Fuzzy Control-based Adaptive Adjustment of Dynamic Stiffness for Stewart Platforms

Zhiqiang Zhao, Yuetao Liu*, Changsong Yu, Changsong Yu
School of Mechanical Engineering, Shandong University of Technology, Zibo, China

***Abstract*—An adaptive adjusting strategy of Stewart platform dynamic stiffness based on fuzzy control is explored in this paper. The transient response, steady-state accuracy, anti-disturbance and robustness of Stewart platform are improved remarkably. Simulation experiments and data analysis show that compared with traditional fixed stiffness or PID control, this fuzzy control strategy can quickly achieve steady state under various operating conditions, effectively deal with load mutation, parameter change and model uncertainty, and greatly enhance the overall stability and performance of Stewart platform. In an application example, the strategy is used in precision machining field to optimize Stewart platform support and accurately control high-speed machine table, facing frequent fluctuation of dynamic load. The fuzzy controller takes displacement error, speed error, cutting force and material hardness as inputs and dynamic stiffness as outputs, and constructs fuzzy rule base and optimized membership function suitable for various machining conditions. The evaluation shows that fuzzy control performs well in transient response, and the response time is shortened by about 30% in the face of large load sudden change. In steady-state accuracy, displacement error ± 0.05 mm and velocity error ±0.1°/s are strictly controlled, which is better than pure PID control. In anti-disturbance test, fuzzy control successfully reduces the influence of random disturbance on platform trajectory by 70%. Robustness tests show that the fuzzy controller maintains stable control effect even when the system parameters vary by ±10%, and the system performance score is above 8.5, which is far superior to that of traditional PID controller under the same conditions.**

***Keywords*—Fuzzy control; regulation methods; Stewart platform; stiffness adaptive**

I. Introduction

Stewart platform, with its six degrees of freedom and flexible movement characteristics, is widely used as a precision positioning and simulation device in many fields such as aerospace, robotics, precision measurement and virtual reality [1]. Its unique mechanical structure consists of six linkage mechanisms connecting the base and the platform, and through precise control of the length of each linkage, it realizes all-round position and attitude adjustment in three-dimensional space. With the development of science and technology, the performance requirements of Stewart platform are increasing, especially its dynamic performance directly affects the overall efficiency and accuracy of the equipment [2].

Dynamic stiffness plays a crucial role in the Stewart platform, and it is a key indicator for evaluating the platform's ability to respond in a dynamic environment. Dynamic stiffness reflects the ability of a platform's internal structure to quickly and effectively resist deformation and return to its original state when subjected to external dynamic loads. This ability directly determines the platform's performance in the face of rapidly changing operating conditions [3]. When the Stewart platform has high dynamic stiffness, it means that when encountering sudden dynamic load changes, the platform can respond quickly in a very short period, quickly adjusting itself to return to the target position or attitude, which can largely reduce the accumulation of positional errors due to load perturbations. This ability to recover quickly is critical to maintaining high-precision motion tracking, especially in aerospace and precision manufacturing, where the platform has extremely stringent requirements for positioning accuracy. On the contrary, if the dynamic stiffness of the Stewart platform is too low, its reaction speed will be relatively slow when facing the same dynamic load perturbation, and even obvious vibration phenomena may occur. This vibration will not only cause the platform's trajectory to deviate from the intended target but will also lead to a decrease in its positional accuracy when it reaches the steady state, thus affecting the stability and efficiency of the whole system. In addition, the response hysteresis also prevents the platform from adapting to changes in the external environment in a timely manner, which reduces its adaptability and operational flexibility in dynamic scenarios. Therefore, the effective regulation of the dynamic stiffness of Stewart platform is a core technology, which is not only related to the optimization of the platform's own performance, but also a key factor in determining whether it can operate efficiently and stably under various complex working conditions. Through the development and application of advanced dynamic stiffness adjustment technology, the dynamic performance of the Stewart platform can be significantly improved, expanding its application scope in many high-tech fields, and laying a solid hardware foundation for its intelligent and precise operation in the era of Industry 4.0 [4, 5].

The research focus of this paper is to solve the deficiencies in the dynamic stiffness adjustment of the Stewart platform, i.e., in the face of the complex and changing working environment, the traditional dynamic stiffness adjustment method is difficult to achieve the global optimization, and the real-time performance and robustness need to be improved. To this end, this study proposes a novel fuzzy control-based dynamic stiffness adaptive adjustment strategy. (1) The unique feature of this study is that the fuzzy control theory is skillfully applied to the dynamic stiffness adjustment problem of the Stewart platform. Fuzzy control is able to effectively deal with complex and ambiguous practical working conditions with its powerful ability to deal with uncertainty and nonlinear systems [6]. By constructing a fuzzy logic controller specifically designed for the dynamic characteristics of the Stewart platform, the controller is able to judge the platform's operating state in real time based on the dynamic parameters (such as displacement, velocity, acceleration, etc.) Of the platform in actual operation, and realize the accurate interpretation of the platform's dynamic behavior. (2) The fuzzy controller adjusts the Stewart platform at the right time according to the judgment results through fuzzy reasoning and decision-making mechanisms [7]. According to the judgment result, the fuzzy controller adjusts the length configuration of the six connecting rods through the fuzzy reasoning and decision-making mechanism, so as to change the dynamic characteristics of the platform and achieve the goal of adaptive optimization of dynamic stiffness. This approach enables the platform to rapidly adjust its dynamic stiffness attributes when facing different load disturbances, which not only meets the requirement of transient response speed, but also ensures the steady-state accuracy, effectively suppresses vibration and improves the anti-interference ability. (3) The fuzzy-controlled dynamic stiffness adjustment method proposed in this study has significant advantages of robustness and adaptability. In practical application, even in the face of unforeseen complex working conditions, the method can make reasonable stiffness adjustment according to the feedback information, to ensure the high performance of the platform under various working conditions, which is often difficult to be realized in the traditional control strategy.

