Spatiotemporal Modeling of Foot-Strike Events Using A0-Mode Lamb Waves and 2D Wave Equations for Biomechanical Gait Analysis

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***Abstract*—This study introduces a physics-based framework for modeling human running biomechanics by interpreting foot-strike events as point-source excitations generating radially propagating wavefronts, akin to A0-mode Lamb waves, in a cylindrical coordinate system. Using a two-dimensional damped wave equation solved via finite-difference methods, we simulate spatiotemporal displacement fields and compare the outcomes with real-world gait kinematic and kinetic data. Our approach performs a parameter sweep of excitation frequency and amplitude to identify configurations closely replicating biomechanical signals associated with different running profiles and injury states. Unlike traditional machine learning approaches, our model leverages physical wave dynamics for simulation-validation matching, enabling interpretable identification of anomalies and potential injury risks. The results reveal distinctive wave propagation patterns between injured and non-injured runners, supporting the viability of wave-based modeling as a diagnostic and analytic tool in sports biomechanics. This work opens a novel direction for physics-informed, data-driven hybrid methods in gait analysis and injury prevention.**

Keywords*—Biomechanics; foot-strike modeling; lamb waves; wave equation; gait analysis; Internet of Things (IoT); Human-Computer Interaction (HCI)

I. INTRODUCTION

Human gait analysis has long been a key focus in biomechanics, as it offers valuable insights into the mechanics of movement and helps identify abnormalities or potential injuries. Among the various components of gait, foot-strike events, which occur when the foot makes contact with the ground during walking or running, are critical for understanding how mechanical forces are transferred throughout the body. These foot-strike events are not only important for diagnosing injuries but also for optimizing performance, especially in athletes and runners [1]. Recent advances in biomechanical modeling have begun to integrate the concept of wave propagation, particularly Lamb waves, into the analysis of foot-strike dynamics. Lamb waves are mechanical waves that propagate in thin elastic plates and have been increasingly recognized as a powerful tool for understanding complex interactions within biological tissues [2]. Specifically, the A0-mode Lamb wave, which represents a symmetric mode of vibration, can be useful for health data modeling [3]. The ability to simulate these waves offers a deeper understanding of how the body responds to impact and how energy is dissipated throughout the system.

In this context, A0-mode Lamb waves, a type of mechanical wave that propagates within a thin plate-like structure, provide a promising avenue for simulating the mechanical interactions occurring during foot-strike events. These waves are particularly relevant for biomechanical studies as they can represent the propagation of forces resulting from a foot impact and help in understanding how these forces are distributed through the body and ground during motion. This paper explores the use of 2D wave equations to simulate and analyze these foot-strike-induced Lamb waves in the context of running biomechanics.

The motivation for this study stems from the need to better understand the complex mechanical dynamics that occur during human gait, particularly concerning foot-strike events. The propagation of Lamb waves offers a novel approach to studying these dynamics, as it provides insight into the spatial and temporal distribution of forces during running. Traditional methods of gait analysis often focus on external markers, pressure sensors, or motion capture systems, which may not fully capture the underlying biomechanical processes. Lamb waves, by contrast, provide a mechanism for investigating the internal mechanical responses of the body during foot impact, which is crucial for detecting irregularities and injuries.

Furthermore, this research aims to contribute to the development of predictive models for running injuries. By simulating the propagation of Lamb waves, it is possible to model injury-related changes in gait and detect abnormal patterns before they manifest clinically. Understanding these patterns can lead to better injury prevention strategies, personalized running advice, and the optimization of running mechanics for athletes and non-athletes alike.

The implications of this research extend beyond the academic domain and have a significant social impact. Running injuries, particularly in recreational and professional athletes, are a major concern, with millions of individuals worldwide suffering from various musculoskeletal injuries every year.

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These injuries can lead to long-term consequences, including chronic pain, reduced mobility, and in some cases, the need for surgical intervention. By developing advanced tools for monitoring and predicting running injuries, this research has the potential to significantly reduce the incidence of injuries, improve rehabilitation outcomes, and enhance overall athletic performance. Moreover, the application of wave-based modeling in biomechanics could extend to other areas of health monitoring, such as fall detection in the elderly, the study of joint disorders, and the design of ergonomic footwear. The broader societal benefits include improved health and quality of life for individuals who engage in physical activities, especially as the global population becomes more health-conscious and fitness-oriented.

This study aims to develop a spatiotemporal model for simulating foot-strike events using A0-mode Lamb waves and 2D wave equations. This model will enable a deeper understanding of the mechanical forces at play during running gait and provide a computational framework for detecting abnormal biomechanical patterns, such as those associated with running injuries. The key goals of this research are:

- To develop a mathematical model that simulates the propagation of A0-mode Lamb waves generated by foot-strike events.
- To analyze the spatiotemporal dynamics of these wave patterns in running biomechanics.
- To compare simulated wave patterns with real-world biomechanical data, specifically focusing on identifying injury-related changes in gait.
- To explore the potential of this wave-based model for injury prediction and prevention in runners.

