Performance Enhancement of Wi-Fi Fingerprinting-based Indoor Positioning using Truncated Singular Value Decomposition and LSTM Model

Duc Khoi Nguyen¹, Thi Hang Duong², Le Cuong Nguyen³, Manh Kha Hoang⁴*
Faculty of Electronics Engineering, Hanoi University of Industry, Hanoi, Vietnam¹, 2, 4
Faculty of Electronic and Telecommunications, Electric Power University, Hanoi, Vietnam³

Abstract—Wi-Fi based indoor positioning has been considered as the most promising approach for civil location-based service due to the widespread availability Wi-Fi systems in many buildings. One of the most favorable approaches is to employ received signal strength indicator (RSSI) of Wi-Fi access points as the signals for estimating the mobile object locations. However, developing a solution to obtain high positioning accuracy while reducing system complexity using traditional methods as well as deep learning based methods is still a very challenging task. This paper presents a proposal to combine the Truncated Singular Value Decomposition (SVD) technique with a Long Short-Term Memory (LSTM) model to enhance the performance of indoor positioning system. Experimental results on a public dataset demonstrate that the proposed approach outperforms other state-of-the-art solutions by means of positioning accuracy as well as computational cost.

Keywords—Indoor positioning; Wi-Fi fingerprinting; Truncated Singular Value Decomposition; LSTM

I. INTRODUCTION

Indoor positioning has attracted significant interest [1, 2, 3, 4] due to its potential applications for various Location-based Service (LBS) in rescue operations, military, medical care, civil activities, etc. While satellite based positioning systems have successfully applied in many outdoor applications, the satellite signal is rarely available inside buildings. Therefore, it is still a very challenging task to develop a solution that achieves accurate position estimates at low cost due to the frequent change of environment, people movement, etc.

Various indoor positioning approaches have been proposed utilizing different types of signals including Wi-Fi, Bluetooth, visible light, acoustic, etc. and their combination [3, 5]. Among them, many approaches utilize Received Signal Strength Indicator (RSSI) from Wi-Fi Access Points (APs) due to widespread deployment of WLANs and Wi-Fi equipped devices [6]. It is worth noting that Wi-Fi RSSI signal can be captured easily by all smart phones which many people own. Therefore, Wi-Fi RSSI based indoor positioning is considered as the most promising approach for civil LSB applications since it requires no extra infrastructures [6, 7].

In indoor environment, traditional localization techniques such as trilateration based and triangulation based often require line-of-sight (LoS) condition between the transmitter and the receiver. Unfortunately, this condition is often false due to obstacles and room partitions in buildings [2]. These approaches also often require some prior knowledge of the infrastructure such as AP locations and additional devices. On the other hand, Wi-Fi RSSI fingerprinting based techniques do not require the mentioned conditions have been become the most promising approach [3, 6, 8, 9], especially for civilian applications. This method operates in two phases, one for training, and the other for online localization/classification [8]. In the training phase, RSSI data are captured at the predetermined reference points (RPs) from available Wi-Fi APs to build the radio map database. In the localization phase, the online captured data are compared to the radio map to determine the target location based on the similarity between online data and training data. The flow of fingerprinting is visually depicted in Fig. 1.

Traditional fingerprinting based approaches being used for estimating position of an object can be classified into deterministic and probabilistic methods [10, 11]. Among those two methods, previous studies have indicated that the probabilistic approaches often deliver better positioning results compared to the deterministic approach [12, 13]. The critical problem with traditional solutions is that their computational cost in the classification phase is often very high when the region of interest is large [8, 9]. This leads to the reduction of positioning accuracy in real time applications due to the movement of the mobile object between Wi-Fi RSSI scanning time and the time the system delivers the positioning result. Therefore, improving the performance of Wi-Fi indoor positioning system (WF-IPS) is a challenge since it needs to satisfy both requirements: reducing positioning error and reducing execution time.

