Capacity Analysis of MIMO Channels Under High SNR Using Nakagami-q Fading Distribution

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***Abstract*—This study explores the capacity of multiple-input multiple-output (MIMO) wireless channels under high signal-to-noise ratio (SNR) conditions, incorporating Nakagami-q fading distribution alongside Rayleigh and Rician fading models. The main objective is to develop an analytical framework that accurately models MIMO channel capacity under high-SNR conditions using Nakagami-q fading and compares its performance with conventional fading models. By employing a robust wireless channel modeling approach, the study examines the impact of various antenna configurations on system performance. The derived framework assesses how different fading conditions affect capacity, showing that MIMO systems effectively mitigate multipath effects. The results reveal that channel capacity improves with an increasing number of antennas and favorable fading parameters, emphasizing the significance of antenna configurations in enhancing performance. The comparative analysis highlights substantial differences in capacity across fading models, offering critical insights to optimize next-generation wireless channel modeling in diverse environments.**

Keywords*—*MIMO systems; Nakagami-q; high-SNR capacity; antenna configurations; wireless channel modeling

I. INTRODUCTION

Due to the explosive expansion of bandwidth-intensive applications like online gaming, video streaming, and the internet of things (IoT), there is an unprecedented demand for dependable, high-capacity wireless communication networks. Multiple-input multiple-output (MIMO) systems have become a key component of next-generation wireless networks, including 5G and beyond, due to many gains in spectral efficiency and stability [1]. MIMO systems use diversity gain and spatial multiplexing to increase capacity by utilizing many antennas at both the transmitter and receiver [2]. However, optimizing performance requires an understanding of capacity under various channel conditions.

One of the critical aspects of MIMO system performance is its capacity, which depends heavily on the characteristics of the wireless channel. While traditional analyses often use Rayleigh or Rician fading models, these are insufficient to capture the variety of fading scenarios encountered in practical environments [3]. This study uses the Nakagami-q distribution, a generalized model that can represent a wide range of channel conditions, from severe fading to near-line-of-sight scenarios, to address this limitation [4]. The Nakagami-q distribution's flexibility makes it particularly suitable for modeling advanced wireless networks.

The capacity of MIMO systems in the high signal-to-noise ratio (SNR) domain is the main emphasis of this work. In situations where power efficiency and dependability are crucial, such as short-range communications, millimeter-wave technology, and backhaul networks, high-SNR analysis is especially pertinent. The high-SNR approximation simplifies capacity expressions, providing valuable insights into key performance factors such as antenna configurations and channel eigenvalue distributions.

Key contributions of this paper include:

- Derivation of the MIMO channel capacity using the Nakagami-q fading model to capture diverse channel conditions.
- Simplification of the capacity expression in the high-SNR regime, facilitating practical insights for system design.
- Analytical evaluation of eigenvalue behavior under Nakagami-q fading, leading to an understanding of the average capacity.
- Practical implications for optimizing antenna configurations and transmission strategies in advanced wireless networks.

This work builds on the existing literature by bridging the gap between theoretical models and practical scenarios. By leveraging the Nakagami-q distribution, it provides a robust framework for capacity analysis, contributing to the development of efficient, high-performance communication systems. Recent research in MIMO capacity under generalized fading models highlights the importance of such studies.

The remainder of the paper is structured as follows. Section II provides a review of related studies, offering an overview of existing research on MIMO channel capacity. Section III presents a detailed capacity formulation, starting with the system model and extending to the derivation of the final capacity expression through determinant analysis and high-SNR approximations. Section IV focuses on performance analysis, examining variations in MIMO channel capacity under different antenna configurations, Nakagami-q parameters, and fading conditions, and concludes with a summary of key findings. Section V provides a discussion of the results, highlighting the implications, addressing research gaps, and highlighting future research directions. Finally, Section VI concludes the paper by summarizing the key outcomes of the research.

II. RELATED STUDIES

Throughout recent times, various distributions have been utilized to model wireless communication channels. From, single-input single-output (SISO) to MIMO, distrbutions like, Rayleigh, Rician, Nakagami-m and others have been incorporated. Nakagami-q, also known as Hoyt-fading distribution, is another distribution model that has considerable potential in this domain, and this research aims to propose a novel MIMO communication channel based on this. Though, different studies have worked on Nakagami-q distribution, important factors like diverse fading, environment, SNR conditions have been overlooked. In this section, existing works in this area are investigated to outline the current progress and areas of improvement that this research aims to address.

As demonstrated by researchers in [5], MIMO systems improve the quality of wireless communication. This paper addresses ways to simplify MIMO while preserving its advantages, addressing technologies such as space-time coding and spatial multiplexing. The need for improved wireless networks with high capacity and data rates has been highlighted globally by researchers in [6]. In terms of data rates and capacity, MIMO systems—which have multiple antennas at both the sending and receiving ends—perform noticeably better than SISO, single-input multiple-output (SIMO), and multiple-input and single-output (MISO) systems. MIMO systems' multiplexing and diversity gains are the main topics of this research, which also points out that adding more antennas increases the systems' capacity.

