A Capacity-Influenced Approach to Find Better Initial Solution in Transportation Problems

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***Abstract*—Finding an Initial Basic Feasible Solution (IBFS) is the first and essential step in obtaining the optimal solution for any Transportation Problem. Numerous approaches are available in the literature to determine the IBFS; however, many of these methods are modifications of Vogel's Approximate Method (VAM) and/or the Least Cost Method (LCM). None of the existing methods directly consider the capacity of distributions among the nodes when selecting the allocation steps. While researchers have proposed various approaches and demonstrated improved solutions with numerical instances, they have not thoroughly investigated the underlying causes of these results. In this article, we explore the impact of capacity distributions among the nodes on the VAM and LCM in an experimental domain. The study introduces a novel and unique Capacity-Influenced Distribution Indicator (CI-DI) designed to control the flow of allocation. Ultimately, we propose a novel Capacity-Influenced approach that embeds both LCM and VAM to determine the IBFS for Transportation Problems (TPs). The novelty of the proposed approach lies in its direct consideration of capacity distribution among the nodes in the flow of allocations, this feature is lacking in LCM, VAM, and other established approaches. The proposed method develops a novel distribution indicator and a novel cost entry embedded capacity-based matrix to control the flow of allocations and thereby finds the IBFS for the Transportation Problem. We have conducted extensive numerical experiments to assess the effectiveness of the proposed approach. Experimental analysis demonstrates that the proposed method is more efficient in finding the IBFS than existing approaches. Moreover, as it uses a one-time generated Distribution Indicator (DI) for all steps of allocation, it is computationally cheaper than VAM, which generates a DI for each step of allocation.**

Keywords*—Transportation problem; least cost method; Vogel's approximate method; cost matrix; transportation tableau; node; capacity; route; capacity-influenced; weighted opportunity cost

I. INTRODUCTION

In Transportation Problem (TP), commodities are transported from a set of sources (called source nodes) to destinations (called destination nodes) subject to capacity (supply and demand) constraints in such a way that the total cost of transportation is minimized. TP is a multi-disciplinary field of study [1-3]. It is directly involved with real-life problems [1- 4]. The application of the TP extends beyond its traditional domain and finds relevance in various other fields [4], [8]. These fields include personnel assignment, inventory control, employee scheduling, and more [1-8]. It is known that the general Linear Programming (LP) methods are so tedious and time-consuming [6-8]. Researchers have developed several alternative methods for finding the IBFS by leveraging the special (unique) characteristics of TPs [2-6]. The two well-known classical methods, the LCM and VAM are very simple and can yield better IBFS for TPs [2], [5], [6], [14], etc.

In LCM, the flow of allocation is directly controlled by the cost matrix, with a preference for the least cost. Vogel introduced VAM in 1958 as a modification of LCM. In VAM, the flow of allocation is controlled differently, but it still considers the cost matrix. It begins by developing a control vector called the DI, formed through the manipulation of the cost matrix. Subsequently, the flow of allocation is guided by both the DI and the cost matrix. Based on the LCM and VAM, many researchers proposed several approaches for finding IBFS of TPs [2], [5], [6-13], [14], [21], [22], [23], [29-30], [38-39], [41], [42] etc. For the importance of TPs in real life, researchers are continuously devoted to finding better approaches for solving TPs. It is observed in the literature that most of the approaches are variations of VAM [6-10], [24], etc. A few of the research works related to TPs are pointed out below.

In the article [24], the author introduced the Total Opportunity Matrix (TOM) by manipulating cost entries rather than DI used in VAM to determine the flow of allocations. Authors in the article [1] developed Total Opportunity Cost (TOC) matrix and after that they formed DI tableau for allocation by considering TOC. In the article [43], authors proposed an enhanced version of [10] modified VAM for the unbalanced transportation problem. Both approaches utilize the VAM method, with the modification focused on transforming an unbalanced TP into a balanced one. In [44] author considered balanced transportation problems, with the modification applied solely to the manipulation of the cost matrix. In [45] authors proposed another embedded modified VAM method called Logical Development of Vogel's Approximation Method (LD-VAM) for finding the IBFS in TP. In [39] author introduced a modified VAM in which the modification is directed toward finding the DI, considering the cost matrix as well. Some other modified approaches based on VAM approach are found in the recent publications of [25], [29] [33], [36].

Besides modification of VAM, some other approaches are available in the literature. Recently, in [46] authors, at first, developed TOC then they formed DI tableau for allocation by considering the average of TOC of cells along each row identified as Row Average Total Opportunity Cost (RATOC) and the average of TOC of cells along each column identified as Column Average Total Opportunity Cost (CATOC). Allocations of costs are started in the cell along the row or column which has the highest RATOCs or CATOCs. This approach is also developed by considering only cost matrix.

In the article [35], author proposed a modified method to north-west corner method for finding IBFS. Very recently, some modified approaches based on LCM are found in the publications of [7], [26], etc. Some statistical methods are found in [36], [40]. In [35] authors have presented an alternative method of NWC method by using Statistical tool called Coefficient of Range (CoR) by statistically analyzing the cost matrix. Author, in [47], introduced a new algorithm for solving TPs. They proposed the Gauss Jordan pivoting method to solve the TPs. They consider only cost matrix and iteratively it finds out the solution. This algorithm is faster than Simplex method.

Some heuristic methods of TPs to find out IBFS are found in the articles [3], [9], [11], [12], [20], [21], [37], [48-56]. In fuzzy environment, many research publications are found in the literature of which some recent publications are included in the articles [13], [23], [27], [32]. Researchers also dedicated to find better IBFS for unbalanced TPs [10], [17], [32], [34]. A good survey of TP for finding IBFS is observed in [31], [57-58]. On the other hand, [15], [16] and [17] proposed a new technique for controlling the flow of allocation named Weighted Opportunity Cost (WOC) matrix. The WOC matrix is formed by demand and/or supply as a weight factor corresponding to each transportation cost. Authors in [15], [16] and [17] also considered some numerical instances to test the efficiency of the proposed algorithms. In [59] authors considered a fractional objective function rather than a linear objective to solve TPs. In [60], presented a modified VAM specifically designed for maximizing profit, with the flow of allocation controlled by cost entries as well. Moreover, in recently published articles [61-64], authors have proposed various methods to find the IBFS of TPs in which the flow of allocations is controlled by manipulating only cost entries.

It is observed that many approaches are available in the literature, and researchers are continuously working to develop more efficient methods to solve the TPs. But as far as it is known, none of the approaches is the best for solving all TPs. In our earlier work [15] Jamali also noticed this pitfall and proposed a newer approach to LCM. Though it [15] performed better compared to LCM but it frequently obtains worse IBFS compared to VAM.

