The Optimization Design of the Pattern Matrix Based on EXIT Chart for PDMA Systems

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***Abstract*—The maximum degree of function node of pattern matrix (PM) dominates the detection complexity of belief propagation algorithm for pattern division multiple access (PDMA) systems. This work proposes a method to search the optimal PM ensemble for PDMA system under constrained detection complexity. This issue is converted to find the optimal variable node (VN) degree distribution (DD) of PM with function node DD concentrated. Utilizing extrinsic information transfer chart (EXIT) techniques, the DD of PM with overload rate of 150% is obtained and its DD is designed by progressive edge growth (PEG) algorithm. The performance of this PDMA system is evaluated and compared with the ones of the same overload rate in literature to verify the effectiveness of the proposed method. Furthermore, for iterative detection and decoding (IDD), the concatenated LDPC code is optimized to enhance the overall performance. EXIT analysis and Monte Carlo simulations confirm that the designed pattern matrix outperforms other pattern matrix about 2.3 dB in bit error rate when both schemes employ the same LDPC code, and 0.2 dB when using the optimized codes respectively.**

***Keywords*—PM optimization; EXIT chart; PDMA system**

I. Introduction

In future 6th Generation Mobile Communication Technology (6G), pattern division multiple access (PDMA) technology, as a non- orthogonal multiple access (NOMA) method, which based on the joint design of transmitters and receivers helps meet the demand for massive user access and the capability to approach the capacity boundary of multi-user communication systems. For PDMA system, the design of the pattern matrix (PM) is crucial as it affects the transmission diversity, overload rate, and the detection complexity at the receiver side. For instance, PM's with high column weight offer higher diversity order, which is enable to reliable data transmission. However, this also increases the detection complexity at the receiver. Therefore, it is necessary to balance between transmission diversity and detection complexity when designing the PM.

In [1], the authors outlines the design criterions for PM's with respect to three typical scenarios in 5G respectively, and search for the PMs with optimization method. The effectiveness of those method is demonstrated by conducting link-level simulations. In [2], the authors investigate the influence of row-weight, column-weight and rotation-factor on the performance of PDMA system. Reference [3] proposes an enhanced PDMA technique called interleaver-based PDMA, which distinguishes users through different bit-level inter-leavers. Reference [4] presents a joint design method for PDMA based on power and beam domains to optimize pattern mapping, achieving power allocation optimization by maximizing overall throughput, and validating the corresponding optimization problem. An iterative algorithm for optimizing power allocation and PMs is also proposed to improve PDMA performance in [5]. Reference [6] proposes a design method for the characteristic matrix of the PDMA system based on the binary particle swarm optimization (BPSO) algorithm. This method models the design of the pattern matrix as a discrete optimization problem, with the goal of maximizing the average mutual information. It generates the optimal binary sparse matrix by dynamically adjusting the parameters of the particle swarm.The research results show that, compared with traditional schemes, the optimized matrix significantly improves the coding efficiency and diversity gain, and effectively solves the problem of mismatch between detection and decoding. This method provides new ideas for the design of the pattern matrix. However, it faces the problems of high computational complexity and a tendency to get trapped in local optimal solutions during matrix optimization.

Extrinsic information transfer (EXIT) chart based on average mutual information measures was originally used to calculate the decoding threshold of low-density parity-check (LDPC) code over binary erasure channel, later in AWGN channel and multiple input multiple output channel. coded PDMA systems with different degree distributions. The optimized system shows improved iterative convergence performance [7]. The author in [8] studies the EXIT characteristics of LDPC decoders in interleaved multiple access systems with LDPC coding to illustrate the convergence of optimized degree distributions. The author in [9] uses EXIT charts to analyze the iterative convergence behavior in decoders and demodulators, aiding in predicting the system's bit error performance. The author in [10] proposes a factor graph-based iterative Multiple-Input Multiple-Output (MIMO) detection scheduling algorithm based on the convergence characteristics of EXIT charts, which accelerates the mutual information exchange between variable nodes and check nodes. In [11], the authors extend the EXIT based method to PDMA channel, and find the optimal (or near-optimal) degree distribution of LDPC codes by a two-stage iterative optimization algorithm based on EXIT for LDPC-coded PDMA systems. EXIT is used to aid the design of the IDD system and the optimization of LDPC code, however the front-end PDMA is also designed empirically.

Overall, existing studies on PDMA system optimization primarily utilize empirically designed pattern matrices, lacking systematic algorithmic approaches. While [6] introduces a pattern design algorithm, it suffers from high computational

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complexity and is limited to single-dimensional optimization: the research focuses solely on offline pattern matrix design without joint optimization with channel codes like LDPC, resulting in a mismatch in mutual information transfer characteristics between the detector and decoder and degrading iterative convergence efficiency.

Therefore, building on the work presented in [11], this study first investigates the PM ensemble under detection complexity constraints for PDMA systems. Since the maximum degree of function node of PM dominate the BP detection complexity, as a result, we consider to find the optimal variable node (VN) degree distribution under constant FN degree by EXIT tool. As the new pattern matrix results in mismatched EXIT between the PDMA detector and the LDPC decoder. To address this issue, with the PM is fixed, the degree distribution of the LDPC code is optimized to further improve the bit error rate (BER) performance of the LDPC-coded PDMA system.

The primary objectives of this research include:

- Complexity constrained PM optimization: Employ the EXIT tool to derive the optimal variable node (VN) degree distribution under a fixed maximum function node (FN) degree, minimizing belief propagation (BP) detection complexity while maintaining system performance.
- EXIT matching and LDPC code refinement: Fix the optimized PM and adjust the degree distribution of the LDPC code to eliminate the EXIT curve mismatches between the detector and the decoder, thus improving the bit error rate (BER) performance of LDPC-coded PDMA systems.
- Complexity-performance tradeoff: Achieve an efficient balance between detection complexity and BER performance through the proposed optimization strategies, providing theoretical support and feasible solutions for practical communication system design.

