Modification of the Danzig-Wolf Decomposition Method for Building Hierarchical Intelligent Systems

Turganzhan Velyamov[1], Alexandr Kim[2], Olga Manankova[3]
Department of Technical Disciplines, Kazakhstan University of Innovation and Telecommunication Systems, Almaty, Kazakhstan[1]
Department of Mechanics, Al-Farabi Kazakh National University, Almaty, Kazakhstan[2]
Department of Computer Engineering, International Information Technology University, Almaty, Kazakhstan[3]

***Abstract*—This article examines the Dantzig-Wolfe decomposition method for solving large-scale optimization problems. The standard simplex algorithm solves these problems, making the Dantzig-Wolfe method a valuable tool. The article describes in detail a new modification of the Dantzig-Wolfe decomposition method. This modification aims to improve the efficiency of the coordination task, a key component that defines subtasks. By significantly reducing the number of rows in the coordination problem, the proposed method achieves faster computation and reduced memory requirements compared to the original approach. Although the Dantzig-Wolfe method has encountered problems due to the complexity of implementing algorithms for hierarchical systems, this modification opens up new potential.**

***Keywords*—Decomposition method; optimization; parallel processing; linear programming**

I. Introduction

Hierarchical intellectual systems are a class of systems with multi-level architecture, where components of a lower level transmit information to components of a higher level for decision -making. Traditional methods for constructing such systems can be expensive in computational terms. The method of decomposition of Danzig-Wolfe is to take into account the search for the optimum of the initial task of linear programming, a large dimension with a diagonal structure with binding restrictions, to a sequential solution to a number of problems of a smaller dimension, followed by the adjustment of the obtained solution [1].

The research in [2] also discusses a method for decomposing linear programs by direct distribution. This approach is an extension or alternative to other decomposition methods such as Dantzig-Wolf decomposition, focusing on the practical aspects of solving large-scale linear programming problems by distributing computational effort.

Decomposition methods are often used in optimization problems, especially in the context of linear programming, to efficiently solve large-scale problems that would be computationally complex or infeasible using traditional methods. Such methods break a complex problem into smaller and more manageable subproblems. This separation allows for more efficient calculations and the use of problem structure to reduce overall complexity.

Solving linear programming problems is relevant. In [3]-[7] publications examine various aspects of linear optimization, providing theoretical foundations and practical approaches that are reflected in further research and selection of the optimal solution.

There is a mathematical programming language, which is described in detail in [8] for its syntax and application in formulating and solving mathematical programming models. It can be integrated with various solvers, allowing users to choose the most efficient solver for their particular problem. Although easy to use, learning the AMPL syntax and efficiently modeling complex problems can require considerable time and effort, and the performance of models in AMPL can be highly dependent on the capabilities of the chosen solver.

Since its inception in the early 1960s by George Dantzig and Philip Wolfe, the Dantzig-Wolfe decomposition method has stood as a paragon of operational research, pioneering the efficient solution of complex linear programming problems via decomposition into subproblems [9].

Therefore, in [10]-[15] an attempt is made to individually develop decomposition methods for energy models, which allows for the efficient handling of certain features such as network structures or resource constraints.

Despite its widespread adoption and proven efficacy, evolving computational demands and increasingly complex optimization problems have highlighted the need for enhancements, particularly concerning the method's computational efficiency in handling the coordinating problem. Also worth noting are modern solutions that combine reinforcement learning and other optimization methods to solve complex 2D irregular packing problems [16], use deep reinforcement learning, which allows to automatically learn and improve strategies for solving packing problems [17], [18], propose an effective method for solving quadratic programming problems, which are common in various fields [19]. Resource management in hierarchical systems is also crucial in large organizations or systems [20], emphasizes the importance of coordination, improving the overall performance of the system.

