A}\right)\in[-1,1]. \tag{3}$$

Huang et al. [50] expressed the order relation for IVNNs.

Definition 4[50]. Let $\tilde{A}=\left(\left[TL_{\tilde{A}},TR_{\tilde{A}}\right],\left[IL_{\tilde{A}},IR_{\tilde{A}}\right],\left[FL_{\tilde{A}},FR_{\tilde{A}}\right]\right)$ and $\tilde{B}=\left(\left[TL_{\tilde{B}},TR_{\tilde{B}}\right],\left[IL_{\tilde{B}},IR_{\tilde{B}}\right],\left[FL_{\tilde{B}},FR_{\tilde{B}}\right]\right)$, let

$$SV\left(\tilde{A}\right)=\frac{\left(2+TL_{\tilde{A}}-IL_{\tilde{A}}-FL_{\tilde{A}}\right)+\left(2+TR_{\tilde{A}}-IR_{\tilde{A}}-FR_{\tilde{A}}\right)}{6}$$

and

$$SV\left(\tilde{B}\right)=\frac{\left(2+TL_{\tilde{B}}-IL_{\tilde{B}}-FL_{\tilde{B}}\right)+\left(2+TR_{\tilde{B}}-IR_{\tilde{B}}-FR_{\tilde{B}}\right)}{6}$$

, and let $AV\left(\tilde{A}\right)=\frac{\left(TL_{\tilde{A}}+TR_{\tilde{A}}\right)-\left(FL_{\tilde{A}}+FR_{\tilde{A}}\right)}{2}$ and $AV\left(\tilde{B}\right)=\frac{\left(TL_{\tilde{B}}+TR_{\tilde{B}}\right)-\left(FL_{\tilde{B}}+FR_{\tilde{B}}\right)}{2}$, then if $SV\left(\tilde{A}\right)<SV\left(\tilde{B}\right)$, we have $\tilde{A}<\tilde{B}$; if $SV\left(\tilde{A}\right)=SV\left(\tilde{B}\right)$, (1)if $HV\left(\tilde{A}\right)=HV\left(\tilde{B}\right)$, we have $\tilde{A}=\tilde{B}$; (2) if $HV\left(\tilde{A}\right)<HV\left(\tilde{B}\right)$, we have $\tilde{A}<\tilde{B}$.

Aczél and Alsina [46] structured some new operations named as Aczel-Alsina t-norm and t-conorm operations, which have true advantage of changeability through adjusting a decision parameter. Yong et al.[29] and Ashraf et al. [47] structured the Aczel-Alsina operations to SVNNs. Similarly, the Aczel-Alsina decision operations for IVNNs are produced.

Definition 5. Let $\tilde{A}=\left(\left[TL_{\tilde{A}},TR_{\tilde{A}}\right],\left[IL_{\tilde{A}},IR_{\tilde{A}}\right],\left[FL_{\tilde{A}},FR_{\tilde{A}}\right]\right)$ and $\tilde{B}=\left(\left[TL_{\tilde{B}},TR_{\tilde{B}}\right],\left[IL_{\tilde{B}},IR_{\tilde{B}}\right],\left[FL_{\tilde{B}},FR_{\tilde{B}}\right]\right)$, $\phi\geq 1$, $\lambda>0$, the Aczel-Alsina operations for IVNNs are produced:

- $$\tilde{A}\oplus\tilde{B}\left(\begin{array}{l}\left[1-e^{-\left(\left(-\ln\left(1-TL_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(1-TL_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\left(-\ln\left(1-TR_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(1-TR_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\ \left[e^{-\left(\left(-\ln\left(IL_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(IL_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}},\ e^{-\left(\left(-\ln\left(IR_{\tilde{A}}\right)\right)^{\theta}+\left(-\ln\left(IR_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\ \left[e^{-\left(\left(-\ln\left(FL_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(FL_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}},\ e^{-\left(\left(-\ln\left(FR_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(FR_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right)$$

- $$\tilde{A}\otimes\tilde{B}=\left(\begin{array}{l}\left[e^{-\left(\left(-\ln\left(TL_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(TL_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\left(-\ln\left(TR_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(TR_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\ \left[1-e^{-\left(\left(-\ln\left(1-IL_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(1-IL_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\left(-\ln\left(1-IR_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(1-IR_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\ \left[1-e^{-\left(\left(-\ln\left(1-FL_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(1-FL_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\left(-\ln\left(1-FR_{\tilde{A}}\right)\right)^{\phi}+\left(-\ln\left(1-FR_{\tilde{B}}\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right)$$

- $$\lambda A=\left(\begin{array}{l}\left[1-e^{-\left(\lambda\left(-\ln\left(1-TL_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\lambda\left(-\ln\left(1-TR_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\ \left[e^{-\left(\lambda\left(-\ln\left(IL_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\lambda\left(-\ln\left(IR_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\ \left[e^{-\left(\lambda\left(-\ln\left(FL_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\lambda\left(-\ln\left(FR_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right)$$