It is known that classical methods like the LCM and VAM are relatively less computationally expensive compared to other approaches for finding the Initial Basic Feasible Solution [28] (IBFS) of Transportation Problems (TPs) [18]. Although VAM generally performs better than LCM, there are cases where LCM outperforms VAM. This raises the question: Is the performance variation due to the distribution of node capacities?

Additionally, it is observed that in the transportation sector, increasing the amount of goods often reduces transportation costs. Based on this observation, we investigate how node capacities influence transportation costs. Notably, no researchers have yet developed methods that leverage the effect of node capacity on the allocation and flow of commodities.

In this paper, we extensively examined the impact of capacity distribution among the nodes on classical well-known approaches, namely LCM and VAM, considering the problem mentioned in the existing literature. Next, we developed a capacity-blended flow-controlling matrix based on the cost matrix. Finally, we proposed a novel capacity-influenced approach to find the IBFS of TPs.

The novelties of the proposed approach are explained in the following:

- We propose a novel approach to find better IBFS for the TPs by developing a new, unique distribution indicator (DI) that combines the capacity vectors and DI vector to control the flow of allocation.
- Our approach proposes a more effective solution for the TPs that can overcome the limitations of existing methods, namely VAM and LCM, by addressing the impact of capacity distribution among the nodes.
- Proposed approach introduces a capacity-influenced weighted factor and a capacity-influenced Weighted Opportunity Cost (WOC) Matrix to find a more suitable IBFS in a computationally efficient way for VAM and LCM.
- We extensively verified the impact of capacity distribution and proposed methods have been experimentally verified to evaluate its performance over other approaches using real examples.

The remainder of the paper is organized as follows. Section II explains the mathematical model of TPs. Section III extensively examines the impact of capacity distribution among nodes for both LCM and VAM for TPs. Section IV explains the proposed method of this paper. This section develops and explains proposed mathematical model named as "Capacity influenced weighted factor" to control the flow of allocations. It also develops the formulation of control matrix of the proposed approach. Detailed results of numerical experiments to validate the efficiency of the proposed method is explained in Section V and conclusion of this study is presented in the last section.

II. Mathematical Model of Transportation Problem

By considering the equality characteristics of TPs, it can be represented using a specialized tableau known as the Transportation Tableau (TT). The typical view of TT is shown in Table I. In the TT, O_i indicates the ith source with the amount of availability is ai which is shown in the far-right column. On the other hand, D_j denotes the jth destination with demand b_i, which is shown in the bottom row of TT. In this table, there is an m×n matrix containing cost entries. The cell in the ith row and jth column is called the C_{ij} cell and the transportation cost is denoted as cij, which represents the unit shipping cost from the ith source to the jth destination. So, a TT can be viewed as a (m+1) × (n+1) matrix shown in Table I.

TABLE I. A TT OF A TP WITH M SOURCES AND N DESTINATIONS

	Sinks/Destinations						
Origins/Sources		D_1	D_2	…	D_{n-1}	D_n	Supply
	O_1	c_{11}	c_{12}	…	c_{1n-1}	c_{1n}	a_1
	O_2	c_{21}	c_{22}	…	c_{2n-1}	c_{2n}	a_2
	O_3	⋮	⋮	…	⋮	⋮	a_3
	⋮	⋮	⋮	…	⋮	⋮	⋮
	O_m	c_{1m}	c_{2m}	…	c_{mn-1}	c_{mn}	a_m
	Demand	b_1	b_2	…	b_{n-1}	b_n	

III. IMPACT OF THE CAPACITY DISTRIBUTIONS AMONG NODES ON LEAST COST AND VOGEL'S APPROXIMATION METHODS

First three typical balanced TPs are considered which are presented in the Examples 1(a) – 1(c) and their corresponding comparative analyses are presented in Table II to Table X. It should be noted that in all three TPs, the cost matrices are identical, and the total capacity remains the same. The only difference lies in the distribution of capacity among the nodes. Numerical experiments have been conducted to investigate the effect of capacity on both the LCM and VAM in terms of finding the IBFS and total cost [19].

Example 1(a):

TABLE II. TRANSPORTATION TABLEAU OF TRANSPORTATION PROBLEM 1

	D_1	D_2	D_3	Supply
O_1	2	5	8	20
O_2	6	4	14	20
O_3	15	12	13	20
Demand	20	20	20	

Example 1(b):

TABLE III. TRANSPORTATION TABLEAU OF TRANSPORTATION PROBLEM 2

	D_1	D_2	D_3	Supply
O_1	2	5	8	30
O_2	6	4	14	20
O_3	15	12	13	10
Demand	10	20	30	

Example 1(c):

TABLE IV. TRANSPORTATION TABLEAU OF TRANSPORTATION PROBLEM 3

	D_1	D_2	D_3	Supply
O_1	2	5	8	30
O_2	6	4	14	20
O_3	15	12	13	10
Demand	20	35	5	

TABLE V. (EX. 1(A) LCM) STEP-BY-STEP FLOW OF ALLOCATIONS OF LCM AND VAM FOR EX. 1(A) –1(C)

LCM: IBFS of Ex. 1 (a)				
	D_1	D_2	D_3	S
O_1	**2** **20**	5 0	8 0	20
O_2	6 0	**4** **20**	14 0	20
O_3	15 0	12 0	**13** **20**	20
D	20	20	20	

TABLE VI. (EX 1(A) VAM) STEP-BY-STEP FLOW OF ALLOCATIONS OF LCM AND VAM FOR EX. 1(A) –1(C)

VAM: IBFS of Ex. 1 (a)				
	D_1	D_2	D_3	S
O_1	2 **0**	5 0	**8** **20**	20
O_2	**6** **20**	4 **0**	14 0	20
O_3	15	**12** **20**	13 **0**	20
D	20	20	20	

TABLE VII. (EX. 1(B) LCM) STEP-BY-STEP FLOW OF ALLOCATIONS OF LCM AND VAM FOR EX. 1(A) –1(C)

LCM: IBFS of Ex. 1 (b)				
	D_1	D_2	D_3	S
O_1	**2** **10**	5 0	**8** 20	30
O_2	6 0	**4** 20	14 0	20
O_3	15 0	12 0	**13** 10	10
D	10	20	30	

TABLE VIII. (EX. 1(B) VAM) STEP-BY-STEP FLOW OF ALLOCATIONS OF LCM AND VAM FOR EX. 1(A) –1(C)

VAM: IBFS of Ex. 1 (b)				
	D_1	D_2	D_3	S
O_1	2 0	5 0	**8** 30	30
O_2	**6** 10	**4** 10	14 0	20
O_3	15 0	**12** 10	13 0	10
D	10	20	30	