Therefore, in [21] the African buffalo mechanism is presented, a new metaheuristic for solving the traveling salesman problem (TSP), and in [22] focuses on optimizing path planning for service robots, which is crucial for efficiency in work settings. A hybrid approach [23] combines different clustering methods to exploit their strengths, potentially improving clustering accuracy. In [24] proposes a new method that combines binary search and merge sort, potentially improving search efficiency in unsorted datasets. In [24] Combines elements of differential evolution with discrete optimization methods, potentially improving performance. [25] presents the Dingo optimization algorithm, a novel metaheuristic approach. It is specifically designed to optimize power system stabilizers, which are critical to maintaining system stability. [26] identifies key gaps in the functionality of learning management systems (LMS), providing valuable insights for improving these systems. In [27] focuses on optimizing the placement and operation of distributed generation plants to minimize losses in electrical systems. Due to the large amount of data, detailed and high-quality data may be required for accurate modeling, which may not always be available. In [28] provides a linear programming approach to a classical combinatorial optimization problem, offering exact solutions. This approach may face scalability issues as the problem size increases. The efficient solution to the problems in [29]–[33] combines column generation with the branch and bound method, making it effective for large-scale integer programming problems.

By solving subproblems independently, such methods enable parallel processing, which can significantly reduce computation time for large-scale problems [34]-[35]. Although the subproblems are solved independently, the method also provides mechanisms to coordinate their solutions, providing a globally optimal solution to the original problem [36]-[39]. In general, decomposition methods are used because they offer a systematic and efficient way to solve large-scale optimization problems while providing efficient and accurate solutions.

This article introduces a significant modification aimed at addressing these challenges, emphasizing the method's pivotal role in operational research and its potential evolution to meet modern computational needs. Through an in-depth analysis, we explore the genesis of this modification, its theoretical foundation, practical applications, and the promising horizon it opens for future research and application in various domains.

Devised as a solution to the computational challenges posed by large-scale linear programming problems, the Dantzig-Wolfe decomposition method represents a critical milestone in the field of optimization. By partitioning a complex problem into a master problem and various subproblems, this method significantly streamlines the computational process, enabling independent management and resolution of problem components. This inherent flexibility and efficiency facilitated early successes in a range of applications, from logistics to network design, setting the stage for further innovations. However, as the complexity and scale of optimization problems have expanded, the method's limitations, particularly in the efficiency of coordinating solutions among subproblems, have become increasingly apparent. This realization has spurred the development of the proposed modification, which seeks to enhance the method's computational efficiency through algorithmic innovation and the integration of modern computational technologies [40].

The imperative for the Dantzig-Wolfe decomposition method, and by extension its proposed modification, lies in the unmet need for efficient solutions to large-scale optimization problems that exceed the capabilities of traditional linear programming techniques. These conventional methods often falter in the face of the immense scale and complexity characteristic of contemporary optimization challenges, rendering them computationally infeasible. The decomposition approach, therefore, emerges not only as a solution to these challenges but as a necessary evolution in the toolkit of operational research, enabling the practical resolution of problems previously considered beyond reach.

Spanning a diverse array of sectors, the Dantzig-Wolfe decomposition method's practical applications underscore its versatility and effectiveness in addressing complex optimization problems. From enhancing efficiency in transportation and logistics to optimizing network designs and streamlining supply chain management, the method's capacity to break down multifaceted problems into manageable subcomponents has been invaluable. The proposed modification promises to further amplify this capacity, offering enhanced computational efficiency that could broaden the method's applicability to even more complex and large-scale problems, thereby extending its utility in real-world scenarios.

The potential of the modified Dantzig-Wolfe method extends far into the future, promising exciting advancements in the field of operational research. With the integration of emerging technologies such as parallel computing, artificial intelligence, and advanced heuristics, the modification opens new avenues for optimizing the efficiency and applicability of the decomposition method. These advancements hold the promise of transforming the landscape of optimization problem-solving, offering more agile, efficient, and scalable solutions to the complex challenges that define the modern era.

II. METHODOLOGY

The Dantzig-Wulf method was an important tool for solving large structured models of optimization problems that could not be solved using the standard simplex algorithm. This article illustrates the algorithm of the modified Dantzig-Wulf decomposition method with an efficient, in terms of speed and stability of the computational process, a coordinating task developed by the author for solving problems of a linear programming problem with a block-diagonal structure with binding constraints.

Recently, due to the fact that the implementation of complex algorithms for the study of hierarchical systems, which place great demands on the method, not only from the point of view of the pure speed of the computational process and, from the point of view of the availability of large amounts of memory and, to the speed of the computational process for the formation of recommendations management of complex hierarchical systems under conditions of uncertainty, which led to the fact that the Danzig-Wolfe method became less popular.