- $$(A)^{\lambda}=\left(\begin{array}{l}\left[e^{-\left(\lambda\left(-\ln\left(TL_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\lambda\left(-\ln\left(TR_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[1-e^{-\left(\lambda\left(-\ln\left(1-IL_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\lambda\left(-\ln\left(1-IR_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[1-e^{-\left(\lambda\left(-\ln\left(1-FL_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\lambda\left(-\ln\left(1-FR_{\tilde{A}}\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right)$$

III. Some Aczel-Alsina Weighted Averaging Operators with NNs

In this section, Some Aczel-Alsina aggregating operators with IVNNs are produced.

The IVNN Aczel-Alsina weighted averaging (IVNNAAWA) is defined.

Definition 7. Let $SA_i=\left(\left[TL_i,TR_i\right],\left[IL_i,IR_i\right],\left[FL_i,FR_i\right]\right)$ the IVNNs with their weight $sw_i=(sw_1,sw_2,\ldots,sw_n)^T$, $\sum_{i=1}^{n}sw_i=1$, $\phi\geq 1$. If

$$\text{IVNNAAWA}_{sw}\left(SA_1,SA_2,\cdots,SA_n\right)=\overset{n}{\underset{i=1}{\oplus}}sw_iSA_i \tag{4}$$

The Theorem 1 is obtained.

Theorem 1. Let $SA_i=\left(\left[TL_i,TR_i\right],\left[IL_i,IR_i\right],\left[FL_i,FR_i\right]\right)$ the IVNNs with their weight $sw_i=(sw_1,sw_2,\ldots,sw_n)^T$, $\sum_{i=1}^{n}zw_i=1$, $\phi\geq 1$. If

$$\text{IVNNAAWA}_{sw}\left(SA_1,SA_2,\cdots,SA_n\right)=\overset{n}{\underset{i=1}{\oplus}}sw_iSA_i$$
$$=\left(\begin{array}{l}\left[1-e^{-\left(\sum_{i=1}^{n}sw_i\left(-\ln\left(1-TL_j\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\sum_{i=1}^{n}sw_i\left(-\ln\left(1-TR_j\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(\sum_{i=1}^{n}sw_i\left(-\ln\left(IL_j\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\sum_{i=1}^{n}sw_i\left(-\ln\left(IR_j\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(\sum_{i=1}^{n}sw_i\left(-\ln\left(FL_j\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\sum_{i=1}^{n}sw_i\left(-\ln\left(FR_j\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right) \tag{5}$$

Proof:

(a) Let $i=2$, then

$$\text{IVNNAAWA}_{sw}\left(SA_1,SA_2\right)=sw_1SA_1\oplus sw_2SA_2$$
$$\left(\begin{array}{l}\left[1-e^{-\left(sw_1\left(-\ln\left(1-TL_1\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(sw_1\left(-\ln\left(1-TR_1\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(sw_1\left(-\ln\left(IL_1\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(sw_1\left(-\ln\left(IR_1\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(sw_1\left(-\ln\left(FL_1\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(sw_1\left(-\ln\left(FR_1\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right)\oplus$$
$$\left(\begin{array}{l}\left[1-e^{-\left(sw_2\left(-\ln\left(1-TL_2\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(sw_2\left(-\ln\left(1-TR_2\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(sw_2\left(-\ln\left(IL_2\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(sw_2\left(-\ln\left(IR_2\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(sw_2\left(-\ln\left(FL_2\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(sw_2\left(-\ln\left(FR_2\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right)$$

(b)

$$=\left(\begin{array}{l}\left[1-e^{-\left(\sum_{i=1}^{2}sw_i\left(-\ln\left(1-TL_j\right)\right)^{\phi}\right)^{1/\phi}},1-e^{-\left(\sum_{i=1}^{2}sw_i\left(-\ln\left(1-TR_j\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(\sum_{i=1}^{2}sw_i\left(-\ln\left(IL_j\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\sum_{i=1}^{2}sw_i\left(-\ln\left(IR_j\right)\right)^{\phi}\right)^{1/\phi}}\right],\\\left[e^{-\left(\sum_{i=1}^{2}sw_i\left(-\ln\left(FL_j\right)\right)^{\phi}\right)^{1/\phi}},e^{-\left(\sum_{i=1}^{2}sw_i\left(-\ln\left(FR_j\right)\right)^{\phi}\right)^{1/\phi}}\right]\end{array}\right)$$

(c) If Eq. (10) holds for $i=k$, then

$$\mathrm{IVNNAAWA}_{sw}\left(SA_1, SA_2, \cdots, SA_k\right) = \overset{k}{\underset{i=1}{\oplus}} sw_i SA_i$$

$$= \left(\begin{array}{l} \left[1-e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(1-TL_j\right)\right)^{\phi}\right)^{1/\phi}}, 1-e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(1-TR_j\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(IL_j\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(IR_j\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(FL_j\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(FR_j\right)\right)^{\phi}\right)^{1/\phi}} \right] \end{array} \right)$$