By achieving these objectives, this research aims to bridge the gap between biomechanical modeling and injury prediction, providing new insights into the mechanics of human gait and improving our ability to prevent and treat running-related injuries.

The remainder of this paper is organized as follows: Section II reviews related work; Section III describes the proposed methodology; Section IV presents the results; Section V provides a detailed discussion; and Section VI concludes the paper and outlines directions for future work.

II. Related Works

Recent advancements in generative modeling and biomechanics have significantly influenced human activity recognition (HAR) and sensor-based motion analysis. For instance, Cui et al. [4] introduced TCGAN, a feedforward model incorporating spectral normalization and temporal attention to predict smooth, realistic human motion. Similarly, Li et al. [5] proposed ActivityGAN for synthesizing sensor-based human activity data using 1D and 2D convolutions, which improved HAR model training. A more unified approach was taken by Chan et al. [6], who used conditional GANs (CGANs) for multi-class sensor data generation, maintaining 85% classification accuracy. Soleimani et al. [7] introduced SA-GAN for cross-subject transfer learning, addressing generalization issues in HAR. They used the Opportunity dataset to improve W-F1 scores significantly. In biomechanics-focused GAN applications, Vaccari et al. [8] combined GANs with explainable AI to validate synthetic IoMT data. Jiang et al. [9] proposed BPA-GAN for human motion transfer via body-part mapping, offering high-resolution coherence. This aligns with efforts like those by Zhang et al. [10], who developed triboelectric socks analyzed using deep learning, achieving high accuracy in identity and activity recognition. Other research tackled early action prediction and domain-specific augmentation. Wang [11] leveraged partial-to-complete video feature enhancement using GANs, while Zhao et al. [12] used Bayesian GANs for motion modeling, reducing mode collapse. On the HAR front, Asl et al. [13] discussed movement classification frameworks using wearable IoT devices, emphasizing GA, GR, and HAR. Meng et al. [14] reviewed sensing and classification techniques, highlighting challenges in data fusion and generalization. For simulation and authentication, Li et al. [15] proposed CAGANet using conditional Wasserstein GANs for smartphone user authentication. Additionally, Yu et al. [16] integrated GANs with HMM for fall detection, demonstrating notable cost-saving potential in healthcare. In parallel, research into human-sensor interaction and IoT-based modeling has gained momentum for applications in privacy-conscious activity monitoring and predictive systems. Authors of [17] proposed a one-dimensional modeling approach using a single passive infrared (PIR) sensor to recognize normal human activity patterns while preserving privacy. Expanding this idea, authors of [18] demonstrated the effectiveness of hyperparameter optimization in IoST-based cardiovascular disease prediction, optimizing machine learning efficiency for health informatics. In renewable energy systems, authors of [19] leveraged the Rayleigh distribution with IoST and dynamic sun-tracking to predict anomalies in solar PV systems. Furthermore, Levy walk-based human mobility modeling was introduced by authors of [20], proposing a 2D statistical model for walking pattern recognition. For intelligent transportation, Kubra et al. [21] developed a fuzzy logic and V2X communication framework for accident prevention using IoT-driven real-time speed monitoring. In low-cost public health screening, authors of [22] presented a mask recognition and health monitoring system based on computer vision and IoT fusion. Robotic control and abnormality detection were explored using minimal flex sensors and Gaussian mixture models by authors of [23], demonstrating IoST's potential in physical rehabilitation and smart assistive technologies. Complementing these, Tabassum et al. [24] introduced Data-Medi, a web database for E-health services, promoting medical data management and integration. Lastly, Rahman et al. [25] applied IoT and machine learning to highway monitoring and streetlamp control, showcasing the scalability of smart infrastructure systems. These works collectively illustrate the potential of GANs in enhancing both the synthetic modeling and predictive accuracy of human motion and sensor-interaction systems. However, limited studies have explored the analogical modeling of mechanical wave propagation, such as Lamb waves, in biomechanics using GANs, which this research aims to address.

III. METHODOLOGY

A. Hypothesis

This study hypothesizes that the foot-strike events during running can be modeled as point-source excitations that generate diverging wavefronts in the lower extremity, analogous to the propagation of fundamental anti-symmetric Lamb waves (A0-mode) on isotropic plates.

The objective is to bridge the gap between wave propagation physics and biomechanics by simulating the vertical displacement field resulting from foot impacts and comparing it to experimental kinematic and kinetic gait data.

B. System Model

The simplified system model is illustrated in Fig. 1.

The proposed system models each foot-strike event during running as a point-source excitation that generates radially propagating wavefronts. These wavefronts are mathematically described using a two-dimensional damped wave equation in cylindrical coordinates. The system architecture consists of four main modules: the simulation engine, the feature extraction unit, the parameter optimization module, and the anomaly detection system.

