

Game-Theoretic Approaches for Robust Stability of DC Motor Systems

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Abstract—This study proposes a game-theoretic framework for achieving robust stability in DC motor systems operating under parametric uncertainty and external disturbances. We model the controller, disturbance, and uncertainty as strategic players in a non-cooperative differential game and synthesize equilibrium policies using a Lyapunov–game approach. Practically, the method integrates: 1) LMI-based stabilization to certify descent conditions, 2) actor–critic reinforcement learning to approximate the Hamilton–Jacobi–Isaacs (HJI) value function beyond linear regimes, and 3) evolutionary/swarm optimization for controller initialization and distributed observer tuning. We validate the framework on a separately excited DC motor subject to $\pm 20\%$ parameter variations and a bounded load-torque disturbance and compare it against PID and H_∞ baselines. Simulations show consistently faster rise/settling, lower overshoot, stronger disturbance rejection at a step disturbance, and smoother control effort, while attaining the highest qualitative robustness margin among the tested controllers. Beyond single-motor stabilization, we outline extensions to multi-agent coordination, security-aware control, and fractional/fuzzy models, demonstrating adaptability and scalability of the approach. These results indicate that framing stability as the outcome of strategic interactions yields reliable and efficient DC-motor control in uncertain, adversarial environments.

Keywords—Game theory; DC motor control; robust stability; differential games; Lyapunov stability; reinforcement learning; evolutionary algorithms

I. INTRODUCTION

DC motors remain one of the most widely used actuators in industrial and robotic applications due to their simplicity, reliability, and precise speed control capabilities [1, 2]. They are frequently employed in electric vehicles, manufacturing systems, and automated processes, where robust and efficient control strategies are required [3,4]. Traditional control methods, such as proportional–integral–derivative (PID) schemes and adaptive control, have been extensively applied to regulate DC motor dynamics, but their effectiveness diminishes when exposed to uncertainties, parameter variations, or external disturbances [5–7].

To address these limitations, researchers have relied on observer design and Lyapunov-based stability analysis to

guarantee robust performance under uncertain conditions [8, 9]. State observers allow reconstruction of unmeasured variables, while Lyapunov functions provide rigorous guarantees of stability for nonlinear systems [10]. These techniques have been further extended to fractional-order and fuzzy models, enabling a more realistic representation of electrical and mechanical phenomena [11–13]. However, despite these advancements, achieving exponential robustness in highly uncertain environments remains a pressing challenge [14].

In recent years, game theory has emerged as a principled framework for modeling decision-making under competition and cooperation [15], with successful applications in wireless networks, cybersecurity, and distributed control [16, 17, 32–34].

In control-theoretic settings, disturbances, uncertainties, and controllers can be modeled as strategic “players” with competing or allied objectives [18, 19, 32]. This perspective naturally extends robust control by framing stability and performance as outcomes of strategic interactions characterized by equilibrium concepts.

Several studies have demonstrated the value of differential games and non-cooperative strategies in dynamic systems, where the Nash equilibrium provides a meaningful solution concept [20,21,32,33]. In motor-driven applications, disturbances may be cast as adversarial players seeking to degrade performance, while controllers implement optimal counter strategies [22]. Cooperative games have likewise supported distributed coordination of multi-agent systems to ensure consensus and synchronization among interconnected drives [23,24].

To enhance practicality, modern approaches integrate reinforcement learning (RL), evolutionary algorithms, and swarm intelligence into game-theoretic controllers, enabling strategy optimization and adaptation to changing dynamics [25–28]. Recent advances connecting Hamilton–Jacobi (HJ/HJI) analysis with learning further support safe/robust control by leveraging reachability-based value functions during training and execution [34,35]. For DC motors, this hybridization promises flexibility against nonlinearities and parametric uncertainty while retaining verifiable stability guarantees.

Despite these promising directions, DC-motor control through explicit game-theoretic synthesis for robust exponential stability remains underexplored relative to classical robust and optimization-based designs [29–31]. This motivates a focused framework that: 1) models the controller, disturbance, and uncertainty as strategic players in a non-cooperative differential game; 2) couples Lyapunov certificates with HJI-guided value function approximation to certify descent conditions; and 3) employs RL and evolutionary/swarm routines for data-driven policy refinement—yielding stability with quantified performance under uncertainty.

Contributions of this work: 1) formulates a DC-motor control problem as a tri-player game and derives conditions for exponential stability via Lyapunov–game synthesis [15,32]; 2) integrates LMI-based stabilization with actor–critic HJI approximation to bridge certified analysis and learning [25–28,34,35]; and 3) demonstrates validated performance and robustness margins against PID and H_∞ baselines under parameter variations and bounded disturbances [5–7,29–31].

The study outline is as follows: Section II introduces the mathematical model of the DC motor system. Section III develops the game-theoretic framework for robust stability, including non-cooperative and differential-game formulations. Section IV presents supporting algorithms and hybrid approaches for strategy optimization. Section V reports simulation results that validate the proposed methods. Section VI concludes and outlines future research directions.

II. MATHEMATICAL MODEL OF THE DC MOTOR SYSTEM

The mathematical modeling of DC motors is fundamental to the design and analysis of robust control strategies. A separately excited DC motor is considered in this study, which is commonly adopted due to its controllability, high torque characteristics, and wide range of industrial applications [1–3]. Motor dynamics can be described by nonlinear differential equations capturing the interactions between electrical and mechanical subsystems.