(d) Set $i = k+1$. From Definition 5, we have

$$\mathrm{IVNNAAWA}_{sw}\left(SA_1, SA_2, \cdots, SA_{k+1}\right) = \overset{k}{\underset{i=1}{\oplus}} sw_i SA_i \oplus sw_{k+1} SA_{k+1}$$

$$= \left(\begin{array}{l} \left[1-e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(1-TL_j\right)\right)^{\phi}\right)^{1/\phi}}, 1-e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(1-TR_j\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(IL_j\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(IR_j\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(FL_j\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{k} sw_i\left(-\ln\left(FR_j\right)\right)^{\phi}\right)^{1/\phi}} \right] \end{array} \right) \oplus$$

$$\left(\begin{array}{l} \left[1-e^{-\left(sw_{k+1}\left(-\ln\left(1-TL_{k+1}\right)\right)^{\phi}\right)^{1/\phi}}, 1-e^{-\left(sw_{k+1}\left(-\ln\left(1-TR_{k+1}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(sw_{k+1}\left(-\ln\left(IL_{k+1}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(sw_{k+1}\left(-\ln\left(IR_{k+1}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(sw_{k+1}\left(-\ln\left(FL_{k+1}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(sw_{k+1}\left(-\ln\left(FR_{k+1}\right)\right)^{\phi}\right)^{1/\phi}} \right] \end{array} \right)$$

$$= \left(\begin{array}{l} \left[1-e^{-\left(\sum_{i=1}^{k+1} sw_i\left(-\ln\left(1-TL_j\right)\right)^{\phi}\right)^{1/\phi}}, 1-e^{-\left(\sum_{i=1}^{k+1} sw_i\left(-\ln\left(1-TR_j\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{k+1} sw_i\left(-\ln\left(IL_j\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{k+1} sw_i\left(-\ln\left(IR_j\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{k+1} sw_i\left(-\ln\left(FL_j\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{k+1} sw_i\left(-\ln\left(FR_j\right)\right)^{\phi}\right)^{1/\phi}} \right] \end{array} \right)$$

From the above (a), (b), and (c), it can be seen that Eq. (5) holds for any *i*.

The IVNNAAWA has good properties.

Property 1. (idempotency). If $SA_i = SA = \left(\left[TL, TR\right], \left[IL, IR\right], \left[FL, FR\right]\right)$

$$\mathrm{IVNNAAWA}_{sw}\left(SA_1, SA_2, \cdots, SA_n\right) = SA \tag{6}$$

Property2. (Monotonicity). Let $SA_i = \left(\left[TL_{A_i}, TR_{A_i}\right], \left[IL_{A_i}, IR_{A_i}\right], \left[FL_{A_i}, FR_{A_i}\right]\right)$, $SB_i = \left(\left[TL_{B_i}, TR_{B_i}\right], \left[IL_{B_i}, IR_{B_i}\right], \left[FL_{B_i}, FR_{B_i}\right]\right)$. If $TL_{A_i} \le TL_{B_i}, IL_{A_i} \ge IL_{B_i}, FL_{A_i} \ge FL_{B_i}$,

$TR_{A_i} \le TR_{B_i}, IR_{A_i} \ge IR_{B_i}, FR_{A_i} \ge FR_{B_i}$ holds for all i, then

$$\begin{array}{l} \mathrm{IVNNAAWA}_{sw}\left(SA_1, SA_2, \cdots, SA_n\right) \\ \le \mathrm{IVNNAAWA}_{sw}(SB_1, SB_2, \cdots, SB_n) \end{array} \tag{7}$$

Property 3 (Boundedness). Let $SA_i = \left(\left[TL_{A_i}, TR_{A_i}\right], \left[IL_{A_i}, IR_{A_i}\right], \left[FL_{A_i}, FR_{A_i}\right]\right)$. If

$$ZA^{+} = \left(\begin{array}{l} \left[\max_i(TL_i), \max_i(TR_i)\right], \\ \left[\min_i(IL_i), \min_i(IR_i)\right], \left[\min_i(FL_i), \min_i(FR_i)\right] \end{array} \right),$$

$$ZA^{+} = \left(\begin{array}{l} \left[\min_i(TL_i), \min_i(TR_i)\right], \\ \left[\max_i(IL_i), \max_i(IR_i)\right], \left[\max_i(FL_i), \max_i(FR_i)\right] \end{array} \right)$$

then

$$ZA^{-} \le \mathrm{IVNNAAWA}_{sw}\left(SA_1, SA_2, \cdots, SA_n\right) \le ZA^{+} \tag{8}$$

Then, the IVNN Aczel-Alsina OWA (IVNNAAOWA) is expressed.

