Selecting the Best Machine Learning Models for Industrial Robotics with Hesitant Bipolar Fuzzy MCDM

A Comprehensive Framework for Evaluating Industrial Robotics

Chan Gu[1], Bo Tang[2,*]
School of Electrical and Control Engineering, Shaanxi University of Science and Technology, Xi'an 710021, China[1, 2]

Abstract*—Machine learning models (MLMs) are used in industry to automate complicated activities, minimize human error, and improve decision-making by evaluating large volumes of data in real time. To managing inventory and quality control in the apparel and auto industries, they provide predictive capabilities such as predicting equipment breakdowns, maintenance and detecting fraud in the finance sector and the major key advantages include cost reduction, higher productivity, better product quality, and tailored client experiences. MLM helps the industries to reduce downtime, prevent errors, and gain a competitive edge through data-driven strategies and processing massive volumes of data in real time. So, there is a need to select the best MLMs for industrial robotics and by considering it, this paper addresses this problem as multiple criteria decision-making (MCDM) by exploiting hesitant bipolar fuzzy information, which takes into account both hesitation and bipolarity in decision-maker preferences. This paper introduced the new aggregation operators (AO) based on geometric and arithmetic procedures to efficiently aggregate the data including the hesitant bipolar fuzzy weighted geometric operator (HBFWGO), which is appropriate for multiplicative relationships, and the hesitant bipolar fuzzy weighted average operator (HBFWAO), which gives weighted importance to qualities. Further, the dual operators including the dual hesitant bipolar fuzzy weighted geometric operator (DHBFWGO) and the dual hesitant bipolar fuzzy weighted average operator (DHBFWAO) have been presented that are further applied to create novel strategies for resolving MCDM issues and offering a methodical manner to assess and combine features. Moreover, the example of selecting the optimal MLMs to show the robustness and efficiency of the suggested methodology has been presented which illustrates the applicability and strength of the proposed methodology in actual decision-making situations.

Keywords—Machine Learning Model (MLM); Hesitant Bipolar Fuzzy Set (HBFS); Dual Hesitant Bipolar Fuzzy Set (DHBFS); Hesitant Bipolar Fuzzy Aggregation Operators (HBFAO); Dual Hesitant Bipolar Fuzzy Aggregation Operators (DHBFAO); Multi-Criteria Decision-Making (MCDM)

I. INTRODUCTION

The development and evaluation of the highest quality (MLMs) [1] for industrial robotics is an important step toward the improvement of current automation and decision-making systems. Industrial robotics [2] has an enormous effect on manufacturing environments by automating challenging activities, enhancing precision, and minimizing human error. As companies transition to smart manufacturing and Industry 4.0 [3], the demand for advanced robotic systems that can intelligently adapt to dynamic and uncertain surroundings grows. Choosing the best MLM is a challenging process as it involves balancing various criteria [4]. In response, MCDM has grown to be a potent technique for handling complexity and of controlling uncertainty. This decision-making strategy is based on fuzzy set theory (FS) [5], which provides a flexible framework for addressing uncertainty. Zadeh developed the FS notion in 1965 to solve the boundaries of conventional set theory and binary logic [6], in which an individual either fully belongs to a set or does not. Many situations in the real world are not black and white, but rather exist in shades of gray, making it difficult to establish clear boundaries. FS permits the depiction of uncertainty by assigning degrees of membership to elements in a set. This significant development established the way for subsequent advances in decision-making under uncertainty, including the introduction of more advanced ideas like hesitant fuzzy sets (HFS) [7], and then bipolar fuzzy sets (BFS) [8].

FS was created to address difficulties where traditional true/false reasoning was insufficient. Since the traditional set theory implies that an element is either a member of a set or not, which is useful for issues having binary solutions. However, in many practical contexts such as robotics, control systems, and decision-making, real-world data is frequently unclear or missing. To address this, Zadeh's FS developed the concept of partial membership, which allows an element to belong to a set to some extent, represented by values ranging from $[0, 1]$. But, as the research developed, it became clear that the concept of membership alone was not necessarily adequate for modeling all types of uncertainty. This resulted in the creation of increasingly advanced extensions of FS. By considering it, Atanassov presented the concept of an intuitionistic fuzzy set (IFS) [9] in 1986, which expanded Zadeh's FS by including both a membership and non-membership function. This set offers an additional structure for dealing with uncertainty by considering an element's degree of non-membership in addition to its membership. The IFS was especially beneficial when decision-makers needed to indicate hesitancy about whether an element should be included in a set. Later on, researchers such as Alcantud [10] constructed on aggregation operator (AO) for IFS that allows for more flexible ways to aggregate and handle IFS, Ali et al. [11] utilized it for material selection, and Ahn et al. [12] utilized this framework for medical diagnosis. This

*Corresponding author.

approach improved the ability to combine data from numerous sources under uncertainty and it into an effective framework for MCDM which is utilized by various researchers for evaluating decision-making problems. Furthermore, for even more freedom in expressing uncertainty, Torra presented the HFS [13]. In an HFS, an element's membership is represented by a set of alternative values rather than a single value, indicating uncertainty in choosing membership. Additionally, he investigated the connection between IFS and HFS, demonstrating that an IFS is fundamentally contained within the envelope of an HFS. Xia and Xu [14] expanded on previous research on HFS by inventing aggregation algorithms specifically intended for hesitant fuzzy information and applying them to decision-making situations. The bipolar fuzzy set (BFS) [15] has emerged as a potential solution to managing uncertainty in MCDM situations and utilized two values to characterize an object i.e. the positive membership degree and the negative membership degree. Unlike IFS, membership degrees in BFS range from $[-1,1]$. BFS has been widely applied in various domains, such as bipolar fuzzy heat equation [16], traditional Chinese medicine [17], bipolar cognitive mapping [18], decision analysis and organizational modeling [19], biosystem regulation, and graph theory [20]. Moreover, MLMs in industrial robots is selected based on a variety of performance parameters, including speed, accuracy, resilience, and computing efficiency. So, a structured strategy for decision-making due to the abundance of MLMs that are tuned for distinct tasks, such as object recognition, navigation, or manipulation. When evaluating the MLMs, the HFS approach allows decision-makers to express their uncertainty that a model may perform well under some conditions but poorly under others, raising questions about its overall applicability. In such circumstances, HFS offers the ability to depict hesitation and BFS approach extends this concept by allowing decision-makers to consider both the positive and negative elements of any MLM which is beneficial in industrial robots, where the trade-offs between speed and precision, or adaptability and computational cost, must be carefully balanced. By integrating HFS and BFS into the decision-making process, it assists the decision-maker to systematically evaluating various factors while balancing competing aims to get the optimum ML model. Robotic systems can be made far more capable, efficient, and adaptable by choosing the best ML model for industrial robots. So, in the past FS, HFS, and BFS have all been adopted to model uncertainty in decision making, but these methodologies are still limited in terms of dealing with complex and conflicting criteria especially when working in changing arenas like industrial robotic systems. So, this paper aims to introduce the HBFWAO and HBFWGO operators that are more flexible and accurate than the existing ones hence fixing the drawback and improving support in decision making on the choice of ML models for industrial applications. Some of the primary benefits are:

- With the help of MLMs, data is optimized and repeat processes are undertaken without errors in performance to learn from past experiences.
- Traditional models are designed to perform only a few specific tasks since they follow prewritten instructions and cannot adapt to changes but ML on the other hand facilitates real-time data analysis, and enhancing efficiency in performing strategies.
- Through the integration of MLMs, robots can now work together with a human teammate and accomplish diverse tasks with a great level of efficiency.

However, the Industrial robotics faced numerous major obstacles that limited their effectiveness and flexibility prior to the inclusion of ML model, including:

- Before the advent of ML, programming and maintaining robots was an expensive affair, and supervision made them inefficient and very impractical for the modern industries.
- Conventional robots were confined to a certain set of tasks and lacked the quality of adaptability and therefore required expensive changes of programming if there were new tasks or new situations emerged.

A. *Motivation of the Research*

When it comes to making decisions that are quite complex in nature, the standard fuzzy sets have a lot of difficulties in capturing the preferences especially for those that have hesitations and bipolar judgments. This is addressed by HBFS but new aggregation operators (AOs) must be introduced to deal with the complexity of the existing data sets effectively thus providing the motivation of this research in enhancing MCDM processes.

- In practice, the making of decisions tends to be marred with uncertainty and ambivalent views, for instance in industrial, financial, resource allocation scopes, etc. The HBFS framework depicts this uncertainty but does not apply well in MCDM without sophisticated aggregation methods.
- In this study, the HBFWAO and HBFWGO are introduced in order to aggregate hesitant bipolar fuzzy information for more effective decision-making outputs.
- The research extends these operators to develop flexible decision-making techniques for HBFS and DHBFS, useful in industrial applications like selecting the best ML model for robots.

B. *Organization of the Study*

For evaluating the MLMs for robot selection, this paper is organized as follows: Section I gives a brief introduction to MLMs and their evaluation as a decision-making problem. Then, the fundamental notions of FS, HFS, BFS, and its operational laws are defined in Section II. Section III proposed the HBF set and DHBF set which are then followed by AOs including averaging and geometric operators. In Section IV, the methodology was proposed by utilizing these AOs to address MCDM concerns and then utilized in evaluating the real-world decision-making problem. Section V provides a comparison between the prior studies and the proposed study and highlights the effectiveness of the proposed operator. In the end, Section VI concludes the whole discussion by defining its limitations and future direction.

II. Preliminaries

This section contains a prior defined definition of FS, BFS, HFS, and its operational laws for the understanding of the readers.

Definition 1 [5]: Let U be a fixed and non-empty set. Then, the FS $\mathcal{B}$ on U is defined as:

which is determined by a membership function MF $\mu_{\mathcal{B}}: \mu_{\mathcal{B}} \in [0,1]$

$$\mathcal{B} = \left\{\left(\sigma_j, \mu_{\mathcal{B}}(\sigma_j)\right): \sigma_j \in U\right\} \quad (1)$$

Definition 2 [15]: Let U be a fixed and non-empty set. Then, the BFSs $\mathcal{B}$ on U is defined as:

$$\mathcal{B} = \{< \sigma_j(\mu_{\mathcal{B}}^{+}(\sigma_j), \nu_{\mathcal{B}}^{-}(\sigma_j) > |\sigma_j \in U)\} \quad (1)$$

The positive MF function, denoted as $\mu_{\mathcal{B}}^{+}(\sigma_j): U \to [0,1]$, represents the degree to which an element σ_j satisfies the property associated with a bipolar fuzzy set (BFS) $\mathcal{B}$. Conversely, the negative membership degree function, $\nu_{\mathcal{B}}^{-}(\sigma_j)$: $U \to [0,1]$, indicates the degree to which an element σ_j meets an implicit counter property related to the same BFS $\mathcal{B}$. For any σ_j in the set U, the combination of these functions, expressed as $\mathfrak{b}(\sigma_j) = \left(\mu^{+}(\sigma_j), \nu^{-}(\sigma_j)\right)$, is referred to as a bipolar fuzzy number (BFN), represented by $\mathfrak{b} = (\mu^{+}, \nu^{-})$, adhering to the conditions $0 \leq \mu^{+} \leq 1$ and $-1 \leq \nu^{-} \leq 0$.

Definition 3 [15]: The following is a description of the basic operations on BFNs.