Researchers in [7] have examined multipath fading propagation, which is typically modeled using the Rayleigh distribution and causes destructive interference of signals at the receiver due to phase discrepancies. Using SIMO and MIMO models under an additive white gaussian noise (AWGN) channel, Rayleigh fading in communication channels is investigated in this article. The bit error rate (BER) performance across different SNR ranges is examined once the systems are modeled in SIMULINK. According to the results, BER performance improves with more receivers in a fading channel, getting closer to the ideal SISO system without fading. Furthermore, as compared to pre-shared key (PSK), frequency-shift keying (FSK), and privileged access management (PAM), the Alamouti space-time block code (STBC) 22 MIMO system with quadrature amplitude modulation (QAM) greatly improves BER performance, particularly in the 0–15 dB SNR band.

Researchers in [8] have investigated current developments in MIMO technology, concentrating on small arrays with multiple antenna elements in order to take advantage of the bandwidth advantages of increased mutual coupling (MC). Two important contributions of this study are the expression of circuit-theoretic models in standard MIMO terminology and the development of a physically-consistent Rician channel model for super-wideband (SW) systems [8]. The new channel model causes bandwidth broadening, as the study shows, and MC alters line-of-sight pathways, which impacts beamforming. It also emphasizes how spatial correlations at low frequencies are diminished by tight coupling.

A new fading distribution known as fluctuating Nakagami-m was presented by researchers in [9] which is based on the ratio of two independent random variables: a power of the uniform distribution and the Nakagami-m distribution. The Nakagami-m and Rayleigh fading models are included in this model as special examples. In order to fit the envelope probability density function (PDF) to empirical data from underwater acoustic, vehicle-to-vehicle, and device-to-device communications, the study offers closed-form formulas for the envelope PDF and cumulative distribution function (CDF). Furthermore, outage probability, average bit error rate, and channel capacity are used to examine the performance of traditional wireless systems, and precise asymptotic equations are obtained for these parameters.

The second-order statistics of the Nakagami-Hoyt (Nakagami-q) fading channel model have been studied by researchers in [10]. They obtained expressions for the average duration of fades (ADF) and level crossing rate (LCR), demonstrating that these analytical findings are in good agreement with measurement data from mobile satellite channels in highly darkened conditions. This implies that actual mobile communication channels can be used with the Nakagami-q model. Strong agreement between simulated, analytical, and experimental data is further shown by describing a deterministic simulation model based on Rice's sum of sinusoids, which successfully emulates the Nakagami-q fading envelope with the required statistics.

Researchers in [11] examined downlink multiuser precoding in massive MIMO systems with optimal channel state information (CSI) using maximum ratio transmission (MRT), zero forcing (ZF), and minimum mean square error (MMSE) algorithms. They discovered that rates often rise with additional base station antennas and higher SNR after deriving precise feasible rate expressions under Rayleigh fading. With the ideal number of users for ZF and MMSE, MRT is more effective at low SNR but less effective at high SNR. Holographic MIMO technology, which integrates several antennas in a small area to achieve great spectral efficiency, has been investigated by researchers in [12]. They investigated channel capacity under realistic angle distribution and array aperture limits and computed spectral density using a wavenumber domain-based technique [24]. The study found that capacity is significantly influenced by angle distribution at high SNR but not at low SNR, and that capacity does not increase eternally with antenna density due to array aperture constraints.

In comparison to 4G networks, millimeter wave (MMW) cellular systems with high bandwidths greatly boost capacity, preventing needless cell splitting in high-density deployments, according to researchers in [13]. This study examines hybrid MIMO capacity in 5G mmW networks by modifying the orthogonal matching pursuit (OMP) algorithm and applying sparse signal processing. The results demonstrate that channel over-saturation causes both the conventional and hybrid MIMO capacity curves to decrease with rising SNR, with hybrid MIMO catching up to conventional MIMO capacity at specific channel gains. To improve 5G performance, researchers in [14] have suggested integrating MIMO technologies, free-space optical (FSO) transmissions, and MMW which uses MIMO for spatial diversity and FSO and MMW networks to handle fading and turbulence, respectively. With closed-form BER formulae and different modulation techniques investigated under varied situations, the study demonstrates enhanced performance and robustness against channel fading in comparison to employing FSO or MMW alone.

The Nakagami-m model for MIMO systems has been updated by researchers in [15], fixing phase distribution errors and expanding its applicability to arbitrary m numbers. Using spatial shift keying (SSK), quadrature spatial modulation (QSM), and spatial multiplexing (SMX) MIMO systems as examples, this new model is thoroughly examined and contrasted with Monte-Carlo simulations. The study emphasizes the model's increased precision and wider range of applications. The performance of cooperative communication systems with direct links and numerous reconfigurable intelligent surfaces (RISs) across Nakagami-m fading channels has been examined by researchers in [16]. The cooperative RIS-D, a double-RIS, system greatly reduces the symbol error probability (SEP) and saves energy when compared to SISO systems without RISs. Performance is further enhanced by adding more RISs or reflecting components.

Using short packets, researchers have investigated ultra-reliable and low-latency communications (URLLC) in multiuser downlink MIMO non-orthogonal multiple access (NOMA) systems over Nakagami-m fading [17]. They suggested antenna-user selection techniques and used minimal blocklength and average block error rate (BLER) to analyze performance. According to the study, MIMO NOMA ensures complete diversity gains by lowering transmission latency and enhancing BLER performance. Binary data transmission in spatial modulation (SM) MIMO systems over Nakagami-m fading channels has been examined by researchers in [18], with an emphasis on pairwise error probability. They discovered that while SM MIMO systems save hardware complexity by reducing the number of radio frequency (RF) chains, performance deteriorates with increasing modulation orders. Both 5G and 6G wireless systems can benefit from these findings.