*Corresponding author.
Recently, various artificial neural network (ANN) based approaches have been developed for WF-IPS. Since the transformation between the observed RSSI values and mobile object location is nonlinear, it is difficult to derive a close form solution. Therefore, ANN is considered a suitable and reliable approach to approximate this transformation. Compared to traditional algorithms, deep learning approaches have proved their effectiveness when applying to WF-IPS [8]. Several solutions for WF-IPS have been proposed utilizing different ANN models such as multilayer perception (MLP) [14], stacked autoencoder (SAE) [15], convolution neural network [3, 9, 16], recurrent neural network (RNN) and its variations [8, 17], etc., or their combinations which are considered as hybrid or ensemble system. In addition to ANN based methods, several solutions combining dimensionality reduction and the use of LSTM models have emerged to enhance the accuracy of indoor positioning systems based on Wi-Fi fingerprints (WF-IPS) [18, 19]. These solutions emphasize the significance of dimensionality reduction in processing RSSI data to improve the performance and efficiency of the system [20].

Although having been extensively investigated in the literature, determining a sufficient neural network model is still of particular interest among the research community. Wi-Fi data often has a high dimensionality, and when applied to machine learning models, processing a large volume of data becomes extremely expensive. Therefore, there have been many studies advocating for the use of combined solutions to reduce data dimensionality including Truncated SVD, Principal Component Analysis (PCA), and autoencoders [20]. Prominent studies in this field have demonstrated that reducing data dimensionality not only improves model performance but also significantly reduces the required computational resources.

Having inspiration from the advantages of data dimensionality reduction in classification and regression problem, this article introduces a solution for WF-IPS which combines Truncated SVD and LSTM models to improve positioning accuracy while reducing computational costs. To ensure a fair performance comparison of the proposed approach with state-of-the-art solutions, the dataset provided in study [33] is utilized in this study. In summary, our contributions are as follows:

- Truncated SVD is utilized for data dimensionality reduction as well as noise removal, demonstrating its superiority over PCA in various scenarios.
- We demonstrate that utilizing truncated SVD for dimensionality reduction reduces the computational load while improving performance in location prediction and execution time compared to the other state-of-the-art approaches on the same dataset.
- We conduct a thorough analysis of the improvements gained from employing our proposed solution in different test scenarios, highlighting its overall effectiveness.

The rest of the article is organized as follows. In Section II, the related works are presented. The proposed approach of combining Truncated SVD and LSTM model is presented in Section III. In Section IV, the experimental results are extensively presented to demonstrate the superiority of the proposed approach. The conclusions of the paper are drawn in Section V.

II. RELATED WORKS

Recently, many researchers have focused on the challenges of indoor positioning systems (IPS) based on Wi-Fi fingerprinting using machine learning and deep learning techniques. Collecting Wi-Fi fingerprint signals often results in high-dimensional data, which poses challenges during both the training and localization phases of machine learning models. Dimensionality reduction is an important solution for high-dimensional indoor positioning problems, although there might be a trade-off between dimensionality reduction and model accuracy [21, 22]. Minimizing computational costs during the position estimation phase is crucial for real-time monitoring systems. Therefore, designing a model with low computational cost makes indoor positioning systems more feasible. In the following content, machine learning and deep learning techniques with data dimensionality reduction are explored.

Various types of neural networks have been utilized to develop solutions for WF-IPS. Among them, Recurrent Neural Network (RNN) seem to be very attractive in many previous research [17, 18, 19, 23, 24]. In study [17], the authors presented the evaluation of RNN and LSTM (a variant of RNN) as the deep learning technique to build a WF-IPS system. The experimental evaluation on a public available dataset showed that their proposed RNN and LSTM model can deliver almost the same accuracy on floor classification (99.7%) as well as position estimates (2.5-2.7 meters). The computation time between the two models was also presented, RNN required less time than LSTM model both on training and testing procedures. In study [23], the authors proposed a local feature-based deep LSTM approach for a WF-IPS. The robust local features are extracted, and the noise is eliminated by a local feature extractor applying sliding windows. The local features are then fed into a deep LSTM for target position estimation. Their proposed approach is conducted in real environments and compared with other state-of-the-art approaches for indoor positioning. The experimental results indicate the mean localization error of their approach has been improved by 18.98% to 53.46% compared to the others.