- $a_1 \oplus a_2 = (\mu_1^{+} + \mu_2^{+} - \mu_1^{+}\mu_2^{+}, -|\nu_1^{-}||\nu_2^{-}|)$
- $a_1 \otimes a_2 = (\mu_1^{+}\mu_2^{+}, \nu_1^{-} + \nu_2^{-} - \nu_1^{-}\nu_2^{-})$
- $\gamma a = (1 - (1 - \mu^{+})^{\gamma}, -|\nu^{-}|^{\gamma}), \gamma > 0$
- $(a)^{\gamma} = ((\mu^{+})^{V}, -1 + |1 + \nu^{-}|^{\gamma}), \gamma > 0$
- $a^{c} = (1 - \mu^{+}, |\nu^{-}| - 1)$
- $a_1 \subseteq a_2, \Leftrightarrow \mu_1^{+} \leq \mu_2^{+}$ and $\nu_1^{-} \geq \nu_2^{-}$
- $a_1 \cup a_2 = (max\{\mu_1^{+}, \mu_2^{+}\}, min\{\nu_1^{-}, \nu_2^{-}\})$.
- $a_1 \cap a_2 = (min\{\mu_1^{+}, \mu_2^{+}\}, max\{\nu_1^{-}, \nu_2^{-}\})$;

Theorem 1 [15]: Let $a_1 = (\mu_1^{+}, \nu_1^{-})$ and $a_2 = (\mu_2^{+}, \nu_2^{-})$ represents for two BFNs, where $\gamma, \gamma_1, \gamma_2 > 0$. In this context, μ_1^{+} and μ_2^{+} represent the positive membership functions, while ν_1^{-} and ν_2^{-} denotes the negative membership functions. Under these conditions, the following operations can be applied to a_1 and a_2.

- $a_1 \oplus a_2 = a_2 \oplus a_1$
- $a_1 \otimes a_2 = a_2 \otimes a_1$
- $\gamma(a_1 \oplus a_2) = \gamma a_1 \oplus \gamma a_2$
- $(a_1 \otimes a_2)^{\gamma} = (a_1)^{\gamma} \otimes (a_2)^{\gamma}$
- $\gamma_1 a_1 \oplus \gamma_2 a_1 = (\gamma_1 + \gamma_2) a_1$
- $(a_1)^{\gamma_1} \otimes (a_1)^{\gamma_2} = (a_1)^{(\gamma_1 + \gamma_2)}$
- $((a_1)^{\gamma_1})^{\gamma_2} = (a_1)^{\gamma_1 \gamma_2}$

III. Hesitant Bipolar Fuzzy Aggregation Operators (HBFAO)

In this part, a set of innovative and specialized aggregation procedures designed exclusively for HBFAO. These operators are developed to effectively integrate and process HBFAO, boosting their utility in various decision-making and analysis settings. Additionally, the important aspects of these operators by applying fundamental operations have been analyzed which allowing us to obtain deeper insights into their behavior and performance and provide more robust methods for managing unpredictable and bipolar data.

Definition 4: Let U be a fixed and non-empty set. Then, the HBFS $\mathcal{B}^{¤}$ on U is defined as:

$$\mathcal{B}^{¤} = \left\{< \sigma_j, \mathcal{H}_{\mathcal{B}^{¤}(\sigma_j)} > |\sigma_j \in U\right\} \quad (2)$$

where, $\mathcal{H}_{\mathcal{B}^{¤}(\sigma_j)}$ is a collection of BFNs in $\mathcal{B}$. Specifically,

$$\mathcal{H}_{\mathcal{B}^{¤}(\sigma_j)} = \cup_{\left(\mu_{\mathcal{B}^{¤}}^{+}(\sigma_j), \nu_{\mathcal{B}^{¤}}^{-}(\sigma_j)\right) \in \mathcal{H}_{\mathcal{B}^{¤}(\sigma_j)}} \left(\mu_{\mathcal{B}^{¤}}^{+}(\sigma_j), \nu_{\mathcal{B}^{¤}}^{-}(\sigma_j)\right)$$

Where $\mu_{\mathcal{B}^{¤}}^{+}(\sigma_j)$ represents the positive MF, indicating the degree to which an σ_j satisfies a given property related to HBFS $\mathcal{B}^{¤}$ and $\nu_{\mathcal{B}^{¤}}^{-}(\sigma_j)$ represents the negative MF which indicates the degree to which σ_j satisfies an opposing or counter-property related to the HBFS $\mathcal{B}^{¤}$.

These membership functions are bounded by the following conditions: $0 \leq \mu_{\mathcal{B}^{¤}}^{+}(\sigma_j) \leq 1$ and $-1 \leq \nu_{\mathcal{B}^{¤}}^{-}(\sigma_j) \leq 0$ for every $\sigma_j \in U$.

For ease of reference, the pair $h(\sigma_j) = \left\{\left(\mu^{+}(\sigma_j), \nu^{-}(\sigma_j)\right)\right\}$ is called a HBFN, denoted as $h = (\mu^{+}, \nu^{-})$, with the constraints: $0 \leq \alpha^{+} \leq 1$ and $-1 \leq \beta^{-} \leq 0$, $(\alpha^{+}, \beta^{-}) \in (\mu^{+}, \nu^{-})$.

To compare HBFNs, the following comparison laws have been used which give a systematic method for evaluating and distinguishing between different HBFNs and allow us to compare their relative strengths in terms of positive and negative membership functions.

Definition 5: Let $h_i = (\mu_i^{+}, \nu_i^{-})$ $(i = 1,2)$ be any two HBFNs. Then,

$$\mathfrak{s}(h_i) = \frac{1}{\# h_i} \sum_{i=1}^{\# h_i} \frac{1 + \alpha^{+} + \beta^{-}}{2}$$

$\mathfrak{s}(h_i)$ represents the score function of $h_i = (\mu_i^{+}, \nu_i^{-})$.

Definition 6: Let $h_i = (\mu_i^{+}, \nu_i^{-})$ $(i = 1,2)$ be any two HBFNs. The accuracy function of $h_i = (\mu_i^{+}, \nu_i^{-})$,

$$\mathfrak{s}^{*}(h_i) = \frac{1}{\# h_i} \sum_{i=1}^{\# h} \frac{\alpha^{+} - \beta^{-}}{2}$$

where $\# h_i$ is the number of elements in h_i.

- If $\mathfrak{s}(h_1) > \mathfrak{s}(h_2)$, then h_1 is considered superior to h_2, which is represent as $h_1 \succ h_2$;
- If $\mathfrak{s}^*(h_1) = \mathfrak{s}^*(h_2)$, then, h_1 is equal to h_2, denoted by $h_1 \sim h_2$;
- If $\mathfrak{s}^*(h_1) > \mathfrak{s}^*(h_2)$, then h_1 is considered superior to h_2, which represent as $h_1 \succ h_2$.

The following operational laws will enable to combine the HBFNs in a variety of ways, making comparisons and analyses easier within the context of HBFS theory and improve the understanding of the links and interactions between various HBFNs.