The performance of MU-massive MIMO systems, which employ enormous antenna arrays to serve several users simultaneously and reduce inter-user interference using orthogonal channel vectors, has been assessed by researchers in [19]. They used CSI at the base station and user terminals to examine several precoding techniques (MMSE, ZF, and MRT) over Nakagami-m fading channels. The study also examined how the shaping parameter and pilot reuse parameters affected system performance. The heterogeneous multiplex relay (HMR) protocol was proposed by researchers in [20] to improve spectrum efficiency in MIMO systems employing half duplex (HD) and full duplex (FD) modes. Simulations demonstrate 80% capacity performance and enhanced BER versus SNR when compared to Rayleigh, Rician, and Nakagami fading channels, demonstrating that this protocol provides diversity and multiplexing advantages. Moreover, massive MIMO increases energy economy, throughput, and channel capacity.

The SIMO framework has been investigated by researchers in [21] to examine wireless communication performance across the Hoyt fading channel. They calculated the channel capacity in high SNR regimes using massive limit argument approximations. The study discovered that SIMO high-speed railway (HSR) outperforms both SIMO low SNR regime (LSR) and SISO HSR systems, and that raising instantaneous SNR greatly increases channel capacity. Using tiny limit argument approximations for low SNR regimes, researchers in [22] have examined the capacity of Nakagami-q fading SIMO wireless communication systems. They discovered that adding more receiver antennas boosts the system's capacity, which may be further increased by modifying specific settings.

The data rate limits of SISO wireless communication systems over Nakagami-q fading channels have been examined by researchers in [23]. They computed channel capacity in both low and high SNR regimes using small and big limit argument approximations. The behavior of channel capacity with regard to SNR and fading parameters was thoroughly examined, and the study discovered that channel capacity increases with SNR in both regimes [23].

The investigation of several fading models, such as the Nakagami-q distribution, has significantly improved the comprehension of wireless communication channels. By filling in the gaps in previous research on various fading environments and SNR situations, this study demonstrates the promise of the Nakagami-q model in MIMO systems. This work offers important insights into improving wireless communication performance by examining the channel capacity under various SNR regimes and the effect of multiple antennas. To further improve data rates and system capacity in real-world situations, future research should keep improving these models and investigating the useful applications. Table I provides an overview of techniques used in MIMO system studies, highlighting the identified constraints and challenges related to various fading models.

III. Capacity Formulation

In this study, the capacity analysis of MIMO wireless channels is investigated within the context of high SNR regimes, specifically utilizing the Nakagami-q distribution to model the channel. The Nakagami-q distribution, known for its flexibility in characterizing fading environments, provides a more generalized framework compared to conventional models. By examining the capacity formulation through this distribution, a more comprehensive understanding of MIMO system performance in high SNR conditions can be achieved. The ensuing sections delineate the system model, derive the capacity expression, and simplify it under high SNR approximations to elucidate the impacts of Nakagami-q fading on the channel capacity.

A. System Model

Figure 1 represents the MIMO system model for this research work considering the fact that the sending and receiving power of each antennas are identical and the sender as well as receiver antennas are mutually independent.

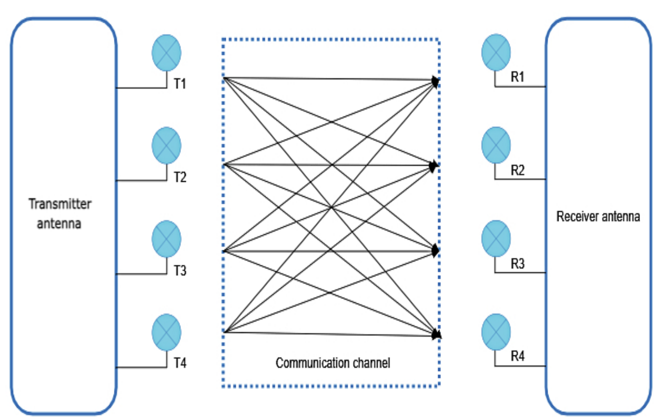

Fig. 1. Nakagami-q Fading MIMO wireless channel system model.