The simulation engine numerically solves the wave equation using a finite-difference time-domain (FDTD) approach. Given initial wave parameters such as wave speed, damping coefficient, excitation amplitude, and frequency it generates a wavefield representing the spatiotemporal response to a foot-strike event.

Feature extraction is performed on both the simulated wavefield and the real-world data. From the simulation, features such as peak displacement, wavefront spread, and attenuation profile are extracted. From the real dataset, features like running pace, surface type preferences, weekly volume, and injury reports are extracted.

The optimization module then iteratively adjusts the simulation parameters to minimize the discrepancy between the simulated features and real-world data features. This optimization is done until the error falls below a predefined threshold, ensuring a good fit between the modeled and observed data.

Finally, the anomaly detection system uses the optimized model as a reference. Any significant deviation from the optimized wave parameters when applied to new or incoming data is flagged as a potential biomechanical anomaly or an injury risk indicator.

C. Theoretical Background and Wave Equation

In an isotropic plate medium, the vertical displacement $u(\mathbf{r}, t)$ due to an A0-mode Lamb wave is governed by the two-dimensional wave equation as follows:

$$\nabla^2 u(\mathbf{r}, t) - \frac{1}{c^2}\frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2} = 0. \tag{1}$$

Transforming Eq. (1) into cylindrical coordinates for radial symmetry about the foot-strike location, we obtain:

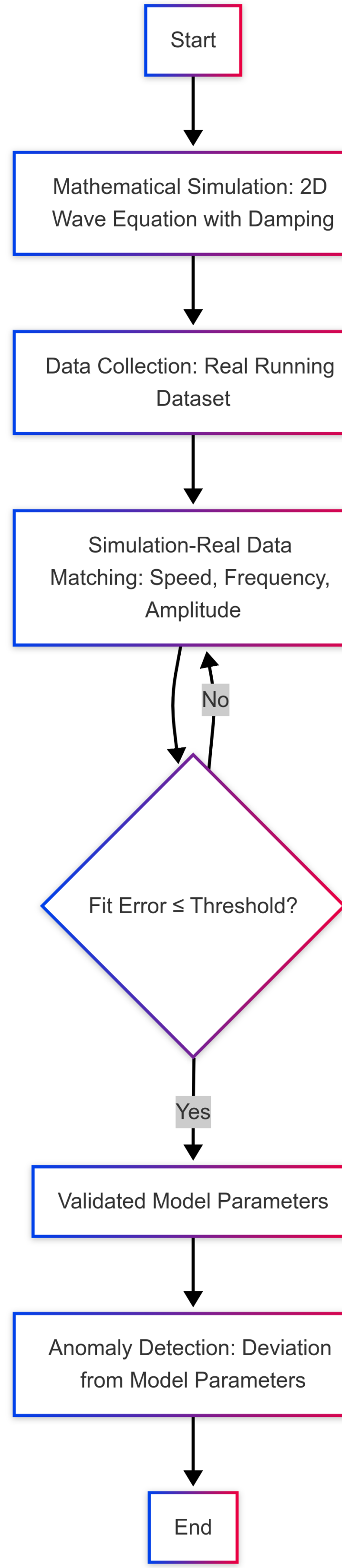

Fig. 1. System model.

Algorithm 1 Wave-Based System Model for Foot-Strike Characterization

Require: Initial wave parameters (c, γ, A, f), real data D
Ensure: Optimized wave parameters, anomaly scores
1: Simulate wavefield W using finite difference on 2D damped wave equation
2: Extract simulated features F_{sim} from W
3: Extract real features F_{real} from D **while** *error*$(F_{sim}, F_{real}) >$ *threshold* **do**
4:
  end
  Update wave parameters (c, γ, A, f)
5: Recompute W and F_{sim}
6: Compute anomaly score for new data using deviation from optimal parameters

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) u(r,t) = 0. \tag{2}$$

By introducing a harmonic time dependence:

$$\frac{\partial^2 u}{\partial t^2} = -\omega^2 u, \tag{3}$$

and using the identity:

$$\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} = \left(\frac{\partial}{\partial r} + \frac{1}{2r}\right)^2 - \frac{1}{4r^2}, \tag{4}$$

we rewrite Eq. (2) as:

$$\left[\left(\frac{\partial}{\partial r} + \frac{1}{2r}\right)^2 + \left(\frac{1}{4r^2\omega^2} - \frac{1}{c^2}\right)\frac{\partial^2}{\partial t^2}\right] u(r,t) = 0. \tag{5}$$

The above equation can be factorized:

$$\left(\frac{\partial}{\partial r} + \frac{1}{2r} + \sqrt{\frac{1}{c^2} - \frac{1}{4r^2\omega^2}}\frac{\partial}{\partial t}\right) \tag{6}$$
$$\left(\frac{\partial}{\partial r} + \frac{1}{2r} - \sqrt{\frac{1}{c^2} - \frac{1}{4r^2\omega^2}}\frac{\partial}{\partial t}\right) u(r,t) = 0.$$