A novel method that transforms RSSI signals into principal components (PCs) using all the effects of APs is proposed in [25]. Instead of selecting APs, this research replaces the captured Wi-Fi RSSI with a subset of PCs to enhance localization accuracy and reduce computational costs. Test results in a real WLAN environment showed that the average distance error decreased by 33.75%, and complexity decreased by 40% in comparison with other methods. Authors in [26] introduced a new technique for clustering location data into subregions using an algorithm named fuzzy C-means. Useful APs were then selected to reduce the dimensionality of RSSI fingerprint data during the training procedure. In the online phase, the Nearest Neighbor (NN) method was used to select subregions and compute location coordinates of the target utilizing the Relative Distance Fuzzy Localization algorithm. Test results demonstrated that their proposed model reduced computation time and improved localization accuracy. In study
[27] a magnetic field indoor fingerprinting system based on CNNs was proposed. The Recurrence Plots were utilized as sequence fingerprints and the localization problem is approaching from a regression framework. The real-world experimental results show the advantages of their proposal compared to the other studies, though its computation cost is high. In study [28], an LSTM network was used to learn high-level representations of extracted local temporal features, then to eliminate the noise impact, a local feature extraction approach was employed to extract powerful local features. In study [29], to avoid quality degradation, spatial features of Wi-Fi signals are extracted by a residual-based network at the same time slice and then an LSTM network is employed to extract temporal features of Wi-Fi signals between successive time slices. Research [30] proposed a data dimensionality reduction technique to enhance performance of Wi-Fi IPS based on Multiple Service Set Identifiers. Test results of the proposed system achieved localization error of less than 0.85 m over an area of 3000 m², with a cumulative distribution function of 88% at a localization error of 2 m.

In general, there is often a trade-off between accuracy and computational speed in indoor Wi-Fi RSSI-based positioning models due to high dimensionality data. However, studies combining dimensionality reduction and machine learning have shown significant effectiveness in both accuracy and computational speed. In this research, we propose the use of dimensionality reduction with Truncated SVD combined with LSTM for indoor Wi-Fi signal-based location estimation. To ensure fairness in performance evaluation, we utilized [17, 19] as a reference document to conduct a comprehensive comparison and assess localization performance on both positioning accuracy and system complexity using the same dataset.

III. PROPOSED APPROACH

The proposed approach which combines Truncated SVD and LSTM model (Truncated SVD-LSTM) for performance enhancement of Wi-Fi fingerprinting based indoor positioning is systematically presented in this section.

A. System Architecture

The structure of the proposed indoor positioning system consists of two main phases as is illustrated in Fig. 2. This block diagram provides a visual representation of the operation of the indoor positioning model, allowing us to understand how data flows from the initial data collection phase to the final estimation of the user's location.

The proposed indoor positioning model is separated into two phases: the offline training phase and the online testing phase. During the offline training phase, data collected from various sources are aggregated and normalized. The data are then passed through the Truncated SVD for dimensionality reduction. The utilization of Truncated SVD helps eliminate unnecessary information and reduce the complexity of the original data. Once the data has been dimensionally reduced, they are ready to be utilized for training the LSTM model. During the offline training phase, the LSTM model learns how to predict the target position based on the reduced training data processed by Truncated SVD and known locations. After the model has been trained, it can be used to estimate the target position in real-time. In the online testing phase, each new data sample collected from a target device is processed by normalization and Truncated SVD in the same way as in the offline phase. Subsequently, the dimensional reduced data sample is fed into the already trained LSTM model to estimate the target real-time position. The result of the testing phase is the predicted position of the target within the area of interest.

![System architecture of the proposed WF-IPS.](image)

B. The Proposed Approach for Combining Truncated SVD and LSTM Model

1) Introduction to Truncated SVD: Truncated SVD [31] is a technique developed for dimensionality reduction. It is commonly utilized to solve the various problems where high-dimensional data are presented. This phenomenon, namely “curse of dimensionality”, often affects the performance of the machine learning based system. Truncated SVD is built upon the concept of SVD, which decomposes a matrix \( A \) into three separate matrices \( \Sigma, U, V \) corresponding to singular values, left and right singular vectors of the matrix \( A \), as presented in Eq. (1).

\[
A_{M \times N} = U_{M \times M} \Sigma_{M \times N} (V_{N \times N})^T
\]

(1)

Truncated SVD retains the top \( k \) singular values and their associate singular vectors. The main concept of Truncated SVD is finding a representation of the original matrix with a much lower dimensionality while preserving the most data information such as data patterns and data relationships. To effectively reduce the data dimensionality according to any specific problem, determining the best value of \( k \) is of important task. Mathematical expression of Truncated SVD is presented in Eq. (2).

\[
A_{M \times N} \approx A_{k \times k} = U_{k \times k} \Sigma_{k \times k} (V_{k \times k})^T
\]

(2)