TABLE I. OVERVIEW OF TECHNIQUES AND IDENTIFIED CONSTRAINTS

Reference	Used Methods	Effectiveness and Challenges
[5]	Simplifies MIMO while preserving advantages such as space-time coding and spatial multiplexing.	Lacks a detailed capacity analysis under generalized fading models like Nakagami-q.
[6]	Investigates data rates and capacity improvements in MIMO systems.	Does not consider the effect of specific fading conditions on capacity.
[7]	Analyzes BER performance of Rayleigh fading using SIMO and MIMO models under an AWGN channel.	Limited to Rayleigh fading and does not evaluate Nakagami-q fading effects.
[8]	Develops a Rician channel model for super-wideband MIMO systems.	Does not explore the impact of Nakagami-q fading on system performance.
[9]	Proposes a fluctuating Nakagami-m fading model and derives its PDF and CDF.	Focuses on Nakagami-m fading but does not analyze its effects on MIMO capacity.
[10]	Studies second-order statistics of Nakagami-q fading, obtaining ADF and LCR expressions.	Does not analyze capacity expressions under high SNR conditions.
[11]	Examines multiuser precoding in massive MIMO using MRT, ZF, and MMSE under Rayleigh fading.	Lacks a comparison with Nakagami-q fading and its effects on capacity.
[12]	Investigates holographic MIMO with angle distribution and array aperture constraints.	Does not consider Nakagami-q fading in high-SNR conditions.
[13]	Analyzes hybrid MIMO capacity in 5G mmW networks using sparse signal processing.	Fails to account for Nakagami-q fading and its capacity implications.
[14]	Studies MIMO, FSO, and mmWave integration for improved 5G performance.	Does not incorporate Nakagami-q fading effects in the analysis.
[15]	Updates the Nakagami-m model for MIMO and compares it using Monte-Carlo simulations.	Limited to Nakagami-m fading; lacks insight into Nakagami-q's influence.
[16]	Evaluates cooperative RIS-assisted communication over Nakagami-m fading.	Does not include Nakagami-q fading model in the analysis.
[17]	Investigates URLLC in multiuser MIMO-NOMA under Nakagami-m fading.	Focuses on Nakagami-m but does not compare with Nakagami-q fading.
[18]	Examines binary data transmission in SM MIMO over Nakagami-m fading.	Lacks evaluation of Nakagami-q fading in the capacity model.
[19]	Assesses massive MIMO precoding techniques over Nakagami-m fading.	Does not include Nakagami-q fading or high-SNR scenarios.
[20]	Proposes HMR protocol for MIMO systems under Rayleigh, Rician, and Nakagami fading.	Lacks a specific capacity analysis under Nakagami-q fading.
[21]	Analyzes Hoyt fading in SIMO channels using high-SNR approximations.	Does not extend the study to MIMO systems or Nakagami-q fading.
[22]	Examines SIMO capacity under Nakagami-q fading in low SNR regimes.	Does not generalize findings for MIMO capacity analysis.
[23]	Studies SISO channel capacity under Nakagami-q fading in both low and high SNR regimes.	Does not analyze MIMO capacity or its performance variations.

Consider a MIMO system with N_t transmit antennas and N_r receive antennas. The received signal vector $\mathbf{y}$ can be written as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where:

- $\mathbf{y}$ is the $N_r \times 1$ received signal vector.
- $\mathbf{H}$ is the $N_r \times N_t$ channel matrix with entries modeled as Nakagami-q distributed random variables.
- $\mathbf{x}$ is the $N_t \times 1$ transmitted signal vector.
- $\mathbf{n}$ is the $N_r \times 1$ noise vector, modeled as independent and identically distributed (i.i.d.) complex Gaussian with zero mean and variance σ^2.

B. Capacity of MIMO Channels

The capacity C of a MIMO channel is given by:

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{P}{N_t\sigma^2}\mathbf{H}\mathbf{H}^H\right) \tag{2}$$

where P is the total transmit power and σ^2 is the noise power.

In the high SNR regime, the SNR per receive antenna is defined as:

$$\rho = \frac{P}{\sigma^2} \tag{3}$$

Thus, the capacity expression becomes:

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right) \tag{4}$$

C. Determinant of the Matrix

The determinant of the matrix $\mathbf{I}N_r + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H$ can be expressed using the eigenvalues λ_i of $\mathbf{H}\mathbf{H}^H$:

$$\det\left(\mathbf{I}N_r + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H\right) = \prod_{i=1}^{N_r}\left(1 + \frac{\rho}{N_t}\lambda_i\right) \tag{5}$$

D. High SNR Approximation

In the high SNR regime ($\frac{\rho}{N_t}\lambda_i \gg 1$), the approximation:

$$\log_2\left(1 + \frac{\rho}{N_t}\lambda_i\right) \approx \log_2\left(\frac{\rho}{N_t}\lambda_i\right) \tag{6}$$

holds because, at high SNR, $\frac{\rho}{N_t}\lambda_i$ is significantly greater than 1, rendering the "1" negligible.

E. Simplifying the Capacity Expression

Initially, the capacity expression for the MIMO channel is given by:

$$C \approx \log_2 \det \left(\mathbf{I}N_r + \frac{\rho}{N_t} \mathbf{H}\mathbf{H}^H \right) \tag{7}$$

This expression involves the determinant of the matrix $\mathbf{I}N_r + \frac{\rho}{N_t} \mathbf{H}\mathbf{H}^H$, which can be simplified by expressing it in terms of the eigenvalues λ_i of the matrix $\mathbf{H}\mathbf{H}^H$. Using the property that the determinant of a matrix equals the product of its eigenvalues, the capacity expression can be rewritten as:

$$C \approx \log_2 \left(\prod_{i=1}^{N_r} \left(1 + \frac{\rho}{N_t} \lambda_i \right) \right) \tag{8}$$

Applying the logarithm property $\log_2(a \cdot b) = \log_2(a) + \log_2(b)$, the expression becomes:

$$C \approx \sum_{i=1}^{N_r} \log_2 \left(1 + \frac{\rho}{N_t} \lambda_i \right) \tag{9}$$