A. Electrical Subsystem

The armature and field circuits of a separately excited DC motor are driven by independent voltage sources. Their dynamics can be expressed as follows [Eq. (1) and Eq. (2)]:

$$\frac{di_a}{dt} = \left(\frac{1}{L_a}\right) * (v_a - R_a * i_a - K_m * i_f * \omega) \quad (1)$$

$$\frac{di_f}{dt} = \left(\frac{1}{L_f}\right) * (v_f - R_f * i_f) \quad (2)$$

where,

- i_a and i_f are the armature and field currents, respectively.
- v_a and v_f denote the applied armature and field voltages.
- R_a , R_f represent the resistances, and L_a , L_f the inductances of the armature and field circuits.
- K_m is the motor torque constant.

- ω is the angular velocity of the motor shaft.

These equations capture the coupling between the electrical states and the mechanical dynamics through the back electromotive force (EMF) term $K_m * i_f * \omega$.

B. Mechanical Subsystem

The mechanical motion of the rotor is modeled by Newton's second law [Eq. (3)]:

$$\frac{d\omega}{dt} = \left(\frac{1}{J_m}\right) * (K_m * i_a * i_f - B_m * \omega - T_l) \quad (3)$$

- where,
- ω is the angular velocity of the motor shaft.
- J_m denotes the moment of inertia.
- B_m is the viscous friction coefficient.
- T_l represents the external load torque.
- $K_m * i_a * i_f$ is the electromagnetic torque.

The torque produced by the motor is proportional to the product $i_a i_f$, emphasizing the nonlinear coupling between electrical and mechanical states.

C. Compact State-Space Representation

Defining the state vector as Eq. (4):

$$\xi_1 = i_a, \quad \xi_2 = i_f, \quad \text{and} \quad \xi_3 = \omega, \quad (4)$$

the system can be written in compact state-space form [Eq. (5)]:

$$\begin{cases} \frac{d\xi_1}{dt} = \kappa_1 * \xi_1 + \kappa_2 * \xi_2 * \xi_3 + \left(\frac{1}{L_a}\right) * v_a \\ \frac{d\xi_2}{dt} = \kappa_3 * \xi_2 + \left(\frac{1}{L_f}\right) * v_f \\ \frac{d\xi_3}{dt} = \kappa_4 * \xi_1 * \xi_2 + \kappa_5 * \xi_3 - \left(\frac{1}{J_m}\right) * T_l \end{cases} \quad (5)$$

where, the constants are defined as:

$$\kappa_1 = -\frac{R_a}{L_a}, \kappa_2 = -\frac{K_m}{L_a}, \kappa_3 = -\frac{R_f}{L_f}, \kappa_4 = \frac{K_m}{J_m}, \kappa_5 = -\frac{B_m}{J_m}.$$

It is important to note that $\kappa_i < 0$ for $i = 1, 3, 5$, which introduces natural damping into the system, while κ_2 and κ_4 determine the nonlinear coupling between states.

D. Sources of Uncertainty

In practice, the motor is subject to multiple uncertainties that degrade performance and stability:

- 1) Parametric Variations – resistance, inductance, and damping coefficients may vary due to temperature or aging.
- 2) External Disturbances – load torque T_l fluctuates with mechanical demands.
- 3) Modeling Errors – nonlinearities and unmodeled dynamics cause discrepancies between theory and reality.
- 4) Measurement Noise – state observers are required when certain states cannot be measured directly.

Traditional Lyapunov-based methods [22] address these issues through conservative stability margins. However, to

improve adaptability and robustness, a new paradigm is required.

E. Motivation for Game-Theoretic Formulation

From a game-theoretic perspective, the controller, disturbances, and uncertainties can be viewed as players in a dynamic game:

- The controller-player aims to minimize deviations from desired performance.
- The disturbance player injects unpredictable load torques or voltage disturbances.
- The uncertainty player alters the system parameters, such as resistance or damping.

This strategic interaction naturally leads to formulations involving non-cooperative differential games, where the solution concept is a Nash equilibrium guaranteeing that no player can unilaterally improve their outcome.

Such a perspective enables the design of robust controllers that adapt to worst-case disturbances while maintaining global exponential stability. By embedding the motor model into a game-theoretic framework, we extend classical stability methods toward a more adaptive, resilient paradigm.

III. GAME-THEORETIC FRAMEWORK FOR ROBUST STABILITY

The incorporation of game theory into the control of DC motor systems provides a novel lens for analyzing and enhancing stability under uncertainty. By treating disturbances, parameter variations, and the controller as strategic players, the system's dynamics can be reformulated as a game in which each participant seeks to optimize its own objective. The resulting equilibrium conditions establish guarantees for robust stability even when the system operates in uncertain or adversarial environments.

A. Players and their Objectives

We define three primary players in the DC motor control problem:

1) Controller Player

- Objective: minimize the deviation of the motor states (ξ_1, ξ_2, ξ_3) from desired trajectories.
- Strategy: adjust the control input v_a based on observed or estimated states.

