A Fuzzy Petri Net Approach with Automated ANFIS Rule Learning for Modelling Real-Time Systems

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***Abstract*—In this paper, we propose a modelling approach for real-time intelligent systems using Fuzzy Petri Nets (FPNs), a formalism that generates dynamic fuzzy rules, supports uncertainty, and enables concurrent reasoning. FPNs offer a well-defined tool for dynamically evaluating Fuzzy Production Rules (FPRs), Certainty Factors (CFs), and truth degrees, and for making real-time decisions. To reduce the complexity of manually constructed or probabilistically modelled fuzzy rules, we extend the modelling toolkit with the Adaptive Neuro-Fuzzy Inference System (ANFIS). ANFIS learns membership functions and Sugeno-type rules from numeric datasets through a feature. This results in a richer and more accurate set of rules. At the novelty level, we propose a rule-integrating scheme that maps Sugeno rules learned by ANFIS into FPN transitions to obtain more clearly explained reasoning and traceable rule execution within a neuro-fuzzy Petri net. Based on these learned rules, FPN executes them within a two-layer real-time (prediction and decision) while maintaining concurrent inference and real-time execution. The hybrid methodology is verified by fitting a real-time expert system for solar collector cleaning. Results from the experiments demonstrate that, in terms of predictive performance, ANFIS-induced rules drastically boost accuracy (from 85% to 93%) and reduce Root Mean Square Error (RMSE) from 4.82 to 2.57 relative to those generated by a single probabilistic FPN model. These results indicate that using neural learning combined with an FPN-based expert system makes real-time decision-making much more accurate and reliable.**

Keywords*—Fuzzy petri net; adaptive neuro-fuzzy inference system; expert systems; fuzzy logic; real-time system; artificial intelligence

I. Introduction

Expert systems support decision-making in uncertain and dynamic environments. Classical fuzzy expert systems rely on expert-defined linguistic rules, CFs, and membership functions, which makes them interpretable but labour-intensive, subjective, and challenging to scale to real-time or highly nonlinear settings [1], [2]. These limits are exacerbated when reasoning must continuously adapt to streaming data. FPNs have recently been proposed as a novel approach to handling expert systems because of their ability to represent FPRs and to support real-time reasoning through concurrency, formal semantics, and structural transparency [1], [3]. In many present-day FPN-based expert systems, the rule base is written offline. It depends on probabilistic derivation, which extracts CF and Thresholds (TH) from conditional probabilities [4]. While the aforementioned rule-generation methods are generally valid in many applications, they do not evolve over time and sometimes fail to describe real-time behaviour, such as that of dynamically changing systems. Alternatively, data-driven neuro-fuzzy learning combines the learning capabilities of neural networks and the interpretability of Fuzzy logic (FL) to offer a new approach [5], [6].

This work proposes a unified real-time modelling methodology. This strategy merges a new rule-based preparation technique that integrates Sugeno-type fuzzy inference representation with ANFIS learning with a new FPN model and execution infrastructure. The new concept is that the rule base is not treated as a static thing but as a dynamic model. A dynamic rule base was created using ANFIS, trained on data, and then integrated into an FPN-based inference and decision process. As well as explicating the whole procedure (rule preparation, mapping, modelling semantics, and real-time implementation), we present a controlled validation under a real-time case study, an expert system for solar collector Cleaning Decision-making (CD), a real-time validation of the proposed solution in the same experiment and comparison with other rule generation and decision-making strategies. This proves that the novelty is not merely a hybrid solution of two existing tools but a consistent modelling paradigm for real-time expert systems that connects data-driven rule learning with interpretable FPN execution. The main contributions of this work are as follows:

- Real-time unified model and execution of FPN: We introduce a unified FPN modelling and execution architecture for real-time expert systems, enabling concurrent reasoning and transparent rule-based inference within a single formal tool.
- Two-layer real-time modelling pattern: We develop a two-layer FPN decision model that separates a predictive layer (system-state estimation from streaming inputs) from a decision layer (action selection under real-time constraints), explicitly linking prediction outputs to decision transitions.
- Novel data-driven rule construction: We propose a rule-base preparation method that uses Sugeno-type fuzzy representation and ANFIS learning to generate a data-driven fuzzy rule base from numerical datasets for interpretable real-time execution.
- Expert-system rule integration pipeline: We define a systematic pipeline that converts ANFIS-learned Sugeno rules into FPRs and embeds them as executable FPN transitions, including explicit modelling of CF and TH for faithful truth propagation and firing behaviour.

- Real-time dynamic rule and model update: We support dynamic learning and dynamic modelling by updating the FPN structure during operation.
- Controlled comparison and real-time validation: We evaluate the proposed approach on real-time solar-collector CD-making and compare it fairly against a probabilistic rule-extraction baseline under identical execution conditions, demonstrating improved predictive accuracy and reduced RMSE using the ANFIS-learned rule base.

To complete the introduction and provide a clear reading guide, the remainder of this paper is organised as follows. Section II reviews representative FPN-, fuzzy-logic-, and ANFIS-based methods and highlights the gap addressed by our work. Section III summarises our unified real-time methodology, comprising a two-layer FPN architecture for prediction and decision-making, a probabilistic baseline, and a Sugeno-ANFIS rule-learning and rule-integration pipeline that explicitly handles CF/TH. Section IV provides an overview of the experimental setup and presents results on solar-collector CD-making regarding accuracy and RMSE under consistent conditions. Section V investigates the implications of the results, justifying the enhancement of ANFIS performance, the trade-off between rule-base size and real-time execution, and the proposed modelling paradigm's propensity to generalise. Finally, Section VI presents the concluding results and future research directions.