The remainder of this paper is organized as follows. Section II presents the system model of the PDMA system, the joint factor graph, and the EXIT calculation analysis for LDPC-coded PDMA systems. In Section III, the proposed EXIT-graph-based pattern matrix optimization algorithm is described in detail. The numerical results are provided in Section IV, followed by discussions and future perspectives in Section V. Finally, Section VI concludes this paper.

II. PDMA System Model

A. System Model of LDPC Code Uplink PDMA

Fig. 1 shows the diagram of a LDPC coded uplink PDMA system scheduled K users. At the transmitter, the information sequence of the ith user is encoded by an LDPC encoder with code rate of R_c. The encoded sequence is then BPSK modulated to produce symbol sequence of length denoted as $X^{(i)} = [x_1^i, x_2^i, \cdots, x_{Ns}^i]^T, 1 \leq i \leq K$. The modulated symbols are mapped to N orthogonal frequency-division multiplexing (OFDM) resource elements (REs) according to the pattern sequence by the PDMA detector and then transmitted.

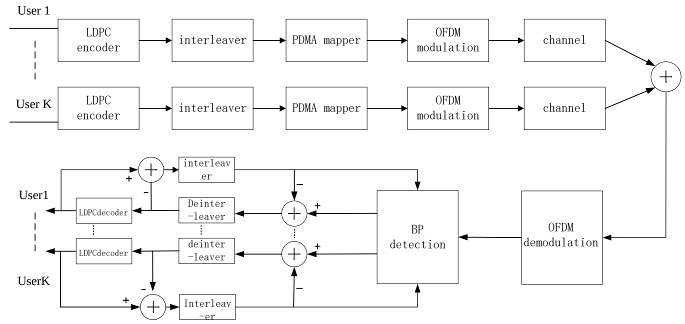


Fig. 1. Block diagram of LDPC coded PDMA system.

At the receiver, the OFDM demodulation is first performed. The demodulated OFDM signals are then passed to a multi-user detector (MUD) based on the belief propagation (BP) algorithm. Additionally, the iterative detection and decoding based on belief propagation algorithm (BP-IDD) is employed at the receiver, which execute three types of iterative operations: the first is the internal BP iteration within the MUD, the number of which is denoted as In_iter, the second is the BP iteration of LDPC decoder, the last is the turbo style processing between the detector and the channel decoder, the number of which is denoted by Out_iter.

B. PM of PDMA System

For each user, the modulated symbols x_j^i for $1 \leq i \leq K$ and $1 \leq j \leq N$ are mapped onto N OFDM REs according to the specific pattern sequence (PS) of user i which corresponds to the ith column of PM. Suppose the ith PS has $d_{s,i}$ non-zero elements, $d_{s,i}$ is defined as the ith column weight of the PM, which means the effective spreading factor for the ith user is $d_{s,i}$. Similarly, define $d_{f,j}$ as the jth row weight of the PM which represents the number of symbols interfering with each other at jth RE. Here, $1 \leq d_{s,i} \leq N$, $1 \leq d_{f,j} \leq K$. The overload factor β of the PDMA system can be expressed as the ratio of the number of users K to the number of resource elements N, i.e.

$$\beta = \frac{K}{N} \tag{1}$$

The overload factor β can also be written as

$$\beta = \frac{\sum_i \alpha_i / d_{s,i}}{\sum_j \gamma_j / d_{f,j}} \tag{2}$$

Where α_i is the fraction of variable node with degree $d_{s,i}$ in terms of edge perspective, and γ_j is the fraction of FN with degree $d_{f,j}$ in terms of edge perspective. Substituting into Eq. (1) and (2), we can obtain:

$$\beta = \frac{d_f}{d_s} \tag{3}$$

In [12], two PM schemes with overload rates of 150% and 200% are proposed, as shown in Fig. 2. Taking the pattern matrix PM150% as an example, its first column represents

$$\mathbf{S}_{2,3}=\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}\Rightarrow \begin{matrix} & U1 & U2 & U3\\ RE1 & x_1 & 0 & x_3\\ RE2 & 0 & x_2 & x_3\end{matrix}$$

$$\mathbf{S}_{3,6}=\begin{bmatrix}1&1&1&0&1&0\\1&1&0&1&0&1\\1&0&1&1&0&0\end{bmatrix}$$

Fig. 2. PMs with overload rates of 150% and 200%.

the PS of user 1, meaning that user 1 spreads its symbol x_1 across resource elements RE1 and RE2 for transmission. The second and last columns represent the PS of users 2 and 3, respectively, with users 2 and 3 spreading their symbols x_2 and x_3 to the same resource elements RE1 and RE2. Additionally, the diversity order (i.e. effective processing gain) for users 1, 2, and 3 is 2, 1, and 1, respectively.

Similarly, the other scheme shows six users mapped to different pattern matrices, which are loaded onto three resources for transmission. As the pattern matrix changes, the overload rate increases, indicating a further improvement in the spectral resource utilization of the PDMA system. However, this also increases the system complexity, making signal detection more challenging. A larger d_s can achieve better diversity gain. But with the increase of the average row weight $\bar{d_f}$, the average column weight $\bar{d_s}$ also increases, leading to stronger interference in the system and higher computational complexity. Therefore, it is crucial to find a proper balance between d_s and computational complexity. Constructing high-overload, low-interference pattern matrices is thus a vital step in PDMA system design.