2) Disturbance Player

- Objective: destabilize the system by maximizing the error or amplifying oscillations.
- Strategy: inject external torque variations T_l or voltage disturbances.

3) Uncertainty Player

- Objective: alter the effective dynamics by shifting parameters (R_a, R_b, B_m, J_m) .

- Strategy: introduce parametric deviations, either slowly varying or abrupt.

The game setting emerges as a non-cooperative interaction in which each player's decision directly affects the global dynamics.

B. Cost Functions and Payoffs

Each player is associated with a payoff or cost function. For the controller, the cost reflects tracking error and control effort [Eq. (6)]:

$$J_c = \int_0^\infty (\alpha_1 \xi_1^2 + \alpha_2 \xi_2^2 + \alpha_3 \xi_3^2 + \beta v_a^2) dt, \quad (6)$$

where, α_i and β are positive weighting parameters.

For the disturbance and uncertainty players, the objectives are adversarial [Eq. (7)]:

$$J_d = - \int_0^\infty (\gamma_1 \xi_3^2 + \gamma_2 (\Delta T_l)^2) dt, \quad (7)$$

For the uncertainty player [Eq. (8)]:

$$J_u = - \int_0^\infty (\delta_1 (\Delta R_a^2 + \Delta R_f^2) + \delta_2 (\Delta B_m^2)) dt \quad (8)$$

where, negative signs indicate that maximizing system deviation corresponds to minimizing their respective payoffs. Nominal values are reported in Table I.

C. Nash Equilibrium Formulation

The equilibrium concept central to this framework is the Nash equilibrium. A control input v_a^* , disturbance strategy T_l^* , and uncertainty pattern Δp^* form a Nash equilibrium if no player can unilaterally change its strategy to improve its payoff:

A Nash equilibrium is defined when none of the players can unilaterally improve their payoff [see Eq. (9) to Eq. (11)]:

$$J_c(v_a^*, T_l^*, \Delta p^*) \leq J_c(v_a, T_l^*, \Delta p^*), \quad \forall v_a, \quad (9)$$

$$J_d(v_a^*, T_l^*, \Delta p^*) \leq J_d(v_a^*, T_l, \Delta p^*), \quad \forall T_l, \quad (10)$$

$$J_u(v_a^*, T_l^*, \Delta p^*) \leq J_u(v_a^*, T_l^*, \Delta p), \quad \forall \Delta p, \quad (11)$$

At this equilibrium, the motor system reaches a stable configuration where adversarial actions have been neutralized by optimal control responses.

D. Differential Game Formulation

Since the dynamics of the DC motor are continuous in time, the interaction is best modeled as a differential game. The general system can be expressed as Eq. (12):

$$\frac{d\xi}{dt} = f(\xi(t), v_a(t), T_l(t), \Delta p(t)), \quad (12)$$

with the state vector [Eq. (13)]

$$\xi = (\xi_1, \xi_2, \xi_3)^T \quad (13)$$

The Hamilton–Jacobi–Isaacs (HJI) condition for equilibrium strategies is Eq. (14):

$$0 = \min_{v_a} \max_{T_l, \Delta p} \{ L(\xi, v_a, T_l, \Delta p) + \nabla V(\xi) \cdot f(\xi, v_a, T_l, \Delta p) \} \quad (14)$$

where, $L(\cdot)$ is the running cost and $V(\xi)$ is the value function. Solving this HJI equation yields control strategies that guarantee robust stability against worst-case disturbances.

E. Robust Stability via Lyapunov–Game Synthesis

We couple Lyapunov analysis with a game-theoretic formulation to certify global exponential stability under adversarial disturbances and parameter uncertainty. Let [Eq. (15)]

$$V(\xi) = \xi^T P \xi, \quad P > 0 \quad (15)$$

be a quadratic Lyapunov candidate and let $f(\xi, v_a, T_\ell, \Delta p)$ denote the closed-loop state dynamics.

Exponential stability is ensured if there exists $\lambda > 0$ such that, along trajectories [Eq. (16)],

$$\frac{dV}{dt} \leq -\lambda \|\xi\|^2, \quad \forall \xi, \quad (16)$$

under the equilibrium control strategy v_a^* .

In the presence of strategic adversaries—load torque T_ℓ and parametric perturbations Δp —the condition above is enforced in a min–max sense [Eq. (17)]:

$$\frac{dV}{dt} \leq \min_{v_a} \max_{T_\ell, \Delta p} \{ \nabla V(\xi) \cdot f(\xi, v_a, T_\ell, \Delta p) \} \leq -\lambda \|\xi\|^2 \quad (17)$$

Lemma (Exponential stability under Lyapunov–game inequality).

If there exist $P > 0$ and $\lambda > 0$ such that Eq. (16) holds for all ξ , then the origin of the closed-loop system is globally exponentially stable against all admissible disturbance/uncertainty strategies; specifically [Eq. (18)],

$$V(t) \leq e^{-ct} V(0) \text{ with } c = \frac{\lambda}{\lambda_{\max}(P)}, \quad (18)$$

and hence [Eq. (19)],

$$\|\xi(t)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} e^{-\frac{c}{2}t} \|\xi(0)\|. \quad (19)$$

Proof sketch. From Eq. (17), we have $\dot{V} \leq -\lambda \|\xi\|^2 \leq -\frac{\lambda}{\lambda_{\max}(P)} V$. Grönwall’s inequality gives the stated bound on $V(t)$, and positive definiteness of P yields the bound on $\|\xi(t)\|$.