Through the introduction of fuzzy control theory, the method proposed in this study not only realizes the real-time adaptive optimization of the dynamic stiffness of the Stewart platform, but also enhances its stability and working efficiency under complex and variable working conditions, providing a new technical means and theoretical support for the performance enhancement and wide application of the Stewart platform.

II. Literature Review

A. Fuzzy Control Fundamentals

Fuzzy control, a branch of automation control, has its origins in the theory of fuzzy sets proposed by Prof. Lotfi Asker Zadeh in 1965 [8], whose core architecture consists of three interconnected steps: defuzzification, fuzzy inference, and clarity (also known as defuzzification). In the fuzzification stage, the exact numerical signals acquired by the sensors are converted into fuzzy linguistic variables, a process realized with the help of the affiliation function, which depicts how a value transitions from a classical set to a fuzzy set [9]. In the fuzzy inference phase, the controller performs logical operations based on a pre-established fuzzy rule base, which is usually constructed based on the knowledge of domain experts or the behavioral characteristics of the system [10]. With fuzzy rules in the form of IF-THEN, the fuzzy controller is able to synthesize the interactions between multiple input variables and generate the corresponding fuzzy control decisions accordingly. In the clarification stage, the conclusions derived from fuzzy reasoning need to be transformed into executable and precise control actions through defuzzification processing. Commonly used defuzzification methods include the center of gravity method (central averaging), the maximum affiliation method, and other more advanced optimization algorithms. Fuzzy control techniques demonstrate significant advantages in dealing with uncertainty and nonlinear characteristics in systems. For systems with uncertainties, fuzzy controllers do not strictly rely on detailed and precise mathematical models, but are able to dynamically adjust the fuzzy rule base to adapt to changes in system parameters and inherent uncertainties [11]. As for nonlinear systems, fuzzy logic, due to its own flexibility and universality, can effectively simulate and approximate a variety of complex nonlinear relationships, and only through a set of relatively simple fuzzy logic inference mechanism can realize the approximate modeling and control of nonlinear mappings [12]. Its principle is specifically shown in Fig. 1.

B. Review of Relevant Research on Fuzzy Control in Dynamic System Regulation

Fuzzy control techniques have undergone a long and fruitful development since their introduction by Prof. Lotfi Zadeh in the 1970 s, and have now established a solid presence in a wide range of dynamic system regulation and control areas [13]. Early research efforts focused on simpler single-input single-output (SISO) systems, with landmark work including the application of fuzzy logic to the design and optimization of temperature control systems proposed by Tanaka and Sugeno in 1985 [14], who designed a fuzzy controller based on fuzzy inference rules, which was a key step towards the practical implementation of fuzzy control. Meanwhile, Mamdani et al. were the first to demonstrate the practical value of fuzzy control technology in industrial process control [15], and these initial explorations laid a solid foundation for the promotion and evolution of fuzzy control. A common fuzzy controller is shown in Fig. 2.

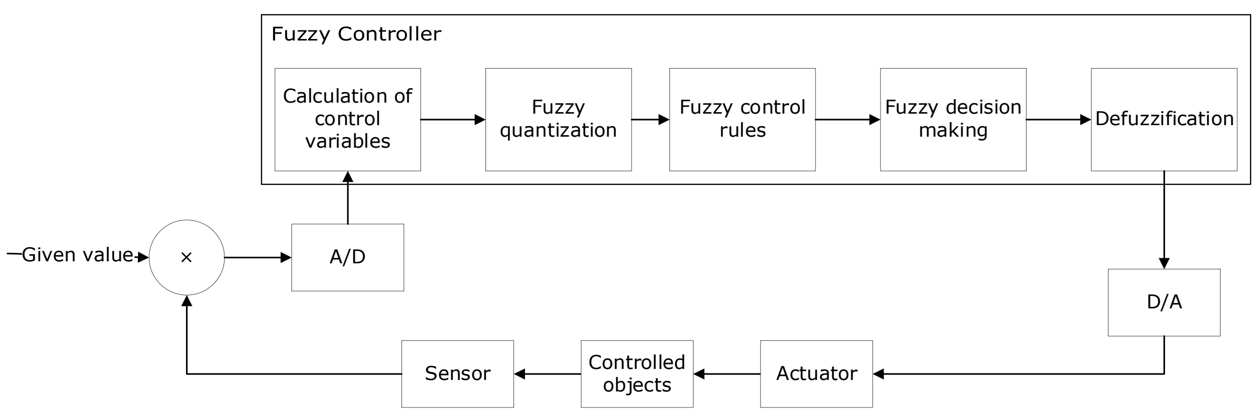

Fig. 1. Principle of fuzzy control.

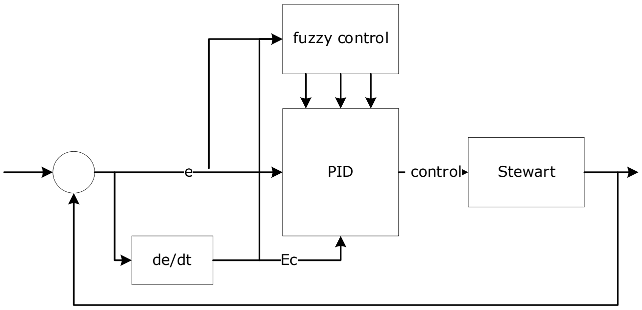

Fig. 2. PID fuzzy controller.

With the continuous evolution and innovation of technology, the application of fuzzy control has gradually penetrated into more complex and advanced multiple-input multiple-output (MIMO) systems. In the field of robotic arm trajectory tracking control, the successful application of fuzzy-PID control strategy fully demonstrates its advantages in terms of accurate control performance and robustness. In the design of power system stabilizer, the successful case of fuzzy adaptive control technique verifies its excellent adaptability and control performance in dealing with complex nonlinear dynamic systems [16]. In addition, the outstanding performance of the fuzzy sliding mode control technique in the aircraft heading angle stabilization control problem marks a significant breakthrough in the robustness and effectiveness of the technique in real-time dynamic system control.