C. Joint Factor Graph of LDPC Coded PDMA System

In the factor graph of the LDPC-coded PDMA system shown in Fig. 3 , there are three types of nodes: function nodes associated with the received signals $y_j (1 \leq j \leq N)$ of each RE, variable nodes corresponding to the transmitted signals $v_i (1 \leq i \leq K)$, and check nodes $c_m (1 \leq m \leq M)$ representing parity-check equations of LDPC code. These nodes correspond to the jth resource for a specific user, the ith transmitted symbol, and the mth parity-check equation, respectively. The variable nodes connect two types of nodes, forming the PDMA detector and LDPC decoder. The edges between the variable nodes and function nodes constitute the pattern matrix $\mathbf{S}_{4,6}$, while the edges between the variable nodes and check nodes form the low-density parity-check matrix. This matrix uses the a priori information received from the other two types of nodes to calculate the extrinsic information that will be sent to the function nodes. Simultaneously, based on the received a priori information, it calculates the extrinsic information that will be sent to the check nodes. Therefore, we divide the variable nodes into two categories: Variable Nodes I (VNDI) in the detector and Variable Nodes II (VNDII) in the decoder. Similarly, the extrinsic information sent from the function nodes and check nodes to the variable nodes is calculated and transmitted in the same way. To facilitate the evaluation of the extrinsic information transfer in the joint factor graph, the function nodes, variable nodes, and check nodes are collectively referred to as Function Node Detector (FND), Variable Node Detector I (VNDI), Variable Node Decoder II (VNDII), and Check Node Decoder (CND), respectively.

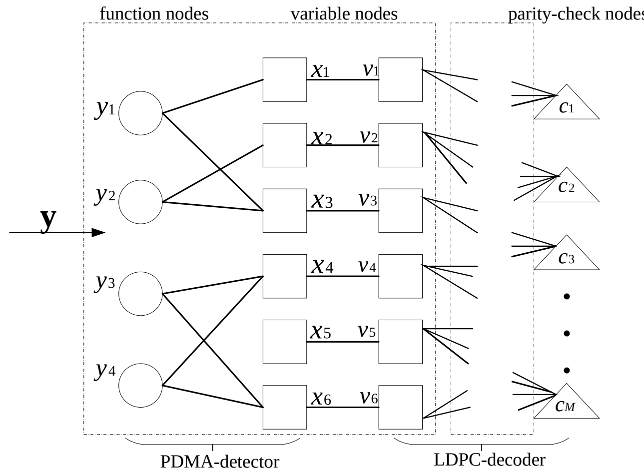


Fig. 3. Joint factor graph of LDPC-Coded PDMA system.

III. PM Design and IDD Receiver Optimization

The EXIT chart is helpful for analyzing the information transfer in the iterative detection/decoding process, and it has been used in the design of systems with iterative operations. Since we need to first determine the degree distribution of the pattern matrix and then use this as the basis to find the degree distribution of the LDPC code, we divide the joint factor graph into three modules for EXIT analysis. Module A consists only of the detector, including the Function Node Detector (FND) and Variable Node Detector I (VNDI). Module B contains both the detector and Variable Node Decoder II (VNDII). Module C consists only of the Check Node Decoder (CND), as shown in Fig. 4.

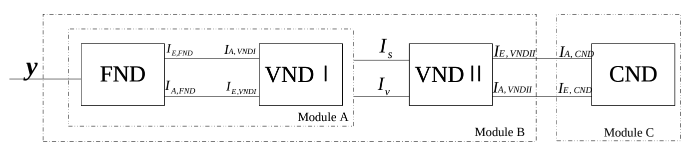


Fig. 4. Three modules of factor graph in LDPC-Coded PDMA system.

A. EXIT Based PM Design

As mentioned above, in this step, we only need to redesign the new pattern matrix and determine the optimal degree distribution of the pattern matrix. Therefore, at this stage, module A is needed to be considered only.

1) EXIT Curve for VNDI: Let $I_{A,VNDI}$ denote to the average mutual information (AMI) between the coded bits and the associated prior Log-Likelihood Ratio (LLR) while $I_{E,NDI}$ be the AMI between those bits and the associated extrinsic LLR at the output of VNDI of the module A. To calculate the EXIT curve of the variable node, for each $I_{E,VNDI} \in [0, 1]$, a-priori LLR corresponding to the coded bits can be modeled as follows:

$$L_A = \mu_A x + n_0 \tag{4}$$

where $\sigma_0 = 2/J^{-1}(I_{A,VND1})$, $n_0 \sim N\left(0, \sigma_0^2\right)$, $\mu_A = \sigma_0^2/2$, $\operatorname{var}(L_A) = \sigma_0^2$, $x \in \{\pm 1\}$

The mutual information $I_{A,VNDI} = I(X; A)$ can be calculated by

$$\begin{aligned} I_{A,VNDI} = \quad & \frac{1}{2} \cdot \sum_{x=-1,+1} \int_{-\infty}^{\infty} p_A\left(\zeta \,|X = x\right) \\ & \cdot \log_2 \frac{2 \cdot_A \left(\zeta \,|X = x\right)}{p_A\left(\zeta \,|X = -1\right) + p_A\left(\zeta \,|X = +1\right))} d\zeta \end{aligned} \tag{5}$$

Since the conditional probability density function n $p_A(\zeta|X = x)$ depends on LLR of L_A, we can write

$$\begin{aligned} I_{A,VNDI}(\sigma_A) = \quad & 1 - \frac{1}{\sqrt{2\pi}\sigma_A} \int_{-\infty}^{+\infty} \exp\left(-\frac{\left(\zeta - \frac{\sigma_A^2}{2}\right)^2}{2\sigma_A^2}\right) \\ \cdot \quad & \log_2\left[1 + e^{-\zeta}\right] \mathrm{d}\zeta. \end{aligned} \tag{6}$$