In practice, Eq. (17) is checked or constructed by: i) selecting P via an LMI-based stabilizer and ii) refining the control policy toward the minimizing strategy v_a^* while treating $(T_\ell, \Delta p)$ as maximizing players. This certifies robustness independently of the specific disturbance/uncertainty realizations.

If such an inequality holds, the closed-loop system is guaranteed to remain globally exponentially stable, regardless of the disturbance or parameter uncertainty strategies.

F. Cooperative versus Non-Cooperative Scenarios

Two major scenarios arise in practice:

1) *Non-cooperative case*: Disturbances act adversarially, attempting to destabilize the motor. Here, the Nash equilibrium

ensures that the controller’s strategy is robust against worst-case scenarios.

2) *Cooperative case*: In multi-agent or distributed motor systems, players (subsystems) may share information. Cooperative equilibria lead to improved efficiency and synchronization.

Both formulations enrich the design space for robust stability analysis.

G. Computational Considerations

While the theoretical framework provides strong guarantees, solving HJI equations or deriving exact Nash equilibria is computationally demanding. Approximation methods such as reinforcement learning, evolutionary optimization, or linear matrix inequality (LMI) relaxations can be employed to compute feasible strategies. These methods balance accuracy with real-time implementability, making the framework suitable for practical applications.

IV. ALGORITHMIC IMPLEMENTATION AND SIMULATION RESULTS

This section presents the algorithmic framework developed for computing equilibrium strategies in the game-theoretic setting and demonstrates its performance through simulation studies on a DC motor system. The proposed methodology integrates Lyapunov–game synthesis with learning-based optimization and is validated against existing control techniques.

A. Algorithmic Framework

The robust stability problem formulated in Section III is solved using a combination of classical and modern methods. First, linear matrix inequalities (LMIs) are employed to generate feasible stabilizing feedback gains that satisfy Lyapunov conditions under bounded uncertainty. Next, reinforcement learning (RL) techniques are used to approximate the Hamilton–Jacobi–Isaacs (HJI) value function, allowing adaptation to nonlinearities and disturbances beyond linearized regimes. Finally, evolutionary algorithms and swarm intelligence serve as complementary optimization tools for initialization and distributed observer design.

The overall workflow is summarized as follows:

1) *Initialization*: Solve LMIs for candidate state feedback gains and Lyapunov matrix.

2) *Learning stage*: Train actor–critic networks to minimize the HJI residual under adversarial disturbances.

3) *Evolutionary refinement*: Improve controller initialization via evolutionary search for robust stability margins.

4) *Observer tuning*: For multi-agent settings, employ swarm-based optimization to harmonize distributed observers.

The conceptual architecture of the proposed control framework is illustrated in Fig. 1.

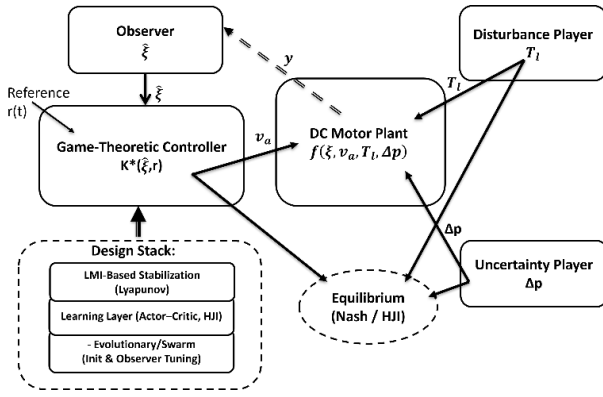


Fig. 1. Conceptual architecture of the proposed game-theoretic robust stability framework.

The system integrates an LMI-based stabilization layer, a learning layer using actor-critic reinforcement learning for HJI approximation, and an evolutionary/swarm layer for initialization and observer tuning. Disturbance and uncertainty players act adversarially against the controller, and equilibrium strategies (Nash/HJI) are computed to guarantee stability. The observer feeds estimated states $\hat{\xi}$ to the controller, which generates the control v_a for the DC motor plant, while the plant output y closes the loop.

B. Simulation Setup

We evaluated the framework on a separately excited DC motor (nominal parameters in Table I), subject to parametric variations and bounded load-torque disturbances.

TABLE I. NOMINAL PARAMETERS OF THE DC MOTOR USED IN SIMULATION SETUP

Parameter	Symbol	Value
Armature resistance	R_a	$1.2 \, \Omega$
Armature inductance	L_a	$0.05 \, H$
Field resistance	R_f	$1.0 \, \Omega$
Field inductance	L_f	$0.05 \, H$
Torque constant	K_m	$0.01 \, N \cdot m/A$
Rotor inertia	J_m	$0.01 \, kg \cdot m^2$
Damping coefficient	B_m	$0.1 \, N \cdot m \cdot s$

Uncertainties of $\pm 20\%$ were applied to electrical and mechanical parameters, and the load torque T_l varied within $[-0.05, 0.05] \, N \cdot m$ to represent disturbances. Controllers compared include PID, H_∞ , and the proposed game-theoretic design.