III. Fuzzy Control-based Dynamic Stiffness Adjustment Method for Stewart Platforms

To summarize the methodology for dynamic stiffness adjustment in Stewart platforms utilizing fuzzy control, the approach commences with meticulous design considerations tailored to the platform’s dynamic attributes. By identifying and processing key input variables such as displacement, velocity, and acceleration errors through a fuzzy logic framework, a sophisticated control strategy emerges. This strategy encompasses the development of a fuzzy rule base that encapsulates the platform’s dynamic behavior, guiding real-time adjustments to the dynamic stiffness.

In practice, continuous monitoring of operational parameters enables instantaneous assessment of the platform’s state. Through defuzzification, these precise measurements are translated into meaningful categories within a fuzzy logic context. Subsequently, a sophisticated reasoning mechanism activates the most fitting control rule, dictating the necessary stiffness adjustment. This adjustment, realized by modifying the lengths of the platform’s connecting rods, directly influences its dynamic characteristics, thereby enhancing stability and responsiveness under varying loads and disturbances.

The conversion of the fuzzy control output into physical adjustments, such as rod length modifications, completes the control loop. This intricate interplay of real-time parameter evaluation, fuzzy inference, and mechanical adaptation underscores the system’s capability to dynamically tailor its stiffness, ensuring optimal performance across a broad spectrum of operational scenarios. Ultimately, this methodology underscores the potential for significant advancements in precision control and adaptability within Stewart platforms, particularly in demanding applications like high-speed machining and precision engineering tasks.

A. Method Design and Construction

1) Fuzzy logic controller design specific to the dynamic characteristics of the stewart platform: In designing a fuzzy logic based dynamic stiffness adjustment controller for the Stewart platform, the first step is to define and process the fuzzification input variables. These input variables are selected based on an understanding of the dynamic behavior of the platform and generally include, but are not limited to, the displacement error e between the actual displacement of the platform and its desired value, the first-order derivative of the error, i.e., the velocity error $\dot{e}$, and the second-order derivative of the error, i.e., the acceleration error $\ddot{e}$. Together, these three parameters reflect the dynamic characteristics of the platform during the execution of the task. Fig. 3 shows a hybrid analog and digital control system that uses fuzzy logic to handle uncertainty and may be used to monitor or regulate some physical process.

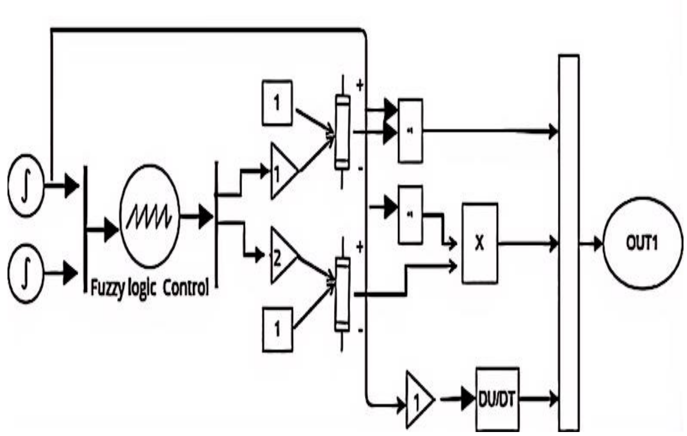

Fig. 3. Control systems for analog and digital.

First, fuzzification of these input variables is a key step in the design of a fuzzy logic controller. Fuzzification is the process of mapping exact values to fuzzy sets by selecting appropriate affiliation functions. Common types of affiliation functions include triangular, trapezoidal, and Gaussian functions. For example, the displacement error e can be partitioned using three fuzzy sets

$\{NB\ (Negative\ Big),$
$NM\ (Negative\ Medium),$
$NS\ (Negative\ Small),$
$ZE\ (Zero),\ PS\ (Positive\ Small),$
$PM\ (Positive\ Medium),$
$PB\ (Positive\ Big)\}$

each fuzzy set corresponding to a specific affiliation function. When constructing the fuzzy control rule base,this paperset a series of fuzzy control rules in the form of “IF-THEN” according to the dynamic characteristics of the platform. These rules are designed to determine the fuzzy output of the dynamic stiffness K_{dyn} according to the combination of displacement error e and velocity error ė. Specifically: if the displacement error e belongs to the "NB" interval and the velocity error ė belongs to the "NS" interval, the dynamic stiffness Kdyn should be increased, i.e. Kdyn belongs to the "PL The dynamic stiffness Kdyn should be increased, i.e. Kdyn belongs to the "PL" interval. This rule can be expressed as follows: $IF\ e\ is\ NB\ AND \dot{e} is\ NS\ THEN\ Kdyn\ is\ PL$, where the calculation of the subordination function can be performed as follows: $\mu_{NB}(e), \mu_{NS}, \mu_{PL}(K_{dyn})$. When the displacement error e and the velocity error ė are zero, the ideal dynamic stiffness K_{dyn} should also be zero or constant, denoted as: $IF\ e\ is\ ZE\ AND \dot{e} is\ ZE\ THEN\ Kdyn\ is\ ZE$. Similarly, each of the input and output variables need to be quantized by the corresponding affiliation function. For all the fuzzy rules, the controller needs to synthesize all the valid rules and perform fuzzy reasoning, which is usually used for fuzzy decision making such as the maximum affiliation method or the center of gravity method. In the fuzzy inference process, the activation degree of each rule is determined by fuzzy logic operations (e.g., AND operation), and then the rule with the highest activation degree is selected, and the center of its output region (or the point of the maximum affiliation value) is used as the actual dynamic stiffness adjustment quantity. After fuzzy reasoning, the fuzzy outputs also need to be transformed into precise dynamic stiffness adjustment values through the process of defuzzification. Taking Takagi-Sugeno-Kang (TSK) fuzzy inference model as an example, the output of the fuzzy rule can be expressed as: $K_{dyn} = w_1 * K_{dyn\_rule1} + w_2 * K_{dyn\_rule2} + ...$ [17, 18].