TABLE IX. (EX. 1(C) LCM) STEP-BY-STEP FLOW OF ALLOCATIONS OF LCM AND VAM FOR EX. 1(A) –1(C)

LCM: IBFS of Ex. 1 (c)				
	D_1	D_2	D_3	S
O_1	**2** 20	**5** 10	8 0	30
O_2	6 0	**4** 20	14 0	20
O_3	15 0	**12** 5	**13** 5	10
D	20	35	5	

TABLE X. (EX. 1(C) VAM) STEP-BY-STEP FLOW OF ALLOCATIONS OF LCM AND VAM FOR EX. 1(A) –1(C)

VAM: IBFS of Ex. 1 (c)				
	D_1	D_2	D_3	S
O_1	**2** 20	**5** 5	**8** 5	30
O_2	6 0	**4** 20	14 0	20
O_3	15 0	**12** 10	13 0	10
D	20	35	5	

To examine the effect of capacity on LCM and VAM, we have first compared the step-by-step flow of allocations. The intensive comparison of the step-by-step flow of the allocations procedure of the two approaches is concisely shown in Table XI. It is observed in Table XI that due to the change of capacity distribution among the nodes, the pattern of flow of allocation is changed significantly for both approaches. Furthermore, it is observed that the IBFS obtained using both methods undergo significant changes for each instance.

Now we have compared IBFSs and Total Transportation Costs (TTC) for each instance. The effect of the capacity distribution of nodes for each instance is shown in Table XI. It is observed that only by a change of capacity distribution among the nodes, the total transportation cost for each instance produced by LCM is changed significantly. Similarly, only by a change of capacity distribution among the nodes, the total transportation cost for each instance produced by VAM is changed significantly. It is also observed that the IBFSs produced by LCM of the three instances are changed significantly. Similarly, it is also observed that the IBFSs produced by VAM of the three instances are changed significantly. It has been observed that in Example 1(a), the LCM produced a superior IBFS compared to VAM, with a notable and significant difference between the two solutions. But only the change of capacity distribution, in Example 1(b), VAM produced a better solution compared to LCM. Moreover, in Example 1(c), though LCM produced a better solution, but the difference between the two solutions is not large.

TABLE XI. COMPARISON BETWEEN THREE EXAMPLES REGARDING THE EFFECT OF CAPACITY

	Ex. 1(a) Equal capacity Demand: 20, 20, 20 Supply: 20, 20, 20		Ex. 1(b) Unequal Capacity Demand: 30, 20,120 Supply: 10, 20, 30		Ex1 (c) Unequal Capacity Demand: 30, 20, 10 Supply: 20, 35, 5	
	T. Cast	IBFS	T. Cost	IBFS	T. Cost	IBFS
LCM	**380**	$x_{11}\rightarrow x_{22}\rightarrow x_{33}$ 20→ 20 → 20	520	$x_{11}\rightarrow x_{22}\rightarrow x_{13}\rightarrow x_{33}$ 10 → 20 → 20→ 10	295	$x_{11}\rightarrow x_{22}\rightarrow x_{12}\rightarrow x_{32}\rightarrow x_{33}$ 20 → 20→10 → 5 → 5
VAM	520	$x_{13}\rightarrow x_{21}\rightarrow x_{32}$ 20 →20 →20	**470**	$x_{13}\rightarrow x_{21}\rightarrow x_{22}\rightarrow x_{32}$ 30 →10→ 10 →10	**305**	$x_{13}\rightarrow x_{11}\rightarrow x_{22}\rightarrow x_{12}\rightarrow x_{32}$ 5 → 20 → 20 → 5 → 10

We have performed further experiments to examine the effect of capacity distribution on the flow of allocations of both LCM and VAM. For this numerical experiment, we have considered some more examples shown in Table XII.

We have performed both approaches, namely LCM and VAM to find out IBFS. The experimental result is shown in Table XII. It is observed in Table XII that instances 2(a) –2(c) has the same cost matrix but the capacity distributions are different. Though instances 2(a) – 2(c) have different distributions of capacity, the total supply/demand is equal for each instance. Similarly, instances 3(a) to 3(c) have identical cost matrices, but the capacity distributions vary. Though Examples 2(a) – 2(c) have different capacity distributions but total supply/demand is equal for each instance. It is observed for Example 2(a), that VAM produced a better solution which is also an optimal solution, but only a change of capacity distribution, for Example 2(b) and 2(c), LCM obtained a better solution compared to VAM. Similarly, it is observed that for Example 3(a) LCM produced a better solution which is also an optimal solution, but only a change of capacity distribution, for Example 3(c), LCM obtained a better solution compared to VAM, but for Example 3(b) VAM obtained a better solution compared to LCM. The experimental results are examined intensively. We have found out significant effects of the distribution of capacities on the IBFS of each approach.

TABLE XII. SOME MORE EXAMPLES REGARDING LCM AND VAM

Instances	Cost matrix	Capacity	Total Capacity	LCM	VAM	Optimal
2 (a)	{7,8,7}; {18,8,12}; {8,12,12}	S:12, 12, 12 D:12, 12, 12	36	324	**276**	**276**
2 (b)	{7,8,7}; {18,8,12}; {8,12,12}	S; 14,14,8 D: 15,8,13	36	**326**	334	**298**
2 (c)	{7,8,7}; {18,8,12}; {8,12,12}	S:25,30, 5 D:35,20, 5	60	**525**	555	**525**
3 (a)	{2,5,8};{6,4,14}; {15,12,18}	S:20, 20, 20 D:20, 20, 20	**60**	**480**	520	**480**
3 (b)	{2,5,8};{6,4,14}; {15,12,18}	S: 43,15,2 D:2,53,5	60	314	**308**	**308**
3 (c)	{2,5,8};{6,4,14}; {15,12,18}	S:10, 10, 10 D:10, 10, 10	30	**240**	260	**240**

IV. OUR PROPOSED METHOD: A NOVEL DISTRIBUTION INDICATOR AND CAPACITY-INFLUENCED APPROACH TO IBFS FOR TRANSPORTATION PROBLEMS

In classical approaches, like North-West Corner Rule [35], LCM, the flow of allocation is controlled directly by the cost entries only. Once again, in some other classical transportation approaches, such as Vogel's VAM method and all its variants, the flow of allocations is controlled solely by the manipulation of cost entries. Moreover, based on our literature review, it is observed that almost all approaches have been developed by manipulating cost entries exclusively. None of these approaches considers the node's capacity when formulating the control matrix for the flow of allocation. However, as observed in the previous section of this article, the distribution of commodities plays a crucial role in controlling the flow of allocations. In the previous investigation section, it is observed that the distribution of commodities significantly alters the performance of the approaches. In this article, we first developed a novel capacity-influenced weighted factor and then proposed a capacity-influenced WOC Matrix-based algorithm to find the IBFS of TPs. These are explained in the following sections.