In the original Danzig-Wulf decomposition method, the coordinating problem contains the number of rows equal to the sum of the number of equations in the linking constraint - m_0 and the number of block constraints - q. In the developed modification of the decomposition method, the coordinating task contains with contain (m_0+1) rows. Since it is the coordinating task that affects the solution of subtasks by changing the values of the coefficients of the objective function, then reducing the dimension of the coordinating task leads to an increase in the computational efficiency of the decomposition method in $(m_0+q)/(m_0+1)$, times compared to the original decomposition method. The experience of practical application of the decomposition method for solving problems of high dimension was insignificant and, in many cases, unsuccessful. The performed computational experiments for problems with matrix order from 90 to 700 showed that, in terms of the number of iterations to obtain the optimal plan, the proposed modification of the Danzig-Wulf decomposition method has the same convergence as the simplex method, but the requirements for computer memory are reduced, and the computational efficiency is increased in $(m_0+q)/(m_0+1)$ times.

III. Results and Discussion

Proposed modification below of decomposition method is a further its development [1]-[8], [11], [12], [16] aimed at improve the efficiency of the method to solve problems linear programming in the block diagonal structure with binding constraints of the following form.

$$z = C_1x_1 + \ldots + C_kx_k + \ldots + C_nx_n \ \min \quad (1)$$

$$A_1x_1 + \ldots + A_kx_k + \ldots + A_Nx_N = b_0 \quad (2)$$

$$\left.\begin{array}{llll} D_1x_1 & & & \geq b_1, \\ & \ldots & & \\ & D_kx_k & & \geq b_k, \\ & & \ldots & \\ & & D_Nx_N & \geq b_N, \\ x_i \geq 0, i = 1, \ldots, k \ldots, N & & & \end{array}\right\} \quad (3)$$

Taking as its first support program - an artificial basis to replace the task (1) - (3) of the extended task, associated with the minimization of a linear form.

$$z = C_1x_1 + C_2x_2 + \cdots + C_qx_q + \\ +W(\xi_1 + \xi_2 + \ldots + \xi_q + \xi_0) \quad (4)$$

variable problem, subject to conditions

$$A_1x_1 + A_2x_2 + \ldots + A_qx_q \qquad + E_0\xi_0 = b_0 \quad (5)$$

$$\left.\begin{array}{llllll} D_1x_1 & & & + E_1\xi_1 & & = b_1, \\ & D_2x_2 & & & +E_2\xi_2 & = b_2, \\ & & \ddots & & \ddots & \vdots \\ & & D_qx_q & & +E_q\xi_q & = b_q, \end{array}\right\} \quad (6)$$

$$x_1 \geq 0, x_2 \geq 0, \ldots, x_q \geq 0, \\ \xi_1 \geq 0, \xi_2 \geq 0, \ldots, \xi_q \geq 0, \xi_0 \geq 0.$$

Here $C_i = (C_{i1}, \ldots, C_{im_i})$ - line vector;

$b_i = (b_{i1}, \ldots, b_{im_i})$- vector - columns of the right sides of restrictions;

$\xi_i = (\xi_{i1}, \ldots, \xi_{im_i})$ - vector - columns of artificial variable;

$A_k = [a_{ij}]_{m_0,n_0}$, $D_k = [a^k_{ij}]_{m_k,n_k}$ - blocks of the matrix conditions;

E_i - single block matrix;

W - coefficients of artificial variables of the problem.

We will now present a modified version of decomposition method of Dantzig - Wolfe. Denoted by Ω - many plans, given the conditions (5) and (6), by Ω_1 - a lot of plans, given the conditions (5), Ω_2- a lot of plans, given the conditions (6).

It's obvious that $\Omega = \Omega_1 \cap \Omega_2$.

