

Fault-Tolerant Control of Nonlinear Delayed Systems Using Lyapunov Approach: Application to a Hydraulic Process

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Abstract—Designing stabilizing controllers for delayed nonlinear systems with control constraints presents a significant challenge. This paper addresses this issue by proposing a fault-tolerant control approach for a specific class of delayed nonlinear systems with actuator faults based on Lyapunov redesign principle. Initially, an assumption is introduced to facilitate the control design for the nominal system. Then, a new control law is developed to resolve the difficulty caused by actuator failures. The proposed nonlinear controller demonstrates the ability to compensate for actuator faults. To validate its effectiveness, the method is applied to a hydraulic system.

Keywords—Delayed nonlinear system; actuator faults; delayed hydraulic process; additive fault tolerant control; redesign Lyapunov approach

I. INTRODUCTION

The primary objective of Fault Tolerant Control (FTC) is to guarantee performance and stability for systems, whether they are operating without faults or in a faulty state. Multiple approaches have been proposed to tackle this problem. FTC can be categorized into two main types: passive (P) and active (A) FTC [1]. In the domain of nonlinear control, extensive theoretical research and practical applications have been conducted for Passive Fault Tolerant Control (PFTC) and Active Fault Tolerant Control (AFTC) [2]. Active FTC is centered on ensuring stability and certain performance aspects for the post-fault model by dynamically adjusting the controller in response to the current fault [3], as detected, isolated, and estimated by the Fault Detection and Diagnosis (FDD) block [4]. A FTC employs real-time adjustment techniques for the regulators to uphold, at a minimum, the system's stability [5]. Another approach involves using a robust controller able of handling all anticipated faults, by eliminating the requirement for online control reconfiguration and an FDD block [6], PFTC methods have been introduced, predominantly grounded in robust theories. These include approaches such as linear-matrix-inequality-based methods [7], quantitative feedback theory [8], pole assignment [9], and nonlinear regulation theory [10, 11].

A. Literature Review

Many recent studies have focused on FTC applied to particularly complex nonlinear processes, such as boundary adaptive fault-tolerant control for a flexible Timoshenko [12] the proposed method ensures robust and adaptive control of a

Timoshenko flexible arm, guaranteeing stability and precision despite actuator faults, hysteresis, and disturbances, while enhancing reliability under variable conditions. Another research addresses a 2 DOF helicopter system, this study proposes an adaptive control strategy for a 2-DOF helicopter, considering actuator faults and an unknown dead zone. A neural network and a quantizer are used to model the uncertainty and reduce system chattering [13].

Such presence of delays may influence the qualitative system properties and may affect the stability of the process control. When dealing with systems involving time delay, two approaches are employed to establish stability, aligning with the conventional Lyapunov stability theory. The first approach relies on Lyapunov-Krasovski functionals, while the second approach uses Lyapunov-Razumikhin functions [14]. Hence, research into control systems with delays in a nonlinear context holds great importance [15]. The literature offers a variety of methods for developing fault-tolerant controllers in the context of nonlinear systems. In their work [16], Liang and Xu introduced a variable structure stabilizing control law to handle actuator faults within a nonlinear system [17].

The concept of Control Lyapunov Functions (CLF) has played a pivotal role in the advancements of robust control for nonlinear systems [18, 19, 20]. A function that is positive definite and radially unbounded qualifies as a CLF if its time derivative become negative definite through appropriate control input selection, regardless of the value of state. Once a CLF is identified, various methods exist to derive control laws that stabilize the nonlinear system [21].

The application of CLF has been extended to systems with disturbances [20] and [22], to systems with delay and to stochastic systems [19].

Furthermore, the issue of faults, loss of effectiveness, and delay has been addressed using the Lyapunov tool, and has also been treated in the context of a stochastic system such as an adaptive fuzzy control strategy for stochastic nonlinear systems with faults and input saturation uses control filtering to reduce computational load. Fuzzy logic systems approximate the unknown nonlinearities and system variations caused by faults [23]. Another fault-tolerant fuzzy control strategy for stochastic nonlinear systems with quantized inputs is proposed. It uses a hysteretic quantizer to avoid chattering and fuzzy logic systems to estimate unmeasurable states and approximate

nonlinearities. The approach guarantees system stability and signal boundedness in the presence of actuator faults and quantization [24]. Furthermore, a fault-tolerant control strategy is proposed for nonlinear strict-feedback systems with actuator saturation, disturbances, and faults. Neural Networks (NNs) are used to approximate the unknown dynamics, and a back-stepping technique is employed to design the controller. The NN weights are updated online using a gradient descent algorithm, thereby improving the approximation accuracy.