- $h^\gamma = \cup_{(\alpha^+,\beta^-)\in(\mu^+,v^-)} \left\{ \begin{matrix} (\alpha^+)^\gamma, \\ -1 + |1+\beta^-|^\gamma \end{matrix} \right\}, \gamma > 0;$
- $\gamma h = \cup_{(\alpha^+,\beta^-)\in(\mu^+,v^-)} \left\{ \begin{matrix} 1-(1-\alpha^+)^\gamma, \\ |\beta^-|^\gamma \end{matrix} \right\}, \gamma > 0;$
- $h_1 \oplus h_2 = \cup_{(\alpha_1^+,\beta_1^-)\in(\mu_1^+,v_1^-),(\alpha_2^+,\beta_2^-)\in(\mu_2^+,v_2^-)} \left\{ \begin{matrix} \alpha_1^+ + \alpha_2^+ - \alpha_1^+\alpha_2^+, \\ -|\beta_1^-||\beta_2^-| \end{matrix} \right\}$
- $h_1 \otimes h_2 = \cup_{(\alpha_1^+,\beta_1^-)\in(\mu_1^+,v_1^-),(\alpha_2^+,\beta_2^-)\in(\mu_2^+,v_2^-)} \left\{ \begin{matrix} \alpha_1^+\alpha_2^+, \\ \beta_1^- + \beta_2^- - \beta_1^-\beta_1^- \end{matrix} \right\}$

A. Hesitant Bipolar Fuzzy Weighted Averaging Operators (HBFWAO)

This part defines the HBFAO, which allow us to combine these HBFV in an organized manner for further analysis and decision making.

Definition 7: Let $h_j = (\mu_j^+, v_j^-)$ $(j = 1,2,3,\ldots,n)$ represent an entire collection of HBFV. The HBFWAO is defined as:

$$HBFWAO_{w}(h_1 h_2,\ldots,h_n) = \sum_{j=1}^{n}(w_j h_j) \tag{3}$$

where, $w = (w_1, w_1, \ldots, w_1)^t$ is the weight vector for each h_j for $j = 1,2,3,\ldots,n$, with $w_j > 0$ and $\sum_{j=1}^{n}(w_j) = 1$. This operator combines the HBFVs by applying their respective weights.

Theorem 2: The HBFWAO provides a HBFV with

$$HBFWAO_{w}(h_1 h_2,\ldots,h_n) = \sum_{j=1}^{n}(w_j h_j)$$

$$HBFWAO_{w}(h_1 h_2,\ldots,h_n) = \cup_{(\alpha_j^+,\beta_j^-)\in(\mu_j^+,v_j^-)} \left\{ \begin{matrix} 1 - \prod_{j=1}^{n}(1-\alpha_j^+)^{w_j}, \\ -\prod_{j=1}^{n}|\beta_j^-|^{w_j} \end{matrix} \right\} \tag{4}$$

B. Hesitant Bipolar Fuzzy Weighted Geometric Operators (HBFWGO)

This section introduced the hesitant bipolar fuzzy geometric operators (HBFGO) by combining hesitant fuzzy and bipolar fuzzy geometric mean principles. These operators are intended to successfully combine HBFNs by capturing the multiplicative relationships inherent in the dataset. This method not only improves the aggregation process, but it also assures that the output values better reflect the underlying interactions between the components.

Definition 8: The HBFWGO is defined as:

$$HBFWGO_{w}(h_1 h_2,\ldots,h_n) = \sum_{j=1}^{n}(h_j)^{w_j} \tag{5}$$

where, $w = (w_1, w_1, \ldots, w_1)^t$ is the weight vector for each h_j for $j = 1,2,3,\ldots,n$, with $w_j > 0$ and $\sum_{j=1}^{n}(w_j) = 1$. This operator combines the HBFV by applying their respective weights.

Utilizing the established definition and mathematical induction methods, the validity of the following theorem can be demonstrate as;

Theorem 3: The HBFWGO provides a HBFV, and

$$HBFWGO_{w}(h_1 h_2,\ldots,h_n) = \sum_{j=1}^{n}(h_j)^{w_j}$$

$$HBFWGO_{w}(h_1 h_2,\ldots,h_n) = \cup_{(\alpha_j^+,\beta_j^-)\in(\mu_j^+,v_j^-)} \left\{ \begin{matrix} \prod_{j=1}^{n}(\alpha_j^+)^{w_j}, \\ -1 + \prod_{j=1}^{n}(1+\beta_j^-)^{w_j} \end{matrix} \right\} \tag{6}$$

where, $w = (w_1, w_1, \ldots, w_1)^t$ is the weight vector for each h_j for $j = 1,2,3,\ldots,n$, with $w_j > 0$ and $\sum_{j=1}^{n}(w_j) = 1$.

C. Dual Hesitant Bipolar Fuzzy Aggregation Operators (DHBFAO)

The Dual hesitant bipolar fuzzy AOs (DHBFAO) combine dual hesitant and bipolar fuzzy sets to deal with uncertainty, hesitation, and both positive and negative information. They are used to combine conflicting or uncertain evidence in decision-making, hence improving analysis in complicated, confusing situations.

Definition 9: Let $\mathfrak{h}_j = (\mu_j^+, v_j^-)$ $(j = 1,2,3,\ldots,n)$ represent an entire collection of dual hesitant bipolar fuzzy values (DHBFV). Then, the DHBFS $\mathcal{B}^*$ on U is defined as:

$$\mathcal{B}^* = \left\{ < \sigma_j, \left(\mu^+{}_{(\sigma_j)}, v^-{}_{(\sigma_j)}\right) > | \sigma_j \in U \right\} \tag{7}$$

where: positive membership function $\mu^+_{\mathcal{B}^*(\sigma_j)}: U \to [0,1]$ denotes the possible satisfaction function of an element σ_j with respect to the property corresponding to DHBFS $\mathcal{B}^*$ and the negative membership function $v^-_{\mathcal{B}^*(\sigma_j)}: U \to [0,1]$ denotes the possible satisfaction function of an element σ_j with respect to some implicit counter property corresponding to $\mathcal{B}^*$. For each $\sigma_j \in U$, the following conditions hold:

$$0 \leq \alpha^+ \leq 1, -1 \leq \beta^- \leq 0$$

where, $\alpha^+ \in \mu^+_{(\sigma_j)}$, $\beta^- \in v^-_{(\sigma_j)}$, and $\alpha^{max} \in \mu^+_{(\sigma_j)} = \cup_{\alpha^+ \in \mu^+_{(\sigma_j)}} max\{\alpha^+\}$, $\beta^{max} \in v^-_{(\sigma_j)} = \cup_{\beta^- \in v^-_{(\sigma_j)}} max\{\beta^-\}$ for all $\sigma_j \in U$. To make things easier, the pair $\mathcal{B}^*(\sigma_j) = \left(\mu^+_{(\sigma_j)}, v^-_{(\sigma_j)}\right)$ is called dual hesitant bipolar fuzzy values (DHBFV) denoted by $\mathcal{B}^*(\sigma_j) = (\mu^+, v^-)$.

Definition 10: The dual hesitant bipolar fuzzy weighted aggregation operator (DHBFWAO) is defined as:

$$DHBFWAO_{w}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n) = \sum_{j=1}^{n} \left(w_j \mathfrak{h}_j\right) \tag{8}$$

where, $w = (w_1, w_1, \dots, w_1)^t$ is the weight vector for each $\mathfrak{h}_j$ for $j = 1,2,3, \dots, n$, with $w_j > 0$ and $\sum_{j=1}^{n}(w_j) = 1$. This operator combines the DHBFV by applying their respective weights.

The basic definition and principle of mathematical induction can be used to show Theorem 4. The following theorem uses the inductive reasoning and ensuring that it applies appropriately in all relevant cases.

Theorem 4: The DHBFWAO provides a hesitant bipolar fuzzy value (HBFV) with

$$DHBFWAO_{w}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n) = \sum_{j=1}^{n} \left(w_j \mathfrak{h}_j\right)$$

$$DHBFWAO_{w}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n) = \cup_{\left(\alpha_j^+ \in \mu_j^+\right), \left(\beta_j^- \in v_j^-\right)} \left\{ \begin{array}{c} \left\{1 - \prod_{j=1}^{n} \left(1 - \alpha_j^+\right)^{w_j}\right\}, \\ \left\{-\prod_{j=1}^{n} \left|\beta_j^-\right|^{w_j}\right\} \end{array} \right\} \tag{9}$$

D. Dual Hesitant Bipolar Fuzzy Geometric Operators

Dual hesitant bipolar fuzzy geometric operators (DHBFGOs) use dual hesitant and bipolar fuzzy sets to deal with uncertainty, reluctance, and both positive and negative information.

Definition 11: The dual hesitant bipolar fuzzy weighted geometric operator (DHBFWGO) is defined as:

$$DHBFWGO_{w}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n) = \sum_{j=1}^{n} \left(\mathfrak{h}_j\right)^{w_j} \tag{10}$$

where, $w = (w_1, w_1, \dots, w_1)^t$ is the weight vector for each $\mathfrak{h}_j$ for $j = 1,2,3, \dots, n$, with $w_j > 0$ and $\sum_{j=1}^{n}(w_j) = 1$. This operator combines the DHBFV by applying their respective weights.

The basic definition and principle of mathematical induction can be used to show Theorem 5. The following theorem uses the inductive reasoning and ensures that it applies appropriately in all relevant cases.

Theorem 5: The DHBFWGO provides a hesitant bipolar fuzzy value (HBFV) with

$$DHBFWGO_{w}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n) = \sum_{j=1}^{n} \left(\mathfrak{h}_j\right)^{w_j}$$

$$DHBFWGO_{w}(\mathfrak{h}_1, \mathfrak{h}_2, \dots, \mathfrak{h}_n) = \cup_{\left(\alpha_j^+ \in \mu_j^+\right), \left(\beta_j^- \in v_j^-\right)} \left\{ \begin{array}{c} \left\{\prod_{j=1}^{n} \left(\alpha_j^+\right)^{w_j}\right\}, \\ \left\{-1 + \prod_{j=1}^{n} \left(1 + \beta_j^-\right)^{w_j}\right\} \end{array} \right\} \tag{11}$$

IV. Evaluation of Best Machine Learning Models for Industrial Robotics

To evaluate the best machine learning models by applying the proposed hesitant bipolar AOs (HBAO), consider the collection of alternatives as $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_m\}$, and the collection of criteria denoted by $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_n\}$. The weight vectors for the criterias are given by $w = \{w_1, w_2, \dots, w_n\}$, where $w_j \geq 0 \ \forall \ j = 1,2, \dots, n$, and $\sum_{j=1}^{n}(w_j) = 1$. Assume $H = [h_{ij}]_{m \times n} = [(\mu_{ij}^+, v_{ij}^-)]_{m \times n}$ which represent the hesitant bipolar fuzzy decision matrix. Here μ_{ij}^+ and v_{ij}^- and represent positive and negative functions, respectively, assessed by the decision-maker for the effectiveness of alternative $\mathcal{A}_i$ meets criteria $\mathcal{C}_j$. These functions lie within ranges $\mu_{ij}^+ \in [0,1]$ and $v_{ij}^- \in [0,1]$, where $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$.

The methodology for using the HBFWAO or HBFWGO operator to solve a MCDM problem is explained below:

Step 1: To evaluate the MCDM problem, formation of decision matrix based on hesitant bipolar fuzzy environment.