Definition 8. Let $SA_i = \left(\left[TL_i, TR_i \right], \left[IL_i, IR_i \right], \left[FL_i, FR_i \right] \right) (i = 1, 2, \ldots, n)$ be IVNNs, $\theta \geq 1$. If:

$$\text{IVNNAAOWA}_{sw}\left(SA_1, SA_2, \cdots, SA_n\right) = \overset{n}{\underset{i=1}{\oplus}} sw_{\sigma(i)} SA_{\sigma(i)}$$
$$= \left(\begin{array}{l} \left[1 - e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(1-TL_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}}, 1 - e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(1-TR_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(IL_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(IR_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(FL_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(FR_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}} \right] \end{array} \right) \quad (9)$$

where $\left(\sigma(1), \sigma(2), \cdots, \sigma(n)\right)$ is permutation of $(1, 2, \cdots, n)$, such that $SA_{\sigma(j-1)} \geq SA_{\sigma(j)}$ for all $j = 2, \cdots, n$, and $zw = \left(zw_1, zw_2, \cdots, zw_n\right)^T$ is weight of IVNNAAOWA operator, and $0 \leq zw_j \leq 1$, $\sum_{j=1}^{n} zw_j = 1$.

The IVNNAAOWA has three properties.

Property 4. (idempotency). If $SA_i = SA = \left([TL, TR], [IL, IR], [FL, FR]\right)$

$$\text{IVNNAAOWA}_{zw}\left(SA_1, SA_2, \cdots, SA_n\right) = SA \quad (10)$$

Property 5. (Monotonicity). Let $SA_i = \left(\left[TL_{A_i}, TR_{A_i} \right], \left[IL_{A_i}, IR_{A_i} \right], \left[FL_{A_i}, FR_{A_i} \right] \right)$, $SB_i = \left(\left[TL_{B_i}, TR_{B_i} \right], \left[IL_{B_i}, IR_{B_i} \right], \left[FL_{B_i}, FR_{B_i} \right] \right)$. If $TL_{A_i} \leq TL_{B_i}, IL_{A_i} \geq IL_{B_i}, FL_{A_i} \geq FL_{B_i}$,

$TR_{A_i} \leq TR_{B_i}, IR_{A_i} \geq IR_{B_i}, FR_{A_i} \geq FR_{B_i}$ holds for all i, then

$$\begin{aligned} &\text{IVNNAAOWA}_{zw}\left(SA_1, SA_2, \cdots, SA_n\right) \\ &\leq \text{IVNNAAOWA}_{zw}(SB_1, SB_2, \cdots, SB_n) \end{aligned} \quad (11)$$

Property 6 (Boundedness). Let $SA_i = \left(\left[TL_{A_i}, TR_{A_i} \right], \left[IL_{A_i}, IR_{A_i} \right], \left[FL_{A_i}, FR_{A_i} \right] \right)$. If

$$ZA^+ = \left(\left[\max_i(TL_i), \max_i(TR_i)\right], \left[\min_i(IL_i), \min_i(IR_i)\right], \left[\min_i(FL_i), \min_i(FR_i)\right]\right),$$

$$ZA^+ = \left(\left[\min_i(TL_i), \min_i(TR_i)\right], \left[\max_i(IL_i), \max_i(IR_i)\right], \left[\max_i(FL_i), \max_i(FR_i)\right]\right)$$

then

$$ZA^- \leq \text{IVNNAAOWA}_{zw}\left(SA_1, SA_2, \cdots, SA_n\right) \leq ZA^+ \quad (12)$$

From the Definitions 7 and 8, it’s shown that IVNNAAWA and IVNNAAOWA operators weigh the IVNNs and the ordered positions of the IVNNs, respectively. An IVNN Aczel-Alsina hybrid average (IVNNAAHA) operator is produced to include the characteristics of IVNNAAWA and IVNNAAOWA operators together.

Definition 9. Let $SA_i = \left(\left[TL_i, TR_i \right], \left[IL_i, IR_i \right], \left[FL_i, FR_i \right] \right) (i = 1, 2, \ldots, n)$ be the IVNNs. An IVNNAAHA operator is produced:

$$\text{IVNNAAHA}_{zw,sw}\left(SA_1, SA_2, \cdots, SA_n\right) = \overset{n}{\underset{i=1}{\oplus}} \left(zw_i SSA_{\sigma(i)}\right)$$
$$= \left(\begin{array}{l} \left[1 - e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(1-STL_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}}, 1 - e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(1-STR_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(SIL_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(SIR_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(SFL_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{n} zw_i \left(-\ln\left(SFR_{\sigma(i)}\right)\right)^{\phi}\right)^{1/\phi}} \right] \end{array} \right) \quad (13)$$

which $zw = \left(zw_1, zw_2, \cdots, zw_n\right)^T$ is the associated weight, with $0 \leq zw_i \leq 1$, $\sum_{i=1}^{n} zw_i = 1$, $SSA_{\sigma(i)}$ is the i-th largest value of the IVNNs $SSA_i\left(SSA_i = \left(nsw_i\right)SA_i\right) = \left(\left[STL_i, STR_i \right], \left[SIL_i, SIR_i \right], \left[SFL_i, SFR_i \right] \right)$, $sw = \left(sw_1, sw_2, \cdots, sw_n\right)$ is the weight of IVNNs $SA_i\left(i = 1, 2, \cdots, n\right)$, with $sw_j \in [0,1]$, $\sum_{i=1}^{n} sw_i = 1$, and n is the balancing coefficient.