In this paper, the approach introduced in recent studies [8] and [25] is adopted, where actuator faults are represented as bounded additive periodic unknown signals added to the control signal. Additionally, a scenario is examined in which the efficiency of the actuator is compromised, represented by a multiplicative factor. This factor, when applied to the control signal, decreases its performance in accordance with the factor's value [6]. The proposed fault-tolerant control (FTC) is applied to water level control of a hydraulic system based on the Lyapunov redesign principle [26]. If a stabilizing closed-loop controller and its corresponding Lyapunov function exist for the nominal plant, an FTC is constructed based on these nominal controllers and the Lyapunov function. This FTC guarantees stability even in the presence of faults in the system.

B. Main Contribution

The main contribution of this work is the development of a control strategy for nonlinear systems that are subject to both actuator faults (including additive faults and loss of effectiveness) and time delays, addressing a complex and challenging control scenario without linearizing the nonlinear system. Unlike many traditional approaches that simplify the problem through linearization.

The proposed control scheme combines a nominal control component, which governs the system under normal operating conditions, with an additive corrective term specifically designed to compensate faults.

The subsequent sections of the paper are organized as follows: Section II will present the system characterization and problem formulation, Section III will outline the main results, Section IV will provide a real application to verify the efficiency of the proposed additive FTC. In final Section V, we give some conclusions about this work.

II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Here, we consider nonlinear time delay system affine in the control of the form:

$$\dot{x}(t) = f(x_d(t)) + g(x_d(t))u(t, x) \quad (1)$$

Where $x \in R^n, x_d \in R^n, u \in R^m$, represent respectively, the state of system, delayed state and the input vectors, the initial condition $x_d(0) = \phi_d$ is given by the continuous function $\phi_d : [-d, 0] \rightarrow R^n$. Vector fields f and g are smooth functional mapping piecewise continuous function in t and locally Lipschitz in x and u [27]. The functions f and g are known precisely.

Hypothesis 1: We propose the existence of a nominal closed-loop control, represented as $u_{nom}(t, x)$, with the anticipation that it guarantees the overall stability of the closed-loop system.

$$\dot{x}(t) = f(x_d(t)) + g(x_d(t))u_{nom}(t, x) \quad (2)$$

The concept of Control Lyapunov Functions (CLF) has gained attention in the literature due to the availability of various CLF based control laws, which aim to stabilize the system and ensure a certain level of robustness for the closed-loop system. The function $V(x)$ is defined as a positive definite Lyapunov function such that for all $x \in B_x$ for the system [Eq. (1)].

The Lyapunov function will be derived along the trajectories of Eq. (2).

$$\dot{V} \leq -\Gamma_1(|x|) \quad (3)$$

with $\Gamma_1(s) > 0$ for $s > 0$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \frac{\partial x}{\partial t} \leq -\Gamma_1(|x|) \quad (4)$$

Using (2), we get

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} [f(x_d(t)) + g(x_d(t))u_{nom}(t, x)] \leq -\Gamma_1(|x|) \quad (5)$$

Which shows that \dot{V} is negative definite. Consequently, the origin of the full system [Eq. (2)] is asymptotically stable.

III. PROPOSED FAULT TOLERANT CONTROL DESIGN

In the upcoming sections, we will outline the main results.

If we assume that the system can be stabilized within the domain B_x and that the state x is accessible for feedback, our objective is to find a control scheme that achieves asymptotic stabilization of the point $x = 0$ of the closed loop nonlinear delayed system despite actuator fault occurrence.

The fault-tolerant control strategy denoted as control input u_{FTC} designed for the purpose of stabilizing the system in the presence of faults, is proposed as:

$$u_{FTC} = \alpha (u_{nom} + F(x, t) + u_{add}) \quad (6)$$

We take into account the reduction in actuator efficiency, represented by a multiplicative matrix α , where $\alpha \in R^{m \times m}$ is a diagonal continuous time variant matrix, with the diagonal elements $\alpha_{ii}(t)$, $i = 1, \dots, m$ s.t $0 < \alpha_{ii} \leq 1$, $u = u_{nom}$ designates the nominal controller responsible for system stabilization in the absence of any actuator faults.