Step 2: Applying the HBFWAO and HBFWG operator to process the information in matrix H. Calculate the overall values $h_i (i = 1,2, \dots, m)$ of alternative $\mathcal{A}_i$.

$$HBFWAO_{w}(h_{i1} h_{i2}, \dots, h_{in}) = \sum_{j=1}^{n} \left(w_j h_{ij}\right)$$

$$= \cup_{\left(\alpha_{ij}^+, \beta_{ij}^-\right) \in \left(\mu_{ij}^+, v_{ij}^-\right)} \left\{ \begin{array}{c} 1 - \prod_{j=1}^{n} \left(1 - \alpha_{ij}^+\right)^{w_j}, \\ -\prod_{j=1}^{n} \left|\beta_{ij}^-\right|^{w_j} \end{array} \right\} \tag{12}$$

$$HBFWGO_{w}(h_{i1}, h_{i2}, \dots, h_{in}) = \sum_{j=1}^{n} \left(h_{ij}\right)^{w_j}$$

$$=\cup_{\left(\alpha_{ij}^{+},\beta_{ij}^{-}\right)\in\left(\mu_{ij}^{+},\nu_{ij}^{-}\right)}\left\{\begin{array}{c}\prod_{j=1}^{n}\left(\alpha_{ij}^{+}\right)^{w_j},\\ -1+\prod_{j=1}^{n}\left(1+\beta_{ij}^{-}\right)^{w_j}\end{array}\right\} \quad (13)$$

Step 3: Determine the score by $\mathfrak{s}(h_i) = \frac{1}{\#h_i}\sum_{i=1}^{\#h_i}\frac{1+\alpha^{+}+\beta^{-}}{2}$, where $\mathfrak{s}(h_i)$ $(i = 1,2,\ldots,m)$.

Step 4: Rank all the alternatives $\mathcal{A}_i$ (for $i = 1,2,\ldots,m$) based on their scores $\mathfrak{s}(h_i)$ (for $i = 1,2,\ldots,m$). If two scores $\mathfrak{s}(h_i)$ and $\mathfrak{s}(h_j)$ are identical, then calculate the accuracy functions $\mathfrak{s}^*(h_i)$ and $\mathfrak{s}^*(h_j)$ to differentiate and rank alternatives $\mathcal{A}_i$ and $\mathcal{A}_j$.

Step 5: Select the most suitable alternatives based on their score values.

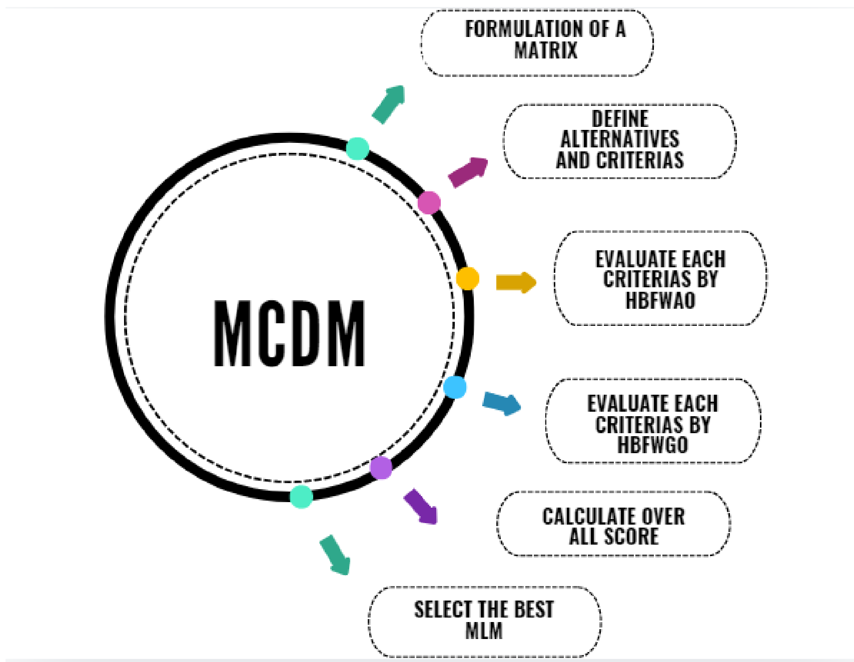

Fig. 1. Methodology of MCDM.

The pictorial representation of methodology to evaluation of best ML models is shown in Fig. 1.

A. Illustrative Example

Consider a manufacturing business that specializes in electronic device assembly. To boost their production efficiency, they decide to adopt an industrial robotic arm that can independently handle duties such as assembly and quality control. To maximize performance, however, choosing the best ML model for the robotic arm's functioning is essential. The main objective is to maximize the robotic arm's performance on the assembly line by selecting the best ML model from a pool of candidates using HBFAO. In this section, an empirical case study to assess the quality of ML model for industrial robots. The objective of the study is to evaluate which ML model, among several options that maximizes robotic performance in assembly line activities. The ML model for industrial robotic systems is shown in Fig. 2.

Fig. 2. Some ML models for industrial robotic system.

So, for the evaluation of the ML model, consider the following machine learning models (alternatives) which are evaluated based on the following criteria including accuracy of the model, training period, robustness, and interpretability which can be formulated as an MCDM problem.

The five machine learning models (alternatives) based on the following criteria are:

- $\mathcal{A}_1$ is Support Vector Machines (SVM): A high-dimensional space classification algorithm that locates the hyperplane dividing distinct classes
- $\mathcal{A}_2$ is Random Forest (RF): An ensemble technique for reliable regression and classification with insights into feature relevance that uses several decision trees.
- $\mathcal{A}_3$ is Deep Neural Networks (DNN): A flexible model with numerous layers capable of learning complicated patterns from vast datasets.
- $\mathcal{A}_4$ is Gradient Boosting Machines (GBM): An ensemble technique that creates models in a step-by-step manner, fixing mistakes in earlier models to increase accuracy.
- $\mathcal{A}_5$ is k-Nearest Neighbors (k-NN): A straightforward technique that relies on the closest training instances in the feature space to classify have been recognized.

These models will be assessed by a panel of experts who will make decisions based on the following four criteria:

- $\mathcal{C}_1$: Accuracy of the model
- $\mathcal{C}_2$: Training time required for the model.
- $\mathcal{C}_3$: Robustness of the model under various operational conditions.
- $\mathcal{C}_4$: Interpretability of the model results.

The weight values assigned by the decision makers (hypothetically) to each criterion represented by weighting vector $w = (0.20,0.10,0.30,0.40)$.

The decision-making problem i.e. evaluation of the ML model is evaluated by utilizing the above-defined methodology as follows;

Step 1: To evaluate the MCDM problem, the formation of a decision matrix from the opinion of the decision-maker based on a hesitant bipolar fuzzy environment is shown in Table I.