2) Introduction to LSTM: In this study, the LSTM model [32] is employed to develop an indoor positioning solution. The target location is predicted via the LSTM linear regression model utilizing the low dimensional data processed by Truncated SVD. LSTM model selectively forgets or remembers information over long data sequences. In the LSTM, long-term dependencies are captured for modeling context and sequential
patterns. There are a memory cell and three gates namely input gate $i_t$, forget gate $f_t$, and output gate $o_t$ in an LSTM cell as shown in Fig. 3. The input gate regulates the information transmitted to the cell. The forget gate decides how much information transmitted to the cell should be retained. Output sequences and hidden state are produced and updated by output gate. The memory cell is responsible for storing information over time in the network. The mathematical expressions for the LSTM network at each time step $t$ are presented in Eq. (3) to Eq. (8).

$$i_t = \sigma \left( W_{ix}x_t + W_{ih}h_{t-1} + b_i \right)$$

$$f_t = \sigma \left( W_{fx}x_t + W_{fh}h_{t-1} + b_f \right)$$

$$C_t = \tanh \left( W_{cx}x_t + W_{ch}h_{t-1} + b_c \right)$$

$$C_t = f_tC_{t-1} + i_tC_t$$

$$o_t = \sigma \left( W_{ox}x_t + W_{oh}h_{t-1} + b_o \right)$$

$$h_t = o_t \tanh \left( C_t \right)$$

Where, $x_t, h_{t-1}, C_t, \tilde{C}_t$ are the input, output, cell state and updated cell state at time step $t$, respectively, $C_{t-1}, h_{t-1}$ are the previous cell state and hidden state. $W_i, W_f, W_c, b_i, b_f, b_o$ are, respectively, the weight matrices and the bias vectors of the input, forget, updated cell state and output gate layers. The activation functions utilized in LSTM cell are $\sigma$ and $\tanh$.

3) Proposed model: Fig. 4 introduces the general model that integrates data dimensionality reduction using Truncated SVD and LSTM neural network to address indoor localization based on Wi-Fi RSSI data.

The original RSSI data consists of a large number of features ($N$), and Truncated SVD is employed to reduce the data dimensionality to $k$ features ($k < N$), thus reducing complexity and enhancing the generalization capability of the model. The low dimensional data samples are then fed into the LSTM model for training and predicting the device's position within the indoor environment. The format of the RSSI data can be seen as follows:

$$\text{RSSI}_{MN} = \left\{ \text{RSSI}_{11}, \text{RSSI}_{12}, \ldots, \text{RSSI}_{1N}, \text{RSSI}_{21}, \text{RSSI}_{22}, \ldots, \text{RSSI}_{2N}, \ldots, \text{RSSI}_{MN}, \right\}$$

This is a collection of RSSI values obtained at each location in the training set, where $M$ and $N$ represent the number of RSSI samples and the number of detected APs in the dataset.

The workflow of our proposal is described in Fig. 5.
IV. RESULTS AND DISCUSSION

The effectiveness of the proposed approach is extensively presented and analyzed in this section. The experimental results are produced by using a public dataset [33]. Localization error and computational cost are the focused performance characteristics for comparison between our proposal and other state-of-the-art methods.

A. Wi-Fi RSSI Dataset

The dataset [33] was collected on the 3rd and 5th floors of a university’s library building. Data collection involved facing specific directions and gathering six fingerprints per location, with six consecutive samples per point to exclude any initial measurements. The training, Test-01, and Test-05 datasets covered "Up" and "Down" directions, while Test-04 and Test-05 focused on "Left" and "Right." Collection followed a sequence: (1) direct 3rd floor, (2) reverse 3rd floor, (3) direct 5th floor, and (4) reverse 5th floor. Training, Test-01, and Test-05 always included data from all directions monthly. Test-04 data were from horizontal corridors. Due to time constraints, Test-02 and Test-03 considered only two directions, covering 308.4 m² on both floors. The datasets were organized into 15 collection months, resulting in 16,704 training and 46,800 test samples, collected comprehensively for Wi-Fi RSSI-based indoor positioning. Table I presents the main characteristics of the dataset. For data preprocessing, the values for undetected APs are replaced by -100 dBm which is the weakest signal in the dataset for the whole work presented in the following content of this paper.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training samples</td>
<td>16,704</td>
</tr>
<tr>
<td>Testing samples</td>
<td>46,800</td>
</tr>
<tr>
<td>Number of measurements at each RP</td>
<td>12</td>
</tr>
<tr>
<td>Number of observable APs</td>
<td>448</td>
</tr>
<tr>
<td>Number of floors</td>
<td>2</td>
</tr>
<tr>
<td>Coverage</td>
<td>308.4 m²</td>
</tr>
<tr>
<td>Number of training RP</td>
<td>48</td>
</tr>
<tr>
<td>Number of test positions</td>
<td>212</td>
</tr>
<tr>
<td>Period of measurement campaign</td>
<td>15 months</td>
</tr>
<tr>
<td>Training RSSI range</td>
<td>-98 dBm to -31 dBm</td>
</tr>
<tr>
<td>Testing RSSI range</td>
<td>-100 dBm to -32 dBm</td>
</tr>
<tr>
<td>Constant value for undetected APs</td>
<td>100 dBm</td>
</tr>
</tbody>
</table>