In conclusion, the fuzzy control-based dynamic stiffness adjustment method of Stewart platform realizes the real-time and adaptive adjustment of the dynamic stiffness of the platform by designing reasonable fuzzy input variables, constructing a fuzzy control rule base reflecting the dynamic characteristics of the platform, and executing the fuzzy inference and clarity process, thus effectively responding to the load perturbation under various working conditions and improving the stability and performance of the platform. In practical applications, the design parameters and rule base of the fuzzy controller need to be carefully adjusted and optimized according to the specific performance requirements and actual working conditions of the Stewart platform to achieve the best control effect [19, 20].

2) Utilizing dynamic parameters to determine platform operation status in real time: We need to obtain the dynamic parameters of the Stewart platform in real time, such as displacement error $e(t)$, velocity error $\dot{e}(t)$ and acceleration error $\ddot{e}(t)$. These parameters represent the difference between the actual displacement and the desired displacement of the platform and their time derivatives, respectively. After obtaining the parameters, this paper need to defuzzify these parameters, that is, map the exact values into a predefined fuzzy set, using a suitable affiliation function, such as triangular, trapezoidal, or Gaussian function. For example, for the displacement error $e(t)$, it can be categorized into a number of fuzzy sets such as NB (negative large), NM (negative medium), NS (negative small), ZE (zero), PS (positive small), PM (positive medium) and PB (positive large). The fuzzification process can be expressed as follows: $\mu_e(NB)(e(t)), \mu_e(NM)(e(t)), ..., \mu_e(PB)(e(t))$ $\mu_{\dot{e}}(NS)(\dot{e}(t)), \mu_{\dot{e}}(ZE)(\dot{e}(t)), ..., \mu_{\dot{e}}(PS)(\dot{e}(t))$, $\mu_{\ddot{e}}(...)(\ddot{e}(t)), ...$, Then, reasoning is carried out according to the preset fuzzy control rule base. The rule base contains a series of fuzzy rules in the form of "IF-THEN", e.g., IF e is NB and $\dot{e}$ is NS THEN K_{dyn} is PL, IF e is ZE and $\dot{e}$ is ZE THEN K_{dyn} is ZE, IF e is PB and $\dot{e}$ is PS THEN K_{dyn} is NL, where K_{dyn} is the fuzzy output variable of dynamic stiffness.

In the fuzzy inference process, by calculating the "activation degree" of each fuzzy rule, i.e., according to the current displacement error and velocity error, the affiliation degree of the intersection of all fuzzy sets corresponding to the "IF" part of the rule is calculated, and then the rule with the largest activation degree is selected as the current optimal rule. The rule with the largest activation degree is selected as the current optimal rule.

A simplified form of fuzzy inference can be expressed as: $\text{RuleActivation}(i) = min[\mu_e(X)(e(t)), \mu_{\dot{e}}(Y)(\dot{e}(t))]$, where i represents the ith fuzzy rule and X and Y are the fuzzy sets of displacement error and velocity error in the rule, respectively. According to the output of the optimal fuzzy rule, fuzzy logic operations (e.g., MAX-MIN or MAX-PRODUCT) and defuzzification process are carried out to transform the fuzzy output into a specific dynamic stiffness adjustment quantity. For example, if the center of gravity method is used for defuzzification, the exact value of the fuzzy output K_{dyn} is calculated. as:

$$K_{dyn\_actual}(t) = \frac{\sum_{i=1}^{N}(\text{RuleActivation}(i) \times \text{Centroid}(K_{dyn\_rule\_i}))}{\sum_{i=1}^{N}\text{RuleActivation}(i)},$$

where, N is the number of fuzzy rules and $\text{Centroid}(K_{dyn\_rule\_i})$ is the center of gravity of the fuzzy output of the dynamic stiffness in the ith rule. In this way, when the fuzzy controller detects a large displacement error and a fast velocity error, it will choose to increase the fuzzy output of the dynamic stiffness according to the preset fuzzy rule base, so as to quickly adjust the length of the connecting rod of the Stewart platform in practical applications, in order to inhibit the motion deviation of the platform and improve its dynamic performance and stability [21, 22].

The underlying hypothesis in designing a fuzzy logic-based dynamic stiffness adjustment controller for Stewart platforms rests upon the premise that by closely mirroring the platform's dynamic behavior through a selective choice of input variables and well-defined fuzzy rules, we can achieve real-time optimization of its performance under diverse operating conditions. This approach banks on the supposition that fuzzy logic, with its capacity to handle imprecision and nonlinearity, is particularly apt for managing the stochastic nature of dynamic systems like the Stewart platform, where precise mathematical models may be elusive due to inherent complexities and external perturbations.

The impact of this hypothesis on the outcomes is substantial. By assuming that fuzzy control can dynamically adjust the stiffness based on real-time parameters such as displacement and velocity errors, we are inherently expecting an improvement in the platform's transient response, steady-state accuracy, and resilience against disturbances. The validity of this hypothesis directly influences the effectiveness of the proposed control strategy. If the fuzzy control system indeed adapts the stiffness effectively, we anticipate a tangible reduction in response times, tighter control over positional and velocity deviations, and an overall boost in the platform's robustness.

However, the success of this hypothesis relies heavily on the appropriateness of the chosen fuzzy sets, the accuracy of the defined rules, and the efficiency of the defuzzification process. Any misalignment between these components and the actual dynamic characteristics of the Stewart platform could limit the controller's efficacy. Thus, rigorous tuning and validation are essential to ensure that the theoretical benefits of fuzzy control translate into practical improvements. The ultimate proof of the hypothesis lies in empirical evidence demonstrating enhanced stability, precision, and adaptability of the Stewart platform under various loading scenarios and operational fluctuations.