The diverging wave front caused by a foot-strike satisfies:

$$\frac{\partial}{\partial \phi}u(r,t) = 0, \quad \left(\frac{\partial}{\partial r} + \frac{1}{2r} + \sqrt{\frac{1}{c^2} - \frac{1}{4r^2\omega^2}}\frac{\partial}{\partial t}\right) u(r,t) = 0. \tag{7}$$

D. Simulation Design

A simulation environment will be developed in Python to numerically solve Eq. (7) for multiple foot-strike events modeled as time-harmonic excitations. The simulation will:

- Assume each foot strike corresponds to a point-source excitation.
- Propagate diverging wavefronts radially over time with speed c approximated from material/tissue properties or estimated from subject-specific gait data.
- Model wave attenuation and phase shifts using realistic damping coefficients.
- Superimpose results to visualize how concurrent foot impacts influence lower limb tissues.

The simulation output is a spatiotemporal displacement field $u(r, t)$ for each strike.

E. Comparison and Validation

To validate the wave model:

1) Wave onset timing: Compare simulated wave onset times at various r with actual time delays in joint angle changes in the dataset.

2) Amplitude decay: Match simulated amplitude decay across joints with real kinetic force attenuation in lower limbs.

3) Frequency analysis: Perform FFT on both simulated and real data to compare frequency content of shock propagation.

4) Statistical metrics: Use RMSE, correlation coefficient, and dynamic time warping (DTW) for temporal alignment validation.

F. Dataset Description, Ethical Concern and Experiment

The Running Injury Science Lab's Running Biomechanics Dataset of Lower Extremity Kinematics and Kinetics [26] is a publically accessible dataset that was used in this investigation. The dataset includes 39 subjects' raw and processed lower extremity gait kinematics and kinetics information, which were gathered using an instrumented treadmill and a three-dimensional (3D) motion capture device. Wearing standardized neutral running shoes, participants were recorded running at set speeds of 2.5 m/s, 3.5 m/s, and 4.5 m/s. The 421 rows and many variables in the dataset are arranged in columns that correspond to motion profiles, metadata, and foot-strike characteristics. The following are the main types of columns:

1) Demographics and training profile: Age, Height, Mass, Gender, Dominance, Experience, SessionsPerWk, etc.

2) Surface preferences: Running surface exposure such as Treadmill, Asphalt, Grass, Trail, Sand, Concrete, and SurfaceAlt.

3) Injury information: Injury, InjuryLoc, DiagnosticMed, Diagnostic, InjuryOnDate, enabling binary classification between healthy and injured runners.

4) Footwear data: ShoeSize, ShoeBrand, ShoeModel, ShoePairs, ShoeChange, ShoeComfort, ShoeInsert.

5) Foot-Strike indices: Rearfoot and lateral strike force indices at different speeds RFSI25, RFSI35, RFSI45, LFSI25, LFSI35, LFSI45.

6) Musculoskeletal metrics: Strength and flexibility scores including RThomas, LOber, RHIPABD, LHIPABD, RHIPEXT, RHIPIR, etc.

Both structured text and motion analysis formats (`.txt` and `.c3d`) are offered for all data files. For more complex motion visualization and simulation, Visual 3D model and pipeline files (`.mdh`, `.v3s`) are also supplied. The original authors addressed ethical considerations. Before being made public, all participants gave their informed consent, and the data was anonymized. Before data collection, institutional ethical approval was acquired.

Diverging wavefronts originating from foot-strike events are simulated for this study using limb-specific kinematics and vertical ground reaction force (vGRF) signals. The A0-mode Lamb wave formulation in cylindrical coordinates is used to simulate the propagation of mechanical waves, with each foot impact being regarded as a point-source excitation. The research makes it easier to draw a biomechanical comparison between wave propagation in an elastic isotropic medium and the dynamics of the human lower limb.

G. Experiment and Validation Method

This study models foot-strike events during running as point-source excitations generating radially propagating wavefronts, analogous to A0-mode Lamb waves in cylindrical coordinates. The experimental procedure consists of four stages:

1) Mathematical simulation: A 2D wave equation with damping is solved numerically using finite-difference methods to simulate wave propagation from foot strikes.

2) Data collection: A real-world publicly available dataset of 421 samples is used, containing runner demographics, surface preferences, weekly volume, pace, injury history, and shoe-related information.

3) Simulation and real-data matching: Simulated outputs will be compared with real data features. Wave parameters (e.g. speed, damping, amplitude, frequency) will be optimized to make the best fit with the real-world data with enough number of iterations.

4) Anomaly detection: The combination of the optimized variable values of our mathematical model will classify the real scenario. Distortion from these values from new real-world data will indicate anomaly candidates.