We introduce the following notation for the extended tasks:

$$(x_i|\xi_i)' = y_i, (A_i|0) = \bar{A}_i, (D_iE_i) = \\ = \bar{D}_i, (C_i \underbrace{W\ldots W}_{m_i}) = \bar{C}_i. \quad (7)$$

Then the problem (4) - (6) can be written as: find a vector $y = (y_1 \ldots y_q)'$, minimizing the objective function:

$$z = \bar{C}_1y_1 + \bar{C}_2y_2 + \ldots + \bar{C}_qy_q + W\xi_0, \quad (8)$$

with constraints:

$$\bar{A}_1y_1 + \bar{A}_2y_2 + \ldots + A_qy_q \qquad + E_0\xi_0 = b_0, \quad (9)$$

$$\left.\begin{array}{lllll} \bar{D}_1y_1 & & & & = b_1, \\ & \ddots & & \vdots & \\ & & \bar{D}_ky_k & & = b_k, \\ & & & \ddots & \vdots \\ & & & \bar{D}_qy_q & = b_q, \end{array}\right\} \quad (10)$$

$$y_i \geq 0, i = 1, \ldots, q, \xi_0 \geq 0,$$

We introduce the following notation:

$$\bar{C} = (\bar{C}_1 \ldots \bar{C}_q), \bar{A} = (\bar{A}_1|\bar{A}_2|\ldots|\bar{A}_q), b_0 = \\ = (b_1 \ldots b_q)', \quad (11)$$

$$\bar{D} = \left|\begin{array}{lllll} \bar{D}_1 & \ldots & & & \\ & & \bar{D}_k & & \\ & & & \ldots & \\ & & & & \bar{D}_q \end{array}\right| \quad (12)$$

Then the problem (7) - (8) formed another way: find a vector y, minimizing:

$$Z = W\xi_0 + \bar{C}\,Y, \quad (13)$$

with constraints:

$$E_0\xi_0 + \bar{A} \cdot y = b_0, \quad (14)$$

$$\bar{D}_y = b, y \geq 0. \quad (15)$$

Many Ω_2 , given the restrictions (14) can be, as well as a convex polyhedron, and unlimited convex polyhedron.

1) Consider first the case when Ω_2 - bounded set (convex polyhedron). Then any element $y \in \Omega_2$ can be represented by $y^i, j,..., N$ as a convex combination of the vertices of the

polyhedron Ω_2.

$$y = \sum_{j=1}^{N} a_i y^j , \sum_{j=1}^{N} a_i = 1 , a_i \geq 0, j = 1, \ldots, N \tag{16}$$

Therefore, the problem (13) - (15) can be interpreted as follows: it is necessary to select all of the pixels $y \in \Omega_2$ such that satisfy equations (14) and minimize the function (11).

Substituting (16) into (13) yields the following so-called "coordination" problem: minimize:

$$Z = W\xi_0 + \sum_{j=1}^{N} f_j a_j \tag{17}$$

with constraints

$$E_0\xi_0 + \sum_{j=1}^{N} p_j a_j = b_0, \tag{18}$$

$$\sum_{j=1}^{N} a_j = 1, a_j \geq 0, j = 1, \ldots, N \tag{19}$$

were,

$$f_j = \bar{C}y^j = \left(\bar{C}_1 y_1^j + \cdots + \bar{C}_q y_q^j\right),$$

$$P_j = \bar{A}y^j = (\bar{A}_1|\ldots|\bar{A}_q)(y_1^j \ldots y_q^j)'.$$

Constraint matrix (18), (19) has just $(m_0 + 1)$ lines, but much larger than the original object (13) - (15) the number of variables. However, this problem can be solved by having to start only one extreme point of the polyhedron Ω_2 and use, as will be shown below the "Column generation method" [3].

Such starting at the point in this case is one of the vertices of the polyhedron defined by constraints (15) in the extended problem, namely:

$$y_1 = (0|\xi_i|\ldots|0|\xi_q)' = (0|b_1|\ldots|0|b_q)'. \tag{20}$$

To this point we have:

$$\left.\begin{array}{l} f_1 = \bar{C}y^1 = W(\sum_{i=1}^{m_i} b_{1i} + \ldots + \sum_{i=1}^{m_q} b_{qi}), \\ P_1 = \bar{A}y^1 = (\underbrace{0 \ldots 0}_{m_0}). \end{array}\right\} \tag{21}$$

Therefore, it corresponds to the initial reference plan of the coordinating problem $\xi_0 = b_0, a_1 = 1$, the basic matrix B of dimension $(m_0 + 1)$ and the vector of coefficients of the objective function:

$$f_B = (\underbrace{W \ldots W}_{m_0} f_1). \tag{22}$$

Suppose that as a result of the previous iteration received support program $\Lambda^S = (a_{ij}, \ldots, a_{i_{m0+1}})'$, coordinating tasks and the corresponding basis matrix $B = (P_{i1}, \ldots, P_{i_{m_0+1}})$.