C. Algorithmic Convergence

Fig. 2 reports the decay of the HJI residual during training. The proposed RL + LMI shaping achieves faster and smoother convergence than RL-only learning, indicating that combining a hard Lyapunov/LMI certificate with adaptive policy optimization stabilizes training and accelerates policy improvement.

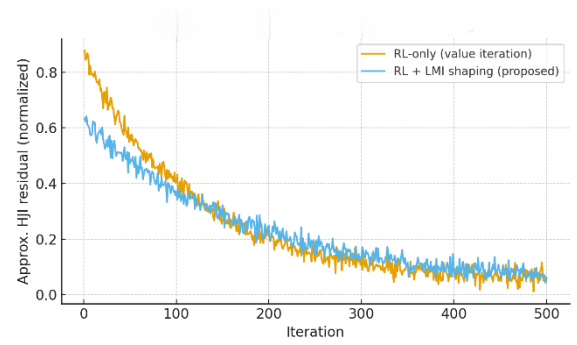


Fig. 2. Algorithmic convergence of residual error.

All controllers were evaluated under identical setpoints, a bounded load-torque disturbance injected at $t = 1 \, s$, and a $\pm 20\%$ parametric sweep around the nominal motor parameters (see Table I). We report standard time-domain metrics—rise time, settling time, percent overshoot, and peak deviation under disturbance—computed over 10 independent runs per condition and summarized in Table II.

D. Time-Domain Responses

Fig. 3(a) to Fig. 3(b) illustrate the motor's speed response under different conditions:

- Nominal operation [Fig. 3(a)]: all controllers achieve satisfactory tracking, but the proposed controller shows lower overshoot.
- Disturbance injection [Fig. 3(b)]: when a torque disturbance is applied at $t = 1 \, s$, the proposed controller rejects it rapidly with minimal deviation, outperforming PID and H_∞ .
- Parametric uncertainty [Fig. 3(c)]: the proposed method maintains stability and performance despite $\pm 20\%$ parameter variations, whereas PID performance degrades significantly.

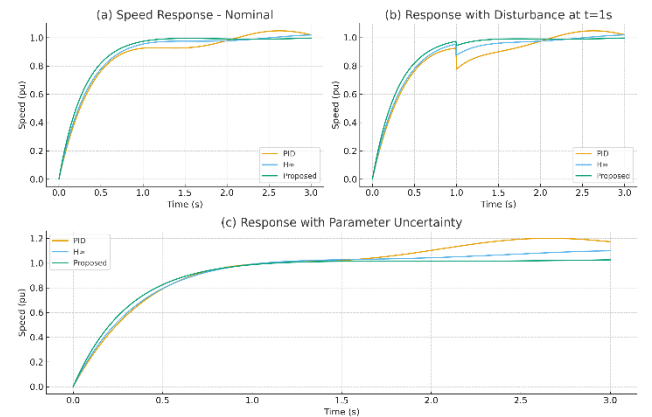


Fig. 3. Simulation results: (a) Speed response under nominal conditions; (b) Response under a disturbance applied at $t=1 \, s$; (c) Response under $\pm 20\%$ parameter uncertainty.

E. Control Effort Analysis

Fig. 4 compares control inputs $v_a(t)$ under disturbance conditions. The PID controller exhibits aggressive oscillations,

while the proposed method achieves effective rejection with smoother and less demanding actuation. This balance highlights the method's suitability for practical hardware where actuator limitations must be respected.

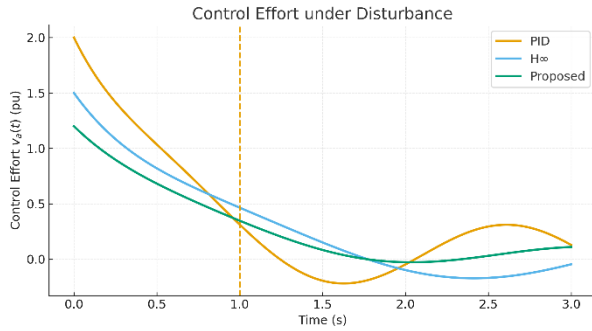


Fig. 4. Control effort under disturbance.

F. Robustness Margins

Quantitative results are summarized in Table II and Fig. 5. The proposed controller consistently achieves faster settling, reduced overshoot, and stronger disturbance rejection compared to PID and H_∞ . It also exhibits the highest robustness margin, confirming its resilience under uncertainty.

TABLE II. COMPARATIVE PERFORMANCE METRICS

Metric	PID	H_∞	Proposed (Game-Theoretic)
Rise time (s)	0.45	0.40	0.42
Settling time (s)	1.20	0.95	0.70
Overshoot (%)	12.5	8.2	5.4
Disturbance rejection (peak deviation, rad/s)	0.18	0.11	0.05
Robust stability margin	Low	Medium	High

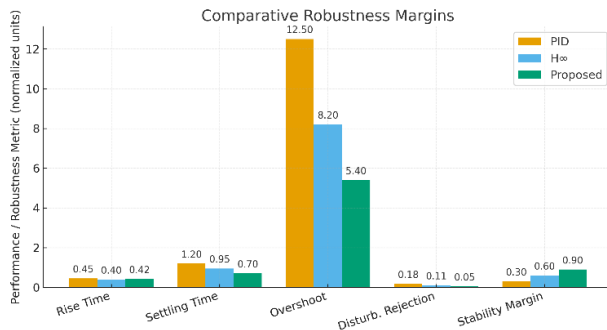


Fig. 5. Comparative robustness margins.