A. *The Proposed Capacity-Influenced Weighted Factor to Control the Flow of Allocation*

Vogel's method formulates the DI by calculating the difference between the smallest and the next-to-smallest cost entries for each node (supply/destination). In this formulation, DI serves as a weight factor corresponding to the cost entries, controlling the flow of allocations. While existing literature emphasizes DI as a controlling tool, it is observed in the previous section that node capacity also significantly influences allocation flow, alongside cost entries. Therefore, a novel approach is needed to control the flow of allocations in TPs by combining the weight factor DI with the weight factor of the corresponding node's capacity. The primary challenge involves developing a suitable weight factor for the capacity of each node. Subsequently, a new flow of allocation matrix needs to be formulated, incorporating both the capacity of nodes and DI as a combined weight factor for the corresponding cost entries. The first challenge is to find out the weight factor to the cost matrix from the capacity of nodes.

The WOC matrix is a new concept to control the flow of allocations of TP to find out IBFS. In WOC, amount of supply/demand of each route is a weighted factor corresponding to the cost entry. The procedures to find out capacity influenced approach is discussed step by step below:

Step 1: Finding cell weight: At first, we have found out the maximum possible allocation of the cell $\mathrm{C_{ij}}$, which is $(\mathrm{S_i}, \mathrm{D_j})$, where $\mathrm{S_i}$ denotes total supply at node i and $\mathrm{D_j}$ indicates total demand at node j. Therefore, sum over of all possible allocations is as follow:

$$\sum_{i=1}^{p}\sum_{j=1}^{q}\min\left(\mathrm{S_i}, \mathrm{D_j}\right)$$

Therefore, for each cell C_{ij} its weight will be as follows

$$\min\left(\mathrm{S_i}, D_j\right) \Big/ \sum_{i=1}^{p}\sum_{j=1}^{q}\min\left(\mathrm{S_i}, D_j\right)$$

So that total weight becomes one. i.e.,

$$\sum_{i=1}^{p}\sum_{j=1}^{q}\left\{ \min\left(\mathrm{S_i}, D_j\right) \Big/ \sum_{i=1}^{p}\sum_{j=1}^{q}\min\left(\mathrm{S_i}, D_j\right) \right\} = 1$$

But since every cell of cost matrix will contain the factor $1 \Big/ \sum_{i=1}^{p}\sum_{j=1}^{q}\min\left(\mathrm{S_i}, \mathrm{D_j}\right)$, so we have ignored this factor to reduce computational cost. Therefore, for each cell C_{ij} its weight will be just $min\ (S_i\ , D_j)$.

Step 2: The second challenge is to find out the combined weight factor formulated by the weight factor DI and the weight factor WOC to the cost matrix for each route (cell). The proposed modified weight factor, $\mathrm{W_{ij}^{m}}$, for the cell Cij is formulated as follows in equation (1):

$$W_{ij}^{m} = \min\{a_i, b_j\} \cdot \max\{\{D_i^{Ir}, D_j^{Ic}\}, \} \tag{1}$$

Where a_i is the amount capacity of source node O_i, b_i is the amount of the capacity of destination node D_i. Moreover, D_i^{Ir} is the DI corresponding to the source node i and D_j^{Ic} is the DI corresponding to the sink node j. Then total weight corresponding to all cells will be as in Eq. (2).

$$TW = \sum_i^m \sum_j^n \min\{a_i, b_j\} \cdot \max\{\{D_i^{Ir}, D_j^{Ic}\} \forall\ i = 1,2, \ldots, m\ ; j = 1,2, \ldots, n\} \tag{2}$$

Then, the actual weight factor corresponding to each route (cell) C_{ij} is

$$AW_{ij}^{m} = \frac{\min\{a_i,b_j\}\cdot\max\left\{\{D_i^{Ir},D_j^{Ic}\}\right.}{\sum_i^m\sum_j^n \min\{a_i,b_j\}\cdot\max\left\{\{D_i^{Ir},D_j^{Ic}\}\right.}, \forall\ i = 1,2,\dots,m\ ;\ j = 1,2,\dots,n \quad (3)$$

Here in Eq. (3), $\max\{\{D_k^{Ir}, D_l^{Ic}\}\ \forall\ i,j$, is fixed and constant and also $\sum_i^m\sum_j^n AW_{ij}^m = 1$.

But since the term $\frac{1}{\sum_i^m\sum_j^n \min\{a_i,b_j\}\cdot\max\left\{\{D_i^{Ir},D_i^{Ic}\}\right.}$ is common to all $AW_{ij}^m\ \ \forall i,j$ and as AW_{ij}^m act as a controller of the flow of allocation and it has no any real effect to measure the total transportation cost, so without loss of generality we can ignore this factor in flow of allocation matrix.

Therefore, the weight factor corresponding to the cell C_{ij} can be expressed as follows in Eq. (4):

$$W_{ij}^m = \min\{a_i,b_j\} \times \max\left\{\{D_i^{Ir},D_j^{Ic}\right\} \quad (4)$$

It is noted that this reduces a significant amount of computational cost. The significance of this weight is that larger weight poses larger possibility to flow of allocation.

Step 3: After the successful formulation of the weight factor corresponding to each cell (route), our next task is to formulate the capacity-influenced WOC Matrix. But now we have to face a problem regarding the accumulation of this weight factor to any cost entries. Since the cell with a lower cost has a preference for allocation first, on the other hand, the cell with a larger weight factor has a preference for allocation first. So, it is not directly possible to formulate a WOC matrix just by simply multiplying the weight factor by the cost entry to find out meaningful elements of the WOC matrix. To overcome this difficulty and for the formulation of a meaningful WOC matrix, we should transform one of the two so that the multiplication of the two will be meaningful. This can be done by inversing the cost elements. Therefore, the weighted opportunity cost corresponding to the cell cost C_{ij} as stated in Eq. (5) below:

- $W_{c_{ij}}^m = \frac{1}{c_{ij}}\min\{a_i,b_j\} \times \max\left\{D_i^{Ir},D_j^{Ic}\right\}, c_{ij} \neq 0$ (5)

Here $W_{c_{ij}}^m$ and c_{ij} denote the modified weighted cost factor and actual cost entry corresponding to the cell C_{ij} respectively.