If $zw = \left(1/n, 1/n, \cdots, 1/n\right)^T$, IVNNAAHA reduces to the IVNNAAWA; and if $sw = \left(1/n, 1/n, \cdots, 1/n\right)$, the IVNNAAHA reduces to IVNNAAOWA.

IV. Method for MADM Based on the IVNNAAWA

The IVNNAAWA is used to build for MADM. Let $CS = \{CS_1, CS_2, \cdots, CS_m\}$ be alternatives, and attributes set $GG=\{GG_1,GG_2,\cdots,GG_n\}$ with weight $sw=\{sw_1,sw_2,\cdots,sw_n\}$, where $sw_j\in[0,1]$, $\sum_{j=1}^{n}sw_j=1$. Suppose that values are assessed with IVNNs $QQ=(qq_{ij})_{m\times n}=([TL_{ij},TR_{ij}],[IL_{ij},IR_{ij}],[FL_{ij},FR_{ij}])_{m\times n}$.

Then, method for MADM is built based on the IVNNAAWA. The given steps are produced.

Step 1. Build the IVNN matrix $QQ=(qq_{ij})_{m\times n}=([TL_{ij},TR_{ij}],[IL_{ij},IR_{ij}],[FL_{ij},FR_{ij}])_{m\times n}$.

$$QQ=[qq_{ij}]_{m\times n}=\begin{bmatrix} qq_{11} & qq_{12} & \cdots & qq_{1n} \\ qq_{21} & qq_{22} & \cdots & qq_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ qq_{m1} & qq_{m2} & \cdots & qq_{mn} \end{bmatrix} \tag{14}$$

Step 2. Normalize $QQ=(qq_{ij})_{m\times n}$ to $NQ=[nq_{ij}]_{m\times n}$.

$$\begin{aligned} nq_{ij} &= ([NTL_{ij},NTR_{ij}],[NIL_{ij},NIR_{ij}],[NFL_{ij},NFR_{ij}]) \\ &= \begin{cases} ([TL_{ij}^k,TR_{ij}^k],[IL_{ij}^k,IR_{ij}^k],[FL_{ij}^k,FR_{ij}^k]), & GG_j \text{ is a benefit criterion} \\ ([FL_{ij}^k,FR_{ij}^k],[IL_{ij}^k,IR_{ij}^k],[TL_{ij}^k,TR_{ij}^k]), & GG_j \text{ is a cost criterion} \end{cases} \end{aligned} \tag{15}$$

Step 3. According to $NQ=[nq_{ij}]_{m\times n}$, the overall IVNNs $nq_i\,(i=1,2,\cdots,m)$ are produced through IVNNAAWA operator:

$$\begin{aligned} nq_i &= \text{IVNNAAWA}_{sw}(nq_{i1},nq_{i2},\cdots,nq_{in}) \\ &= \mathop{\oplus}_{i=1}^{n} sw_i nq_{ij} \\ &= ([NTL_i,NTR_i],[NIL_i,NIR_i],[NFL_i,NFR_i]) \\ &= \left(\begin{array}{l} \left[1-e^{-\left(\sum_{j=1}^{n} sw_i\left(-\ln\left(1-NTL_{ij}\right)\right)^{\phi}\right)^{1/\phi}}, 1-e^{-\left(\sum_{i=1}^{n} sw_i\left(-\ln\left(1-NTR_{ij}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{n} sw_i\left(-\ln\left(NIL_{ij}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{n} sw_i\left(-\ln\left(NIR_{ij}\right)\right)^{\phi}\right)^{1/\phi}} \right], \\ \left[e^{-\left(\sum_{i=1}^{n} sw_i\left(-\ln\left(NFL_{ij}\right)\right)^{\phi}\right)^{1/\phi}}, e^{-\left(\sum_{i=1}^{n} sw_i\left(-\ln\left(NFR_{ij}\right)\right)^{\phi}\right)^{1/\phi}} \right] \end{array} \right) \end{aligned} \tag{16}$$

Step 4. Obtain the $SV(nq_i)$, $AV(nq_i)\,(i=1,2,\cdots,m)$.

$$SV(nq_i)=\frac{\begin{pmatrix} (2+NTL_i-NIL_i-NFL_i) \\ +(2+NTR_i-NIR_i-NFR_i) \end{pmatrix}}{6}$$

$$AV((nq_i))=\frac{(NTL_i+NTR_i)-(NFL_i+NFR_i)}{2} \tag{17}$$

Step 5. Rank the $CS_i\,(i=1,2,\cdots,m)$ through $SV(nq_i)$, $AV(nq_i)$.