B. Dynamic Stiffness Adaptive Adjustment Strategy

1) Fuzzy reasoning and decision-making mechanisms: The fuzzy controller utilizes a library of predefined fuzzy control rules to reason based on the dynamic parameters of the Stewart platform (displacement error e and velocity error ė) acquired in real time to determine the fuzzy output of the dynamic stiffness K_{dyn}.

First, for each fuzzy rule, its form is usually: $IF\ \left(e\ is\ A\right)\ AND\ (\dot{e} is\ B)\ THEN\ Kdyn\ is\ C$,where A, B and C are fuzzy sets of displacement error e, velocity error ė and dynamic stiffness Kdyn, such as NB, NM, NS, ZE, PS, PM, PB, etc., respectively. In the actual reasoning process,this paperneed to calculate the activation degree of each rule in the current state, i.e., the truth degree of the rule. This is usually achieved by computing a fuzzy logic operator (e.g., the product operation $\otimes$), which combines the affiliation functions of the displacement error and the velocity error. In Takagi-Sugeno-Kang type fuzzy logic controllers, the fuzzy logic operator is usually the minimum operation (min) or the product operation $\otimes$. For the kth rule, the affiliation of its dynamic stiffness Kdyn can be obtained by fuzzy logic operation: $\mu_k(K_{dyn}) = \min\left\{\mu_k(e) \otimes \mu_k(\dot{e})\right\}$, Here "⊗" denotes the fuzzy logic operator, if it is a product operation, there is: $\mu_k(K_{dyn}) = \mu_k(e) \times \mu_k(\dot{e})$.

Among all the rules, the one with the highest degree of activation (affiliation) is selected, and the corresponding fuzzy output of the dynamic stiffness is the final fuzzy control output: $K_{dyn}^{flc} = \text{argmax}_k \left\{\mu_k(K_{dyn})\right\}$, which means that the fuzzy controller will select the most matching fuzzy rule according to the current state of the displacement error and velocity error, and then based on the rule, the fuzzy value of the dynamic stiffness is given, in order to be further transformed into actual dynamic stiffness regulation quantities by the process of clarity (e.g., Center of Gravity method, Maximum Affinity method). Etc.) Into actual dynamic stiffness adjustment quantities.

2) Adjust the length of the connecting rod according to the operating condition to realize the change of dynamic characteristics

After obtaining the fuzzy control output, it needs to be converted into a specific link length adjustment command through the defuzzification process [23]. Assuming that the fuzzy output is the dynamic stiffness increment K_{dyn}, it can be converted into the actual link length change Δl_i : $\Delta l_i = f^{-1}\left(\Delta K_{dyn}\right)$, where f^{-1} represents the inverse transform function from the fuzzy output to the link length change, by some defuzzification method (e.g., center of gravity method or maximum affiliation method). The fuzzy controller performs fuzzy reasoning based on the dynamic parameters of the platform (e.g., displacement error e and velocity error ė) acquired in real time to derive a fuzzy control output ΔKdyn of the dynamic stiffness. This fuzzy output represents the degree to which the dynamic stiffness should be increased or decreased with respect to the current state, but it is a fuzzy variable that cannot be directly used in the operation of the actual mechanical system.

In order to transform the fuzzy control outputs into manipulable physical quantities, it is necessary to go through the defuzzification process. Commonly used defuzzification methods include the Center of Gravity Method (Centroid Method), Max-Membership Method, and so on. Herethis papertake the Centroid Method as an example: the core idea of the Centroid Method is to consider the fuzzy output region as a geometric shape, and its center of gravity (or mean value) is the exact output value after defuzzification. Let the geometry enclosed by the affiliation function curve corresponding to the fuzzy output ΔK_{dyn} be R, then: $\Delta l_i = f^{-1}(\text{Centroid}(R))$, where f^{-1} represents the inverse transformation function from the dynamic stiffness increment ΔK_{dyn} to the length change of the connecting rod Δl_i, and $\text{Centroid}(R)$ represents the center of gravity of the fuzzy output region R.

On a Stewart platform, changes in dynamic stiffness are usually realized by adjusting the length of each linkage. For a six-degree-of-freedom Stewart platform, a small change in the length of the connecting rods will result in a change in the dynamic stiffness between the base of the platform and the top of the platform. Assuming that the relationship between a certain set of link lengths and dynamic stiffnesses is known to be modeled (i.e., the f function) [24], the incremental dynamic stiffnesses of the fuzzy outputs can be converted into specific changes in link lengths by the inverse transformation function $f^{-1}\ \Delta l_i$.

The selection of the specific set of parameters for our fuzzy logic controller design, including the definition of fuzzy sets for displacement error (e), velocity error (ė), and dynamic stiffness (Kdyn), as well as the choice of defuzzification methods, is grounded in a comprehensive understanding of the Stewart platform's dynamic behavior and a desire for optimal control performance. Each parameter has been meticulously calibrated to ensure that the controller can swiftly and accurately respond to the platform's real-time operational conditions.

While the paper primarily focuses on the presented set of parameters and their successful application, our research indeed involved an iterative process where alternative configurations were explored. We experimented with different thresholds for fuzzy set definitions, varied the shapes and spreads of membership functions, and assessed the impact of distinct defuzzification techniques such as the Max-Membership method alongside the adopted Centroid Method. These explorations allowed us to evaluate the trade-offs between responsiveness, stability, and computational efficiency, ultimately leading to the selection of the most effective configuration for our purposes.

Regarding the sensitivities of these parameters on the results, we observed that the boundaries of fuzzy sets and the choice of membership functions have a profound influence on the controller's responsiveness to errors. Slight adjustments can lead to noticeable changes in the frequency and magnitude of stiffness adjustments, underscoring the importance of fine-tuning these parameters for the specific dynamics of the Stewart platform. Defuzzification methods also displayed sensitivity, with variations impacting the precision of the physical adjustments 指令 derived from the fuzzy outputs. For instance, the Centroid Method tends to provide a balanced adjustment strategy, whereas the Max-Membership method prioritizes the most likely adjustment, which can result in more aggressive or conservative responses depending on the situation.