II. Related Works

Research on intelligent decision-making systems has long focused on how to represent uncertainty, model dynamic environments, and formalise knowledge-based reasoning. Within this context, FPN have emerged as an influential modelling formalism, capable of representing fuzzy rules, degrees of truth, and causal relationships while supporting concurrency and interpretability. Foundational work by Chen [1], [3] showed how FPNs can encode FPRs and provide transparent mechanisms for both reasoning and verification. Building on these foundations, later studies extended FPNs to applications such as fault diagnosis, production rescheduling, exception handling, and complex rule-based automation [7], [9], [10]. Further contributions introduced reversed reasoning for diagnostic analysis [11], invariant-based structural control [12], and learning-enabled FPN architectures [13], [14], improving the adaptability of Petri Net-based reasoning in uncertain environments. In contrast, our work targets three missing elements that make prior hybrid solutions difficult to compare: 1) an explicit rule-integration pipeline that converts ANFIS-learned Sugeno rules into FPRs and embeds them as FPN transitions with CF/TH handling, 2) a two-layer real-time modelling pattern within one FPN execution engine (prediction of reflectivity followed by decision-making), and 3) a controlled comparison between probabilistic rule extraction and ANFIS learning under identical execution conditions. Table I summarises these distinctions against representative FPN, probabilistic, and ANFIS-based approaches.

Verifying and validating fuzzy rule systems is essential to ensure their accuracy. For instance, Kouzehgar et al. [15] developed a method that gathers insights from experts through questionnaires to create rule bases. They then transform these into hierarchical fuzzy rules and connect them to FPN models. To identify semantic errors, they use reachability graphs, demonstrating how FPNs can serve as practical tools for reasoning and verification. Beyond this, Petri nets have been applied in various fields, such as modelling the behaviour of neural networks [16], optimising business processes [17], analysing power consumption in wireless sensor networks [18], and managing evolving workflows in healthcare [19]. FL remains a vital approach for addressing uncertainty across a range of applications, including diabetes detection [20], educational assessments [21], intelligent traffic control [22], and renewable energy management [23]. These studies highlight the importance of developing data-driven fuzzy systems to reduce our reliance on manually constructed rules. To overcome these limitations, ANFIS proposes a model that automatically optimises membership functions and rule parameters, combining neural learning with Sugeno-type fuzzy inference [6], [24], which was presented by Jang [5]. ANFIS has been used for nonlinear, interpretable modelling in various fields [25], [26], [27], [28] and is effective for predicting and managing renewable energy [29], [30], [31]. ANFIS has thus demonstrated a method that can provide data-driven fuzzy rules while also retaining interpretability. Reflectors/heliostats of solar energy systems are very sensitive to soiling, as reported reflectivity losses of up to 35% within weeks in dusty environments [32], [33], [34]. Thus, cleaning strategies and predictive maintenance have been investigated, including effective restoration methods [35] and learning-based reflectivity prediction [36], but black-box models pose explainability issues [37], [38]. Others have employed state-space methods in CSP plants [39], [40] and studied soiling and cleaning economics in a PV/Saharan area [41], [42], [43]. However, there is still a potential gap between interpretable fuzzy rule systems such as FPNs and adaptive neuro-fuzzy models such as ANFIS: FPNs represent precise causal semantics and have manual rule bases, whereas ANFIS recognises nonlinear behaviour but lacks a real-time, concurrent execution tool for decision support. There are, however, relatively few studies on hybrid neuro-fuzzy Petri Nets [13], [14]. In addition, probabilistic FPNs construct CF as conditional probabilities [44], [45], [46], and more recent work has assessed confidence values from errors in ANFIS training to optimise rules based on data-derived evidence.

Motivated by these observations, the present study integrates ANFIS-generated fuzzy rules into a formal FPN reasoning tool. This hybridisation combines the adaptability of neuro-fuzzy learning with the interpretability, concurrency, and structural rigour of Petri Nets. In doing so, it supports the construction of a real-time expert system capable of predicting solar reflectivity and determining optimal CDs under rapidly changing environmental conditions.

III. Methodology for Modelling Real-Time Expert Systems Using ANFIS and Probabilistic Methods Integrated within a Fuzzy Petri Net Approach

This section presents the methodological tool used to model a real-time intelligent decision system with the FPN formalism. FPNs provide a transparent, interpretable, and concurrent reasoning mechanism that can represent fuzzy knowledge, handle uncertainty, and dynamically propagate degrees of truth. To

TABLE I. Related Work Comparison (Multi-layer Refers to the Decision Pipeline, Not the Internal Layers of ANFIS)