For abbreviation we define:

$$J(\sigma) := I_A(\sigma_A = \sigma), \tag{7}$$

where $\lim_{\sigma \to 0} J(\sigma) = 0, \lim_{\sigma \to \infty} J(\sigma) = 1, \sigma \geq 0.$

After BP-detection or LDPC decoding, the extrinsic information LLR L_E are obtained for $1 \leq i \leq N_s$. The corresponding output AMI can be evaluated as

$$I_E = 1 - E\{\log_2(1 + e^{-L_E})\} \approx 1 - \frac{1}{N_s} \sum_{i=1}^{N_s} \log_2(1 + e^{-x_i \cdot L_{E,j}}) \tag{8}$$

The output AMI of degree d_s variable node of VNDI can be expressed as

$$\begin{aligned} I_{E,VNDI}(I_{A,VNDI}, d_s) &= f_1(I_{A,VNDI}) \\ &= J\left(\sqrt{d_s - 1} \cdot J^{-1}\left(I_{A,VNDI}\right)\right) \end{aligned} \tag{9}$$

Fig. 5 shows the AMI curves for variable node with different degree d_s from 2 to 6. For the same input $I_{A,VNDI}$, the variable node with larger d_s output the larger $I_{E,VNDI}$. This is coincide with the principles that variable node with higher diversity order has more reliable information.

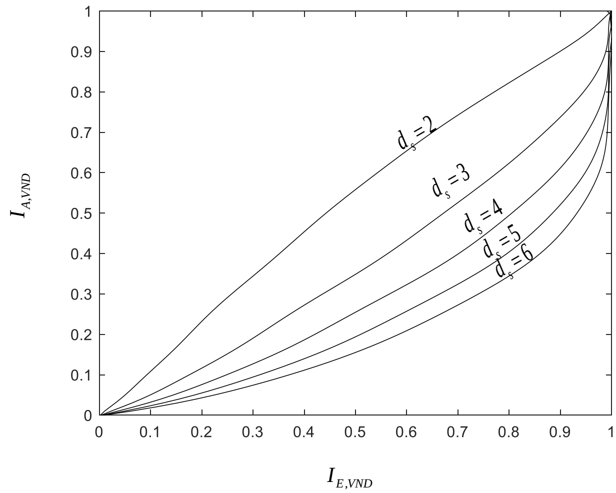


Fig. 5. EXIT curves of detector variable nodes with different degrees.

2) EXIT Curve for FND: Let $I_{A,FND}$ represents the average mutual information between the input bits at the edge of the Function Node Detector (FND) and the prior LLR, and $I_{E,FND}$ represent the average mutual information between the output bits at the edge of the FND and the extrinsic LLR. The function node receives incoming messages from the connected variable nodes and the OFDM demodulator, and its output extrinsic information LLR is modeled as the output of an AWGN channel, where the input corresponds to the transmitted bits using BPSK modulation. The mutual information of the output is then calculated with respect to the actual values on the edge of the node. Due to the complexity of the calculations at the function node, the EXIT curve is simulated under the AWGN channel. The probability density function (PDF) of the extrinsic information is determined through Monte Carlo simulations and histogram measurements, and then the mutual information between the extrinsic information and the bits on the joint graph edges is be evaluated according to Eq. (8). The expression is as follows:

$$I_{E,FND} = f_2(I_{A,FND}, d_f, E_b/N_0) \tag{10}$$

Fig. 6 shows the EXIT curves for the detector with different numbers of users (overload conditions), where d_s=2 and d_f =2/3/4/5/6. From Fig. 6, it can be observed that the EXIT curve of the function node detector starts from a non-zero point, which is due to the input from the OFDM demodulator. The EXIT curve of the variable node detector starts from the zero point. The EXIT curves of the function node detector and the variable node detector intersect, and this intersection marks the termination point of the iterative detection process.

To design a pattern matrix with better performance, the EXIT chart is plotted for different d_s values to evaluate the impact of the intersection of the EXIT curves on the convergence behavior of the detector. Fig. 7 illustrates the EXIT chart for PM designs with different processing gains for d_f = 6 and E_b/N_0=5.5dB. From the figure, as d_s increases, the intersection point gradually shifts to the right, as larger d_s provides greater frequency diversity. In information

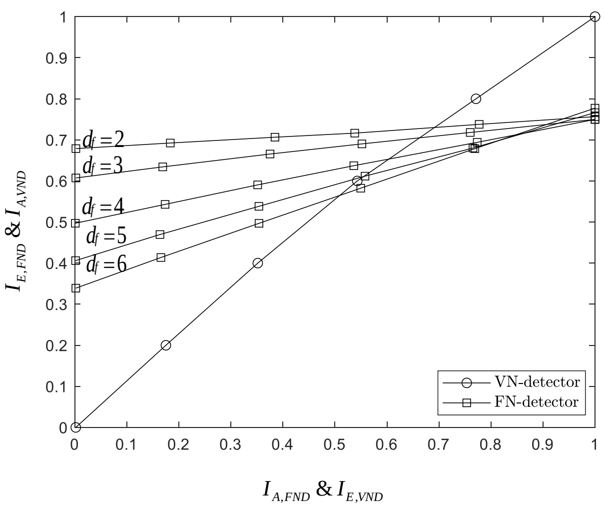


Fig. 6. EXIT curves of detector check nodes with different degrees.