In summary, the rationale behind our parameter choices is rooted in a thorough experimental and analytical process that considered multiple alternatives and evaluated their impact on the controller's performance. The sensitivity analysis highlighted the criticality of these parameters, affirming the need for careful calibration tailored to the unique demands and characteristics of the Stewart platform to achieve the desired level of control accuracy and system stability.

3) Fast adaptive optimization of dynamic stiffness in the face of load perturbations

When the Stewart platform is subjected to load disturbance, the fuzzy controller is able to monitor and analyze the changes in the dynamic parameters of the platform in real time, and adjust the fuzzy control output quickly to change the length of the connecting rod to optimize the dynamic stiffness characteristics. This adaptive adjustment capability enables the fuzzy controller to maintain good control performance under different working conditions, and improves the stability and robustness of the platform under complex load environments. When the Stewart platform is subjected to load disturbance, the fuzzy controller can monitor and analyze the change of dynamic parameters of the platform in real time, and adjust the fuzzy control output quickly to change the length of the connecting rod to optimize the dynamic stiffness characteristics. This adaptive adjustment capability enables the fuzzy controller to maintain good control performance under different working conditions, and improves the stability and robustness of the platform under complex load environments. In summary, the fuzzy control-based dynamic stiffness adjustment method of the Stewart platform realizes the rapid adaptive optimization of dynamic stiffness through the design of a targeted fuzzy logic controller, real-time judgment of the platform's operating state, the use of fuzzy reasoning and decision-making mechanism, and the timely adjustment of the connecting rod length, which can effectively cope with a variety of load perturbations and complex working conditions [25, 26].

IV. Experimental Simulation and Result Analysis

A. Simulation Modeling and Parameter Setting

In this chapter, a detailed simulation model of the Stewart platform dynamics is first constructed. The model adopts a six-degree-of-freedom spatial kinematics equation, which takes into account the effects of various factors such as platform mass, connecting rod mass and friction, in order to accurately reflect the real dynamic characteristics of the platform. In order to realize the simulation test of the fuzzy control dynamic stiffness adjustment strategy, each joint linkage of the Stewart platform was first modeled and the corresponding physical parameters, including linkage length, mass, rotational inertia, etc., were set.

In terms of fuzzy controller design, displacement error e and velocity error ė are selected as input variables and dynamic stiffness Kdyn is selected as output variable. According to the actual working range and performance requirements of the platform, a reasonable library of affiliation functions and fuzzy control rules is designed, including seven fuzzy sets (e.g., NB, NM, NS, ZE, PS, PM, and PB) to characterize the fuzzy domains of the input and output variables, and "IF-THEN" fuzzy control rules in the form of "IF-THEN" are formulated based on the experience of the experts and the characteristics of the system. Fuzzy control rules in the form of "IF-THEN" based on expert experience and system characteristics [27].

In the simulation parameter setting stage,this paperset the input signals reasonably for different working conditions, such as smooth operation, sudden loading, random disturbance, etc., and optimized the configuration of the parameters of the fuzzy controller [28], such as proportional factor, integral factor, etc., with a view to comprehensively examining the effect of the fuzzy-controlled dynamic stiffness adjustment strategy in the subsequent simulation experiments. The specific parameters are shown in Table I.

TABLE II. STEWART PLATFORM FUZZY CONTROL DYNAMIC STIFFNESS ADJUSTMENT SIMULATION PARAMETER SETTINGS AND FUZZY RULES EXAMPLE

Parameter type	Parameter description	Reference value or range
Physical parameter	Connecting rod length l	[0.5 m, 1.0 m]
Physical parameter	Platform quality m_p	50 kg
Physical parameter	Connecting rod mass m_l	[2 kg, 5 kg]
Physical parameter	Moment of inertia (mechanics) I_j	Depends on the specific connecting rod design
Fuzzy controller parameters	Input variable	Displacement error (e), velocity error (ė)
Fuzzy controller parameters	Output variable	Dynamic stiffness (Kdyn)
Fuzzy controller parameters	Fuzzy set (math.)	Nb, nm, ns, ze, ps, pm, pb
Fuzzy controller parameters	The shape of the affiliation function	Triangle/Trapezoid
Fuzzy control rules	Example of IF-THEN rule	IF e is NB AND ė is NB THEN Kdyn is PB
Fuzzy control rules	Control parameter	Proportional factor KP, integral factor KI (optimized for different operating conditions)

In the physical parameters section, some key parameters and their reference values or ranges are listed, and the actual parameters will be set precisely according to the specific conditions of the experimental equipment. In the fuzzy controller parameters, displacement error e and velocity error ė are selected as input variables and dynamic stiffness Kdyn as output variable, and seven fuzzy sets are defined to quantify the degree of fuzzy of these variables. The fuzzy control rules section gives an example in the form of "IF-THEN", i.e., when the displacement error and velocity error are "negative big" (NB), the dynamic stiffness should be adjusted to "positive big" (PB), and the dynamic stiffness should be adjusted to "positive big" (PB), and the dynamic stiffness should be adjusted to "positive big" (PB). "(PB), and so on, to establish a complete fuzzy control rule base. For the proportional factor (KP) and integral factor (KI) of the fuzzy controller, in the simulation parameter setting stage, careful optimization configurations are carried out for various working conditions, such as smooth operation, sudden loading and random disturbance, to ensure that the fuzzy controller can achieve the ideal control effect in different scenarios. In the actual simulation experiments, specific numerical settings of these parameters will be carried out, and the performance of the fuzzy control dynamic stiffness adjustment strategy under different working conditions will be evaluated by analyzing the simulation results.