Using Eq. (6), the system [Eq. (1)] becomes:

$$\dot{x}(t) = f(x_d(t)) + g(x_d(t))\alpha (u_{nom} + F(x, t) + u_{add}) \quad (7)$$

Hypothesis 2:

$F(x, t)$ signifies an actuator fault that satisfies the condition $\|F(x, t)\| \leq L(x, t)$ and $L(x, t)$ is non-negative continuous function satisfying

$$L(x, t) = \frac{1 - \alpha}{\alpha} u_{nom} \quad (8)$$

with $\alpha \in R$

Proposed FTC control: In this context, we will design an additive control u_{add} represents the additional component to compensate the actuator fault's impact as:

$$u_{add} = -\gamma(V) \frac{\partial V}{\partial x} g(x_d(t)) \quad (9)$$

For any $\gamma > 0$ there exists a smooth, positive dominating function γ such that all trajectories of the closed loop system Eq. (2) and (9) satisfy $\lim_{t \rightarrow \infty} |x(t)| < \gamma$

So, the following theorem is proposed to achieve control of the studied nonlinear delayed system Eq. (6) in the presence of malfunctioning actuators.

Theorem: The fault-tolerant control law Eq. (6) ensures asymptotic stability of the closed loop nonlinear delayed system described in Eq. (7) defined by Eq. (8) and (9) under the previous assumptions 1 and 2 even in cases of abnormal operation of the actuators.

Proof:

According to Eq. (6), it follows for the closed-loop faulty nonlinear delayed system Eq. (7), a Control Lyapunov Function (CLF) candidate will be designed such as $V(x_d)$ is a positive definite function as:

$$\dot{V}(t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \frac{\partial x}{\partial t} \quad (10)$$

Using Eq. (7), we get

$$\dot{V}(t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} (f(x_d(t)) + g(x_d(t)) \alpha (u_{nom} + F(x, t) + u_{add})) \quad (11)$$

The establishment of the derivative of $V(t)$ will occur in accordance with the trajectories defined by system Eq. (7), this leads to

$$\dot{V}(t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x_d(t)) + \frac{\partial V}{\partial x} g(x_d(t)) \alpha u_{nom} + \frac{\partial V}{\partial x} g(x_d(t)) \alpha F(x, t) + \frac{\partial V}{\partial x} g(x_d(t)) \alpha u_{add} \quad (12)$$

where

$\beta = 1 - \alpha \geq 0$, Eq. (12) can be expressed as

$$\begin{aligned} \dot{V}(t) = & \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x_d) + \frac{\partial V}{\partial x} g(x_d(t)) u_{nom} \\ & - \beta \frac{\partial V}{\partial x} g(x_d) u_{nom} + \alpha \frac{\partial V}{\partial x} g(x_d) F(x, t) \\ & + \alpha \frac{\partial V}{\partial x} g(x_d) u_{add} \end{aligned} \quad (13)$$

Using Eq. (8) we obtain:

$$\dot{V}(t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x_d) + \frac{\partial V}{\partial x} g(x_d(t)) u_{nom} + \alpha \frac{\partial V}{\partial x} g(x_d) u_{add} \quad (14)$$

By substituting Eq. (9) into Eq. (14), the desired stability can be achieved.

$$\dot{V}(t) \leq -\Gamma_1 (|x|) - \alpha \gamma (V) \left(\frac{\partial V}{\partial x} g(x_d(t)) \right)^2 \quad (15)$$

Such as $\alpha > 0$ and γ is strictly increasing function.

For this purpose

$$\dot{V}(t) \leq -\Gamma_2 (|x|) \quad (16)$$

where $\Gamma_2 > 0$

Hence the system Eq. (1) can be stabilized with the control law Eq. (9). Thus the derivative \dot{V} , is negative along the trajectory of the closed-loop system.

In view of the control law Eq. (9) and taking into account the assumptions about the fault, it is obvious that $\dot{V} \leq 0$.