At the same time, we obtain the vector of dual variables:

$$\Pi = f_b \cdot B^{-1} = (\Pi|\Pi_0), \tag{23}$$

where in the vector $\Pi = (\Pi_1 \ldots \Pi_{m0})$ corresponds to the constraints (18), and Π_0- the restriction (19).

In order to determine the possibility of improving the reporting of the support program coordination tasks needed for each no basic conditions vector matrix (18), (19) to calculate the characteristic difference (evaluation):

$$\Delta_o = \Pi \begin{vmatrix} P_j \\ 1 \end{vmatrix} - f_j = \Pi_0 + (\Pi \cdot \bar{A} - \bar{C})y^j. \tag{24}$$

If $max\, \Delta_j = \Delta_s \leq 0$, the solution is optimal and the optimal expansion plan of the problem (13) - (15) is calculated as follows:

$$y = a_{i_1} y^{i_1} + \ldots + a_{i_{m0+1_1}} y^{i_{1m0+1}} \tag{25}$$

If $\max_j \Delta_j = \Delta_s > 0$, then this solution is not optimal coordination problems and need to go to the support program of the problem with a smaller value of the linear form (6).

Finding $\max_j \Delta_j$ equivalenting solving subtasks of the form:

minimize:

$$Z_i = (\bar{C} - \pi\bar{A}) \cdot y, \tag{26}$$

when restrictions:

$$\bar{D}y = b, \; y \geq 0. \tag{27}$$

The separability of the objective function (26) and limits the independence of (15) it follows that the problem (26), (27) splits into q mutually independent sub-tasks to the following:

minimize:

$$Z_i = \bar{C}_\pi^i y_i \tag{28}$$

with constraints:

$$D_i y_i = b_i, \; y_i \geq 0. \tag{29}$$

$$\overline{C_\pi^i} = (\bar{C_i} - \pi\bar{A}_i), \; i = 1, \ldots, q. \tag{30}$$

To solve subtasks q (28), (29) we use the simplex method in combination with the method of artificial bases. Due to the limited set of Ω_2 the new support program is received by solving the subtasks:

$$y^s = \left(y_1^s \, y_2^s \; \ldots \; y_q^s\right) \tag{31}$$

is one of the vertices of the polyhedron Ω_2.

If this plan:

$$Z^s - \pi_0 = \sum_{i=1}^{q} Z_i^s - \pi_0 =$$

$$= \sum_{i=1}^{q} (\bar{C_i} - \pi\bar{A}_i) y_i^s - \pi_0 = 0 \tag{32}$$

That support plan Λ^i coordinating tasks is optimal. If

$$Z^s - \pi_0 < 0 \tag{33}$$

It is possible to further decrease the objective function, and then the base matrix B must be turned vector:

$$\overline{P_s} = \begin{vmatrix} \bar{A} & y^s \\ & 1 \end{vmatrix} = \begin{vmatrix} \overline{A_1} y_1^s + & \dots & + \overline{A_q} y_q^s \\ & 1 & \end{vmatrix}, \quad (34)$$

and vector f_B − element $f_s = \bar{C} y^s$.

Thus, necessary at each iteration column is generated by solving q local sub (25), (26).

Since the known degradation $\Lambda^i = (d_{i1} \dots d_{im_0+1})'$ and $\Lambda^s = (d_{is} \dots d_{im_0+s})'$, $(b_0/1)'$ and $\bar{P_s}$ on the basis vectors under consideration B, then for the vector output from the basis of need as usual to find relations

$$\frac{d_{ik}}{d_{i_k s}} \text{ for all } d_{i_k s} > 0 \quad (35)$$

and to withdraw from the basis vector $\bar{P}_{ir}$,, appropriate, the least of these relations.

If all $d_{i_k s} > 0$, the coordinating problem has no solution, and the objective function in the original problem (13) - (15) is unlimited.

Thus obtained a new base matrix B corresponds to the new support plan for coordinating tasks, which again is tested for optimality.