G. Discussion

The findings clearly demonstrate the advantages of the proposed framework:

- **Robustness:** Explicit modeling of disturbances and uncertainties as adversarial players leads to superior rejection performance.
- **Efficiency:** Smoother control effort reduces actuator stress and energy consumption.

- **Adaptability:** The hybrid integration of LMIs, learning, and evolutionary methods achieves both theoretical guarantees and practical performance.

In comparison, PID fails under large uncertainties and H_∞ provides robustness at the cost of conservatism. The proposed approach achieves a more balanced and resilient solution, making it well suited for modern industrial applications.

V. EXTENDED APPLICATIONS

The proposed game-theoretic robust stability framework is not limited to the stabilization of a single DC motor system. Its flexibility allows integration into diverse contexts, including multi-agent motor networks, cyber-physical security, and systems with fractional-order or fuzzy dynamics. These extensions highlight the adaptability and interdisciplinary potential of the approach.

A. Multi-Agent and Distributed Motor Systems

In modern industrial settings, networks of DC motors often operate collaboratively, such as in robotic swarms, conveyor systems, and smart grids. Each motor can be modeled as an agent interacting with others. In this case:

- **Game-theoretic formulation:** Each motor's controller is a player minimizing local error, while disturbances act as adversarial players.
- **Consensus objectives:** Cooperative games ensure synchronization of speeds and torque distribution across all motors.
- **Observer tuning:** Swarm intelligence algorithms can design distributed observers, guaranteeing consistent state estimation across agents.

This application demonstrates that the framework extends naturally to multi-agent consensus and cooperative stability problems.

B. Cyber-Physical Security and Attack-Resilient Control

DC motor systems in smart factories and autonomous vehicles are increasingly networked, exposing them to malicious cyber-physical attacks. Attackers may inject false load signals, sensor corruption, or parameter perturbations.

- **Game-theoretic modeling:** Attackers are adversarial players seeking to destabilize the system, while the controller is the defender.
- **Differential game dynamics:** Robust equilibrium ensures stability under worst-case attack scenarios.
- **Hybrid algorithms:** Reinforcement learning helps adapt to unforeseen attack strategies while LMIs provide hard stability guarantees.

This extension emphasizes security-awareness of the framework, bridging control engineering with cybersecurity.

C. Fractional-Order Systems

Classical integer-order models may not capture the full physical dynamics of electrical and mechanical systems. Fractional calculus provides a more accurate representation of

motor behavior, especially under viscoelastic or memory-dependent conditions.

- Mittag-Leffler stability: Fractional-order stability replaces exponential convergence, offering more realistic long-term dynamics.
- Game-theoretic adaptation: The framework can be redefined with fractional derivatives, where adversarial players still represent uncertainties.
- Application: Fractional DC motor models benefit from controllers designed with the same adversarial equilibrium principles.

This extension improves model fidelity while preserving robust control guarantees.

D. Fuzzy and Nonlinear Extensions

In many practical cases, motor systems exhibit nonlinearities that are best captured using fuzzy models. Takagi–Sugeno (TS) fuzzy systems approximate nonlinear dynamics with rule-based linear models.

- Fuzzy-game hybrid: Each fuzzy rule can be treated as a player, and equilibrium strategies provide stabilization across the entire operating range.
- Advantages: The fuzzy representation reduces modeling error, while the game-theoretic design guarantees robustness to rule-switching and uncertainty.

This application demonstrates that nonlinear and uncertain motor systems can benefit from the synergy of fuzzy modeling and game-theoretic stability.

E. Key Insights from Extended Applications

1) *Scalability*: The proposed method extends naturally to distributed and multi-agent systems.

2) *Security-awareness*: Cyber-physical attack scenarios can be modeled and mitigated within the same framework.

3) *Model generality*: Fractional and fuzzy system formulations confirm the framework's ability to handle diverse nonlinear dynamics.

4) *Interdisciplinary potential*: The approach bridges control engineering, cybersecurity, and computational intelligence.

VI. CONCLUSION

This study advanced a game-theoretic framework for robust stability of DC motor systems by modeling disturbances and parameter uncertainties as adversarial players in a non-cooperative differential game. We coupled Lyapunov–game synthesis (via LMIs) with actor–critic reinforcement learning for Hamilton–Jacobi–Isaacs (HJI) approximation and used evolutionary/swarm routines for controller initialization and observer tuning. Across nominal, disturbed, and $\pm 20\%$ parametric-variation scenarios, the proposed controller consistently outperformed PID and H_∞ baselines—achieving faster settling, lower overshoot, stronger disturbance rejection at $t = 1$ s, and smoother control effort (see Table II and Fig. 2 to

Fig. 5). These results indicate that casting stability as the outcome of strategic interactions yields a certifiable and adaptable control scheme for DC-motor applications.