So, it can be concluded that the origin $x = 0$ of the faulty overall system is a asymptotically stable equilibrium point for studied system Eq. (7) under the fault tolerant control law Eq. (9) regardless of the presence of delay.

IV. FTC CONTROL OF A HYDRAULIC SYSTEM

A. Description of a Single-Tank Hydraulic System

Adjusting a hydraulic level in a tank is the main objective of this work by developing fault-tolerant control for this nonlinear time-delay system based on Lyapunov approach. The structure of the entire system is as shown in Fig. 1. The system is built about a water tank, a liquid level sensor, coil and pump. The tank is supplied by two water inputs: the first is located at the top while the second is at the bottom to compensate for any faults if it exists. Furthermore, the tank has two outputs, the first one for liquid discharge and the second one for leakage (disturbance), the coil affects a pure delay.

B. Modeling of a Single-Tank Hydraulic System

In Fig. 2, the tank is supplied by two water inputs: the first $Q_{I1}(t)$ is located at the top while the second $Q_{I2}(t)$ is at the bottom to compensate for any faults if it exists. Furthermore, the tank has two outputs, the first one for liquid discharge and the second one for leakage (disturbance), the coil affects a pure delay.



Fig. 1. Real hydraulic system at MACS laboratory.

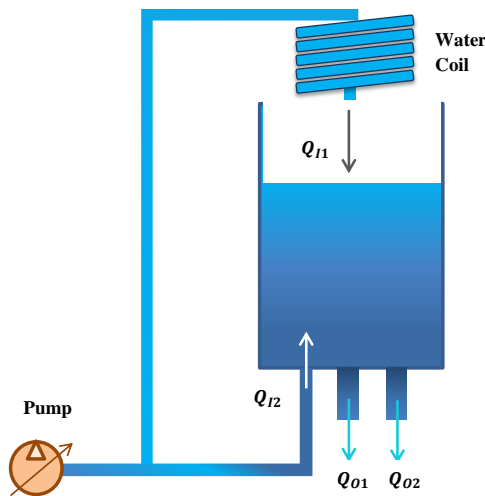


Fig. 2. Model of the hydraulic system at MACS laboratory.

Then, the tank system can be modeled by the following differential equation:

$$\frac{\partial V}{\partial t} = \frac{\partial (S \cdot h(t))}{\partial t} = Q_I(t) - Q_O(t) \quad (17)$$

$V(t)$: Water speed [m/s]

S : Discharge output section [m²]

$h(t)$: Water level [m]

$Q_I(t)$: Input flow rate [m³/s]

$Q_O(t)$: Output flow rate [m³/s]

In our case, the tank section is constant, so we can write

$$\frac{\partial h}{\partial t} = \frac{Q_I(t) - Q_O(t)}{S} \quad (18)$$

We set $x(t) = h(t)$, we obtain

$$\dot{x}(t) = \frac{Q_I(t) - Q_O(t)}{S} \quad (19)$$

with

$$Q_I(t) = Q_i(t) (1 + A(x_d, t)) \quad (20)$$

As long as we have two inputs, $Q_{I1}(t) = Q_i$ is considered the main input and $Q_{I2}(t) = Q_i(t) A(x_d, t)$ is used to compensate faults

We obtain by identification

$$A(x_d, t) = 0.7857x_d^2(t) - 0.6614x_d(t) + 0.68 \quad (21)$$

To simplify, we take

$$g'(x_d) = 1 + A(x_d, t) \quad (22)$$

Therefore,

$$Q_I(t) = Q_i(t) g'(x_d) \quad (23)$$

The output flow rate is divided into two parts: the first part $Q_{O1}(t)$ concerns the main output flow rate, and the second $Q_{O2}(t)$ represents the leakage flow rate which is considered here as a fault.

$$Q_O(t) = Q_{O1}(t) + Q_{O2}(t) \quad (24)$$

Knowing the relationship between flow rate, section and velocity, we can write

$$Q_O(t) = S_{O1}V_1(t) + S_{O2}V_2(t) \quad (25)$$

By replacing Eq. (23) and (25) in Eq. (19), we obtain

$$\dot{x}(t) = \frac{Q_i(t) g'(x_d, t) - (S_{O1}V_1(t) + S_{O2}V_2(t))}{S} \quad (26)$$