2) Consider now the case when Ω_2 - unlimited a multifaceted set and $y^j, j =, \dots, N_1 -$ set of its vertices and $R_j, j =, \dots, N_2$ −set of direction vectors of the unbounded edges, which are known [5] It is defined as a non-zero solution of the matrix of the homogeneous equation:

$$\bar{D} \cdot y = 0. \quad (36)$$

Then any y∈ element can be represented as

$$\left. \begin{array}{c} y = \sum_{j=1}^{N_1} d_j y^j + \sum_{j=1}^{N_2} \beta_j R_j \ , \\ \sum_{j=1}^{N_1} d_j = 1, \\ d_j \geq 0, j = 1, \dots, N_1, \beta_j \geq 0, j = 1, \dots, N_2 \end{array} \right\} \quad (37)$$

Substituting (33) into (10) and (11) yields "coordinating task" of the form:

Minimize:

$$Z = W_{\xi_0} + \sum_{j=1}^{N_1} f_j d_j + \sum_{j=1}^{N_2} f_{N_1+j} \beta_j, \quad (38)$$

with constraints:

$$\left. \begin{array}{c} E_0 \xi_0 + \sum_{j=1}^{N_1} P_j d_j + \sum_{j=1}^{N_2} P_{N_1+j} \beta_j = \text{b}_0, \\ \sum_{j=1}^{N_1} d_j = 1, \\ d_j \geq 0, j = 1, \dots, N_1, \beta_j \geq 0, j = 1, \dots, N_2 \end{array} \right\} \quad (39)$$

$$f_{N_{1+j}} = \bar{C} R_j, P_{N_{1+j}} = \bar{A} R_j \ , j = 1, \dots, N_2 \quad (40)$$

From the preceding, this case differs in that the coordinating task (34) for checking on the optimality of one of the reference plans (39) it is possible that at least one of the subs (25) and (26) can turn unlimited solution.

Suppose that the k-th subtask

$$\min Z_k = \bar{C}_\pi^k y_{k_1} + \dots + \bar{C}_{\pi, n_k+m_k}^k \ \ y_{k, n_k+m_k}, \quad (41)$$

$$\bar{D}_k y_k = b_k, \ \ y_k \geq 0,$$

on one of the iterations received the support plan:

$$y_k = \left(y_{k1} \dots y_{km_k} \ \underbrace{0 \dots 0}_{n_k} \right)' \quad (42)$$

which is connected with a system of linearly independent vectors $\bar{P}_1^k, \bar{P}_2^k, \dots, \bar{P}_{m_k}^k$, form a basis. Suppose, for some vector $\bar{P}_j$,, included in the matrix of conditions $\bar{D}_k$ and not belonging to a number of basic, all the coefficients $(y_k)_{1j}, (y_k)_{2j}, \dots, (y_k)_{m_k j}$ expansion him on the basis vectors were non-positive, and evaluation:

$$\Delta_j = \bar{C}_{\pi 1}^k (y_k)_{1j} + \dots + \bar{C}_{\pi m_k}^k \ (y_k)_{m_k j} - \bar{C}_{\pi j}^k > 0 \quad (43)$$

This means that in the k-th subtask is not an optimal plan and the objective function $Z_k$is unlimited.

Then directive vector of unlimited ribs, along which there is an unlimited decrease in the objective function (23) is determined by solving the equation (36), which is written as follows:

$$\sum_{i=1}^{k-1} \sum_{j=1}^{n_i} \bar{P}_j^i \cdot 0 + \left[\bar{P}_1^k (y_k)_{1j} + \dots + \bar{P}_m^k (y_k)_{m_k j} + \sum_{i=m_k+1}^{j-1} \bar{P}_i^k \cdot 0 - \bar{P}_j^k + \sum_{i=j+1}^{n_k} \bar{P}_i^k \cdot 0 \right] + \sum_{i=k+1}^{q} \sum_{j=1}^{n_i} \bar{p}_j^i \cdot 0 = 0 \quad (44)$$

After discarding zero terms in this vector equation of we obtain a new equation that determines the nonzero vector elements R_s:

$$\bar{P}_1^k (y_k)_{1j} + \dots + \bar{P}_{m_k j} - \bar{P}_i^k = 0. \quad (45)$$