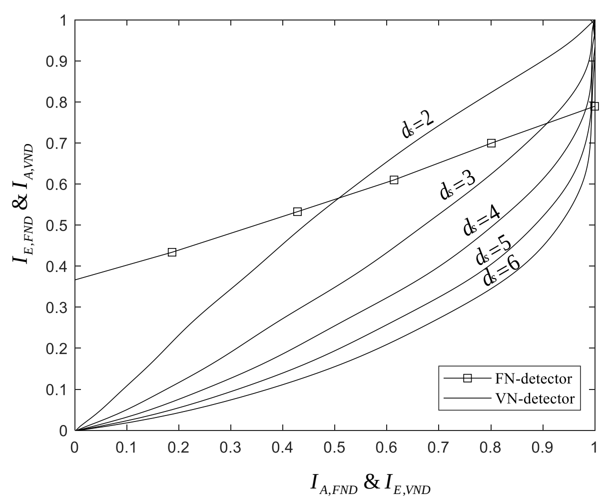


Fig. 7. EXIT chart for PM designs with different processing gains at E_b/N_0=5.5dB.

theory, mutual information measures the dependency between variables. Ideally, to exchange extrinsic information between modules until convergence, thereby achieving arbitrarily low BER, the EXIT curves should not intersect before reaching the point (I_A, I_E)=$(1, 1)$. This implies that given I_A=1, I_E should also be 1, and if this condition is met, an open convergence tunnel appears in the EXIT chart. However, if the two curves intersect below the point (1,1), it forms a semi-convergence tunnel, which will result in a higher BER compared to the scenario where they intersect at (1,1). Hence, it is ideal to design this intersection point as far to the right as possible in the EXIT chart. We will focus on the design of the pattern matrix using the EXIT chart to minimize the detection error performance.

B. Optimization Algorithm for Degree of PM

In the design of the joint factor graph, there are various parameters that influence the performance of the factor graph, one of the most important being the degree distribution of the nodes. The degree of a node is the number of edges connected to other nodes, and the degree distribution is the probability distribution of these degrees within the model. Let $\overline{\alpha} = \left[\alpha_2, \cdots, \alpha_{D_{s,max}}\right]$, $\overline{\gamma} = \left[\gamma_2, \cdots, \gamma_{D_{f,max}}\right]$, $\overline{\lambda} = \left[\lambda_2, \cdots, \lambda_{D_{\nu,max}}\right]$ and $\overline{\rho} = \left[\rho_2, \cdots, \rho_{D_{c,max}}\right]$ represent the degree distribution coefficient vectors for the detector variable nodes, function nodes, decoder variable nodes, and check nodes, respectively. Let $D_{FND}(x)$,$D_{VNDI}(x)$,$D_{VNDII}(x)$,$D_{CND}(x)$ then represent the degree distribution polynomials for the function nodes, variable nodes, and check nodes, defined as:

$$D_{FND}(x) = \sum_{j=2}^{D_f} \gamma_j x^{j-1} \tag{11}$$

$$D_{VNDI}(x) = \sum_{i=2}^{D_s} \alpha_i x^{i-1} \tag{12}$$

$$D_{VNDII}(x) = \sum_{p=2}^{D_\nu} \lambda_p x^{p-1} \tag{13}$$

$$D_{CND}(x) = \sum_{q=2}^{D_c} \rho_q x^{q-1} \tag{14}$$

where $0 \le \gamma_j \le 1, \quad \sum_j \gamma_j = 1$; $0 \le \alpha_i \le 1, \quad \sum_i \alpha_i = 1$; $0 \le \lambda_p \le 1, \quad \sum_p \lambda_p = 1$; $0 \le \rho_q \le 1, \quad \sum_q \rho_q = 1$

This work proposes a method to search the optimal PM ensemble for PDMA system under constrained detection complexity. This issue is converted to find the optimal variable node degree distribution (DD) of PM with function node DD concentrated. The joint factor graph is divided into a detector and a decoder. During optimization, we mainly focus on the detector and redesign the pattern matrix according to the node degree distribution of the detector. In the process of designing the pattern matrix, there are three important parameters: variable nodes d_s, function nodes d_f, and the overload ratio β. In the iterative update operation of the PDMA receiver, the computational complexity of the update rule for function nodes is much higher than that for variable nodes. Therefore, based on the parameter d_f, as the complexity metric of the detector factor graph, this paper proposes a method to search for the optimal set of pattern matrices (PMs) in a PDMA system under the condition of restricted detection complexity. For a given E_b/N_0 with the degree distribution set of function nodes fixed, the optimal variable node degree distribution of the PM is sought to achieve maximum overload. The main implementation method is to optimize the shape of the EXIT tunnel based on EXIT chart analysis, approaching the point (1, 1). The main idea is to adjust the degree distribution of the variable node degree (VNDI) while keeping the function node degree (FND) unchanged, and find the EXIT chart that is closest to the point (1, 1). Equation (2) can be transformed into:

$$\beta = d_f \sum_{i=2}^{D_s} \alpha_i / i \tag{15}$$

As can be seen from the previous analysis, if the bit - error rate of the redesigned pattern matrix is to be minimized, the intersection point of the EXIT chart should be as close as possible to the point (1, 1). Therefore, the loss function can be defined as: As can be seen from the previous analysis, if the bit - error rate of the redesigned pattern matrix is to be minimized, the intersection point of the EXIT chart should be as close as possible to the point (1, 1). Therefore, the loss function is defined as:

$$\Delta\left(\alpha, E_b/N_0\right) = \min\left(f_1\left(I\right) - f_2^{-1}\left(I\right)\right) \tag{16}$$

The degree distribution optimization task can be transformed into solving the following linear programming problem.

$$\max \frac{\sum_{i=2}^{D_i} \alpha_i}{i} \text{s.t.} \begin{cases} \Delta\left(\alpha, E_b/N_0\right) > 0 \\ \sum_i \alpha_i = 1 \\ 0 \leq \alpha_i \leq 1 \end{cases} \tag{17}$$

In principle, by solving the above equation, an optimal distribution can be found under a certain E_b/N_0, achieving the appropriate overload rate and constructing the pattern matrix. Here, D_s represents the maximum allowed variable node degree, and in this paper, it is set to D_s= 6. It initiates from the maximum point of the extrinsic information at the variable nodes, employing a linear programming algorithm to identify the degree distribution of the variable nodes. The process progressively reduces the number of points, with the objective of converging to a fitted curve of the variable node degree distribution function that intersects the EXIT curve of the function nodes at a point as far to the right as possible. The goal is to achieve an intersection point that is maximized in its rightward position.