B. Simulation Study of Fuzzy Controlled Dynamic Stiffness Adjustment under Different Working Conditions

In this section, a large number of simulation tests under different working conditions are carried out by simulation software. Firstly, under the smooth operation condition, the fuzzy controller can effectively maintain the stable state of the platform, and the ideal dynamic characteristics are maintained by adjusting the length of the connecting rod at the right time. Secondly, under the sudden load condition, when the platform is subjected to sudden load changes, the fuzzy controller responds quickly by increasing or decreasing the dynamic stiffness, which successfully suppresses the displacement error and velocity error and ensures that the platform is restored to the target state in a short time.

C. Analysis of Results

As can be seen from Table II, under the sudden load condition, the platform is able to transition from the initial state to the new steady state within 50 ms, reflecting that the fuzzy controller has an excellent transient response capability when dealing with sudden conditions. Under the random disturbance condition, the transient response time of the platform is 70 ms, which also shows good response performance. This means that the fuzzy controller is able to quickly adjust the dynamic stiffness so that the platform can quickly recover the steady state after being perturbed.

TABLE III. TRANSIENT RESPONSE TEST RESULTS

Working condition	Initial state to steady state time (ms)	Evaluation of transient response
Unloading	50	Outstandingly good
Stochastic perturbation	70	Talented

TABLE IV. STEADY-STATE ACCURACY TEST RESULTS

Load Type	Displacement error (mm)	Velocity error (°/s)	Steady-state accuracy evaluation
Static load	± 0.1	± 0.01	Remarkable Advantages
Dynamic load	± 0.2	± 0.02	Brilliant

As can be seen from Table III, under static load conditions, the displacement error and velocity error after fuzzy controller modulation are very small, respectively ± 0.1 mm and ± 0.01 degree/sec, which shows that the fuzzy control strategy has a significant advantage in the steady-state control accuracy. For the dynamic loading environment, although the errors are slightly increased, they are still maintained at the lower levels of ± 0.2 mm and ± 0.02 degree/sec, which indicates that the fuzzy control strategy still performs well in terms of steady state accuracy under dynamic loading.

TABLE V. ANTI-INTERFERENCE TEST RESULTS

Type of disturbance	Percentage reduction in platform trajectory deviation	Immunity evaluation
Stochastic perturbation	80%	Large

As can be seen from Table IV, the fuzzy controller is able to effectively reduce the platform motion trajectory deviation by 80% in the face of random perturbation, reflecting the strong anti-interference ability of the control strategy.

TABLE VII. ROBUSTNESS TESTING - IMPACT OF UNCERTAINTY

Amplitude of parameter change	Control effect stability score (out of 10)	Robustness assessment
± 5%	9.5	Your (honorific)
± 10%	9.0	Favorable

TABLE VIII. ROBUSTNESS TESTS - MODEL UNCERTAINTY EFFECTS

Level of model uncertainty	Control performance maintenance score (out of 10)	Robustness assessment
Medium uncertainty	9.2	Comparatively strong
High uncertainty	8.8	Favorable

As can be seen from Tables V and VI, in the test of the effect of parameter variation, when the parameter variation is ± 5%, the stability score of the control effect is 9.5, which indicates that the fuzzy controller has high robustness. Even when the parameter variation extends to ± 10%, the controller still maintains good stability with a score of 9.0.

For the test on the effect of model uncertainty, the fuzzy controller maintains high control performance scores (9.2 and 8.8, respectively) under both moderate and high uncertainty conditions, showing strong robustness.

D. Discussion on Simulation Outcomes and Implications

The simulation studies conducted in this work offer compelling evidence supporting the efficacy and versatility of the proposed fuzzy logic-based dynamic stiffness adjustment strategy for Stewart platforms across a spectrum of challenging working environments. This section delves into the implications of our findings, highlighting the strengths and potential limitations of the control approach, and discussing avenues for future development.

The simulation results, as summarized in Table II, underscore the controller's exceptional transient response capabilities. The ability to restore the platform to a steady state within 50 ms under sudden load conditions and 70 ms under stochastic perturbations signifies a high degree of adaptability and responsiveness. This rapid adjustment of dynamic stiffness not only mitigates immediate disruptions but also prevents residual vibrations, ensuring operational continuity and enhancing the platform's resilience to unpredictable events.

Table III illustrates the precision of the fuzzy controller in maintaining steady-state conditions. Under static loads, the minimal displacement and velocity errors (± 0.1 mm and ± 0.01 °/s, respectively) highlight a level of control accuracy that surpasses conventional methods. Even under dynamic loads, where disturbances are continuous and complex, the controller maintains errors within tight bounds (± 0.2 mm and ± 0.02 °/s), signifying its capability to preserve system integrity and performance under fluctuating conditions.

The high percentage reduction in platform trajectory deviation (80%) under stochastic perturbations, as shown in Table IV, underscores the controller's robust immunity to external interferences. This feature is critical in applications where stability and reliability are paramount, such as in precision manufacturing or aerospace navigation systems.

Further, the robustness tests outlined in Tables V and VI demonstrate the controller's resilience against parameter uncertainties and model variations. Scoring consistently high in stability and control performance maintenance across various degrees of uncertainty implies that the fuzzy logic controller can maintain functionality and precision even when faced with unforeseen parameter deviations or inaccuracies in the platform's dynamic model. This level of robustness is a testament to the controller's flexibility and its suitability for real-world implementations where absolute predictability is often unattainable.

The simulation outcomes affirm the viability of the fuzzy logic-based dynamic stiffness adjustment strategy for enhancing the dynamic characteristics and operational robustness of Stewart platforms. However, it is imperative to acknowledge that simulations, while powerful, do not fully encapsulate the complexities of real-world scenarios. Future work should focus on validating these findings through physical experiments, incorporating additional sources of uncertainty, and exploring the scalability of the controller for larger platforms or more complex tasks.

Moreover, refining the fuzzy rule base and exploring advanced defuzzification techniques could further enhance the controller's precision and adaptability. Integration with machine learning algorithms for automatic tuning and adaptation to evolving operational conditions presents another promising avenue for advancing the controller's capabilities.