Paper / Approach	Rule learning (how rules are obtained)	How rules are embedded in FPN	Real-time evaluation	Multi-layer	What our work adds beyond it
FPN decision-making [2]	Manual fuzzy rules, certainty values attached to propositions/tokens	Rules encoded as places/transitions with fuzzy truth tokens	Not evaluated	No	Adds data-driven rule learning + explicit CF/TH handling + prediction→decision
FPN knowledge representation [1], [3]	Manual FPR (expert-defined rules)	FPR encoded in FPN structure (places/transitions)	Not evaluated	No	Adds learned-rule generation (ANFIS) + systematic rule-to-transition integration pipeline
Learning on FPN [7]	Supervised learning by adapting transition thresholds in a layered FPN	Learning changes FPN parameters (thresholds/weights)	Not evaluated	No	Adds explicit Sugeno-rule extraction (ANFIS) + rule-to-FPN transition embedding + controlled baseline comparison
Probabilistic FPN (reflectivity prediction) [4]	Rule base extracted by an expert system (association rules), CF/TH derived from conditional probability	Extracted rules mapped to FPN transitions with CF/TH	Yes (real-time prediction stated)	Yes (prediction only)	Adds ANFIS-learned rules + richer rule boundaries + full prediction→decision pipeline under the same execution tool
Probabilistic FPN (prediction + cleaning decision) [8]	Probabilistic / expert-system rule preparation (no neuro-fuzzy learning)	Rules mapped to two FPN expert systems (prediction and decision)	Yes (real-time cleaning decision setting)	Yes (two-stage pipeline)	Adds ANFIS(Sugeno) learning, automatic rule derivation, and dynamic enrichment by adding new transitions when rules are learned
ANFIS / Sugeno neuro-fuzzy [5], [6]	ANFIS learns Sugeno-type rule parameters and membership functions (hybrid learning), supports online updates in principle	Not FPN-based (standalone inference model)	Not evaluated as a real-time execution framework	No	Adds formal FPN execution + CF/TH semantics + concurrent rule execution + prediction→decision
Our approach (Sugeno–ANFIS + FPN)	**ANFIS learning + probabilistic baseline**	**ANFIS rules → FPR → FPN transitions (with CF/TH)**	**Yes**	**Yes**	**Unified comparative tool, learned-rule embedding/execution, supports online enrichment by adding transitions as new rules are learned**

validate the proposed approach, we model a complete real-time decision-making system that integrates expert reasoning, dynamic data streams, and multi-layer fuzzy inference. The modelling process relies on two distinct expert systems: 1) a probabilistic expert system based on conditional probabilities, and 2) a neuro-fuzzy expert system based on the ANFIS. Both expert systems generate fuzzy rules, expressed as FPR, which are then embedded into a unified multi-layer FPN architecture for real-time execution. This shared tool enables a direct, rigorous comparison of manually engineered probabilistic rules and automatically learned ANFIS rules under identical operating conditions.

Fig. 1 summarises the overall modelling workflow of the real-time intelligent decision system. The diagram shows how raw environmental data are progressively transformed into actionable CDs by coupling the probabilistic and ANFIS-based expert systems within a multi-layer FPN tool. The workflow commences with continuous meteorological data from sensors (direct normal irradiance (DNI), temperature, humidity, precipitation, and wind speed). The measurements are kept in an active database. The noise reduction module removes inaccurate readings and reconstructs missing values, producing consistent, accurate data. In addition to this cleaned dataset, a Markov Chain-based prediction block generates short-term predictions for key environmental variables, enabling the system to predict reflectivity degradation rather than simply responding to it. Then, numerical sensor measurements and predicted values are altered into linguistic variables via a fuzzification step by using trapezoidal membership functions. This creates the fuzzy input space needed by both expert systems. Herefrom, two paths running together for rule generation are established:

- In the probabilistic pathway, fuzzy rules are also introduced by abstracting away conditional probability distributions and subjected to human evaluations. Every rule will have a CF and a TH. To maintain interpretability and consistency, only rules with CF ≥ 0.5 and TH ≥ 0.3 are kept.
- In the ANFIS pathway, we use Least Squares Estimation and Gradient Descent as hybrid learning algorithms to automatically learn fuzzy IF–THEN rules, membership functions, and associated parameters from historical meteorological and reflectivity data. Every ANFIS rule has a confidence level defined by its RMSE, and only high-confidence rules are exported for reintegration.

All probabilistic and ANFIS rules are first written as FPR and then encoded in the FPN. Places represent fuzzy propositions with truth degrees (α), while transitions represent rules parameterised by CF and TH. Tokens carry truth values through the net: a transition fires when antecedent places satisfy TH constraints, and consequent places are updated. This mechanism ensures transparent inference and consistent uncertainty propagation. The decision system is implemented as a two-layer FPN. Layer 1 predicts reflector reflectivity in real time from meteorological inputs using rules generated by the two expert methods. Layer 2 combines the predicted reflectivity with real-time sensor data to infer the CD using a separate rule set. Token propagation across both layers supports dynamic tracking of reflectivity and estimation of appropriate cleaning time. Overall, the methodology unifies interpretable probabilistic rules with scalable ANFIS learning within an executable FPN tool for real-time operation, and is validated on an intelligent solar-collector CD system.

A. Data Preparation and Fuzzification

The data used in this study originates from five years of meteorological observations collected at one-hour intervals

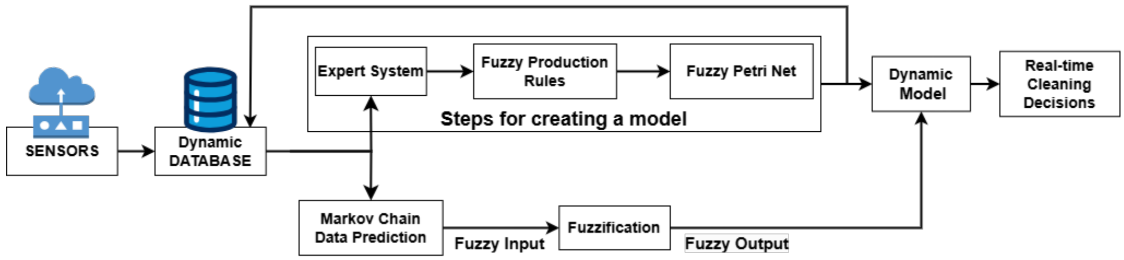

Fig. 1. Crafting and deploying our real-time decision model: step-by-step guidance for seamless implementation.