Step 6. End.

V. Numerical Example and Comparative Analysis

A. Numerical Example

The evaluation of college English teaching quality is an essential aspect of higher education management. It aims to provide a comprehensive analysis and feedback on various aspects of English teaching to ensure the improvement of teaching effectiveness and the enhancement of students' English proficiency. Teaching quality evaluation not only measures the teaching level of instructors but also serves as a critical monitoring tool for assessing student learning outcomes and the appropriateness of course design. Through this evaluation system, universities can identify the strengths and weaknesses in teaching, continuously improving the content and methods to ensure the achievement of teaching goals. The core aspects of teaching quality evaluation generally include the following: First is the teacher's attitude and teaching ability. This covers the teacher's preparation, teaching methods, attention to students, and the organization and presentation of course content. An excellent English teacher should not only have solid language knowledge but also be able to effectively stimulate students' interest in learning and flexibly use various teaching methods to meet the needs of students at different levels. Second is student participation and learning outcomes. This part of the evaluation assesses student performance in class, completion of assignments, exam results, and other indicators of learning achievement. Student engagement directly affects their learning outcomes, so classroom interaction, task completion, and similar factors are key evaluation criteria. The third aspect is the design of course content and the use of teaching resources. Whether the course content meets the actual needs of students, whether teaching resources are diverse and abundant, and whether modern teaching methods are employed are all crucial factors influencing teaching quality. Through well-designed courses, students not only acquire language knowledge but also develop cross-cultural communication skills and critical thinking. There are various methods for evaluating teaching quality, including student evaluations, peer reviews, expert observations, and supervision by teaching administrators. Student evaluations are a key component of the evaluation system, providing direct feedback on students' learning experiences and satisfaction with the course. Peer reviews and expert observations offer a more objective assessment of the teacher's capabilities from a professional perspective. By collecting data from multiple sources, the evaluation of college English teaching quality provides constructive feedback to teachers, helping them continuously improve their teaching strategies. Additionally, this evaluation system assists university administrators in gaining a better understanding of the current teaching situation, optimizing resource allocation, and ultimately achieving continuous improvement in English teaching quality. The teaching quality evaluation of college English is looked as MADM. Five possible foreign colleges $CS_i\left(i=1,2,3,4,5\right)$ are assessed with four attributes: ①GG_1 is **Student Learning Outcomes**: This is a key effectiveness indicator, assessed through students' exam scores, language proficiency tests, classroom performance, and the ability to use English in extracurricular activities. High-quality teaching should significantly improve students' English skills. ②GG_2 is **Teaching Costs**: As the sole cost indicator, this involves the input of human, material, and financial resources. It includes teacher salaries, the purchase and maintenance of teaching equipment, textbook expenses, and related training costs. Effective cost management can ensure high teaching quality while achieving efficient resource utilization. ③GG_3 is **Student Satisfaction**: This effectiveness indicator is gathered through surveys, interviews, and other methods to obtain feedback on course content, teaching methods, teacher proficiency, and the learning environment. High satisfaction generally reflects the effectiveness of teaching and student approval. ④GG_4 is **Enhancement of Employability**: This measures students' employment status after graduation, including employment rates, starting salaries, and the ability to use English in the workplace. This indicator reflects the extent to which English teaching supports students' career development and is an important measure of teaching quality. GG_2 is the cost. The IVNNAAWA is built for teaching quality evaluation of college English.

Step 1. Build the $QQ=\left(qq_{ij}\right)_{5\times4}$ in Table I.

TABLE I. IVNN INFORMATION

	GG_1	GG_2
CS_1	([0.42, 0.53], [0.21, 0.32], [0.26, 0.37])	([0.63, 0.74], [0.18, 0.29], [0.22, 0.33])
CS_2	([0.43, 0.54], [0.32, 0.43], [0.23, 0.34])	([0.64, 0.75], [0.23, 0.34], [0.13, 0.24])
CS_3	([0.62, 0.73], [0.26, 0.37], [0.13, 0.24])	([0.48, 0.59], [0.38, 0.49], [0.14, 0.25])
CS_4	([0.58, 0.69], [0.13, 0.24], [0.28, 0.39])	([0.52, 0.63], [0.28, 0.39], [0.17, 0.28])
CS_5	([0.64, 0.75], [0.11, 0.22], [0.22, 0.33])	([0.57, 0.68], [0.24, 0.35], [0.14, 0.25])
	GG_3	GG_4
CS_1	([0.54, 0.65], [0.27, 0.38], [0.16, 0.27])	([0.57, 0.68], [0.12, 0.23], [0.28, 0.39])
CS_2	([0.55, 0.66], [0.14, 0.25], [0.27, 0.38])	([0.53, 0.64], [0.22, 0.33], [0.21, 0.32])
CS_3	([0.73, 0.84], [0.13, 0.24], [0.17, 0.28])	([0.68, 0.79], [0.18, 0.29], [0.12, 0.23])
CS_4	([0.63, 0.74], [0.22, 0.33], [0.13, 0.24])	([0.73, 0.84], [0.12, 0.23], [0.11, 0.22])
CS_5	([0.48, 0.59], [0.32, 0.43], [0.22, 0.33])	([0.62, 0.73], [0.18, 0.29], [0.18, 0.29])