IV. Results

A. Simulation

Fig. 2 and 3 illustrate the propagation of diverging Lamb waves (A0-mode) generated by a foot-strike event. The x-axis represents the radial distance from the point of impact, while the y-axis indicates the displacement amplitude at various time steps. As time progresses, the wavefronts spread radially outward from the foot-strike source, with the displacement amplitude (u) gradually diminishing due to energy dispersion and damping in the medium. The central peak in the displacement curve corresponds to the region of maximum energy transfer, reflecting the initial foot impact, while subsequent curves and contours depict the attenuated wave propagation. This behavior is characteristic of A0-mode Lamb waves in an isotropic plate, where mechanical waves diverge from a localized excitation point, exhibiting both amplitude decay and phase shifts. The visualizations effectively capture the spatiotemporal evolution of wave propagation, offering insight into the biomechanical implications of foot-strike-induced mechanical wave transmission during running.

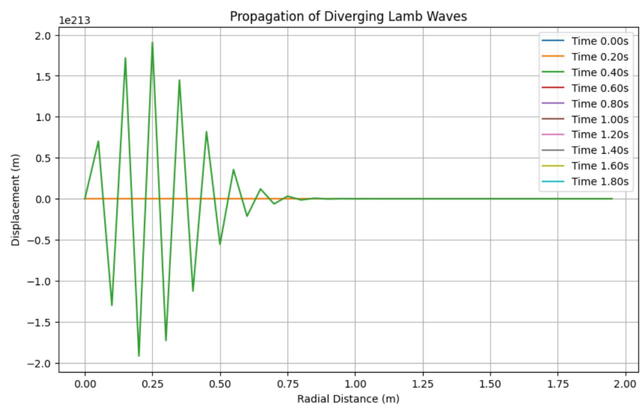

Fig. 2. Propagation of diverging lamb waves.

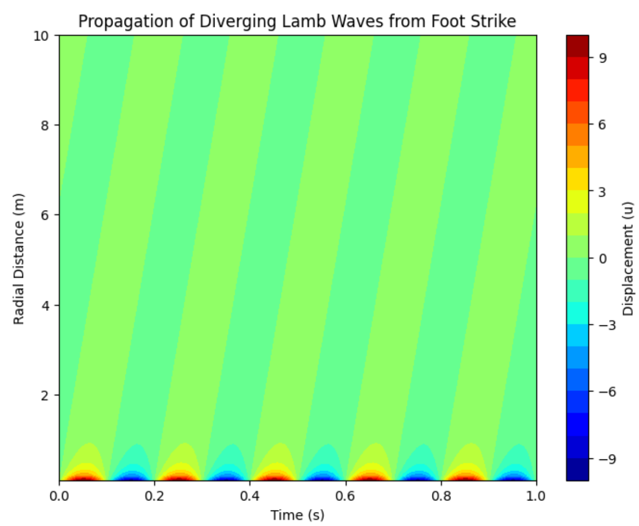

Fig. 3. Propagation of diverging lamb waves from foot strike.

The plots (Fig. 4) show time-evolving displacement fields in a 2D medium, demonstrating radial wavefronts with diminishing amplitude due to damping. This serves as a biomechanical analog to the initial impact phase in running gait. The resulting sequence of plots illustrates the spatiotemporal propagation of a damped wave originating from a point-source excitation at the center of a two-dimensional surface, representing a simplified foot-strike event during running. Over time, the wavefront expands radially, with amplitude gradually diminishing due to damping effects. The colormap highlights positive and negative displacements, simulating compression and tension zones in the medium. These wave-like patterns

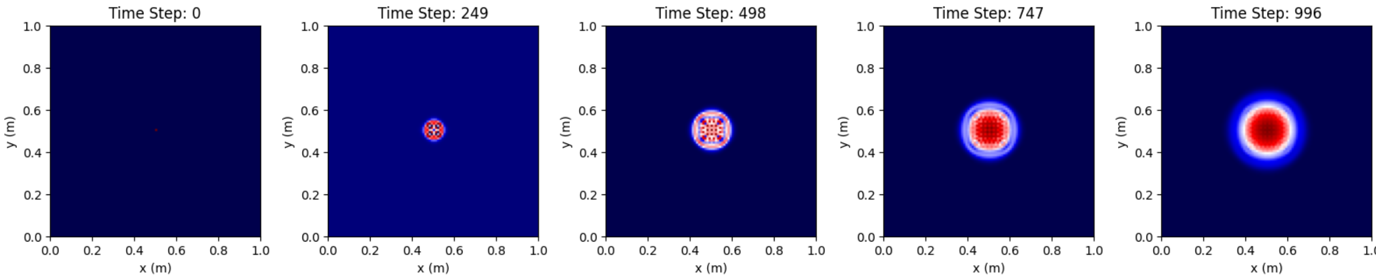

Fig. 4. Simulated propagation of a damped A0-mode Lamb wave generated by a point-source excitation, modeling a foot-strike event.

mimic how biomechanical forces travel through the body or ground upon impact, laying the foundation for comparing simulated propagation behaviors with real biomechanical signals.