B. Data Normalization

The tricky problems in the characteristics of Wi-Fi RSSI data that affect the performance of Wi-Fi IPS are the variation over time and the fluctuation due to the quick changes of indoor environment as well as the behavioral of the devices. To deal with such the problems, data normalization is considered the necessary step which reduces the data variation while maintaining information. Consequently, it helps to enhance the performance of dimensionality reduction techniques and learning capacity of deep learning model. In this study, two common normalization techniques, namely standard normalization and max-min normalization, were evaluated to come up with the best normalization solution. Each Wi-Fi RSSI sample is normalized as presented in Eq. (9) or Eq. (10) according to the chosen normalization technique. It is noted that all the RSSI values of undetected APs were replaced by -100 dBm.

\[
RSSI_{\text{StdNorm}} = \frac{RSSI_j - RSSI_\mu}{RSSI_\sigma} 
\]

(9)

\[
RSSI_{\text{MaxMinNorm}} = \frac{RSSI_j - RSSI_{\min}}{RSSI_{\max} - RSSI_{\min}} 
\]

(10)

where, \( RSSI_{\text{StdNorm}} \), \( RSSI_{\text{MaxMinNorm}} \), and \( RSSI_j \) are the normalized values corresponding to standard normalization and max-min normalization and the raw value of the RSSI of the \( j \)-th AP in each RSSI sample. \( RSSI_\mu, RSSI_\sigma, RSSI_{\min} \), and \( RSSI_{\max} \) are the mean, standard deviation, maximum and minimum values of each RSSI sample.

C. Determination of the Number of Dimensions for Truncated SVD

An important task when using Truncated SVD technique to reduce data dimensionality is determining the number of dimensions to retain preserve data information. We conducted a survey to identify the number of dimensions to be kept. Fig. 6 illustrates the relationship between the number of Truncated SVD dimensions and the amount of preserved information. As can be seen, when the number of dimensions is reduced to 100, the cumulative explained variance ratio almost reaches 100%, it means that the information loss after truncation is negligible. It is noted that the number of features of original data is 448, hence 100 kept dimensions meet the target of dimensionality reduction. Therefore, in this study, the number of dimensions to be kept is set to 100.

![Fig. 6. The relationship between the number of dimensions and the preserved information.](www.ijacsa.thesai.org)
D. Model Optimization

For optimizing our proposed model, the LSTM model presented in study [19] is first utilized as presented in Table II. It is worth noting that in study [19] the authors presented a solution for Wi-Fi fingerprinting based IPS by combining PCA with LSTM, their model was also evaluated on the dataset provided in study [33]. That explained the reason why their LSTM model was chosen as the starting point for our model optimization procedure.

![Table II. General Model](image)

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of LSTM units</td>
<td>100</td>
</tr>
<tr>
<td>Drop rate for LSTM layer</td>
<td>0.3</td>
</tr>
<tr>
<td>Activation function for LSTM layer</td>
<td>sigmoid</td>
</tr>
<tr>
<td>Number of units for Dense layer</td>
<td>100</td>
</tr>
<tr>
<td>Activation function for Dense layer</td>
<td>sigmoid</td>
</tr>
<tr>
<td>Dropout rate for Dense layer</td>
<td>0</td>
</tr>
<tr>
<td>Number of units for Output layer</td>
<td>2</td>
</tr>
<tr>
<td>Activation function for Output layer</td>
<td>linear</td>
</tr>
<tr>
<td>Learning rate</td>
<td>0.001</td>
</tr>
<tr>
<td>Optimizer</td>
<td>Adam</td>
</tr>
<tr>
<td>Batch size</td>
<td>32</td>
</tr>
<tr>
<td>Training epoch</td>
<td>100</td>
</tr>
</tbody>
</table>