After some algebraic manipulations, Eq. (26) can be expressed as

$$\dot{x}(t) = \frac{-S_{O1}V_1(t)}{S} + \frac{Q_i(t) g'(x_d, t) - S_{O2}V_2(t)}{S} \quad (27)$$

In the fault-free case i.e. $Q_{O2}(t) = 0 \rightarrow S_{O2}V_2(t) = 0$, our system can be represented as follows:

$$\dot{x}(t) = \frac{-S_{O1}V_1(t)}{S} + \frac{Q_i(t) g'(x_d, t)}{S} \quad (28)$$

According to Torricelli's theorem,

$$V_1(t) = \sqrt{2gx_d(t)} \quad (29)$$

with $g = 9.81m/s^2$

The Eq. (28) becomes:

$$\dot{x}(t) = \frac{-S_{O1}\sqrt{2gx_d(t)}}{S} + \frac{Q_i(t)g'(x_d,t)}{S} \quad (30)$$

By referring to Eq. (2), we can express Eq. (30) in this form:

$$\dot{x}(t) = f(x_d,t) + g(x_d,t)Q_i(t) \quad (31)$$

Along with

$$f(x_d,t) = \frac{-S_{O1}\sqrt{2gx_d(t)}}{S} \quad (32)$$

And

$$g(x_d,t) = \frac{g'(x_d,t)}{S} \quad (33)$$

In our case, the Q_i Input flow rate of the hydraulic system corresponds to the nominal control u_{nom}

$$Q_i(t) = u_{nom}(t) \quad (34)$$

C. Experimental Validation on a Hydraulic System

1) *Stabilization of nonlinear delayed water tank in fault-free case:* The description of the nominal closed-loop system is as follows:

$$f(x_d,t) + g(x_d,t)u_{nom}(t) = K(x_{ref}(t) - x(t)) \quad (35)$$

with $K > 0$

Let the Lyapunov function be

$$V(t) = \frac{1}{2}(x_{ref}(t) - x(t))^2. \quad (36)$$

However, to ensure the stability of the hydraulic system, it is necessary that $\dot{V}(t) < 0$. So, for this reason, we can choose

$$u_{nom}(t) = \frac{K(x_{ref}(t) - x(t)) - f(x_d,t)}{g(x_d,t)} \quad (37)$$

Consequently, the stability of the hydraulic system is achieved. By examining Eq. (7) and (30), the fault free system is obtained by $\alpha = 1$ and $F(t,x) = 0$, where F represents the fault corresponding to a flow leakage in our real system $Q_{O2}(t) = 0 \rightarrow S_{O2}V_2(t) = 0$. So, it's described as follows:

$$\dot{x}(t) = \frac{-S_{O1}\sqrt{2gx_d(t)}}{S} + \frac{Q_i(t)g'(x_d,t)}{S} \quad (38)$$

The desired level of the water $x_{ref} = 0.4$ m.

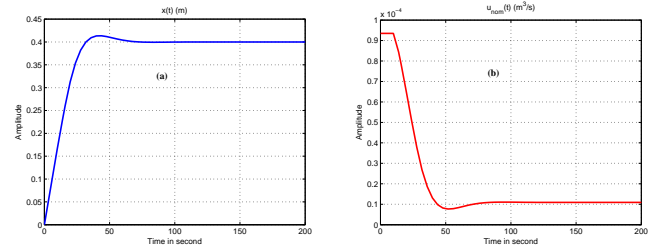


Fig. 3. State trajectory in fault-free case (a) and nominal control (b).

We obtain the next hydraulic system responses in Fig. 3.

Discussion: Initially, in Fig. 3(a) there is a rapid rise to approximately 0.4 m, followed by a stabilization near this desired value. This stabilization shows that the system is able to reach and maintain the desired level without excessive oscillation or instability despite the delay introduced by the tank coil.

In Fig. 3(b) the application of a nominal control Eq. (37) is essential to compensate for the delay and swiftly bring the system to the desired level.

2) *Stabilization of delayed hydraulic system with actuator faults:* In practice, hydraulic systems are susceptible to a various of faults, including loss of effectiveness and additives faults, both of which can significantly reduce their optimal performance.