C. IDD Receiver Optimization

In the newly designed pattern matrix, there will inevitably be a mismatch between the EXIT charts of the front-end detector and the decoder, resulting in suboptimal system performance. Therefore, it is necessary to reconstruct the new LDPC code based on new PM to ensure matching at both ends and improve system performance. In this section, we will briefly introduce the EXIT chart of subsequent modules and the algorithms used for optimizing LDPC codes.

1) EXIT curve for overall module a: Let I_v and I_s refer to AMI of input and output, through prior channel modeling,I_v can be obtained. Since the detector is nonlinear, I_s cannot be expressed in a closed form and must be obtained through Monte Carlo simulations. The expressions for I_v and $I_{A,VNDII}$ are as follows:

$$I_\nu = J\left(\sqrt{d_\nu} \cdot J^{-1}\left(I_{A,VNDII}\right)\right) \tag{18}$$

2) EXIT curve for VNDII: Let $I_{A,VNDII}$ and $I_{E,VNDII}$ represent the average mutual information at the input and output of the VNDII, respectively. The expressions for the average mutual information at the input and output can be obtained through calculation as follows:

$$\begin{aligned} I_{E,VNDII} &= f_3\left(I_{A,VNDII}\right) \\ &= J\left(\sqrt{\left(d_\nu - 1\right)\left(J^{-1}\left(I_{A,VNDII}\right)\right)^2 + \left(J^{-1}\left(I_s\right)\right)^2}\right) \end{aligned} \tag{19}$$

3) EXIT curve for CND: Let $I_{A,CND}$ and $I_{E,CND}$ represent the average mutual information at the input and output of the CND, respectively. The expressions for the average mutual information at the input and output can be obtained through calculation as follows:

$$\begin{aligned} I_{E,CND} &= f_4\left(I_{A,CND}\right) \\ &= 1 - J\left(\sqrt{\left(d_c - 1\right)\left(J^{-1}\left(1 - I_{A,CND}\right)\right)^2}\right) \end{aligned} \tag{20}$$

In the iterative detection algorithm, the update rule for variable nodes in the decoder factor graph depends on the external information from both the detector and the decoder, which is relatively complex. Therefore, the parameter d_c is used as a complexity metric for the decoder factor graph. In IDD receiver optimization, we employ an algorithm to find the degree distribution of variable nodes for LDPC encoding within a given constraint set in [13].For a given E_b/N_0 and the degree of d_c the check node, the goal is to find the optimal variable node degree distribution to achieve the appropriate code rate. The main analysis approach is to use EXIT chart analysis to optimize the tunnel between the EXIT curves, minimizing the tunnel area. This can be achieved by adjusting the degree distribution of the VNDII while keeping the degree distribution of the CND fixed, so that the EXITs of module B and module C match.

IV. Simulation Results Analysis

A. Optimization Result Based on EXIT Chart

According to the algorithm mentioned in the previous section, the degree distribution polynomials for the pattern matrix with an overload factor of 150% can be obtained as follows:

$$\begin{aligned} D_{VNDI}(x) &= 0.055764x^2 + 0.94424x^3 \\ D_{FND}(x) &= x^5 \end{aligned} \tag{21}$$

To better verify the effectiveness of the optimization algorithm, a pattern matrix was constructed using the degree distribution polynomials obtained through optimization and the PEG algorithm as follows:

$$\mathbf{P}_{6,9} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{22}$$

Assuming the target code rate is $R = 1/2$, the node degree distribution of the LDPC code optimized under the above algorithm at $\mathbf{P}_{6,9}$ can be obtained as follows:

$$\begin{aligned} D_{VNDII}(x) &= 0.41832x + 0.30246x^2 + 0.22864x^8 \\ &\quad + 0.050585x^{62}; \\ D_{\text{CND}}(x) &= x^5 \end{aligned} \tag{23}$$

Assuming the target code rate is $R = 3/4$, the node degree distribution of the LDPC code optimized under the above algorithm at $\mathbf{P}_{6,9}$ can be obtained as follows:

$$\begin{aligned} D_{VNDII}(x) &= 0.3336x + 0.40449x^2 + 0.087239x^5 \\ &\quad + 0.018176x^{36}; \\ D_{\text{CND}}(x) &= x^{11} \end{aligned} \tag{24}$$

Fig. 8 shows the EXIT chart of the detector for the designed pattern matrix $\mathbf{P}_{6,9}$ with the optimized degree distribution and $\mathbf{S}_{4,6}$ at E_b/N_0 = 5.5dB under fading channels. The results indicate that, compared to $\mathbf{S}_{4,6}$, the intersection point of the optimized PDMA system moves further to the right, closer to the (1,1) point, suggesting that the BER performance of the designed pattern matrix $\mathbf{P}_{6,9}$ will outperform that of $\mathbf{S}_{4,6}$. Fig. 9 illustrates the EXIT chart for a code rate of 0.5 under fading channels for both the LDPC in World Interoperability for Microwave Access (WiMAX) protocol code and the optimized code. The results show that the threshold SNR of the WiMAX -LDPC code at a code rate of 0.5 significantly decreases after EXIT optimization, with the optimized code achieving 1 dB gain over the WiMAX-LDPC code.