TABLE I. DECISION MATRIX

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{A}_1$	$\{(0.7,0.8,0.1),(-0.7,-0.4,-0.1)\}$	$\{(0.4,0.6,0.8),(-0.4,-0.3,-0.2)\}$	$\{(0.6,0.8,0.7),(-0.6,-0.3,-0.1)\}$	$\{(0.3,0.8,0.1),(-0.6,-0.4,-0.1)\}$
$\mathcal{A}_2$	$\{(0.6,0.7,0.2),(-0.6,-0.2,-0.7)\}$	$\{(0.6,0.7,0.1),(-0.6,-0.5,-0.1)\}$	$\{(0.5,0.7,0),(-0.3,-0.7,-0.1)\}$	$\{(0.5,0.7,0),(-0.3,-0.6,-0.2)\}$
$\mathcal{A}_3$	$\{(0.8,0.6,0),(-0.4,-0.3,0)\}$	$\{(0.5,0.6,0),(-0.4,-0.3,0)\}$	$\{(0.4,0.6,0.8),(-0.2,-0.5,-0.2)\}$	$\{(0.7,0.6,0.3),(-0.2,-0.4,-0.2)\}$
$\mathcal{A}_4$	$\{(0.6,0.7,0.8),(-0.4,-0.3,-0.3)\}$	$\{(0.7,0.8,0.2),(-0.2,-0.5,-0.4)\}$	$\{(0.6,0.7,0),(-0.2,-0.4,-0.1)\}$	$\{(0.6,0.7,0.8),(-0.2,-0.2,-0.4)\}$
$\mathcal{A}_5$	$\{(0.8,0.5,0),(-0.3,-0.4,-0.1)\}$	$\{(0.6,0.8,0),(-0.3,-0.5,-0.4)\}$	$\{(0.4,0.5,0.8),(-0.5,-0.4,-0.6)\}$	$\{(0.4,0.5,0),(-0.5,-0.3,-0.4)\}$

Step 2: By following above step 2, applying the HBFWAO and HBFWGO to process the information in a decision matrix H. Calculate the overall values $h_i(i = 1,2, \ldots, m)$ of each alternative $\mathcal{A}_i$ corresponds to the criteria, shown in Table II.

TABLE II. AGGREGATION OF DECISION MATRIX

	HBFWAO	**HBFWGO**
$\mathcal{A}_1$	$\{(0.5081,0.7856,0.4431),(-0.5942,-0.3565,-0.1072)\}$	$\{(0.4503,0.7773,0.2207),(-0.6067,-0.3618,-0.1105)\}$
$\mathcal{A}_2$	$\{(0.5324,0.7000,0.0537),(-0.3693,-0.4953,-0.1947)\}$	$\{(0.5281,0.7000,0.0),(-0.4082,-0.5690,-0.3108)\}$
$\mathcal{A}_3$	$\{(0.6416,0.6000,0.4650),(-0.2462,-0.3923,0)\}$	$\{(0.5877,0.6000,0.4650),(-0.2661,-0.4050,-0.1446)\}$
$\mathcal{A}_4$	$\{(0.6113,0.7119,0.6277),(-0.2297,-0.2927,-0.2491)\}$	$\{(0.6093,0.7094,0.6277),(-0.2447,-0.3183,-0.3012)\}$
$\mathcal{A}_5$	$\{(0.5375,0.5438,0.3830),(-0.4290,-0.3646,-0.3424)\}$	$\{(0.4785,0.5241,0.0),(-0.4469,-0.3734,-0.4238)\}$

Step 3: Compute the score function of the evaluated decision matrix by step 3 and display in Table III.

TABLE III. SCORE VALUE

	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$
HBFWAO	0.1679	0.1227	0.2068	0.2179	0.1328
HBFWGO	0.1369	0.0940	0.1372	0.1455	0.0758

Step 4: Rank all the ML model $\mathcal{A}_i$ (for $i = 1,2, \ldots ,5$) in according with the score function $\mathfrak{s}(h_i) = h_i(i = 1,2, \ldots ,5)$ and demonstrate in Table IV.

TABLE IV. RANKING OF THE BEST ML MODEL FOR INDUSTRIAL ROBOTIC

	Ranking Value
HBFWAO	$\mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2$
HBFWGO	$\mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_5$

Step 5: The most suitable alternatives based on their score values are shown in Table V.

TABLE V. SUITABLE ML MODEL FOR INDUSTRIAL ROBOTIC

	Ranking Value	Suitable ML (Alternative)
HBFWAO	$\mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2$	$\mathcal{A}_4$
HBFWGO	$\mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_5$	$\mathcal{A}_4$

The graphical representation of the ranking of alternatives is shown in Fig. 3(a) and Fig. 3(b).

Fig. 3. Ranking of alternatives.

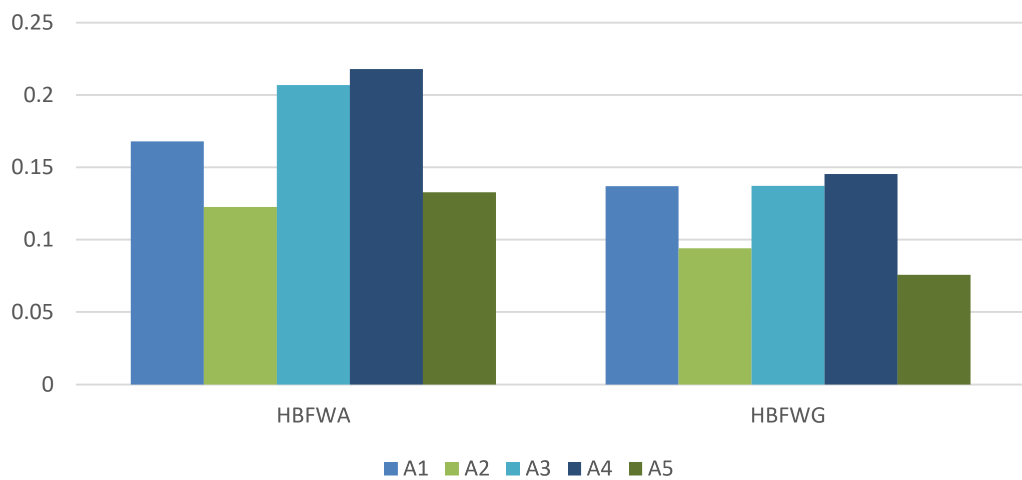

Fig. 4. Ranking of alternatives.