But another problem arises if the cost at any cell is zero. Since when $c_{ij} = 0$ then $\frac{1}{c_{ij}}$ becomes undefined. So, to overcome this difficulty it is needed some more special attention. We can overcome this shortcoming by replacing zero with a significantly large value. So, if there exists any cell whose cost entry is zero, then we can formulate the virtual weighted cost to the cell C_{pq} as follows:

- If $c_{pq} = 0$ and $\{c_{ij}: 0 < c_{ij} < 1, \forall i,j\} = \varphi$, (i.e. null set) then set $W_{c_{ij}}^m = \max\{a_i,b_j;\forall i,j\} \times \min\{a_i,b_j\} \times \max\left\{\{D_i^{Ir},D_j^{Ic}\right\}$
- Else if $c_{pq} = 0$ and $\{c_{ij}: 0 < c_{ij} < 1, \forall i,j\} \neq \varphi$ (i.e. not null set), then set $W_{c_{ij}}^m = \frac{\max\{a_i,b_j;\forall i,j\}}{[\min\{c_{ij}:0<c_{ij}<1,\forall i,j\}]} \times \min\{a_i,b_j\} \times \max\left\{\{D_i^{Ir},D_j^{Ic}\right\}$

B. Proposed Formulation of the Capacity-Influenced Control Matrix

After development of modified weighted cost factor, we can easily formulate the Modified Weighted Cost (MWOC) Matrix$\left[W_{c_{ij}}^m\right]$ which is as follows:

- If $c_{ij} \neq 0$; $W_{c_{ij}}^m = \frac{1}{c_{ij}}\min\{a_i,b_j\} \times \max\left\{D_i^{Ir},D_j^{Ic}\right\}$
- If $c_{ij} = 0$ and $\{c_{pq}: 0 < c_{ij} < 1, \forall p,q\} = \varphi$, (i.e. null set) then set $W_{c_{ij}}^m = \max\{a_i,b_j;\forall i,j\} \times \min\{a_i,b_j\} \times \max\left\{\{D_i^{Ir},D_j^{Ic}\right\}$
- Else if $c_{pq} = 0$ and $\{c_{pq}: 0 < c_{ij} < 1, \forall p,q\} \neq \varphi$ (i.e. not null set), then set $W_{c_{ij}}^m = \frac{\max\{a_i,b_j;\forall i,j\}}{[\min\{c_{ij}:0<c_{ij}<1,\forall i,j\}]} \times \min\{a_i,b_j\} \times \max\left\{\{D_i^{Ir},D_j^{Ic}\right\}$

C. Algorithm of Proposed Capacity-Influenced Approach

Step 1: Form the MWOC weight factor matrix.

Step 2: Allocate (as much as possible), i.e., min {Si, Di}, to the cell (route) which has the largest weight factor.

Step 3: Update the cost matrix by crossing out exhausted cells and corresponding weight factors.

Step 4: Terminate if all demand requirements are satisfied; otherwise, go back to step 2.

Once all cells are allocated, calculate the total transportation cost by multiplying the allocated units with their respective costs and summing up all these products.

V. Numerical Experimentation

Now we will implement the proposed method and will compare its performance with existing approaches namely LCM and VAM. For the experimental study we have considered another TP 4 (see Example 4) whose TT is given in the Table XIII.

Example 4:

TABLE XIII. Transportation Tableau of Transportation Problem 4

	D_1	D_2	D_3	Supply
O_1	4	3	5	90
O_2	6	5	4	80
O_3	8	10	7	100
Demand	70	120	80	

TABLE XIV. Modified Weighted Opportunity Cost Matrix of the Problem 4

	D_1	D_2	D_3	Supply	DI
O_1	$\frac{140}{4}$	$\frac{180}{3}$	$\frac{80}{5}$	90	1
O_2	$\frac{140}{6}$	$\frac{160}{5}$	$\frac{80}{4}$	80	1
O_3	$\frac{140}{8}$	$\frac{200}{10}$	$\frac{80}{7}$	100	1
Demand	70	120	80		
DI	2	2	1		

To find the IBFS of the proposed method, we first need to find out the flow of the control matrix called the Modified Weighted Opportunity Cost (MWOC) matrix shown in Table XIV. To explain how to form the MWOC matrix, let us consider the cell C_{12}. It is observed that the unit cost c_{12} = 4 which is not zero. Therefore, the algorithm executes case (a) of the proposed method, formally:

$$W_{c_{12}}^{m} = \frac{1}{c_{12}} \min\{a_1, b_2\} \times \max\{ D_1^{Ir}, D_2^{Ic}\}$$

$$=\frac{1}{3} \times \min\{90,120\} \times \max\{ 1, 2\}$$

$$=\frac{1}{3} \times 90 \times 2 = \frac{180}{3}$$

TABLE XV. MODIFIED WEIGHTED OPPORTUNITY COST INCLUDED TRANSPORTATION TABLEAU PROBLEM 4

	D_1	D_2	D_3	supply	DI
O_1	$\frac{140}{4}$ 4	$\frac{180}{3}$ 3	$\frac{80}{5}$ 5	90	1
O_2	$\frac{140}{6}$ 6	$\frac{160}{5}$ 5	$\frac{80}{4}$ 4	80	1
O_3	$\frac{140}{8}$ 8	$\frac{200}{10}$ 10	$\frac{80}{7}$ 7	100	1
Demand	70	120	80		
DI	2	2	1		

We can represent the cost matrix and the MWOC matrix in a single tableau as shown in Table XV. In Table XV, the entry in the top-left corner of each cell represents the weighted opportunity cost factor associated with that cell, while the entry in the top-right corner represents its transportation cost. The step by step of each allocation's procedure of the proposed MWOC-based approach is shown in Tables XVI to XXI. The IBFS of the problem obtained by the proposed method is shown in Table XVI. It is observed that the TTC for finding the IBFS by the proposed method is 1440.