To further simplify this expression in the high SNR regime, the approximation $\log_2 \left(1 + \frac{\rho}{N_t} \lambda_i \right) \approx \log_2 \left(\frac{\rho}{N_t} \lambda_i \right)$ is utilized, leading to:

$$C \approx \sum_{i=1}^{N_r} \log_2 \left(\frac{\rho}{N_t} \lambda_i \right) \tag{10}$$

Expanding the logarithm using the property $\log_2(a \cdot b) = \log_2(a) + \log_2(b)$, the capacity expression is:

$$C \approx \sum_{i=1}^{N_r} \left[\log_2 \left(\frac{\rho}{N_t} \right) + \log_2(\lambda_i) \right] \tag{11}$$

Separating the summation yields:

$$C \approx \sum_{i=1}^{N_r} \log_2 \left(\frac{\rho}{N_t} \right) + \sum_{i=1}^{N_r} \log_2(\lambda_i) \tag{12}$$

Simplifying the first term, which is a constant sum, results in:

$$\sum_{i=1}^{N_r} \log_2 \left(\frac{\rho}{N_t} \right) = N_r \log_2 \left(\frac{\rho}{N_t} \right) \tag{13}$$

Thus, the capacity expression can be written as:

$$C \approx N_r \log_2 \left(\frac{\rho}{N_t} \right) + \sum_{i=1}^{N_r} \log_2(\lambda_i) \tag{14}$$

F. Expected Value of $\log_2(\lambda_i)$

To compute the average capacity, the expected value of $\log_2(\lambda_i)$ under Nakagami-q fading is required. This expected value is given by:

$$\mathbb{E} \left[\log_2(\lambda_i) \right] = \int_0^\infty \log_2(\lambda) f_{\lambda_i}(\lambda) d\lambda \tag{15}$$

where $f_{\lambda_i}(\lambda)$ is the probability density function (PDF) of the eigenvalues λ_i. The PDF's exact form is intricate but can be evaluated through numerical methods or approximations based on the moments of λ_i.

G. Final Capacity Expression

Combining these results, the high SNR capacity of the MIMO channel under Nakagami-q fading can be expressed as:

$$C \approx N_r \log_2 \left(\frac{\rho}{N_t} \right) + N_r \mathbb{E} \left[\log_2(\lambda) \right] \tag{16}$$

Here, $\mathbb{E} \left[\log_2(\lambda) \right]$ reflects the average behavior of the eigenvalues under Nakagami-q fading.

This formulation provides a detailed capacity analysis of MIMO wireless channels in high SNR regimes, leveraging the Nakagami-q distribution. The derived expressions facilitate a deeper understanding of channel behavior and performance, offering valuable insights for the design and optimization of MIMO systems in environments characterized by high SNR.

IV. Performance Analysis

This section evaluates the MIMO channel capacity under varying system parameters, including antenna configurations, Nakagami-q fading parameters, and different fading distributions (Nakagami-q, Rayleigh, and Rician). The results are discussed in detail, supported by numerical simulations and visualizations.

A. MIMO Channel Capacity vs. SNR for Different Antenna Configurations

Figure 2 illustrates the MIMO channel capacity as a function of the SNR for different transmit antenna configurations, with a fixed Nakagami-q parameter of $q = 0.5$ and $N_r = 2$ (receive antennas). The configurations analyzed include $N_t = 1$, $N_t = 2$, and $N_t = 4$ (transmit antennas).

1) Observations: The channel capacity increases with the number of transmit antennas (N_t) for any given SNR. This increase is attributed to the spatial diversity and multiplexing gains provided by additional transmit antennas. For instance, the $N_t = 4$ configuration exhibits a significantly higher capacity compared to $N_t = 1$ and $N_t = 2$, highlighting the advantage of using more transmit antennas in MIMO systems.

2) Performance: Among the analyzed configurations, $N_t = 4$ achieves the highest channel capacity across the entire SNR range, followed by $N_t = 2$ and $N_t = 1$. The performance difference is most notable at moderate to high SNR values, where the advantages of spatial multiplexing and diversity are maximized.

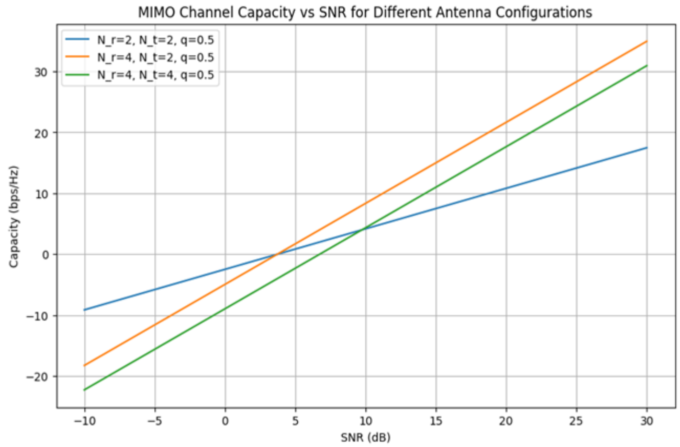

Fig. 2. MIMO Channel capacity vs. SNR for different antenna configurations ($q = 0.5$).