B. Result and Discussion

To evaluate the MLMs that are best suited for industrial robotics, the following Aos HBFWAO and HBFWGO have been employed, in this study. The proposed operators show that the Gradient Boosting Machines (GBM) model i.e. A_4 ranks higher than the other models, as shown by Table IV i.e. ranking of the ML model for industrial robotics based on these operators. By utilizing both AOs i.e. HBFWAO and HBFWGO, the results show that $\mathcal{A}_4$ is the most appropriate model, followed by $\mathcal{A}_3, \mathcal{A}_2$, and so on. The ultimate rankings in Table 5 indicate that $\mathcal{A}_4$ is the best-fit ML model based on both operators. The constancy of these operators' rating findings demonstrates their competence in decision-making, guaranteeing that the best model is chosen for industrial robotics jobs.

V. COMPARATIVE ANALYSIS

To check the validity and effectiveness of the proposed operator, this comparison study demonstrates the benefits and drawbacks of several fuzzy-based operators, from the simpler FS to the more sophisticated HBFAO. While HFS adds the capacity to model uncertainty but lacks flexibility, FS are limited in their ability to handle complicated attribute interactions. Although BFS introduces both positive and negative attribute dimensions, they are still insufficient for parametric flexibility, which hinders decision-making. Flexibility is further increased by operators like HBFWAO and HBFWGO, which consider the weighted relationships between criteria. We have compared the proposed AOs with the prior operators as shown in Table VI, which demonstrates how inadequate and ineffective the previous approaches are at handling connections between attribute values. To close this gap, we developed the HBFWAO and HBFWGO, which support optimal decision-making by thoroughly addressing these constraints.

TABLE VI. COMPARISON BETWEEN PRIOR APPROACHES AND THE PROPOSED APPROACH

Approaches	Connection Between Two Attributive Values	Relationships between Various Attributive Values	Reduced Adverse Effects	Parametric Method Increases Flexibility	Scalability	Robustness
FS [5]	✗	✗	✗	✗	✗	✗
HFS [13]	✓	✗	✗	✗	✗	✗
BFS [15]	✓	✓	✗	✓	✗	✗
HFAO [21]	✓	✓	✓	✗	✗	✗
BFAO [22]	✓	✓	✓	✓	✓	✗
HFGO [21]	✓	✓	✓	✗	✓	✗
BFGO [22]	✓	✓	✓	✓	✓	✗
HBFAWO (Proposed Operator)	✓	✓	✓	✓	✓	✓
HBFGWO (Proposed Operator)	✓	✓	✓	✓	✓	✓

So, the above Table VI demonstrate the proposed operators has the ability to highlights the relation between the various attributive values and reducing the adverse effects which make it flexible, efficient and versatile operator which assists the decision makers in making decisions. The proposed HBFWAO and HBFWGO operators are advanced and less laborious approaches to decision making under uncertainty, conflict, and incompleteness. Compared to earlier works, they afford a superior incorporation of uncertainty and flexibility with respect to complex with many criteria and objectives problems, especially those relating to industrial robotics, which have been the focus of this study.

VI. CONCLUSION

The MLMs are increasingly utilized in industrial applications to automate the complex activities, reduce human error, and enhance decision-making by analyzing large volumes of data in real-time. In this paper, a comprehensive novel approach for evaluating the best MLMs in industrial robots has been developed by utilizing the hesitant bipolar fuzzy and dual hesitant bipolar fuzzy AOs within the averaging and geometric framework. i.e. HBFWAO, HBFWGO, DHBFWAO, and DHBFGO. These operators, inspired by arithmetic and geometric operations, effectively address MCDM challenges and capturing the uncertainties associated with hesitancy and bipolarity which enabling a robust evaluation of positive and negative attributes. To demonstrate the effectiveness and robustness of proposed operator, an exemplary case study has been defined which is evaluated by utilizing the proposed decision-making algorithm. The proposed operators demonstrated their practical utility, providing precise and adaptable solutions for real-world applications in industrial robotics.

Moreover, a rigorous comparative analysis demonstrates the superiority of the proposed approach over existing methods and highlighting its robustness, accuracy, and flexibility. The parametric adaptability of the framework ensures its broad applicability across various decision-making scenarios, minimizing errors and optimizing the outcomes in complex industrial environments.

A. Limitations and Future Direction

To demonstrate the thorough evaluation and defines the balanced perspective, it is necessary to discuss the limitations of the proposed approach.

- The proposed approach offers complexity in handling a large data set, resulting in a high processing time and memory consumption.
- The proposed operators may be sensitive towards the various parameters and then improper selection may affect the accuracy and effectiveness.
- The complex nature of integrating the hesitant and bipolar environment can lead to complex situations in evaluating decision-making problems.
- Although the proposed operators show flexibility and robustness, however, it is not applicable in a highly dynamic framework and demands further modification.

So, to improve the precision and adaptability of decision-making, future research could concentrate on merging the proposed operators with sophisticated fuzzy logic systems [23], Intuitionistic fuzzy framework [24], and Pythagorean fuzzy set (PyFS) [25] framework, Neutrosophic framework [26] that may handle hesitation and more complex decision scenarios precisely. Furthermore, it can be extended by utilizing the AOs [27], [28], [29], [30] that can be useful for dealing with noisy or incomplete data. More resilient and adaptable decision-making models can be created by fusing these operators with sophisticated fuzzy techniques. Moreover, it could expand the applicability of these models beyond industrial robots to industries where uncertainty is crucial, such as healthcare, finance, and autonomous systems. Including different sectors, such as healthcare in diagnostic decision support systems, finance in risk assessment tools and autonomous systems for vehicle navigation or resource allocation, in the research activities or case studies developed would serve to indicate the extensibility of the methods suggested. This wider perspective reveals not only the usefulness of the methods in practice across different sectors, but also the desire to appeal to a larger audience. Such examples would emphasize how these types of models can be adapted to different industries yet remain internally consistent and accurate in the face of uncertainty when making decisions.

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