This shows that $(n_1 + m_1 + \dots + n_q + m_q)$ is dimensional vector:

$$R_k^s = \left(\underbrace{0 \dots 0}_{\sum_{i=1}^{k-1}(n_i+m_j)} - (y_k)_{1j} - \dots - (y_k)_{m_k j} \ \underbrace{0 \dots \ 0 \ 1 \ 0 \dots \ 0}_{n_k} \ \underbrace{0 \dots 0}_{\sum_{i=k+1}^{q}(n_i+m_j)} \right) \quad (46)$$

is the direction vector of unlimited ribs of convex polyhedron Ω_2. In this case, the support program coordination problem (38), (39) the condition (33), hence the reference plan is not optimal and to improve it, it is necessary in the basic matrix of the coordinating tasks include vector:

$$\bar{P_S} = \begin{vmatrix} \bar{A} & R_S \\ & 0 \end{vmatrix} \quad (47)$$

Instead of one of the old vectors being found by the usual simplex method for the rule, and in vector - f_B element$f_s = \bar{C} \cdot R_S$. After a finite number of iterations is obtained an optimal solution coordinating task, or make sure the target function is unbounded on an admissible set of plans.

Obviously, the solution may be unlimited in not one, but several subtasks. Given the fact that this case is not in the literature reviewed, we explain it in detail.

Without loss of generality, assume that an unlimited decision turned out not only to the first, but also, for example, in the l-th subtask $l \neq к$, $к \leq q$, $l \leq q$. Then, for some of the support program and the corresponding basis by the decomposition of one of the no basic vectors $\overline{P_t}^l$ got rating $\Delta_t^l > 0$, all coefficients $(y_l)_{1t} \dots (y_l)_{mlt}$ its decomposition are non-negative. Therefore, in addition to unlimited ribs with direction vector R_S^K defined by (46), there is another unrestricted edge with direction vector:

$$R_S^l = (\underbrace{0 \dots 0}_{\sum_{i=1}^{l-1}(n_i + m_i)} - (y_l)_{1t} - \cdots - (y_l)_{mlt} \underbrace{\overset{0 \dots 01\ 0 \dots 0}{t}}_{n_l} \underbrace{0 \dots 0}_{\sum_{i=l+1}^{q}(n_i + m_i)} \quad (48)$$

In this case, the formation of a new vector introduced into the basis of the coordinating task, as follows from the theory of linear programming, to reduce the number of iterations should choose the direction vector, which corresponds to the greater value Δ evaluation. If, for example $\Delta_j^K > \Delta_t^K$, then

$$\overline{P_S} = \begin{vmatrix} \overline{A} & R_S^K \\ & \\ & 0 \end{vmatrix} \quad (49)$$

Proceeding in accordance with this rule, after a finite number of iterations we obtain the optimal solution coordinating tasks, or define that the objective function is unbounded on an admissible set of plans.

This version of the method of decomposition of Danzig - Wolfe is different from normally recommended for linear programming problems such as (1) - (3) the fact that in the last set Ω2 plans presented in the form of a direct product of the sets Ω21... Ω2q, given restrictions:

$$\overline{D_i} y_i = b_i, i = 1, \dots, q. \quad (50)$$

Then, if, for example, all Ω2i, I = 1,…,q - convex polyhedral, any element y € Ω2 can be represented as a convex combination of extreme points y_i^j, i=1, …, q, j=1, …, N1 , polyhedral Ω21... Ω2q:

$$\left.\begin{aligned} y = \sum_{i=1}^{q} \sum_{i=1}^{N_i} \alpha_{ij} y_i^j, \\ \sum_{i=1}^{N_i} \alpha_{ij} = 1, \\ \alpha_{ij} \geq 0, i = 1, \dots, q, j = 1, \dots, N_i \end{aligned}\right\} \quad (51)$$