3) Implication: The results demonstrate that increasing the number of transmit antennas is an effective strategy to enhance channel capacity in MIMO systems. This finding is particularly relevant for the design of high-SNR communication systems, where antenna configuration becomes a critical factor in performance optimization.

B. MIMO Channel Capacity vs. SNR for Different Nakagami-q Parameters

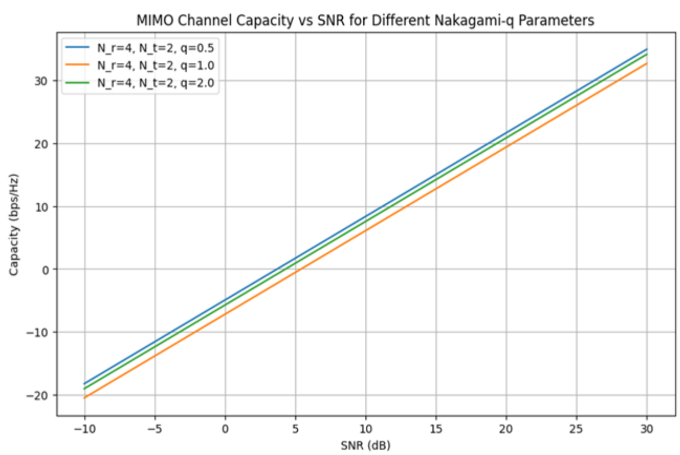

Fig. 3. MIMO Channel Capacity vs. SNR for different Nakagami-q parameters.

Figure 3 shows the impact of the Nakagami-q fading parameter on MIMO channel capacity as a function of SNR. The simulations consider $q = 0.5$, $q = 1$, and $q = 2$, with a fixed antenna configuration of $N_t = 2$ and $N_r = 2$.

1) Observations: The channel capacity improves as the Nakagami-q parameter (q) increases. Specifically, the $q = 2$ configuration achieves the highest capacity, followed by $q = 1$ and $q = 0.5$. The parameter q represents the severity of the fading environment, where lower q values indicate more severe fading.

2) Performance: At low SNR values, the capacity difference between the q configurations is pronounced, with $q = 2$ offering a clear advantage. However, as SNR increases, the capacity curves converge, indicating that the Nakagami-q parameter has a diminishing effect at high SNR.

3) Implication: The Nakagami-q parameter plays a crucial role in determining channel capacity, particularly in low to moderate SNR regimes. Accurate modeling of the fading environment is therefore essential in MIMO system analysis, as it directly influences performance predictions and design decisions.

C. MIMO Channel Capacity vs. SNR for Nakagami-q, Rayleigh, and Rician Fading

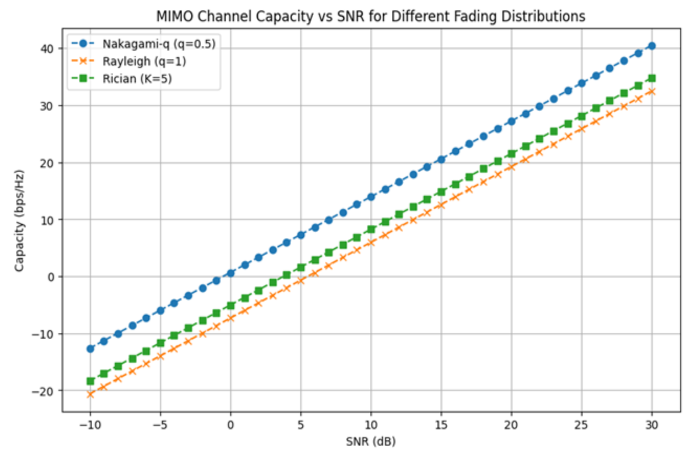

Fig. 4. MIMO Channel Capacity vs. SNR for Nakagami-q, Rayleigh, and Rician fading.

Figure 4 compares the channel capacity of MIMO systems under Nakagami-q fading ($q = 1$), Rayleigh fading, and Rician fading ($K = 3$), with a fixed antenna configuration of $N_t = 2$ and $N_r = 2$.

1) Observations: Among the three fading models, Nakagami-q fading exhibits the highest channel capacity, followed by Rician and Rayleigh fading. Nakagami-q fading's flexibility in modeling a wide range of channel conditions provides it with a performance edge. At lower SNR values, the capacity difference between the fading models is more prominent, while the curves converge as SNR increases.

2) Performance: Rician fading benefits from the presence of a line-of-sight component, resulting in a higher capacity than Rayleigh fading. However, Nakagami-q fading's ability to model diverse environments allows it to outperform both Rician and Rayleigh fading under the analyzed conditions.

3) Implication: The choice of fading model significantly impacts the predicted channel capacity, especially in low SNR scenarios. This highlights the importance of selecting an appropriate fading model based on the specific application and environmental conditions for accurate performance analysis.