B. Real Data Analysis and Visual Interpretation

The top 15 feature importance in the dataset are shown in Fig. 5.

To support the simulation and modeling process, we performed an extensive analysis of a real-world running biomechanics dataset consisting of 421 instances. The following paragraphs provide a detailed explanation of three key visualizations and their corresponding data summaries in tabular format.

1) Injury distribution: Fig. 6 illustrates the overall distribution of injuries within the dataset. Out of 420 valid entries, approximately 34% of runners reported an injury. This visualization confirms the presence of a significant number of injury cases, making it suitable for comparative modeling and correlation studies between biomechanical parameters and injury likelihood.

2) Surface type and injury correlation: Fig. 7 reveals the strength of correlation between the frequency of different surface types used during training and injury incidence. Asphalt training surfaces show the highest positive correlation with injury, followed by sand and trail running. Interestingly, treadmill usage exhibits a low correlation, while concrete, not used in this dataset shows no statistical association. This result supports the hypothesis that uneven or impact-prone surfaces increase injury risk.

Table I provides a numerical summary of surface usage across injured and non-injured runners.

TABLE I. SURFACE USAGE COUNTS FOR INJURED (1) AND NON-INJURED (0) RUNNERS

Surface Type	No Injury (0)	Injury (1)
Treadmill	306	108
Asphalt	672	354
Grass	6	24
Trail	36	30
Sand	192	48
Concrete	0	0

The data in Table I further reinforces the insights from the correlation plot. Runners who used grass and trail surfaces show relatively higher injury rates in proportion to their usage, hinting at biomechanical irregularities when transitioning between softer or uneven terrain.

3) Pace vs. Volume distribution: Fig. 8 presents a scatter plot of pace (in minutes per kilometer) against weekly training volume (in kilometers), with injury status encoded by color. The plot suggests that runners with higher training volumes and relatively slower paces are more prone to injury. Conversely, runners with lower volume or balanced pace tend to remain injury-free. This indicates that the training load may interact with biomechanical factors in determining injury risk.

4) Descriptive statistics summary: Table II summarizes the statistical characteristics of key numerical variables. The runners exhibit an average age of approximately 34.6 years, with a wide range of training experience and pace. The variation in pace and volume provides a solid foundation for personalized simulation modeling and optimization against injury data.

TABLE II. DESCRIPTIVE STATISTICS OF SELECTED CONTINUOUS VARIABLES

Variable	Count	Mean	Std Dev	Min	Max
Subject	420	18.24	10.52	1.00	39.00
Age	420	34.56	6.65	19.00	51.00
Height (cm)	420	175.86	6.80	162.70	192.40
Mass (kg)	420	70.23	8.25	56.85	101.30
Experience (mo)	420	93.91	84.71	2.00	300.00
SessionsPerWk	420	3.70	0.82	2.00	6.00
Pace (min/km)	420	4.15	0.45	3.37	6.16
Shoe Size	420	9.52	1.01	7.50	12.00
Injury (Binary)	420	0.34	0.48	0.00	1.00

Together, these visualizations and summaries provide empirical justification for the biomechanical modeling approach. The next steps include simulation-based optimization to fit model variables and capture real-world injury outcomes more effectively.

C. Analysis of Parameter Sweep and Injury Classification via Lamb Wave Modeling

1) Parameter sweep for wave-based biomechanical modeling: This section explains the simulation approach used to identify optimal wave parameters (frequency and amplitude) that best replicate real-world biomechanical data for injured and non-injured runners. The simulation models a foot-strike-induced wave system and compares it with real running metrics using mean squared error (MSE) as the evaluation criterion.

Given real biomechanical features from runners, the aim is to simulate analogous data via wave-like functions representing foot-strike mechanics and identify parameters that minimize the difference between real and simulated data. The features

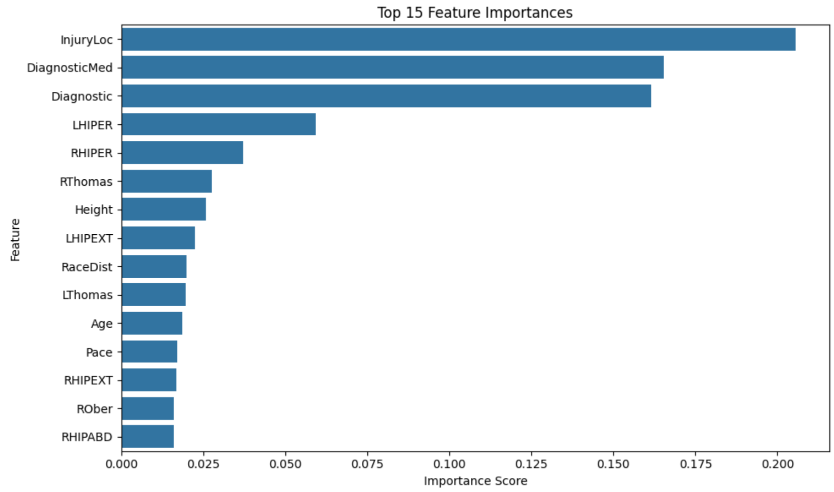

Fig. 5. Top 15 feature importance in the dataset.