Before tuning hyperparameter of the LSTM model, data normalization techniques as presented in subsection B are first evaluated since it strongly affects the performance of dimensionality reduction as well as deep learning. As shown in Table III, standard normalization technique yields better Mean Distance Error (MDE) result compared to the min-max normalization. Therefore, for further hyperparameter tuning of the LSTM model, standard normalization was selected. It is noted that during the tuning process, the min-max normalization is still checked each time a hyperparameter is evaluated and all the results confirmed standard normalization technique is the better one.

![Table III. Mean Distance Error with Different Data Normalization Methods](image)

<table>
<thead>
<tr>
<th>Normalization</th>
<th>MDE (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min-max</td>
<td>2.231</td>
</tr>
<tr>
<td>Standard</td>
<td>2.087</td>
</tr>
</tbody>
</table>

For hyperparameter tuning, different configurations of LSTM were evaluated as illustrated in Table IV. Comparing the values between column “Value delivered the best MDE” in Table IV and column “Value” in Table II, it is obvious that the main structure of the LSTM model such as number of LSTM units and number of units for Dense layer remain unchanged. However, the optimized values of drop rate, activation function, batch size and training epoch were different. These changes make the optimized model operate faster during training on the same dataset as demonstrated in the next subsection.

![Table IV. Hyperparameter Tuning](image)

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value options for tuning</th>
<th>Value delivered the best MDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of LSTM units</td>
<td>[40:20:140]</td>
<td>100</td>
</tr>
<tr>
<td>Drop rate for LSTM layer</td>
<td>[0.2, 0.25, 0.3]</td>
<td>0.2</td>
</tr>
<tr>
<td>Activation function for LSTM layer</td>
<td>[relu, tanh, sigmoid]</td>
<td>relu</td>
</tr>
<tr>
<td>Number of units for Dense layer</td>
<td>[40:20:200]</td>
<td>100</td>
</tr>
<tr>
<td>Activation function for Dense layer</td>
<td>[relu, tanh, sigmoid]</td>
<td>sigmoid</td>
</tr>
<tr>
<td>Dropout rate for Dense layer</td>
<td>[0.0, 0.1, 0.2, 0.3]</td>
<td>0</td>
</tr>
<tr>
<td>Number of units for Output layer</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Activation function for Output layer</td>
<td>linear</td>
<td>linear</td>
</tr>
<tr>
<td>Learning rate</td>
<td>[0.01, 0.001, 0.0001]</td>
<td>0.001</td>
</tr>
<tr>
<td>Optimizer</td>
<td>Adam</td>
<td>Adam</td>
</tr>
<tr>
<td>Batch size</td>
<td>[16, 32, 64, 128]</td>
<td>64</td>
</tr>
<tr>
<td>Training epoch</td>
<td>[20:10:100]</td>
<td>30</td>
</tr>
</tbody>
</table>

E. Positioning Performance Evaluation

To evaluate the localization error of our proposal, for a fair comparison, some state-of-the-art works conducted on the same dataset presented in [32] were selected as the benchmark deep learning-based models. Mean Distance Error (MDE) and Root Mean Squared Error (RMSE) are selected among typical evaluation metrics for comparing the positioning accuracy of different approaches. Denoting \( d_i \) as the localization distance error of the \( i \)-th RSSI test sample, and the coordinates of the true and the predicted position are \( (x_{i,true}, y_{i,true}) \) and \( (x_{i,pred}, y_{i,pred}) \), respectively, the localization distance error measured by Euclidean distance is computed by Eq. (11). MDE and RMSE are then correspondingly determined by Eq. (12) and Eq. (13).

\[
d_i = \sqrt{(x_{i,true} - x_{i,pred})^2 + (y_{i,true} - y_{i,pred})^2} \tag{11}
\]

\[
MDE = \frac{\sum_{i=1}^{N_{test}} d_i}{N_{test}} \tag{12}
\]

\[
RMSE = \sqrt{\frac{1}{N_{test}} \sum_{i=1}^{N_{test}} d_i^2} \tag{13}
\]