D. Summary of Results

The performance analysis presented in this section highlights several critical insights for MIMO system design and optimization:

- Increasing the number of transmit antennas (N_t) significantly enhances channel capacity by leveraging spatial diversity and multiplexing gains, particularly in high-SNR scenarios.
- The Nakagami-q fading model, with its flexibility to represent a wide range of fading environments, demonstrates superior performance, particularly under diverse and challenging channel conditions. The higher capacity achieved in Nakagami-q fading compared to Rayleigh and Rician models highlights its potential for realistic modeling of wireless channels.
- The novel capacity equation derived in this research provides an accurate and simplified representation of MIMO channel behavior in high-SNR conditions, offering critical insights into system performance. This equation not only enhances the understanding of channel capacity under Nakagami-q fading but also serves as a valuable tool for system design and optimization.
- The validation of the proposed model is conducted by analyzing channel capacity concerning key system parameters, such as the number of antennas at both the transmitter and receiver, as well as the fading severity parameter. The performance evaluation demonstrates that the proposed model effectively adapts to varying configurations, exhibiting improved capacity trends. Furthermore, as illustrated in Figure 4, a comparative analysis with existing Rayleigh and Rician fading models highlights the superior performance of the proposed approach, further validating its effectiveness in MIMO systems.

These findings indicate the importance of adopting the Nakagami-q fading model as a robust and generalized framework for analyzing MIMO systems. The novel equation developed in this study, tailored for high-SNR scenarios, enables precise capacity evaluations, paving the way for designing high-performance, next-generation wireless communication systems.

V. Discussion

The results of this study highlight the significant impact of Nakagami-q fading on MIMO channel capacity in high-SNR conditions. Higher Nakagami-q values enhance channel conditions, particularly in low-to-moderate SNR regimes. Compared to Rayleigh and Rician models, Nakagami-q provides superior capacity, demonstrating its flexibility in modeling diverse wireless environments. These findings validate its relevance for next-generation networks requiring high reliability and adaptability.

Despite the advancements in MIMO capacity analysis, several gaps remain in existing research. Traditional studies predominantly focus on Rayleigh and Rician fading models, which fail to accurately capture the full range of fading conditions encountered in real-world wireless communication systems. Furthermore, many prior studies do not optimize capacity expressions specifically for high-SNR regimes, which limits the applicability of these models to practical high-performance systems. Another key limitation in previous research is the lack of eigenvalue-based capacity evaluation, which is crucial for understanding how fading characteristics impact system performance. Additionally, computational efficiency has often been overlooked, making it challenging to implement these models in real-world scenarios.

This study addresses these limitations by incorporating Nakagami-q fading into MIMO capacity analysis, extending beyond conventional models to provide a more comprehensive and adaptable framework. By deriving a high-SNR capacity expression and integrating eigenvalue-based evaluation, this approach enhances theoretical insights and improves model applicability. Moreover, by identifying these research gaps, it becomes possible to prioritize areas that require further exploration and develop strategies for filling those gaps in a targeted and effective way.

Future research should focus on experimental validation using real-world channel measurements and extending the analysis to correlated and non-i.i.d. channels. Investigating Nakagami-q fading in low-SNR conditions and developing energy-efficient transmission strategies would further refine its practical relevance. Additionally, leveraging machine learning for adaptive transmission and exploring the integration of Nakagami-q fading with advanced MIMO technologies such as massive MIMO and RIS-assisted systems could provide further advancements. By addressing these gaps, this study ensures that future research efforts are directed towards practical and high-impact improvements in wireless communication systems.

VI. Conclusion

This study examined the capacity of MIMO wireless systems under high SNR conditions using the Nakagami-q fading model and compared it with Rayleigh and Rician fading models. The results highlighted the significance of antenna configurations and fading models in determining system performance. Increasing the number of antennas, particularly balanced configurations of N_r and N_t, was shown to significantly enhance channel capacity by exploiting spatial diversity and multiplexing gains, particularly in high-SNR scenarios.

The Nakagami-q fading model emerged as a robust and flexible framework for characterizing diverse fading environments, outperforming Rayleigh and Rician fading models in terms of channel capacity, especially under low to moderate SNR conditions. Its adaptability to model varying degrees of fading severity underscores its relevance for practical wireless system analysis and optimization. Moreover, the derived novel capacity equation tailored for high-SNR conditions provided an accurate and simplified tool for understanding and predicting MIMO system behavior. This equation offers valuable insights into the impact of fading parameters and antenna configurations, making it a practical resource for the design and optimization of next-generation wireless communication systems.

These findings underline the importance of the Nakagami-q fading model and the derived capacity equation in advancing the analysis of MIMO systems. By providing a deeper understanding of channel capacity under high-SNR conditions, this research paves the way for the development of high-performance, robust, and efficient wireless communication technologies for future networks, including 5G and beyond.

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References

[1] J. Kumar, A. Gupta, S. Tanwar, and M. K. Khan, "A Review on 5G and Beyond Wireless Communication Channel Models: Applications and Challenges," *Phys. Commun.*, vol. 67, p. 102488, Elsevier, 2024.

[2] S. Taruna and I. Kaur, "Analysis of Multiple-Input-Multiple-Output (MIMO) System with Transmit and Receive Diversity," *Int. J. Comput. Appl.*, vol. 79, no. 12, Citeseer, 2013.

[3] K. N. Le, "A Review of Selection Combining Receivers Over Correlated Rician Fading," *Digit. Signal Process.*, vol. 88, pp. 1–22, Elsevier, 2019.

[4] N. Kumar, A. Dixit, and V. Vijay, "q-Generalization of Nakagami Distribution with Applications," *Jpn. J. Stat. Data Sci.*, pp. 1–24, Springer, 2024.