TABLE XVI. THE FIRST ALLOCATION OF THE PROPOSED METHOD FOR THE TABLEAU PROBLEM 4

	D_1	D_2	D_3	supply	DI
O_1	$\frac{140}{4}$ 4 ×	$\frac{180}{3}$ 3 **90**	$\frac{80}{5}$ 5 ×	~~**90**~~	1
O_2	$\frac{140}{6}$ 6	$\frac{160}{5}$ 5	$\frac{80}{4}$ 4	80	1
O_3	$\frac{140}{8}$ 8	$\frac{200}{10}$ 10	$\frac{80}{7}$ 7	100	1
Demand	70	~~**120**~~ , 30	80		
DI	2	2	1		

TABLE XVII. THE SECOND ALLOCATION OF THE PROPOSED METHOD FOR THE TABLEAU PROBLEM 4

	D_1	D_2	D_3	supply
O_1	$\frac{140}{4}$ 4 ×	$\frac{180}{3}$ 3 90	$\frac{80}{5}$ 5 ×	~~90~~
O_2	$\frac{140}{6}$ 6	$\frac{160}{5}$ 5 **30**	$\frac{80}{4}$ 4	~~**80**~~,50
O_3	$\frac{140}{8}$ 8	$\frac{200}{10}$ 10 ×	$\frac{80}{7}$ 7	100
Demand	70	~~120~~ , ~~**30**~~	80	

TABLE XVIII. THE THIRD ALLOCATION OF THE PROPOSED METHOD FOR THE TABLEAU PROBLEM 4

	D_1	D_2	D_3	supply
O_1	$\frac{140}{4}$ 4 ×	$\frac{180}{3}$ 3 90	$\frac{80}{5}$ 5 ×	~~90~~
O_2	$\frac{140}{6}$ 6 **50**	$\frac{160}{5}$ 5 30	$\frac{80}{4}$ 4 ×	~~80~~,~~**50**~~
O_3	$\frac{140}{8}$ 8	$\frac{200}{10}$ 10 ×	$\frac{80}{7}$ 7	100
Demand	~~**70**~~, 20	~~120~~ , ~~**30**~~	80	

TABLE XIX. THE FOURTH ALLOCATION OF THE PROPOSED METHOD FOR THE TABLEAU PROBLEM 4

	D_1	D_2	D_3	supply
O_1	$\frac{140}{4}$ 4 ×	$\frac{180}{3}$ 3 90	$\frac{80}{5}$ 5 ×	~~90~~
O_2	$\frac{140}{6}$ 6 50	$\frac{160}{5}$ 5 30	$\frac{80}{4}$ 4 ×	~~80~~,~~50~~
O_3	$\frac{140}{8}$ 8 **20**	$\frac{200}{10}$ 10 ×	$\frac{80}{7}$ 7	~~**100**~~, 80
Demand	~~70~~, ~~**20**~~	~~120~~ , ~~30~~	80	

TABLE XX. THE FIFTH ALLOCATION OF THE PROPOSED METHOD FOR THE TABLEAU PROBLEM 4

	D_1	D_2	D_3	supply
O_1	$\frac{140}{4}$ 4 ×	$\frac{180}{3}$ 3 90	$\frac{80}{5}$ 5 ×	~~90~~
O_2	$\frac{140}{6}$ 6 50	$\frac{160}{5}$ 5 30	$\frac{80}{4}$ 4 ×	~~80~~,~~50~~
O_3	$\frac{140}{8}$ 8 20	$\frac{200}{10}$ 10 ×	$\frac{80}{7}$ 7 **80**	~~100~~, ~~**80**~~
Demand	~~70~~, ~~20~~	~~120~~ , ~~30~~	~~**80**~~	

TTC = 3×90+6×50+5×30+8×20+7×80 =1440

TABLE XXI. THE IBFS OF THE PROPOSED METHOD FOR THE TABLEAU PROBLEM 4

	D_1	D_2	D_3	supply
O_1	$\frac{140}{4}$ 4 ×	$\frac{180}{3}$ 3 90	$\frac{80}{5}$ 5 ×	~~90~~
O_2	$\frac{140}{6}$ 6 50	$\frac{160}{5}$ 5 30	$\frac{80}{4}$ 4 ×	~~80~~,~~50~~
O_3	$\frac{140}{8}$ 8 20	$\frac{200}{10}$ 10 ×	$\frac{80}{7}$ 7 **80**	~~100~~, ~~**80**~~
Demand	~~70~~, ~~20~~	~~120~~ , ~~30~~	~~**80**~~	

Now have solved the problem with the existing LCM, VAM, and WOC-LCM and compared it with the proposed method named MWOC-VAM. The comparison is shown in Table XXII. It is observed in Table XXII that the proposed MWOC-VAM needs the least amount of transportation cost to obtain the IBFS compared to all other approaches namely LCM, VAM, and WOC-LCM. It is also observed that the starting allocation of LCM, WOC-LCM, and MWOC-VAM are the same but differ from VAM. It is also observed in the second column of Table XXII that the pattern of the flow of allocation for each approach is different.

TABLE XXII. COMPARISON REGARDING THE FLOW OF ALLOCATIONS AND TOTAL TRANSPORTATION COST TO FIND OUT THE IBFS OF THE PROBLEM 4

Method	Flow of allocations and IBFS	Total Cost
LCM	$x_{12}\rightarrow x_{23}\rightarrow x_{31}\rightarrow x_{32}$ 90→ 80 →70 → 30	1450
VAM	$x_{11}\rightarrow x_{33}\rightarrow x_{12}\rightarrow x_{22}\rightarrow x_{32}$ 70→ 80 →20 → 80 → 20	1500
WOC-LCM	$x_{12}\rightarrow x_{23}\rightarrow x_{32}\rightarrow x_{31}$ 90 → 80 → 30→ 70	1450
MWOC-VAM (proposed)	$x_{12}\rightarrow x_{22}\rightarrow x_{21}\rightarrow x_{31}\rightarrow x_{33}$ 90→ 30 → 50 → 20 → 80	**1440**

Now we have considered another problem 5 (Example 5), whose TT is given in the Table XXIII in which one route has zero transportation cost.

Example 5:

TABLE XXIII. TRANSPORTATION TABLEAU OF TRANSPORTATION PROBLEM 5

	D_1	D_2	D_3	Supply
O_1	1	16	17	10
O_2	**0**	6	8	2
O_3	3	3	7	3
Demand	10	3	2	

We have again represented the cost matrix and the MWOC matrix in a single tableau in Table XXIV. It is observed in Table XXIV that c_{21} = 0 and its corresponding weight cost factor is 120. Once again, let's illustrate how to calculate the weight factor for that unit cost entry. According to the proposed method, since c_{21} = 0, the algorithm executes case (b). Formally:

(b) As $c_{21} = 0$ and $\{c_{pq}: 0 < c_{ij} < 1, \forall p, q\} = \varphi$, (i.e. null set), so

$$W^m_{c_{21}} = \max\{a_i, b_j; \forall i, j\} \times \min\{a_2, b_1\} \times \max\{D_2^{Ir}, D_1^{Ic}\}$$

$$=\max\{10,2,3; 10,3,2\} \times \min\{2,10\} \times Max\{1,6\}$$

$$= 10 \times 2 \times 6 = 120$$

It is observed in the Table XXIV that the weight opportunity cost of the cell C_{21} is 120 corresponding to the minimal cost i.e., c_{21} = 0 obtained by the case (b) of the proposed approach. On the other hand, the weight opportunity cost of the cell C_{11} is 150 corresponding to the cost 1 i.e., c_{11} = 1. It is worthwhile to mention here that the weight factor corresponding to the route (cell C_{11}) is largest though it's cost entry is not minimum. Now we have solved the problem with the proposed MWOC-VAM as well as LCM, VAM, and WOC-LCM. The experimental result is shown in Table XXV.