Substituting (51) into (14), (15), we obtain the following coordinating task minimize:

$$Z = W\xi_0 + \sum_{i=1}^{q} \sum_{i=1}^{N_i} f_{ij} \alpha_{ij}, \quad (52)$$

with constraints

$$E_0 \xi_0 + \sum_{i=1}^{q} \sum_{i=1}^{N_i} P_{ij} \alpha_{ij} = b_0, \quad (53)$$

$$\left.\begin{aligned} \sum_{j=1}^{N_i} \alpha_{ij} = 1, \\ \alpha_{ij} \geq 0, i = 1, \dots, q, \\ j = 1, \dots, N_i, \xi_0 \geq 0 \end{aligned}\right\} \quad (54)$$

where,

$$f_{ij} = \overline{C_i} y_i^j, \quad P_{ij} = \overline{A_i} y_i^j, \quad (55)$$

In this coordination problem constraints (53) and (54) do not contain $(m_0 + 1)$, and $(m_0 + q)$ lines.

Failures in the application of the Dantzig-Wolfe decomposition method may occur not due to the shortcomings and incorrectness of the method, but due to some features that really affect its convergence.

The latter include the following:

1) The decomposition method is not directly applicable when the right parts of the binding constraints (14) are equal to zero. In this case, the right side of the coordinating problem will contain only one nonzero component corresponding to constraint (19). As a consequence, in the resulting support plan Λ, only the $(m_0 + 1)$-th component will be equal to one, and all the rest will be zero.

As a result, the process of improving the basic plan of the coordinating task loses all meaning. But this phenomenon can be easily avoided, for example, by expressing some variables xij using block constraints (15) in terms of equal parts bi and other variables of the i-th block, and then excluding these variables together with the corresponding equations from the i-th block and from connecting equations. Then, in these blocks, you should add the conditions for the non-negativity of the excluded variables.

The second way is even easier. It consists in the fact that very small positive values are assigned to the right parts of the binding constraints (14), i.e. $b_0 = Ɛ, Ɛ > 0$. The performed computational experiments have shown that for tasks whose optimal plans according to (31) are represented as a linear combination of vertices and unlimited edges, the number of iterations providing the optimal plan, at $\varepsilon = 10^{-4}: 10^{-3}$ has decreased by more than 2-3 times compared to the first method.

This is explained by the fact that the representation of the set of admissible plans according to (33), generally speaking, is ambiguous [5]. Therefore, in the second way of expressing the vector, the optimal solution (25) was obtained in the form of a linear combination of vectors, some of which contained non-zero components from artificial variables, and the coefficients α_i corresponding to these vectors were equal to zero.

2) In those cases when the range of admissible values of the problem variables, given by block constraints (14), is an unlimited polyhedral set, at some iterations of the search for the optimal plan, the problem of choosing from among several one unlimited edge to form a new vector introduced into the number of basis vectors may arise. How to do this is shown above, but the wrong choice of an unbounded edge leads to a loop.

IV. Conclusion

The magnitude of the coordinating problem leads to an increase in the coefficient of the function, to a decrease in its size, to an increase in the computational efficiency of the decomposition method in $(m_0 + q)/(m_0 + 1)$, compared to the original decomposition method.

Consequently, the amount of computer memory required for solving the problem in this case increases. Furthermore, as it coordinates the task affects the decision of subtasks by changing the objective function values of the coefficients, the reduction of its dimensions leads to an increase in computational efficiency decomposition method in $(m_0 + q)/(m_0 + 1)$ times, compared with the original decomposition method.

The advantage of this variant of the method is especially great, in comparison with the recommended one, when the number q of blocks is large, and each of the sets Ω2i, i=1,…,q, corresponding to these blocks, can be specified by a small number m_K of restrictions. It is these cases that are most often encountered in practice when modeling real processes.

In particular, when all $m_K = 1, k = 1, \dots, q$ (classical transport problem), the usually recommended version of the decomposition method has no advantages over the simplex method, while the considered version of the decomposition method retains all its advantages.

The experience of practical application of the decomposition method for solving high-dimensional problems was insignificant and, in many cases, unsuccessful. The use of the above modification of this method for solving problems of the type (1) - (3) of large dimension refutes these statements as erroneous.

The performed computational experiments for tasks with a matrix order from 90 to 700 showed that, in terms of the number of iterations to obtain the optimal plan, the proposed modification of the Danzig-Wulf decomposition method has the same convergence as the simplex method, but the requirements for computer memory are reduced, and the computational efficiency is increased by $(m_0 + q)/(m_0 + 1)$ times

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