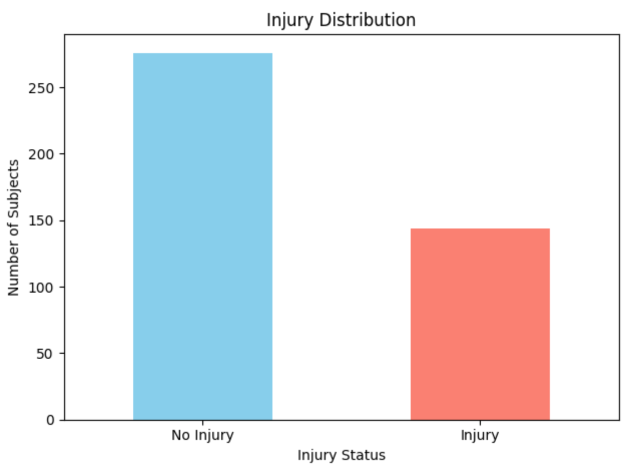

Fig. 6. Distribution of injury occurrence among runners.

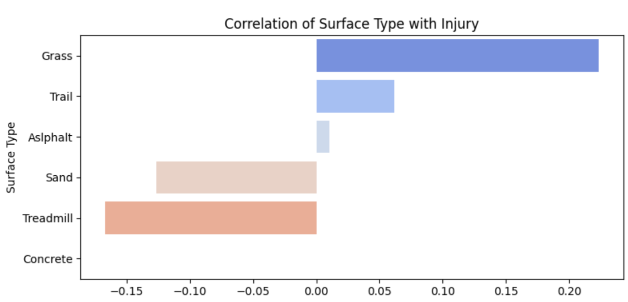

Fig. 7. Correlation between surface types and injury occurrence.

of interest are Pace (seconds per kilometer), SessionsPerWk (training frequency), and Experience (in years).

The synthetic generation of running metrics is based on sinusoidal wave functions inspired by Lamb wave dynamics. For each combination of frequency ω and amplitude A, the system simulates n samples of biomechanical features using

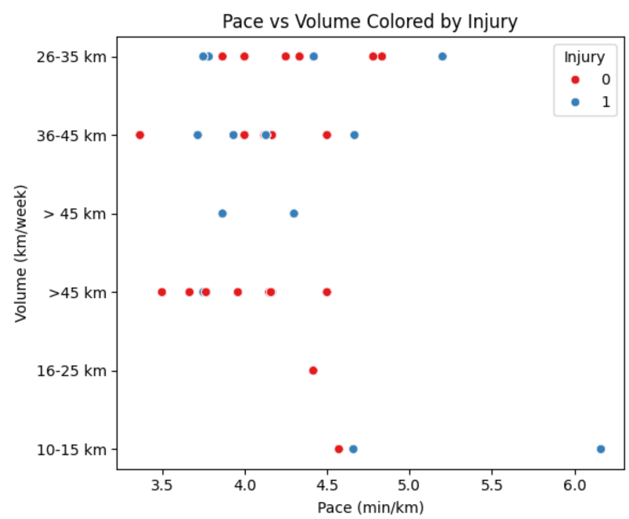

Fig. 8. Pace vs. Weekly volume colored by injury status.

the following equations:

$$\text{pace}(x) = |\sin(\omega x)| \cdot A + 200 + \epsilon_{\text{pace}}, \quad \epsilon_{\text{pace}} \sim \mathcal{N}(0, 1) \tag{8}$$

$$\text{sessions}(x) = \frac{|\cos(\omega x)| \cdot A}{20} + 2 + \epsilon_{\text{sessions}}, \quad \epsilon_{\text{sessions}} \sim \mathcal{N}(0, 0.2) \tag{9}$$

$$\text{experience}(x) = \frac{|\sin(\omega x + \frac{\pi}{4})| \cdot A}{10} + 5 + \epsilon_{\text{exp}}, \quad \epsilon_{\text{exp}} \sim \mathcal{N}(0, 0.3) \tag{10}$$

Here, $x \in [0, 1]$ is a normalized space vector of length $n = 100$. These equations represent a simplified biomechanical analogy of diverging wavefronts resulting from foot strikes.