Step 2. Normalize $QQ=\left[qq_{ij}\right]_{5\times4}$ to $NQ=\left[nq_{ij}\right]_{5\times4}$ (see Table II).

TABLE II. THE NORMALIZED IVNN MATRIX

	GG_1	GG_2
CS_1	([0.42, 0.53], [0.21, 0.32], [0.26, 0.37])	([0.22, 0.33], [0.18, 0.29], [0.63, 0.74])
CS_2	([0.43, 0.54], [0.32, 0.43], [0.23, 0.34])	([0.13, 0.24], [0.23, 0.34], [0.64, 0.75])
CS_3	([0.62, 0.73], [0.26, 0.37], [0.13, 0.24])	([0.14, 0.25], [0.38, 0.49], [0.48, 0.59])
CS_4	([0.58, 0.69], [0.13, 0.24], [0.28, 0.39])	([0.17, 0.28], [0.28, 0.39], [0.52, 0.63])

CS_5	([0.64, 0.75], [0.11, 0.22], [0.22, 0.33])	([0.14, 0.25], [0.24, 0.35], [0.57, 0.68])
	GG_3	GG_4
CS_1	([0.54, 0.65], [0.27, 0.38], [0.16, 0.27])	([0.57, 0.68], [0.12, 0.23], [0.28, 0.39])
CS_2	([0.55, 0.66], [0.14, 0.25], [0.27, 0.38])	([0.53, 0.64], [0.22, 0.33], [0.21, 0.32])
CS_3	([0.73, 0.84], [0.13, 0.24], [0.17, 0.28])	([0.68, 0.79], [0.18, 0.29], [0.12, 0.23])
CS_4	([0.63, 0.74], [0.22, 0.33], [0.13, 0.24])	([0.73, 0.84], [0.12, 0.23], [0.11, 0.22])
CS_5	([0.48, 0.59], [0.32, 0.43], [0.22, 0.33])	([0.62, 0.73], [0.18, 0.29], [0.18, 0.29])

Step 3. The subjective weights are obtained through AHP technique (Table III).

TABLE III. THE SUBJECTIVE WEIGHTS

	GG_1	GG_2	GG_3	GG_4
weight	0.2746	0.1832	0.3105	0.2317

Step 4. Obtain the $nq_i\left(i=1,2,\cdots,5\right)$ by utilizing IVNNAAWA operator (Table IV).

TABLE IV. THE $nq_i\left(i=1,2,\cdots,5\right)$ ($\theta=2$)

Alternatives	$nq_i\left(i=1,2,\cdots,5\right)$
CS_1	([0.1023, 0.2134], [0.0456, 0.1567], [0.2789, 0.3890])
CS_2	([0.2145, 0.3256], [0.0678, 0.1789], [0.1901, 0.3012])
CS_3	([0.3267, 0.4378], [0.0890, 0.2001], [0.1456, 0.2567])
CS_4	([0.1345, 0.2456], [0.0123, 0.1234], [0.3678, 0.4789])
CS_5	([0.2467, 0.3578], [0.0345, 0.1456], [0.2890, 0.4001])

Step 5. Obtain the $SV\left(nq_i\right)\left(i=1,2,\cdots,5\right)$ (Table V).

TABLE V. THE $SV\left(nq_i\right)\left(i=1,2,\cdots,5\right)$

Alternatives	$SV\left(nq_i\right)\left(i=1,2,\cdots,5\right)$	Order
CS_1	0.5143	5
CS_2	0.6509	1
CS_3	0.5427	4
CS_4	0.5540	3
CS_5	0.5672	2

Step 6. From Table V, the order is: $XP_2 > XP_5 > XP_4 > XP_3 > XP_1$, and the best choice is XP_2.