from a solar power plant in Morocco. The dataset includes five key environmental parameters-Direct Normal Irradiation (DNI), Temperature (T), Humidity (H), Wind Speed (WS), and Precipitation (P)-and one performance variable, Reflectivity (R). To prepare the data:

1) Data Cleaning: Outliers and erroneous readings (e.g., negative irradiance or reflectivity) are removed.
2) Normalisation : Each parameter is normalised into the range [0,1] to standardise the learning process.
3) Fuzzification: Quantitative data are transformed into linguistic variables (e.g., “Low,” “Medium,” “High”) using trapezoidal membership functions. Each input variable is divided into fuzzy intervals defined by TH (a, b, c, d) describing the shape of each fuzzy set.

The trapezoidal membership function is defined mathematically as:

$$\alpha_A(x) = \begin{cases} 0, & x \leq a, \\ \frac{x-a}{b-a}, & a < x \leq b, \\ 1, & b < x \leq c, \\ \frac{d-x}{d-c}, & c < x \leq d, \\ 0, & x > d. \end{cases} \tag{1}$$

where $\alpha_A(x)$ is the membership degree of element x in the fuzzy set A.

The linguistic representation of variables ensures interpretability and forms the foundation of the FPR used in both expert systems.

B. Expert System Based on Conditional Probability

In the probabilistic expert system, fuzzy rules are generated based on conditional probability relationships between input and output variables. The probability of an event A given B is expressed as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}. \tag{2}$$

Using this concept, the CF for each rule is derived as:

$$CF_i = \frac{P(R|d_j)}{\max(P(R))}. \tag{3}$$

The TH is set based on the frequency or confidence level required for the rule to be considered valid:

$$TH_i = \frac{N_{valid}}{N_{total}}, \tag{4}$$

where N_{valid} is the number of successful occurrences satisfying the rule condition.

The resulting fuzzy rules are then represented in FPR format and encoded into the FPN. This approach ensures interpretability and transparency but lacks dynamic learning, requiring manual updates as conditions change.

C. Expert System Based on ANFIS

ANFIS, introduced by Jang [5] and based on the Sugeno-type model [6], merges the clarity of FL (interpretable IF-THEN rules) with the adaptation of neural networks. This system is crucial for generating rule-based, interpretable insights in data-heavy, real-time environments, automatically tuning both membership functions and rule parameters.

ANFIS employs a five-layer structure (see Fig. 2) based on the first-order Sugeno fuzzy model.

A first-order Sugeno fuzzy rule is defined as: [31], [5]

$$\text{If } x \text{ is } A_i \text{ and } y \text{ is } B_i, \text{ then } f_i = p_i x + q_i y + r_i \tag{5}$$

Where: x, y are inputs, A_i, B_i are fuzzy sets, and p_i, q_i, r_i are consequent parameters.

Each layer (see Fig. 2) performs a specific inference step [31], [5], [6]:

1) Layer 1: Fuzzification (Antecedent Parameters) Calculates the membership degree $\mu_{A_i}(x)$ for input x in fuzzy set A_i. Trapezoidal functions, defined by parameters a, b, c, d, are common [see Fig. 2]:

$$\mathcal{O}_{1,i} = \mu_{A_i}(x) \tag{6}$$

$$\mu(x, a, b, c, d) = \max\left(0, \min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right)\right) \tag{7}$$

2) Layer 2: Rule Firing Strength Computes the strength ω_i of the rule using the product operator:

$$\mathcal{O}_{2,i} = \omega_i = \mu_{A_i}(x) \cdot \mu_{B_i}(y) \tag{8}$$

3) Layer 3: Normalization Normalizes the firing strength to ensure proportional rule contribution ($\sum \bar{\omega}_k = 1$):

$$\mathcal{O}_{3,i} = \bar{\omega}_i = \frac{\omega_i}{\sum_{k=1}^{N} \omega_k} \tag{9}$$

4) Layer 4: Defuzzification (Partial Output) Calculates the weighted output of each rule using the normalised strength $\bar{\omega}_i$ and the Sugeno function f_i:

$$\mathcal{O}_{4,i} = \bar{\omega}_i f_i = \bar{\omega}_i (p_i x + q_i y + r_i) \tag{10}$$

The linear output can be expressed as:

$$f_i = \mathbf{C}_i \cdot \mathbf{Z} \quad (11)$$

where $\mathbf{C}_i = [p_i, q_i, r_i]$ and $\mathbf{Z} = [x, y, 1]$.

5) Layer 5: Aggregation (Final Output) Sums all weighted outputs to produce the final crisp system output $\mathcal{O}$:

$$\mathcal{O}_{5,i} = \mathcal{O} = \sum_{i=1}^{N} \bar{\omega}_i f_i \quad (12)$$

ANFIS training relies on a **hybrid algorithm** combining LSE and Gradient Descent [47]. The objective is to minimize the total error E between the predicted output $\mathcal{O}$ and the actual target $\mathcal{T}$.

The total squared error across M data points is:

$$E = \frac{1}{2} \sum_{k=1}^{M} (\mathcal{T}_k - \mathcal{O}_k)^2 \quad (13)$$

The hybrid training algorithm has two primary phases:

- Forward Pass (LSE): Input propagates forward. $\mathbf{C}_i$ (consequent, linear parameters) are optimized using Least Squares Estimation to minimize error.
- Backward Pass (Gradient Descent): Error propagates backward. a, b, c, d (premise, nonlinear parameters) are optimized using the Gradient Descent method to reduce the error gradient.