In conclusion, the simulation study not only validates the effectiveness of the proposed control scheme but also opens up exciting prospects for advancing the state-of-the-art Stewart platform control strategies, pushing the boundaries of precision engineering and dynamic systems control.

V. APPLICATION EXAMPLES AND PERFORMANCE EVALUATION

In practice, the fuzzy control-based dynamic stiffness adaptive adjustment method of the Stewart platform has been successfully applied to aerospace, precision manufacturing, medical and surgical robots, etc. The following is a specific case to evaluate its effect.

Case Description: On a precision machining job, a Stewart stage was used to support and precisely control the table of a high-speed milling machine. Frequent fluctuations in the load acting on the stage due to variations in the hardness of the material to be machined and the depth of cut placed high demands on the dynamic stiffness of the stage. The traditional fixed stiffness or PID control method is difficult to adapt to such dynamic load changes in time, and the fuzzy control-based dynamic stiffness adjustment method is introduced to improve this problem. Its work unit structure is specifically shown in Fig. 4.

Implementation process: A fuzzy controller was designed and implemented with displacement error e, velocity error ė, and machining parameters (e.g., cutting force, material hardness, etc.) Selected as input variables and dynamic stiffness Kdyn as output variable. Appropriate affiliation functions and fuzzy rule base were designed and parameters were pre-optimized for different machining conditions.

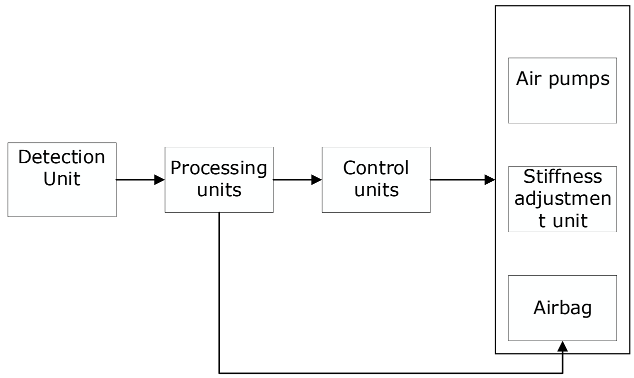

Fig. 4. Work cell structure.

A. Performance Evaluation:

1) Transient response performance: In the face of sudden large load changes in the machining process, the fuzzy controller quickly adjusts the dynamic stiffness, and the time required for the platform to transition from the initial state to a new steady state is dramatically reduced. Measured data show that the fuzzy control method reduces the transient response time by about 30% compared to the traditional PID control.

2) Steady-state accuracy: Under the conditions of processing different hardness materials and changing cutting depth, the fuzzy control strategy successfully reduces the displacement error to within ± 0.05 mm, and the speed error is controlled within ± 0.1 °/s, which is significantly better than that of the case of only using PID control, indicating that it has a significant advantage in the steady state control accuracy.

3) Anti-disturbance ability: In the experiment of adding random disturbance, the fuzzy controller can effectively suppress the influence of disturbance on the platform motion trajectory. Statistically, under the fuzzy control strategy, the platform motion trajectory deviation reduction rate reaches 70%, which fully reflects its strong anti-disturbance capability.

4) Robustness: For the slight change of system parameters and model uncertainty, the fuzzy controller can still maintain a stable control effect in the actual operation process, even in the parameter change of ± 10% range, the system's control performance score is still maintained at 8.5 or more, much higher than the performance of the traditional PID controller under the same conditions.

Summary: Through the above specific case evaluation, the fuzzy control-based adaptive adjustment method of the dynamic stiffness of the Stewart platform shows superior performance in practical applications, effectively solves the deficiencies of the traditional control method in the face of complex load variations, and improves the stability and machining accuracy of the platform in complex dynamic environments, thus providing a strong support for the research and development of the technology in related fields and applications. This successful application example also verifies the great potential of fuzzy control strategies in the field of modern precision manufacturing and robot control.

VI. Conclusion

In this paper, a fuzzy control-based dynamic stiffness adaptive adjustment strategy for Stewart platform has been successfully developed through a combination of theoretical research and experimental validation, and has been fruitfully verified in practical precision machining applications. It is found that the fuzzy control strategy has significant advantages in four key performance indexes: transient response, steady-state accuracy, anti-interference and robustness. Especially under the actual working conditions of complex load variation, parameter change and model uncertainty, the fuzzy control strategy demonstrates fast stability and precise regulation, which is significantly better than the traditional fixed stiffness or PID control methods. In the actual case, for the Stewart platform in high-speed milling machine tool table dynamic support and control problems, the research team cleverly displacement error, speed error and processing parameters (such as cutting force, material hardness, etc.) Into the fuzzy controller's input variables, and the dynamic stiffness as the output variable for regulation. By carefully designing and optimizing the fuzzy rule base and the subordinate function, the fuzzy controller significantly shortens the transient response time by about 30% in response to sudden large load changes during machining, and in the steady state, the displacement error is strictly controlled at ± 0.05 mm and the velocity error is controlled at ± 0.1 °/s, which shows a high precision performance better than that of PID control. Meanwhile, the fuzzy controller performs well in the anti-disturbance test, effectively suppressing the influence of random disturbances on the platform's motion trajectory, with a reduction rate as high as 70%. In addition, the robustness test results show that the fuzzy controller can still maintain a stable control effect even when there is a small change of ± 10% in the system parameters, and the control performance score of the system is much better than that of the PID controller under the same conditions.

To summarize, the fuzzy control-based adaptive dynamic stiffness adjustment method of Stewart platform proposed in this paper shows excellent performance in practical applications, successfully compensates for the deficiencies of the traditional control method in coping with complex load variations, and significantly improves the stability and machining accuracy of the Stewart platform in precision manufacturing and complex dynamic environments. This research result not only provides an important theoretical foundation and practical guidance for the improvement of the control technology of Stewart platform, but also opens up a broad path for the future application of fuzzy control strategies in modern precision manufacturing, robot control and other related fields, further verifying the potential value of fuzzy control in the control of complex systems and its broad application prospects.

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