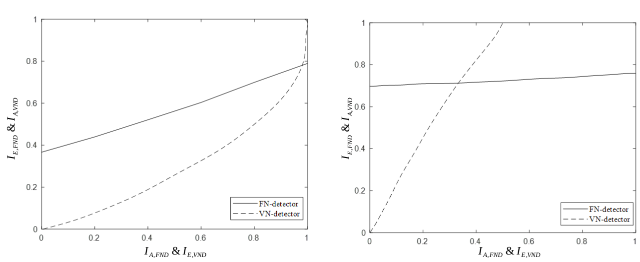


Fig. 8. EXIT chart of detectors $\mathbf{P}_{6,9}$ and $\mathbf{S}_{4,6}$.

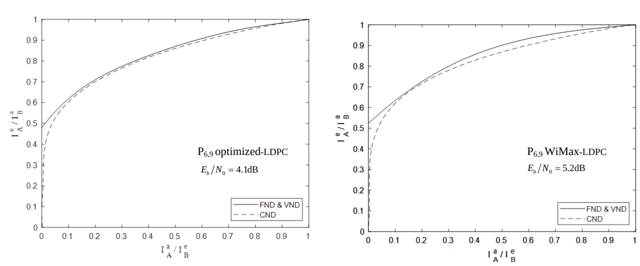


Fig. 9. EXIT chart of the $\mathbf{P}_{6,9}$ PDMA system with WiMAX-LDPC code and optimized LDPC code.

B. BER Performance Comparison

This section primarily focuses on comparing and analyzing BER performance of the proposed pattern matrix $\mathbf{P}_{6,9}$ with other matrices $\mathbf{S}_{4,6}$, utilizing WiMAXLDPC codes as the error-correcting coding scheme. Specifically, two coding rate scenarios are considered: one with a code rate R set to 0.5, corresponding to a codeword length of 2304; the other with an increased code rate R of 0.75, resulting in a codeword length of 2400. In both scenarios, the LDPC decoder employs the standard BP algorithm for decoding, with a unified BP iteration count set to 30. To better describe the iterative process, we introduce the concepts of Out_Iter and In_Iter, where the former refers to the number of iterations between the BP detector and the LDPC decoder, and the latter specifically denotes the number of iterations within the BP detector. Additionally, the simulation experiments in this section adopt BPSK modulation and assume an independent and identically distributed (i.i.d) fading channel, with ideal channel estimation, meaning that channel state information is only available at the receiver and not at the transmitter.

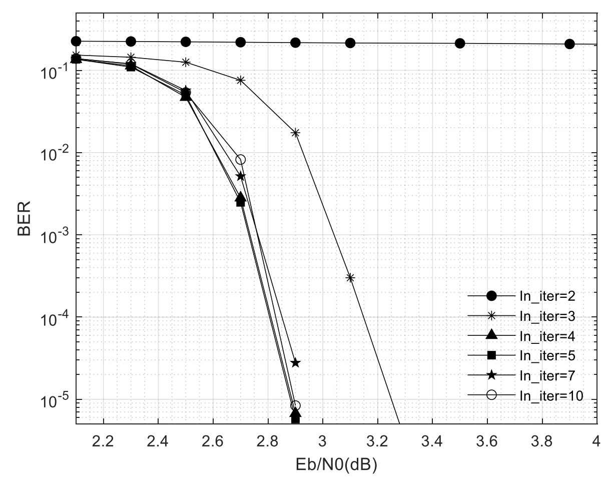


Fig. 10. BER performance of PM $\mathbf{P}_{6,9}$ with different In_Iter iterations as Out_Iter=5 is fixed.

Fig. 10 demonstrates the impact of different In_Iter (In_Iter = 2, 3, 4, 5, 7, 10) on the BER performance of PM $\mathbf{P}_{6,9}$ when the Out_Iter is fixed at 5. The experimental results show that the BP detector converges essentially after 4 inner iterations, with further increases in inner iteration count yielding insignificant performance improvements. Fig. 11 further reveals the influence of varying outer iteration counts (Out_Iter = 1, 2, 3, 4, 5, 10) on the BER performance of the proposed pattern matrix when the inner iteration count (In_Iter) is fixed at 4. Based on the simulation data, when In_Iter= 4, the performance loss associated with Out_Iter of 5 compared to Out_Iter of 10 is negligible, amounting to just 0.1 dB .Therefore, it is reasonable to conclude that selecting Out_Iter of 5 is sufficient and justified, with the resulting performance loss virtually ignorable.

Fig. 12 and Fig. 13 show the BER performance simulation results for the proposed pattern matrices $\mathbf{P}_{6,9}$ and $\mathbf{S}_{4,6}$ using WiMAX-LDPC codes and optimized codes at code rates of 0.5 and 0.75, respectively, under the conditions of $In\_Iter = 4$

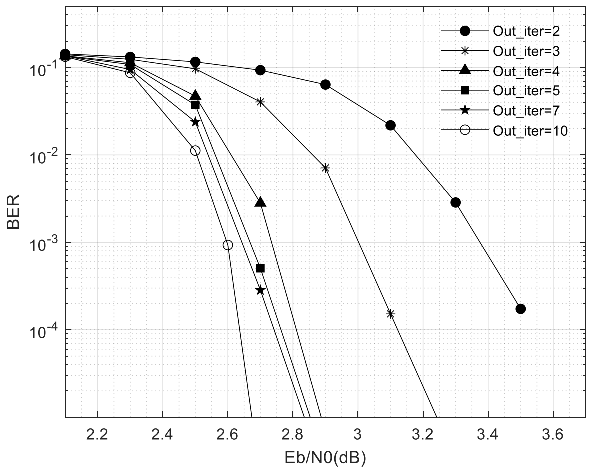


Fig. 11. BER performance of PM $\mathbf{P}_{6,9}$ with different Out_Iter iterations as In_Iter=4 is fixed.