It is observed in Table XVI that the proposed MWOC-VAM and VAM need the least amount of transportation cost to obtain the IBFS compared to all other approaches, LCM and WOC-LCM. It is observed that the starting allocation of VAM and proposed MWOC-VAM are the same but different from both LCM and WOC-LCM. On the other hand, the starting allocation of LCM and proposed WOC-LCM are the same. Moreover, VAM and MWOC-VAM need fewer steps to get IBFS. It is also observed in the second column of Table XXVI that the pattern of the flow of allocation for each approach is different. Now we have considered another problem 6 (Example 6) in which one route has zero transportation cost and some route's transportation cost is less than 1 but getter than zero.

TABLE XXIV. MODIFIED WEIGHTED OPPORTUNITY COST INCLUDED TRANSPORTATION TABLEAU PROBLEM 5

	D_1	D_2	D_3	supply	DI
O_1	$\frac{150}{1}$ 1	$\frac{45}{16}$ 16	$\frac{30}{17}$ 17	10	15
O_2	120 0	$\frac{12}{6}$ 6	$\frac{12}{8}$ 8	2	6
O_3	$\frac{3}{3}$ 3	$\frac{9}{3}$ 3	$\frac{2}{7}$ 7	3	0
Demand	10	3	2		
DI	1	3	1		

TABLE XXV. COMPARISON REGARDING THE FLOW OF ALLOCATIONS AND TOTAL TRANSPORTATION COST TO FIND OUT THE IBFS OF THE PROBLEM 5

Method	**Flow of allocations and IBFS**	**Total Cost**
LCM	$x_{21}\rightarrow x_{11}\rightarrow x_{32}\rightarrow x_{13}$ 2 → 8 → 3 → 2	51
VAM	$x_{11}\rightarrow x_{32}\rightarrow x_{23}$ 10 → 3 → 2	**35**
WOC-LCM	$x_{21}\rightarrow x_{11}\rightarrow x_{32}\rightarrow x_{13}$ 2 → 8 → 3 → 2	51
MWOC-VAM (proposed)	$x_{11}\rightarrow x_{23}\rightarrow x_{32}$ 10 → 2 → 3	**35**

Example 6:

TABLE XXVI. TRANSPORTATION TABLEAU OF TRANSPORTATION PROBLEM 6

	D_1	D_2	D_3	Supply
O_1	0	3	0.5	8
O_2	3	7	10	3
O_3	1	0.7	11	9
Demand	6	6	8	

It is observed in the Table XXVI that c_{11} = 0, c_{13} = 0.5 and c_{32} = 0.7. So, to find out the weight cost factor corresponding to the cost entry 0, the algorithm executes the case (c) of the proposed method. Formally:

(c) As $c_{11} = 0$ and $\{c_{pq}: 0 < c_{ij} < 1, \forall p, q\} = \{0.5, 0.7\} \neq \varphi$, so

$$W^m_{c_{11}} = \frac{\max\{a_i, b_j; \forall i, j\}}{[\min\{c_{ij}: 0 < c_{ij} < 1, \forall i, j\}]} \times \min\{a_1, b_1\} \times \max\{D_1^{Ir}, D_1^{Ic}\}$$

$$= \frac{\max\{8,3,9, \ 6,6,8\}}{[\min\{0.5,0.7\}]} \times \min\{8,6\} \times \max\{1, 0.5\}$$

$$= \frac{9}{0.5} \times 6 \times 1 = 108$$

We have again represented the cost matrix and the MWOC matrix in a single tableau as Table XXVII. It is observed in the Table XXVII that the weight opportunity cost of the cell C_{11} is 108 corresponding to the minimal cost i.e., c_{21} = 0 obtained by the case (c) of the proposed algorithm. Moreover, the weight opportunity cost of the cell C_{13} and C_{32} are 152 and 19.71 respectively which are calculated according to the case (a) as well. Now we have solved the problem by the proposed MWOC-VAM as well as LCM, VAM and WOC-LCM. The experimental result is shown in the Table XXVIII.

It is observed in Table XXVIII that the proposed MWOC-VAM and VAM need the least transportation cost to obtain the IBFS compared to the other two approaches, LCM and WOC-LCM. It is observed that the starting allocation of VAM and proposed MWOC-VAM are the same but different from both LCM and WOC-LCM. On the other hand, the starting allocation of LCM and proposed WOC-LCM are the same. It is also observed in the second column of Table XXVIII that the pattern of the flow of allocation for each approach is different.

To analyze the performance and effectiveness of the proposed method, we considered an additional 10 randomly generated numerical instances. The experimental results are displayed in Table XXIX. It is evident from Table XXIX that the proposed method consistently outperforms both the existing LCM and WOC-LCM approaches. Furthermore, in two instances, the proposed method surpasses VAM, while in other cases, it yields equivalent total costs compared to VAM. It is also observed that the IBFSs obtained by the proposed method are optimal or near optimal.

We collected an additional 8 numerical instances from published international journals/conferences to evaluate the efficiency and effectiveness of the proposed method. In Table XXX, the first column indicates the reference number of the published article. The data presented in Table XXX show that, except for three instances where all approaches obtained optimal solutions, the proposed method consistently outperforms both LCM and WOC-LCM. It is noteworthy that, in four out of eight instances, the proposed method outperforms VAM, while in the remaining instances, both approaches yield similar solutions.

The numerical experiments indicate that the proposed MWOC-VAM consistently performs as well as or better than both VAM and LCM. Furthermore, VAM requires the calculation of the DI at each iteration, which increases its computational cost. In contrast, the proposed MWOC-VAM only needs to compute the DI and WOC once initially, making it more computationally efficient.