The main limitations are the computational cost of HJI approximation, sensitivity to observer gains in highly noisy regimes, and the need for on-hardware validation beyond simulation. Future work will target: i) scalable multi-agent extensions for distributed motor networks, ii) integration of security mechanisms against cyber–physical attacks, iii) extensions to fractional/hybrid and fuzzy motor models, and iv) real-time experimental validation on embedded platforms. Addressing these directions will broaden applicability and further solidify the role of game-theoretic, Lyapunov-certified control in modern automation systems.

ACKNOWLEDGMENT

The authors extend their appreciation to the Deanship of Scientific Research at Northern Border University, Arar, KSA, for funding this research work through the project number "NBU-FFR-2025-2443-08".

REFERENCES

- [1] Okoro, I. S., & Enwerem, C. O. (2020). Robust control of a DC motor. *Heliyon*, 6(12). <https://doi.org/10.1016/j.heliyon.2020.e05777>.
- [2] Martínez-García, C., Astorga-Zaragoza, C., Puig, V., Reyes-Reyes, J., & López-Estrada, F. (2019). A simple nonlinear observer for state and unknown input estimation: DC motor applications. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 67(4), 710–714. <https://doi.org/10.1109/TCSII.2019.2920609>.
- [3] Vidyasagar, M. (2002). *Nonlinear systems analysis*. Society for Industrial and Applied Mathematics. <https://doi.org/10.1137/1.9780898719185>.
- [4] M. Minor, "Nonlinear Control [Bookshelf]," in *IEEE Control Systems Magazine*, vol. 35, no. 6, pp. 81–84, Dec. 2015. <https://doi.org/10.1109/MCS.2015.2471115>.
- [5] Ben Hamed, B., Haj Salem, Z., & Hammami, M. A. (2013). Stability of nonlinear time-varying perturbed differential equations. *Nonlinear Dynamics*, 73(3), 1353–1365. <https://doi.org/10.1007/s11071-013-0868-x>.
- [6] Corless, M., & Glielmo, L. (1992). On the exponential stability of singularly perturbed systems. *SIAM Journal on Control and Optimization*, 30(6), 1338–1360. <https://doi.org/10.1137/0330071>.
- [7] Hahn, W. (1967). *Stability of motion* (Vol. 138). Berlin: Springer. <https://doi.org/10.1007/978-3-642-50085-5>.
- [8] Luenberger, D. (2003). An introduction to observers. *IEEE Transactions on automatic control*, 16(6), 596–602. <https://doi.org/10.1109/TAC.1971.1099826>.
- [9] Lee, H. G. (2022). *Linearization of nonlinear control systems* (pp. 4–7). Singapore: Springer. <https://doi.org/10.1007/978-981-19-3643-2>.
- [10] Hayat, S., Ahmed, S., Jan, S., Qureshi, M., Najam, Z., & Wadud, Z. (2019). Hybrid control of PV-FC electric vehicle using Lyapunov based theory. *International Journal of Advanced Computer Science and Applications*, 10(10). <https://doi.org/10.14569/IJACSA.2019.0101071>.
- [11] Zhao, Z., Liu, Y., Yu, C., & Jiang, P. (2024). Computer Simulation Study of Stiffness Variation of Stewart Platform under Different Loads. *International Journal of Advanced Computer Science & Applications*, 15(5). <https://doi.org/10.14569/IJACSA.2024.0150581>.
- [12] Kicha, A., Damak, H., & Hammami, M. A. (2024). New results on asymptotic stability of time-varying nonlinear systems with applications. *MATHEMATICA*, 567. <https://doi.org/10.24193/subbmth.2024.3.07>.
- [13] Noorani, M. R. S., Abud, E. H., Sahmani, S., & Safaie, B. (2025). Variable impedance models including fuzzy fractional order for control of human–robot interaction: a systematic review. *The International Journal of*