Table V presents the achieved values based on the evaluation criteria, including Mean Squared Error (RMSE) and Mean Distance Error (MDE), for various localization solutions. The results clearly demonstrate that the proposed solution exhibits the lowest RMSE and MDE values, e.g., the MDE of the proposed model is reduced by approximately 6% and 21% compared to the results presented in study [19] and [17], respectively. Furthermore, Table VI highlights the superiority of
our proposed approach in terms of computational complexity. Specifically, our solution reduces training time by more than 80% compared to both benchmark solutions. When using the proposed solution, the prediction time is improved by roughly 20% compared to using LSTM without dimensionality reduction. It is noted that the testing time of the proposed solution is the same as the study presented in study [19] since the two models are very similar as mentioned in previous subsection. This underscores the efficiency and effectiveness of our approach in indoor localization scenarios.

### TABLE V. POSITIONING ERROR COMPARISON

<table>
<thead>
<tr>
<th>Models</th>
<th>MDE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTM [17]</td>
<td>2.5-2.7</td>
<td>-</td>
</tr>
<tr>
<td>PCA-LSTM [19]</td>
<td>2.18</td>
<td>1.95</td>
</tr>
<tr>
<td>Proposed</td>
<td>2.05</td>
<td>1.75</td>
</tr>
</tbody>
</table>

### TABLE VI. MODEL COMPLEXITY COMPARISON

<table>
<thead>
<tr>
<th>Models</th>
<th>Number of trainable parameters</th>
<th>Training time [s]</th>
<th>Testing time [s] (whole test dataset)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTM [17]</td>
<td>NA(^b)</td>
<td>581.3599(^b)</td>
<td>10.1721(^b)</td>
</tr>
<tr>
<td>PCA-LSTM [19]</td>
<td>90,702</td>
<td>Approx. 500(^c)</td>
<td>8.1(^f)</td>
</tr>
<tr>
<td>Proposed</td>
<td>90,702</td>
<td>85(^e)</td>
<td>8.1(^e)</td>
</tr>
</tbody>
</table>

\(^a\) Not available
\(^b\) NVIDIA GeForce GTX 1080 Ti as Graphical Processing Unit (GPU)
\(^c\) NVIDIA Quadro P2200 as Graphical Processing Unit (GPU)
\(^d\) 100 training epoch

![Fig. 7. Comparison of positioning error.](image)

Fig. 7 illustrates the cumulative error function based on Euclidean distance for different LSTM models. The solid line represents the prediction probability using the proposed Truncated SVD and LSTM solution. The dashed line and the dash dot line depict the CDF of distance error of the two benchmark approaches, [19] and [17], respectively. According to the data presented on Fig. 7, it is obvious that data dimensionality reduction based approaches outperform the other in which data preprocessing technique is not implemented. In addition, employing suitable data preprocessing and dimensional reduction techniques can lead to further enhancement of location prediction accuracy.

The experimental results presented above illustrate the important role of Truncated SVD in the proposed approach. This is the main difference between our work and the one presented in [19]. In addition, fine tuning the parameters of LSTM model is also very essential during the development of the concrete solution for a specific indoor positioning system.

### V. CONCLUSIONS

In this study, an approach called Truncated SVD-LSTM for indoor localization based on Wi-Fi fingerprints is introduced. To the best of our knowledge, this is the first time an indoor positioning solution has been built upon the fusion of Truncated SVD and LSTM model. Our solution focuses on reducing the dimensionality of the data to enhance positioning accuracy and computational cost of the model. We conducted experiments on a publicly available dataset and achieved impresssive results. The experimental outcomes have unequivocally demonstrated that the integrated LSTM structure in our solution has attained an average localization error of 2.05 meters, with nearly 60% of cases having errors below 2 meters. This signifies an enhancement of approximately 6% and 21% compared to state-of-the-art studies, [19] and [17], respectively, utilizing LSTM on the same dataset. The results also indicate that the proposed solution significantly reduces computational costs, especially for the training procedure. Compared to the state-of-the-art approach, the evaluation results demonstrated the superiority of the proposed solution. In the future, the supervised techniques for data dimensionality reduction should be investigated in order to extract information in a supervised manner which may help the localization model perform more efficiently.

### CONFLICTS OF INTEREST

The authors declare no conflict of interest.

### REFERENCES