So, designing the control law according to Eq. (6) and (9) is essential for stabilizing the nonlinear delayed hydraulic system Eq. (27) in the presence of faults. Based on the condition of Eq. (9), which states that $\gamma(V)$ must be a strictly increasing function, we can therefore choose:

$$\gamma(V) = |x_{ref}(t) - x(t)|^2 \quad (39)$$

Consider the nonlinear delayed system (27) with intermittent fault between $t = 80s$ and $t = 90s$ and 3 cases of loss of effectiveness fault $\alpha = 20\%$, $\alpha = 30\%$ and $\alpha = 40\%$.

$$\dot{x}(t) = \frac{-S_{O1}V_1(t)}{S} + \alpha \left(\frac{Q_i(t)g'(x_d,t) - S_{O2}V_2(t)}{S} \right) \quad (40)$$

We start by applying nominal control u_{nom} , and subsequently, we implement fault-tolerant control $u = u_{FTC}$ to compensate any potential faults.

Discussion: Fig. 4(a) shows that when nominal control is employed, the system state deviates from its reference trajectory in the presence of faults. The deviation is notably pronounced in the case of an additive and loss of effectiveness faults, suggesting that nominal control fails to effectively compensate for this specific type of fault.

In Fig. 4(b), it is observed that the control corresponding to this case shows a downward peak, reflecting the need for fault compensation.

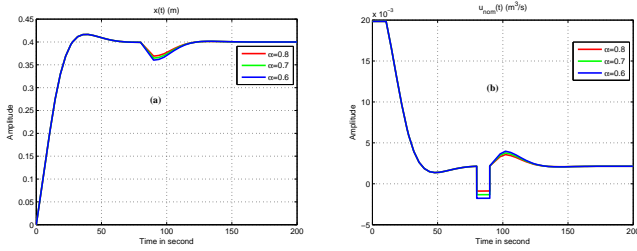


Fig. 4. State trajectory with an additive and loss of effectiveness faults (a) in case of nominal control (b).

To solve the above problem, the control law will be modified to include a new term that represents the component capable of eliminating the impact of the fault.

Hence, we suggest a fault-tolerant control strategy, denoted by u_{FTC} and expressed as:

$$u_{FTC} = \alpha (u_{nom} + F(x, t) + u_{add}) \quad (41)$$

With

$$u_{add} = -|x_{ref} - x|^2 (x_{ref} - x) g(x_d, t) \quad (42)$$

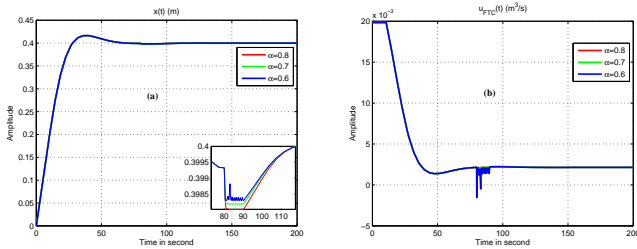


Fig. 5. State trajectory with an additive and loss of effectiveness faults (a) in case of fault tolerant control (b).

Discussion: Fig. 5(a) illustrates that the fault-tolerant control strategy successfully achieves stability and maintains a well-behaved system, even in the presence of faults.

The fault-tolerant control strategy, which accounts for both factors and a delay, compensates the negative impact on the system's behavior and maintains stability. In Fig. 5(b), a shorter-duration peak is observed, highlighting the effect of the newly adopted FTC control, which successfully compensated the system fault.

V. CONCLUSION

In conclusion, this paper aimed to stabilize a nonlinear delayed system affected by actuator faults, focusing on both additive faults and loss of effectiveness. To achieve this objective, we developed a fault-tolerant control strategy based on the Lyapunov redesign approach. Our research was grounded in practical experimentation, conducted on a real hydraulic system within our laboratory : Modeling, Analysis and Control of Systems (MACS).

The results demonstrated that applying nominal control alone to the faulty system leads to performance degradation. However, by integrating an additive control term, the proposed approach successfully compensates for the faults, ensuring system stability and fault tolerance. This study provides valuable insights and contributes to the advancement of fault-tolerant control strategies for nonlinear delayed systems with practical validation.

DECLARATION OF CONFLICTING INTERESTS

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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