This parallel approach ensures efficiency and scalability, managing large datasets effectively [48].

The structured process for building an ANFIS expert system involves:

1) Data Collection and Preprocessing: Compile high-quality, preprocessed data.
2) ANFIS Design: Establish input-output associations and select appropriate membership functions.
3) Training ANFIS: Execute the hybrid algorithm to optimise all parameters.
4) Rule Extraction: Generate and prune fuzzy rules from the converged model [49]. Example rule:
If Speed is Medium and Load is Normal, then Fuel Consumption

$$f = 0.6x + 0.4y + 1.2 \quad (14)$$

5) System Implementation: Integrate rules into the inference engine. Validate performance using metrics like **RMSE**.

D. FPN Representation of FPR in Knowledge Base Construction

The fuzzy knowledge base is represented through FPR, which encode expert knowledge or learned relationships between input and output variables. Each rule follows the general structure:

$$R_i : \text{IF } d_{j_1} \wedge d_{j_2} \wedge \ldots \wedge d_{j_m} \text{ THEN } d_k\,(CF = \mu_i,\, TH = \lambda_i), \quad (15)$$

where:

- $d_{j_1}, \ldots, d_{j_m}$ are fuzzy antecedent propositions (inputs),
- d_k is the fuzzy consequent proposition (output),
- $CF = \mu_i$ is the CF of the rule, representing its reliability,
- $TH = \lambda_i$ is the activation TH required for the rule to fire.

The firing condition for each rule is defined by:

$$y_k = \begin{cases} \min(y_{j_1}, \ldots, y_{j_m}) \times \mu_i, & \text{if } \min(y_{j_1}, \ldots, y_{j_m}) \geq \lambda_i, \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

The complete fuzzy rule base $\mathcal{R}$ is therefore a collection of all valid rules:

$$\mathcal{R} = \{R_1, R_2, \ldots, R_n\} \quad (17)$$

E. Formal Definition of Fuzzy Petri Net

An FPN is a bipartite-directed graph with two types of nodes: places and transitions. Circles represent places, and bars represent transitions (see Fig. 3). A token with a truth value between zero and one may or may not be present in each place in an FPN. A CF value between zero and one is assigned to each transition in an FPN. Directed arcs represent the relationships between places and transitions in an FPN. A collection of eight tuples can define a generalised FPN [1] structure

$$Fuzzy\ Petri\ Net = (P,\ T,\ D,\ I,\ O,\ f,\ \alpha,\ \beta) \quad (18)$$

Where

P = $\{p_1,\ p_2,\ \ldots,\ p_n\}$ denotes a finite set of places, represented by circles and corresponding to the propositions of FPR (see Fig. 3). T = $\{t_1,\ t_2,\ \ldots,\ t_m\}$ denotes a finite set of transitions, represented by bars and corresponding to the execution of FPR (see Fig. 3). D = $\{d_1,\ d_2, \ldots,\ d_n\}$ denotes a finite set of propositions of FPR which may contain some fuzzy linguistic variables such as "high," "low," "hot," etc. I: $T \longrightarrow P^{\propto}$ is the input function, a mapping from transitions to bags of places, represented by arcs directed from proposition (P) to rules (T). O: $T \longrightarrow P^{\propto}$ is the output function, a mapping from transitions to bags of places, represented by arcs directed from the rule (T) to propositions (P). f : $T \longrightarrow$ [0, 1] is an association function, a mapping from transitions to real values between zero and one, $f = \mu_1, \mu_2, ..., \mu_i$ where $\mu_i \in [0, 1]$ denotes (CF = μ_i) of rules R_i,$\alpha = \alpha_1, \alpha_2, ..., \alpha_n$. $\alpha : P \longrightarrow$ [0, 1] is an association function, a mapping from places to real values between zero and one, $\alpha = \alpha_1, \alpha_2, ..., \alpha_n$ where $\alpha_i \in [0, 1]$ denotes the degree of truth of propositions represented by the token in a place. β: $P \longrightarrow D$ is an association function, a bijective mapping from places to a set of propositions.

In an FPN, if $p_j \in I(t_i)$, a directed arc a_{ji} exists from the place p_j to the transition t_i. If $p_k \in O(t_i)$, there exists a directed arc a_{ik} from the transition t_i to the place p_k.The CF is a real value μ_i of a rule associated with its corresponding transition if $f(t_i) = \mu_i$ where $\mu_i \in [0, 1]$.The place p is said to be related to the proposition d if $\beta(p_i) = d_i$, where $d_i \in D$.