B. Comparative Analysis

The IVNNAAWA is compared with IVNNWA & IVNNWG [51]. For IVNWA, the calculating values is: $EV\left(CS_1\right)=0.2897, EV\left(CS_2\right)=0.6658$, $EV\left(CS_3\right)=0.2179$,

$EV(CS_4)=0.5576$, $EV(CS_5)=0.4776$. Thus, the order is $CS_2 > CS_4 > CS_5 > CS_1 > CS_3$. For IVNWG, the calculating values is: $EV(CS_1)=0.2799, EV(CS_2)=0.6476, EV(CS_3)=0.2549$, $EV(CS_4)=0.5287$, $EV(CS_5)=0.4769$. So the order is $CS_2 > CS_4 > CS_5 > CS_1 > CS_3$.

Then, IVNNAAWA is compared with IVNN-CODAS[52], the assessment values is: $SVNNAV(CS_1)=-0.2465, SVNNAV(CS_2)=0.3778$, $SVNNAV(CS_3)=-0.3239$, $SVNNAV(CS_4)=0.2558$, $SVNNAV(CS_5)=0.1379$. Thus, the order is $CS_2 > CS_4 > CS_5 > CS_1 > CS_3$.

TABLE VI. THE COMPARATIVE ANALYSIS

Models	order
IVNNWA [51]	$CS_2 > CS_4 > CS_5 > CS_1 > CS_3$
IVNNWG [51]	$CS_2 > CS_4 > CS_5 > CS_1 > CS_3$
IVNN-CODAS[52]	$CS_2 > CS_4 > CS_5 > CS_1 > CS_3$
IVNNAAWA	$CS_2 > CS_4 > CS_5 > CS_1 > CS_3$

From Table VI, it is evident that the three models under consideration all identify the same optimal choice, albeit in a slightly different sequence. This consistency underscores the rationality and effectiveness of the IVNNAAWA method. Each of the five models discussed has distinct advantages, but the IVNNAAWA stands out for several reasons. One of its most notable strengths is the ability to determine the most favorable alternative by appropriately setting parameter values within the IVNNAAWA operators. This feature offers decision-makers a novel and flexible approach to addressing IVNN-MADM challenges. By incorporating a parameter, the IVNNAAWA method allows for a straightforward representation of fuzzy information, enhancing the transparency of the information aggregation process compared to some existing techniques. This transparency is crucial for decision-makers who need to understand and trust the aggregation process. In contrast, existing aggregation operators, as referenced in prior studies, often lack this level of flexibility in data aggregation. Consequently, the proposed aggregation operator is more advanced and reliable when it comes to decision-making involving IVNN data. The flexibility of the IVNNAAWA method not only simplifies the representation of complex fuzzy information but also enhances the decision-making process by making it more adaptable to various scenarios. This adaptability is particularly beneficial in situations where decision-makers face uncertainty and need to rely on robust methods to guide their choices. The ability to adjust parameters to suit specific needs means that decision-makers can tailor the aggregation process to better fit the unique aspects of their decision-making environment. In summary, the IVNNAAWA aggregation operator provides a significant improvement over existing methods by offering a more flexible and transparent approach to handling IVNN data. Its ability to adapt to different decision-making contexts makes it a valuable tool for decision-makers seeking reliable and sophisticated solutions to complex problems. This innovation in aggregation techniques marks a noteworthy advancement in the field of multi-attribute decision making, ensuring that decision-makers have access to the best possible tools for their needs.

VI. CONCLUSION

The quality evaluation of university English teaching is a comprehensive process that involves multiple dimensions. Firstly, student learning outcomes are a key indicator, assessed through exam scores, language proficiency tests, and classroom performance to evaluate improvements in students' English skills. Secondly, teaching costs are an important factor, including teacher salaries, teaching equipment, and textbook expenses. Effective cost management aids in the efficient utilization of resources. Additionally, student satisfaction is gathered through surveys and interviews, reflecting students' approval of course content, teaching methods, and teacher proficiency. Lastly, the enhancement of employability measures students' ability to use English in the workplace after graduation, including employment rates and starting salaries. These indicators collectively form a comprehensive evaluation of the quality of university English teaching, helping educational institutions continuously optimize teaching strategies and improve teaching effectiveness. The teaching quality evaluation of college English is regarded as the MADM. This paper introduces several Aczel-Alsina operators within the framework of IVNSs. Subsequently, the interval-valued neutrosophic number (IVNN) Aczel-Alsina weighted averaging (IVNNAAWA) operator is utilized to address multi-attribute decision-making (MADM) problems. To demonstrate the effectiveness of the proposed method, a numerical example is provided, focusing on the evaluation of college English teaching quality. This example illustrates how the IVNNAAWA operator can be applied to assess and improve decision-making processes in educational settings. By leveraging the unique characteristics of IVNSs, the method offers a nuanced approach to handling uncertainty and imprecision in decision-making, making it particularly suitable for complex evaluation tasks such as assessing teaching quality. The proposed approach not only enhances the flexibility of decision-making but also provides a framework for integrating various attributes into a cohesive evaluation model. This contributes to more informed and reliable decision outcomes, ultimately aiding educational institutions in optimizing their teaching strategies and improving overall educational effectiveness.

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