[5] B. Kumbhani and R. S. Kshetrimayum, *MIMO Wireless Communications Over Generalized Fading Channels*. CRC Press, 2017.

[6] A. K. Sarangi and A. Datta, "Capacity Comparison of SISO, SIMO, MISO & MIMO Systems," in *Proc. 2018 Second Int. Conf. Comput. Methodol. Commun. (ICCMC)*, pp. 798–801, IEEE, 2018.

[7] N. W. Hlaing, A. Farzamnia, M. K. Haldar, and T. Yousefi Rezaii, "BER Analysis of SIMO and MIMO Systems with Rayleigh Fading Using SIMULINK," in *Proc. 12th Nat. Tech. Semin. Unmanned Syst. Technol. (NUSYS'20)*, pp. 769–782, Springer, 2022.

[8] S. C. Bandara, P. J. Smith, E. Khordad, R. Evans, and R. Senanayake, "Rician Channel Modelling for Super Wideband MIMO Communications," *arXiv preprint arXiv:2411.01878*, 2024.

[9] O. S. Badarneh and D. B. da Costa, "Fluctuating Nakagami-m Fading Distribution," *IEEE Wirel. Commun. Lett.*, IEEE, 2024.

[10] N. Youssef, C.-X. Wang, and M. Patzold, "A Study on the Second Order Statistics of Nakagami-Hoyt Mobile Fading Channels," *IEEE Trans. Veh. Technol.*, vol. 54, no. 4, pp. 1259–1265, IEEE, 2005.

[11] W. Tan, W. Huang, X. Yang, Z. Shi, W. Liu, and L. Fan, "Multiuser Precoding Scheme and Achievable Rate Analysis for Massive MIMO System," *EURASIP J. Wirel. Commun. Netw.*, vol. 2018, pp. 1–12, Springer, 2018.

[12] Y. Zhang, J. Zhang, Y. Zhang, Y. Yao, and G. Liu, "Capacity Analysis of Holographic MIMO Channels with Practical Constraints," *IEEE Wirel. Commun. Lett.*, vol. 12, no.

[13] S. Chopra and A. Kakkar, "Capacity Analysis of Hybrid MIMO Using Sparse Signal Processing in mmW 5G Heterogeneous Wireless Networks," *Wirel. Pers. Commun.*, vol.

[14] S. Derouiche, S. Kameche, and H. E. Adardour, "5G Network Performance Using Novel MIMO Mixed FSO/MMW Communication Systems Under Pointing Errors Effect: Test Analysis Using Image Transmission," *J. Opt.*, vol. 26, no. 11, p. 115707, IOP Publishing, 2024.

[15] R. Mesleh, O. Badarneh, and A. Younis, "Nakagami-m MIMO Channel Model," in *Proc. 9th Int. Conf. Electr. Electron. Eng. (ICEEE)*, pp. 280–284, IEEE, 2022.

[16] V.-D. Phan, B. C. Nguyen, T. M. Hoang, T. N. Nguyen, P. T. Tran, B. V. Minh, and M. Voznak, "Performance of Cooperative Communication System with Multiple Reconfigurable Intelligent Surfaces Over Nakagami-m Fading Channels," *IEEE Access*, vol. 10, pp. 9806–9816, IEEE, 2022.

[17] D.-D. Tran, S. K. Sharma, S. Chatzinotas, I. Woungang, and B. Ottersten, "Short-P

[18] M. Premkumar, V. Sachan, and B. R. Singh, "Data Transmission and Reception in Spatial Modulation MIMO Wireless Systems and

[19] T. B. Bashu, S. Feisso, and M. M. Tulu, "Performance Evaluation of Precoding Schemes for Multi User Massive MIMO System Over Nakagami-m Fading Channel," *Wirel. Pers. Commun.*, vol. 138, no. 1, pp. 29–40, Springer, 2024.

[20] D. Joann and V. Rajamani, "Evaluating MIMO and Massive MIMO Performance with Rayleigh, Rician, and Nakagami Fading Channels Along with Comparing Half-Duplex and Full-Duplex Modes Using HMR Protocol," IntechOpen, 2024.

[21] B. S. Sonok, M. S. Islam, and M. Mazid-Ul-Haque, "Hoyt Wireless Fading Channel Capacity Analysis Using Large Limit Argument Approximation," in *Proc. 2nd Int. Conf. Comput. Advancements*, pp. 18–24, 2022.

[22] S. B. Shawkat, M. Mazid-Ul-Haque, M. S. Islam, and B. S. Sonok, "Fundamental Capacity Analysis for Identically Independently Distributed Nakagami-q Fading Wireless Communication," *Int. J. Adv. Comput. Sci. Appl.*, vol. 11, no. 9, 2020, Science and Information (SAI) Organization Limited.

[23] M. Mazid-Ul-Haque and M. S. Islam, "Data Rate Limit in Low and High SNR Regime for Nakagami-q Fading Wireless Channel," *Int. J. Adv. Comput. Sci. Appl.*, vol. 11, no. 7, 2020, Science and Information (SAI) Organization Limited.

[24] M. Qian, L. You, X.-G. Xia, and X. Gao, "On the Spectral Efficiency of Multi-User Holographic MIMO Uplink Transmission," *IEEE Trans. Wirel. Commun.*, IEEE, 2024.