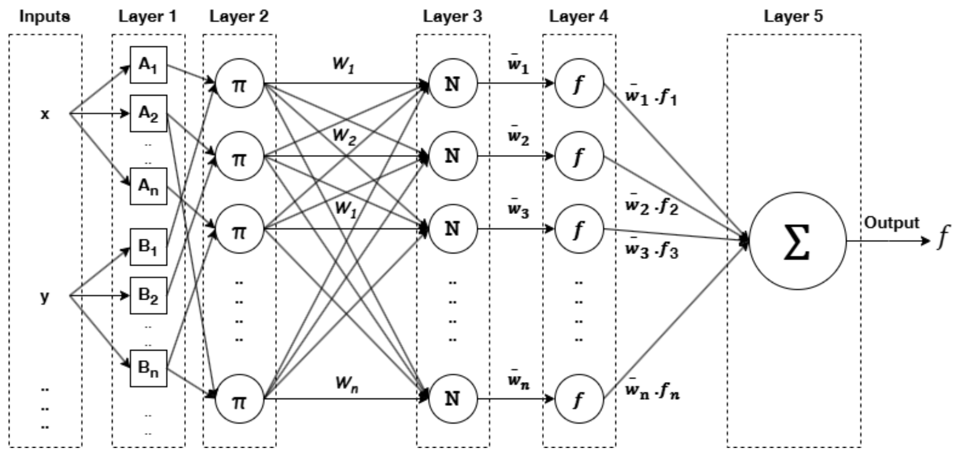

Fig. 2. Overall workflow of ANFIS-based expert systems integrated into a multi-layer FPN.

A marked FPN (see Fig. 1) is one in which a labeled dot represents some tokens in some places, a labeled dot represents the token in place p_i, and the token value in a place $p_i \in P$, is denoted by $\alpha(p_i)$, where $\alpha(p_i) \in [0,1]$. if $\alpha(p_i) = y_i$, where $y_i \in [0,1]$ and $\beta(p_i) = d_i$, then we can see that proposition d_i has a degree of truth of y_i. To reinforce the evolution capacity of our model, we can improve it by including a function $Th = \lambda_1, \lambda_2, ...\lambda_n$ where $\lambda_n \in [0,1]$ denotes the TH of transitions. When the values of tokens $\alpha(p_j)$ in all input places $p_j \in I(t_i)$ of the transition are greater than the TH $\alpha(p_j) \geq \lambda$, the transition t_i is enabled and can be fired.

F. A Multi-layer Approach to Dynamic Prediction and Decision-making

When $\alpha(p_i) \geq TH$, the transition is active, revealing a procedural adherence, identified adequately in our manuscript's Section III-D, where we present a hierarchical structure of the functionality of our system, emphasising the need for maintaining temporal constraints to prevent both computational and system-wide failures. FPN was selected because it comprises a rule-based component and readily supports simulation of real-time system dynamics. The truth-degree tokens are calibrated using data from a model's fuzzification system, with the twofold dependence on CF for concurrent FL and PN calculation for real-time prediction highlighted. This feature of our system, further illustrated with a real-life example in a conference publication [4], [8], demonstrates our approach's ability to forecast R, an important variable in renewable energy applications. In the construction of our two-layer predictive CD system, rigorous validation has been implemented to ensure that the model is robust and effective. As is often the case with machine learning model validation, 30% of the input is used to validate that the system's predictions and decisions stem from a reliable empirical base. This phase of validation is conducted on the remaining 70% of the initial dataset to ensure the dataset is valid and deployable for the model. In a post-validation version of the algorithm, based on the model's successful validation, specific data-collection and analysis intervals are determined. We adopt a Markov Chain probability model to predict new value-generating statistics to derive information for hourly CD-making (see Fig. 1). This methodology maintains a rigorous system to facilitate the execution of the entire model, from initial data processing to final decision-making and the visualisation of outputs in a structured manner. At this stage, every hour, a system performs a detailed computational analysis. The first step is to compute the average of each CD fuzzy interval, since the goal is to find the interval with the highest mean. The system's method approaches its goal by defining the interval's mean, which is then selected as the end case. For example, once the system has been running for a specific period and has been assessing data, like 24 hours, it makes one of its last adjustments to ensure that at least that interval has the highest average. This approach is heavily conditioned by predictive analytics. It means predicting forecasts at a scheduled time frame - typically 24 to 48 hours ahead to keep them in tip-top condition, such as maintaining mirrors well-kept after cleaning jobs. So, the system aims to forecast outcomes a day or two in advance, with a final decision within two days. It will allow experts to make rational, timely, and practical decisions based on this predictive process, leveraging insights from the entire system and its comprehensive analyses and forecasting. One of the aspects that sets our model apart is its ability to automatically extract outputs and present them, enabling real-time performance measurement with dynamic feedback.

G. Algorithmic Workflow

Algorithm 1 summarises the real-time inference process implemented by the FPN expert system. The system operates continuously, starting with the provision of real-time meteorological data (DNI, T, H, WS, P), which is immediately fuzzified to update the α degrees of truth at the FPN input locations. This step directly connects the current environmental conditions to the underlying fuzzy rule base. Central to this is the FPN's iterative firing cycle. The system scans for transitions t_i in every cycle and selects transitions whose input places in question have fuzzy markings α at least as large as their respective TH. Transitions are designated as enabled when they are executed. When an enabled switch fires, the output truth degree (y_{out}) is determined by applying the $\min$ operator to the input truth degrees and multiplying by the rule's CF. The value achieved is then propagated to the output points by the $\max$ operator to capture the contribution from multiple rules, and the confidence of each conclusion is updated at each