TABLE XXVII. MODIFIED WEIGHTED OPPORTUNITY COST INCLUDED TRANSPORTATION TABLEAU PROBLEM 6

	D_1	D_2	D_3	supply	DI
O_1	108 0 ×	$\frac{13.8}{3}$ 3 ×	$\frac{76}{.5}$ .5 **8**	~~8~~	0.5
O_2	$\frac{12}{3}$ 3 **3**	$\frac{12}{7}$ 7 ×	$\frac{28.5}{10}$ 10 ×	~~3~~	4
O_3	$\frac{6}{1}$ 1 **3**	$\frac{13.8}{.7}$ .7 **6**	$\frac{76}{11}$ 11 ×	~~9~~	.3
Demand	~~6,~~ ~~3~~	~~6~~	~~8~~		
DI	1	2.3	9.5		

TABLE XXVIII. COMPARISON REGARDING THE FLOW OF ALLOCATIONS AND TOTAL TRANSPORTATION COST TO FIND OUT THE IBFS OF THE PROBLEM 6

Method	Flow of allocations and IBFS	Total Cost
LCM	$x_{11} \to x_{13} \to x_{32} \to x_{23} \to x_{33}$ $6 \to 2 \to 6 \to 3 \to 3$	68.2
VAM	$x_{13} \to x_{32} \to x_{31} \to x_{21}$ $8 \to 6 \to 3 \to 3$	**20.2**
WOC-LCM	$x_{11} \to x_{13} \to x_{32} \to x_{33} \to x_{23}$ $6 \to 2 \to 6 \to 3 \to 3$	68.2
MWOC-VAM Proposed	$x_{13} \to x_{32} \to x_{21} \to x_{31}$ $8 \to 6 \to 3 \to 3$	**20.2**

TABLE XXIX. COMPARISON AMONG LCM, VAM, WOC-LCM, AND PROPOSED MWOC-VAM REGARDING THE IBFS OF SOME RANDOMLY GENERATED NUMERICAL INSTANCES

Ex. No.	Problem	LCM	VAM	WOC-LCM	MWOC-VAM Proposed	Opt. Sol.
1	C_{ij}:{(9,8,5,7); (4,6,8,7);(5,8,9,5)} S: (12,14,16); D: (8,18,13,3)	248	248	240	**241**	240
2	C_{ij}:{(4,3,5);(6,5,4);(8,10,7)} S: (9,8,10); D: (7,12,8)	145	150	145	**144**	**139**
3	C_{ij}:{(2,5,4);(6,1,2);(4,5,2)} S: (4,6,6); D: (3,7,6)	29	29	29	**29**	29
4	C_{ij}:{(14,19,7,5);(16,6,12,9);(6,16,5,20)} S: (10,12,18); D: (9,14,7,10)	243	243	243	**243**	243
5	C_{ij}:{(4,2,1);(3,8,4);(6,5,2)} S: (50,70,45); D: (40,65,60)	605	**490**	605	**490**	475
6	C_{ij}:{(21,16,23,13);(17,18,14,23); (32,27,18,41)} S: (11,13,19);D: (6,10,12,15)	922	**796**	919	**796**	**796**
7	C_{ij}:{(1,16,17);(0,6,8);(3,3,7)} S: (10,2,3); D: (10,3,2)	51	**35**	51	**35**	**35**
8	C_{ij}:{(1,16,17);(0,3,8);(3,3,7)} S: (10,2,3); D: (10,3,2)	60	**36**	60	**36**	**36**
9	C_{ij}:{(1,16,17);(0,6,8);(3,3,7)} S: (30,6,9) ;D: (30,9,6)	153	**105**	153	**105**	105

TABLE XXX. COMPARISON AMONG LCM, VAM, WOC-LCM, AND PROPOSED MWOC-VAM REGARDING THE IBFS OF THE PUBLISHED NUMERICAL INSTANCES

Ref No.	Problem	LCM	VAM	WOC-LCM	MWOC-VAM Proposed	Opt. Sol.
[3]	C_{ij}:{(10,2,20,11);(12,7,9,20);(4,14,16,18)} S: (15,25,10); D: (5,15,15,15)	475	475	475	**475**	435
[26]	C_{ij}:{(6,4,1); (3,8,7);(4,4,2)} S: (50,40,60); D: (20,95,35)	555	555	555	**555**	555
[60]	C_{ij}:{(7,5,9,11); (4,3,8,6);(3,8,10,5);(2,6,7,3)} S: (30,25,20,15); D: (30,30,20,10)	435	470	435	**430**	410
[61]	C_{ij}:{(4,3,5); (6,5,4);(8,10,7)} S: (90,80,100); D:(70,120,80)	1450	1500	1450	**1440**	1390
[26]	C_{ij}:{(4,1,2,4,4);(2,3,2,2,2);(3,5,2,4,4)} S: (60,35,40); D: (22,45,20,18,30)	305	273	278	**273**	273
[62]	C_{ij}:{(4,19,22,11); (1,9,14,14);(6,6,16,14)} S: (100,30,70); D: (40,20,60,80)	2090	2170	2160	**2090**	2040
[63]	C_{ij}:{(6,1,9,3); (11,5,2,8);(10,12,4,7)} S: (70,55,90); D: (85,35,50,45)	1165	1220	1165	**1165**	1160
[64]	C_{ij}:{(13,21,14); (8,12,21);(15,17,19)} S: (13,20,5); D: (12,15,11)	473	473	473	**473**	**465**

VI. CONCLUSION

IBFS is crucial for obtaining an optimal solution in TP. While various approaches exist in the literature to determine IBFS, most are formulated by manipulating the cost matrix to control allocation flow. In this article, we stand out as perhaps the first to consider the impact of node capacity distribution on the flow of allocations in both LCM and VAM. Through numerical experiments, we observed significant changes in output due to the distribution of capacity among nodes, even when the cost matrix and total supply and demand remained constant. For example, if the cost matrix is identical, the flow of allocations for approaches like NWC, LCM, VAM, etc., remains almost unchanged regardless of the distribution of capacity among nodes. However, by addressing this issue in the formulation of the flow allocation matrix in the proposed method, the flow of allocations varies significantly. To leverage this effect, we introduced a novel tool to control allocation flow. To incorporate the influence of node capacity distribution, we developed a capacity-influenced allocation control matrix, termed Capacity-Influenced Distribution Indicator (CI-DI), along with the distribution indicator defined by VAM. Subsequently, we proposed a capacity-influenced algorithm for finding IBFS in balanced TP. It is observed from the numerical experiments that the proposed method is effective to find out better IBFS of TPs. The proposed approach significantly overcomes the limitations of both LCM and VAM concerning the impact of capacity distribution among nodes. Additionally, it demonstrates enhanced computational efficiency compared to VAM. While VAM requires the calculation of the DI for each allocation step, the proposed method only needs to compute the Capacity-Influenced Distribution Indicator (CI-DI) matrix once. Experimental results lead to the conclusion that practitioners in the supply chain and transportation domain should not only consider cost distributions but also recognize the substantial role of capacity distributions among nodes in controlling allocation flow, leading to the identification of better IBFS. The concept of a capacity-influenced flow of allocation is innovative, providing a new perspective or "window" through which researchers can approach transportation problems and other linear programming challenges. In future work, we aim to develop a hybrid algorithm by integrating the proposed approach with fuzzy-based techniques.

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