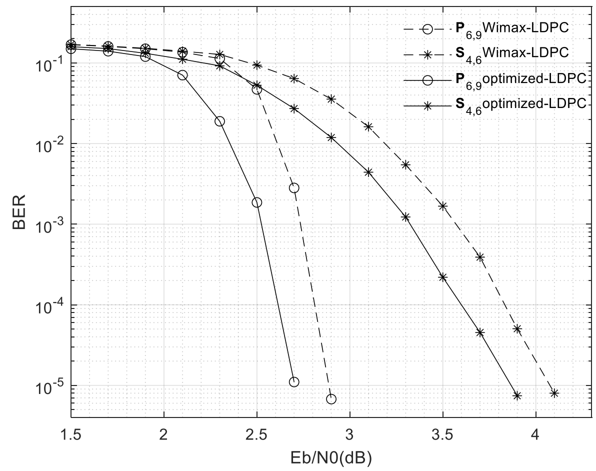


Fig. 12. Out_Iter=5, In_Iter=4, BER Comparison of the $\mathbf{P}_{6,9}$ PDMA System and $\mathbf{S}_{4,6}$ at Code Rate 0.5

and $Out\_Iter = 5$. The dashed lines represent the BER curves for WiMAX-LDPC codes, while the solid lines indicate the BER curves for the optimized codes. The simulation results reveal that for the proposed pattern matrix $\mathbf{P}_{6,9}$, there is approximately a 2.3 dB gain at a BER of 10^{-4} compared to $\mathbf{S}_{4,6}$ when using WiMAX LDPC coding. Furthermore, for itself, the optimized code achieves about a 0.2 dB gain at a BER of 10^{-4} compared to using the LDPC code.

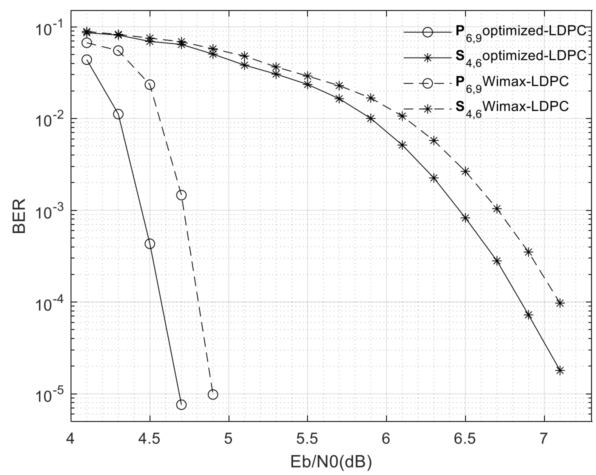
V. Discussion

A. Theoretical Contributions

This study proposes an EXIT-chart-based optimization algorithm that establishes a novel theoretical framework for performance enhancement in PDMA systems. By introducing a new PDMA scheme, it enriches the diversity of future PMDA system mapping strategies. Through the design of the PDMA mapping process, the algorithm effectively balances the diversity gain of the pattern matrix with detection complexity. Furthermore, by optimizing the degree distribution of LDPC codes, it significantly improves the performance of LDPC-coded PDMA systems. Theoretical analysis demonstrates that by adjusting the degree distributions of variable nodes and check nodes, the proposed algorithm achieves a substantial BER gain compared to existing PDMA schemes [13]. However, existing algorithms assume ideal channel state information during the design process. In practical systems, channel estimation errors, the Doppler effect, and interference may affect the optimization results, limiting their applicability in non-ideal scenarios.

B. Future Research Directions

With the advent of the 6G era, wireless communication technologies will face increasingly complex requirements. The current study on PDMA remains insufficient, and further investigations are needed to address the following critical issues:



[h!]

Fig. 13. Out_Iter=5, In_Iter=4, BER Comparison of the $\mathbf{P}_{6,9}$ PDMA System and $\mathbf{S}_{4,6}$ at Code Rate 0.75.

1) Detection and decoding algorithms: The BP-IDD algorithm currently used for PDMA detection and decoding achieves a balance between decoding complexity and system performance in low-order modulation scenarios. However, its complexity exhibits exponential growth under high-order modulation, highlighting the need for novel detection algorithms. Future research could incorporate the approximate message passing (AMP) algorithm into the PDMA framework for in-depth analysis, aiming to reduce complexity while maintaining performance gains.

2) Transmission architecture expansion: This study is limited to single-antenna PDMA systems, a constraint that poses significant challenges to improving transmission efficiency. Integrating PDMA with multiple-input multiple-output (MIMO) technology represents a key research direction to achieve substantial enhancements in data transmission rates and user support capabilities. Such a combination would enable the exploitation of spatial diversity gains and alleviate the limitations of single-antenna systems.

VI. Conclusion

This paper focuses on the design of PDMA systems, with a particular emphasis on the design of the pattern matrix. The objective is to design the degree distribution of the pattern matrix to achieve optimal system load. To this end, a degree distribution optimization algorithm is proposed, which utilizes the EXIT chart technique to search for the optimal PM set for the PDMA system under constrained detection complexity, thereby obtaining the set of variable node degree distributions. Furthermore, the PEG algorithm is employed to design a pattern matrix $\mathbf{P}_{6,9}$ with an overload rate of 150% based on the degree distribution polynomial. BER simulation results demonstrate that with 4 inner iterations and 5 outer iterations, the system with $\mathbf{P}_{6,9}$ can achieve satisfactory performance. Under the same number of iterations, the designed pattern matrix $\mathbf{P}_{6,9}$ improves the BER by approximately 2.3 dB compared to existing PM $\mathbf{S}_{4,6}$ schemes (when both use the same LDPC code), and by 0.2 dB when using an optimized code. In future work, the detection algorithm will be improved to reduce detection complexity and enhance system performance.

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