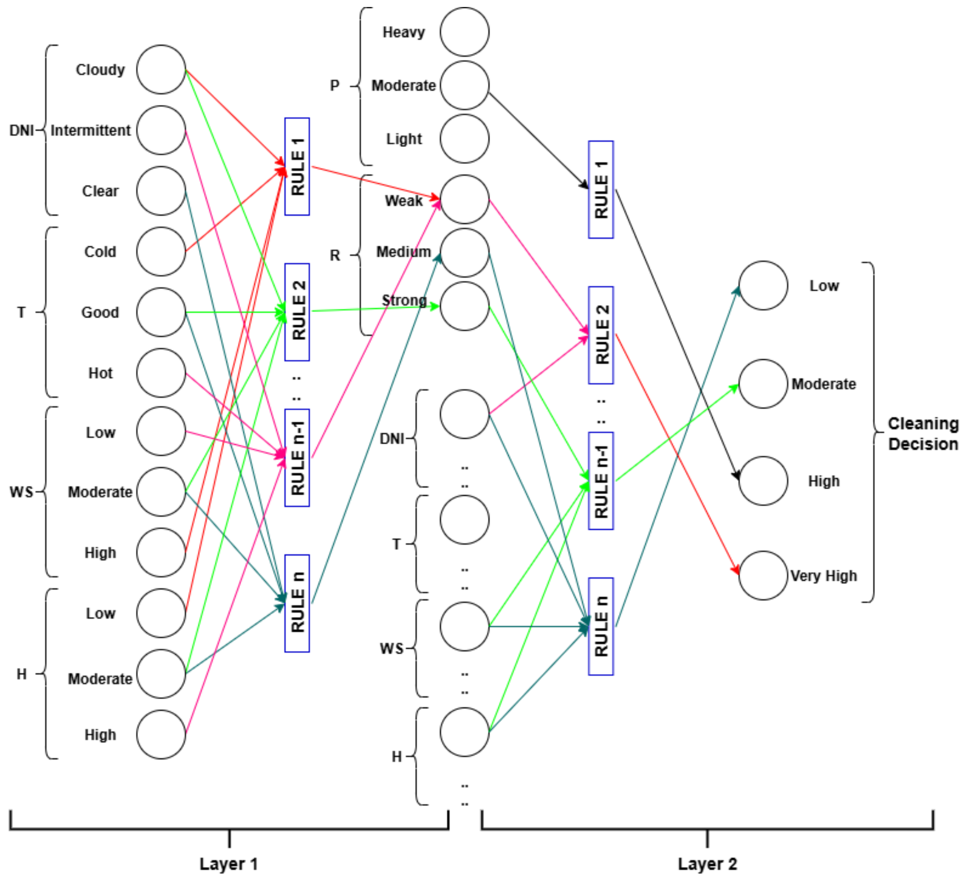

Fig. 3. A model of a solar collector's cleaning system using FPN grids to provide real-time CDs.

moment. After that, no further transitions can be tried. At the final steps of each cycle of this mechanism, the fuzzy marking of the relevant output places is derived from this number to obtain the CD. In this way, when the crisp control signal is needed to trigger or delay a cleaning action, the fuzzy output is defuzzified to reflect the situation. The resulting decision can be logged, and the process resumes with the next wave of real-time data. The FPN expert system's decision-making model is dynamic and adaptive, responding to fluctuating environmental conditions as they change.

IV. Results

In this section, the experimental validation of our proposed hybrid expert system is presented, along with an assessment of the potential value of the ANFIS-based fuzzy rule generation methodology compared with conventional probabilistic FL. We aim to demonstrate the superiority of the ANFIS-augmented model in terms of prediction accuracy, rule generation capabilities, and real-time inference efficiency. All simulations were performed with five years of real meteorological data from a solar power plant in Morocco. The data was split into 70% for training and 30% for validation. To ensure system responsiveness in real-time operation, the model was developed using a multithreaded architecture. Every fuzzy rule, such as one formed by probabilistic estimation or ANFIS training, is passed through an independent thread to the FPN engine. This architecture allows rules to be executed concurrently and also speeds up the inference phase, which is very important when dealing with large rule sets, such as those generated by ANFIS. Simulations were created to mimic natural operational scenarios comprising severe weather variability (temperature, wind speed, humidity), variable soiling conditions, and erratic sensor noise. Two expert system configurations were tested and benchmarked:

- Probabilistic FPN (Baseline): A rule base consisting of 43 FPR, derived from expert knowledge and conditional probability. Rules are associated with ($CF \geq 0.5$) and ($TH \geq 0.3$).
- ANFIS-Enhanced Fuzzy Expert System (Proposed): A data-driven expert system based on the ANFIS. ANFIS was trained on historical input-output data pairs to automatically generate 125 rules. Each rule is formulated in the Sugeno-style and converted into an FPN-compatible FPR. CFs are calculated based on RMSE: $CF = \frac{1}{1+\text{RMSE}}$.

The comparison was made over two system layers: reflectivity prediction and final CD generation.

A. Performance of Reflectivity Prediction Models

Table II compares three models: probabilistic FPN, classical FL, and ANFIS-based fuzzy inference, based on prediction accuracy and RMSE for solar reflectivity.

Algorithm 1 Real-Time Inference with FPN-based Expert System

Input: Real-time meteorological data (DNI, T, H, WS, P)
Initialize: Fuzzy rule base $\mathcal{R}$, FPN structure $(P, T, D, I, O, f, \alpha, \beta)$
while System active **do**
    Acquire and preprocess data
    Fuzzify inputs $\rightarrow$ update $\alpha(p_i)$ for all input places
    repeat
        Identify enabled transitions $\{t_i \mid \forall p_j \in I(t_i), \alpha(p_j) \geq TH_i\}$
        for each enabled t_i **do**
            Compute $y_{out} = \min(\alpha(p_j)) \times CF_i$
            Update output places: $\alpha(p_k) = \max(\alpha(p_k), y_{out})$
        end for
    until No transition can fire
    Aggregate outputs at Layer 2 $\rightarrow$ compute CD
    Defuzzify output if crisp value is required
    Log and display results
end while

TABLE II. COMPARISON OF REFLECTIVITY PREDICTION MODELS (LAYER 1)

Model	Rule Count	Validation Accuracy (%)	RMSE
Probabilistic FPN	43	85	4.82
Classical FL	30	60	6.15
ANFIS-Based FL	125	**93**	**2.57**

The ANFIS-based model clearly outperformed the others, achieving the highest accuracy and the lowest RMSE. This result validates the effectiveness of ANFIS in learning complex, nonlinear dependencies between environmental conditions (temperature, wind speed, humidity, and DNI) and reflectivity levels. The reduction in RMSE indicates a more precise reflectivity estimate, which is critical for determining the optimal timing for solar panel cleaning.