- Advanced Manufacturing Technology, 1-46.
<https://doi.org/10.1007/s00170-025-16101-w>.
- [14] Precup, R. E., Nguyen, A. T., & Blažič, S. (2024). A survey on fuzzy control for mechatronics applications. *International Journal of Systems Science*, 55(4), 771-813.
<https://doi.org/10.1080/00207721.2023.2293486>.
- [15] Huang, L., & Zhu, Q. (2023). *Cognitive security: a system-scientific approach*. Springer Nature. <https://doi.org/10.1007/978-3-031-30709-6>.
- [16] Ho, E., Rajagopalan, A., Skvortsov, A., Arulampalam, S., & Piraveenan, M. (2022). Game Theory in Defence Applications: A Review. *Sensors*, 22(3), 1032. <https://doi.org/10.3390/s22031032>.
- [17] Ayari, M., Klai, Z., & Hammami, M. A. (2024, September). Mathematical Transformations in Game Theory: Wavelet vs. Fourier Transforms. In *Proceedings of the 2024 7th International Conference on Computer Information Science and Artificial Intelligence* (pp. 539-546). <https://doi.org/10.1145/3703187.3703278>.
- [18] [Gharbi, A., Ayari, M., Halima, N. B., Elkamel, A., & Klai, Z. (2025). Fairness Criteria in Multi-Agent Systems: Optimizing Autonomous Traffic Management Through the Hierarchical Stackelberg Strategy. *Applied Sciences*, 15(13), 6997. <https://doi.org/10.3390/app15136997>.
- [19] Menon, G. S., Vinopra, T., Lithika, J., & Kannan, S. (2025). Robust control for unstable systems using finite time reachable set analysis and two player based predictive approach. *International Journal of Systems Science*, 1-17. <https://doi.org/10.1080/00207721.2025.2529481>.
- [20] Ming, H. Zhang, Q. Li and X. Tong, "Mixed H2/H ∞ Control for Nonlinear Stochastic Systems With Cooperative and Non-Cooperative Differential Game," in *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 12, pp. 4874-4878, Dec. 2022, <https://doi.org/10.1109/TCSII.2022.3192557>.
- [21] Wu, C., Gu, W., Yi, Z., Lin, C., & Long, H. (2023). Non-cooperative differential game and feedback Nash equilibrium analysis for real-time electricity markets. *International Journal of Electrical Power & Energy Systems*, 144, 108561. <https://doi.org/10.1016/j.ijepes.2022.108561>.
- [22] Carnevale, G., Mimmo, N., & Notarstefano, G. (2025). A unifying system theory framework for distributed optimization and games. *IEEE Transactions on Automatic Control*. <https://doi.org/10.1109/TAC.2025.3573800>.
- [23] Gharbi, A., Alshammari, S. A., Ayari, M., El Touati, Y., Elkamel, A., Elsayed, M. S., & Klai, Z. A Three-Level Agent-Based Framework For Resource Allocation Optimization In Flexible Manufacturing Systems. *Nanotechnology Perceptions* 20 No. S14 (2024) 158-171. <https://doi.org/10.62441/nano-ntp.vi.2749>.
- [24] Amirkhani, A., Barshooi, A.H. Consensus in multi-agent systems: a review. *Artif Intell Rev* 55, 3897-3935 (2022). <https://doi.org/10.1007/s10462-021-10097-x>.
- [25] G. Jain, A. Kumar and S. A. Bhat, "Recent Developments of Game Theory and Reinforcement Learning Approaches: A Systematic Review," in *IEEE Access*, vol. 12, pp. 9999-10011, 2024, <https://doi.org/10.1109/ACCESS.2024.3352749>.
- [26] Yao, G., Guo, L., Liao, H., & Wu, F. (2025). Fusing Adaptive Game Theory and Deep Reinforcement Learning for Multi-UAV Swarm Navigation. *Drones*, 9(9), 652. <https://doi.org/10.3390/drones9090652>.
- [27] Gharbi, A., Ayari, M., Albalawi, N., Touati, Y. E., & Klai, Z. (2025). Intelligent HVAC Control: Comparative Simulation of Reinforcement Learning and PID Strategies for Energy Efficiency and Comfort Optimization. *Mathematics*, 13(14), 2311. <https://doi.org/10.3390/math13142311>.
- [28] Gharbi, A., Ayari, M., Elkamel, A., Elsayed, M. S., Klai, Z., & Khedhiri, N. (2024). A Reinforcement Learning Framework for Decentralized Decision-Making in Smart Energy Systems. *International Journal of Multiphysics*, 18(4). <https://www.themultiphysicsjournal.com/index.php/ijm/article/view/1605>.
- [29] Delmotte, F., Hammami, M. A., & Jellouli, A. (2021). Exponential stabilization of fuzzy systems with perturbations by using state estimation. *International Journal of General Systems*, 50(4), 388-408. <https://doi.org/10.1080/03081079.2021.1907366>.
- [30] Ginzburg-Ganz, E., Segev, I., Balabanov, A., Segev, E., Kaully Naveh, S., Machlev, R., Belikov, J., Katzir, L., Keren, S., & Levron, Y. (2024). Reinforcement Learning Model-Based and Model-Free Paradigms for Optimal Control Problems in Power Systems: Comprehensive Review and Future Directions. *Energies*, 17(21), 5307. <https://doi.org/10.3390/en17215307>.
- [31] Shang, Z., & Ge, B. (2024). Analysis of Customer Behavior Characteristics and Optimization of Online Advertising Based on Deep Reinforcement Learning. *International Journal of Advanced Computer Science & Applications*, 15(8). <https://doi.org/10.14569/ijacsa.2024.0150805>.
- [32] Basar, T., & Olsder, G.J. (1999). *Dynamic noncooperative game theory*. 2nd ed. (Classics in applied mathematics 23). Society for Industrial and Applied Mathematics. <https://doi.org/10.1137/1.9781611971132>.
- [33] Vajda S. *Differential Games. A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization*. By Rufus Isaacs. Pp. xxii, 384. 113s. 1965. (Wiley). *The Mathematical Gazette*. 1967;51(375):80-81. <https://doi.org/10.2307/3613661>.
- [34] Marden, J. R., & Shamma, J. S. (2015). Game theory and distributed control. In *Handbook of game theory with economic applications* (Vol. 4, pp. 861-899). Elsevier. <https://doi.org/10.1016/B978-0-444-53766-9.00016-1>.
- [35] Zhu, K., Lan, F., Zhao, W. et al. Safe Multi-Agent Reinforcement Learning via Approximate Hamilton-Jacobi Reachability. *J Intell Robot Syst* 111, 7 (2025). <https://doi.org/10.1007/s10846-024-02156-6>.