B. Cleaning Decision Validation (Layer 2)

The second layer of the system uses this to infer the best cleaning solution based on real-time sensor output and the predicted reflectivity of Layer 1. Evaluation results for the CD layer are summarised in Table III, which include precision, recall, F1-score, and RMSE. The ANFIS-enhanced classification model achieves significant gains across all metrics. Precision and recall scores show that the model can distinguish between fuzzy categories (Low, Moderate, High, Very High cleaning priority). In addition, the lower RMSE indicates that ANFIS-based decision-making yields results closer to the ground truth.

V. DISCUSSION

The results indicated that the ANFIS model can better fit the nonlinear relationship between meteorological variables and reflectivity, achieving higher accuracy and lower RMSE. Improvements to the CDs made at Layer 1, based on reflectivity predictions, enable cleaner cleaning at Layer 2 with greater reliability and less uncertainty about when and how the changes were applied. ANFIS has a cleaner, more adaptive decision boundary, as it learned all membership functions and Sugeno-type rules from the data, compared to manually decided or probabilistic ones. While the system is data-driven, it is interpretable: ANFIS produces well-defined IF–THEN rules. In contrast, the rules of the FPN remain interpretable by explicitly stating what information is accurate and which firing transition instructions to execute. This enables real-time responsiveness through concurrent execution and parallel rule evaluation, but requires more computation for larger rule bases.

The main comparative observations are as follows:

- Rule Base Expansion: A larger ANFIS produced finer-grained decision boundaries (125 rule-base vs. 43 rule-base), allowing us to consider more context-sensitive cleaning strategies.
- Higher Prediction Accuracy: The RMSE for the reflectivity prediction was reduced by almost 50%, and cleaner choices were more secure and resources allocated effectively.
- Scalability and Automation: In contrast to the static and expert-centric probabilistic rule base, the ANFIS model automatically learns new data through retraining to scale up effectively in evolving environments.
- Transparency and Interpretability: FPN architecture enables traceability of reasoning paths, while ANFIS supports flexibility, representing a complementary hybrid approach that provides a good balance of an interpretable and learned model.
- Real-Time Efficiency: The implementation of a multithreaded approach to generate the rules will help us optimise response times, even for a massive rule base, as this is an absolute necessity for a deployed system.

Therefore, pruning, merging, or prioritising may be required to prevent latencies. A hybrid approach that finds a balance between interpretability, scalability, and learning ability. Such rules are transparent, and the ANFIS is agile, providing better predictive capabilities. Such an approach could be extended to other real-time expert solutions, where transparency and adaptability are crucial, such as fault diagnosis or predictive maintenance. The need, however, arises for more validation through various datasets and operating conditions. Further work on explicit latency benchmarking,

TABLE III. VALIDATION METRICS FOR CLEANING DECISION OUTPUT

Model	Accuracy (%)	Precision Avg. (%)	Recall Avg. (%)	F1 Score (%)	RMSE
Probabilistic FPN	85	83	84	83	4.51
ANFIS-Fuzzy System	**93**	**90**	**91**	**90**	**2.57**

scalability testing under varied rule-base sizes, and long-term retraining analysis to tackle concept drift is needed.

VI. CONCLUSION

This work proposed a new methodology for modelling real-time intelligent systems by integrating FPN with data-driven rule learning via ANFIS. The primary methodological contribution, apart from the reported performance improvements, is a single model and execution scheme that integrates learned Sugeno-type fuzzy rules with an interpretable FPN reasoning engine, enabling concurrent, traceable inference across two layers (prediction and decision). From a functional standpoint, this paper illustrates how symbolic expert knowledge (FPR with CF/TH semantics) and data-driven learning can be combined into a single executable expert system that remains transparent and suitable for real-time operation. For this purpose, the proposed approach was tested in a real-time expert system for solar collector CD-making with five years of operational data collected from a Moroccan solar power plant under variable weather and soiling conditions. The results reveal that the improved prediction and decision performance of the ANFIS-enhanced setting is better than a probabilistic FPN baseline and a hand-tuned fuzzy-logic model due to the enhanced interpretability via FPN execution, as well as to a consistent inference mechanism by the decision quality using interpretable inference and the ability of responsive operation through concurrent decision making from the rule evaluation process under concurrent examination.

Data-Driven Neuro-Fuzzy learning becomes functional and interpretable in practice by mapping learned rules to a formal FPN execution model that provides traceability to reasoning and multi-layer decision. The tool will be further tested on other plants and climates, and expanded for new real-time applications in fault diagnosis and predictive maintenance. Real-time benchmarking (latency, throughput, scalability) as the rule base grows will also be planned, alongside investigation of scheduling strategies under limited resources. Ultimately, we seek formal verification and consistency checking of the learned rule base generated by the FPN tool to enhance its robustness.

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