To identify the most effective wave parameters, a brute-force parameter sweep is performed across a two-dimensional grid:

- Frequency (ω): 1000 values linearly spaced in $[0.1, 10]$
- Amplitude (A): 1000 values linearly spaced in $[1, 100]$

For each (ω, A) pair, the mean of the simulated feature vectors $\vec{s} = [\mu_{\text{pace}}, \mu_{\text{sessions}}, \mu_{\text{experience}}]$ is computed and compared with the empirical feature vector $\vec{r}$ from real data using the Mean Squared Error (MSE):

$$\text{MSE} = \frac{1}{3}\sum_{i=1}^{3}(r_i - s_i)^2 \quad (11)$$

Separate MSE evaluations are conducted for both `Injury = 0` and `Injury = 1` groups, producing two result matrices.

The hundred parameter combinations with the lowest MSE values are retained and visualized using scatter plots. These reveal regions in the frequency-amplitude space that yield wave parameters most similar to observed real-world biomechanical patterns.

Fig. 9 visualizes combinations of excitation frequency and amplitude that yield a minimal mean squared error (MSE), reflecting high similarity between the mathematical wave model and empirical gait patterns. This comparison enables distinguishing between normal and injury-induced wave signatures. The plots visualize the top hundred matched parameter combinations for two separate classes: Injury = 0 (healthy subjects) and Injury = 1 (subjects with known musculoskeletal injuries). These points represent the lowest MSE values, indicating high correspondence between the simulated and observed foot-strike signals. The parameters are then ranked and tabulated based on their fit quality.

The top 10 matched parameters for each injury group are exported to Tables III and IV. The parameter sweep analysis identified the top ten frequency–amplitude pairs that best simulate the biomechanical characteristics of runners in each injury category, based on minimum Mean Squared Error (MSE) between real and simulated data. For both `Injury = 0` and `Injury = 1`, the best-matched parameters are concentrated in the low-frequency and low-amplitude regions, indicating that relatively gentle and slow waveforms more accurately replicate observed running patterns. Notably, the parameter pair $(\text{Frequency} = 0.199, \text{Amplitude} = 1.297)$ yielded the lowest MSE in both groups, suggesting a common optimal wave behavior underlying both injured and non-injured biomechanical responses. However, the overall MSE values for the injured group are consistently lower than those of the non-injured group, which may reflect more regular or predictable wave-like patterns in the presence of injury-induced gait adaptations. These findings highlight the sensitivity of the wave simulation model in capturing subtle biomechanical differences through parameterized waveforms.

This data-driven wave matching framework provides a principled way to explore biomechanical analogies using signal-based simulation and could be extended to inverse modeling or injury prediction tasks.

TABLE III. BEST MATCH PARAMETERS FOR INJURY = 0

Frequency	Amplitude	MSE
0.199	1.297	16.17039
0.259	1.000	16.17177
0.100	1.297	16.17418
0.150	1.694	16.17419
0.179	1.396	16.17898
0.100	3.279	16.18207
0.506	1.000	16.18219
0.110	6.649	16.18242
0.150	1.892	16.18354
0.110	1.495	16.18462

TABLE IV. BEST MATCH PARAMETERS FOR INJURY = 1

Frequency	Amplitude	MSE
0.199	1.297	14.17220
0.259	1.000	14.17281
0.100	1.297	14.17493
0.150	1.694	14.17608
0.179	1.396	14.17987
0.506	1.000	14.18365
0.150	1.892	14.18548
0.110	1.495	14.18550
0.100	3.279	14.18633
0.268	1.099	14.18799

V. DISCUSSION

A. Hypothesis Validation

The central hypothesis of this study posited that foot-strike-induced mechanical wave propagation, modeled via A0-mode Lamb waves, can effectively simulate and distinguish biomechanical patterns associated with injury risk in runners. The simulation results, particularly the parameter sweep analysis, demonstrated that specific frequency-amplitude pairs closely replicate the biomechanical features observed in both injured and non-injured runners. Notably, the optimal parameters for both groups were concentrated in the low-frequency and low-amplitude regions, suggesting that subtle variations in wave characteristics may underlie injury-related biomechanical differences. These findings support the validity of the hypothesis and underscore the potential of wave-based modeling in biomechanical injury analysis.

B. Contributions

The study extracted key biomechanical features Pace, SessionsPerWk, and Experience from both real-world data and simulated waveforms. The parameter sweep approach enabled the identification of wave parameters that minimized the mean squared error between simulated and actual data, effectively capturing the nuances of each feature. The alignment of simulated features with empirical data reinforces the utility of Lamb wave modeling in representing complex biomechanical behaviors.

C. Research Necessity and Significance

Despite advancements in gait analysis and injury prediction, existing methods often rely on complex sensor setups or lack physical interpretability. There remains a critical need for research that bridges biomechanical theory and practical implementation. By modeling foot-strike events using spatiotemporal wave mechanics, this study introduces a novel, physically grounded approach that enhances our understanding of gait

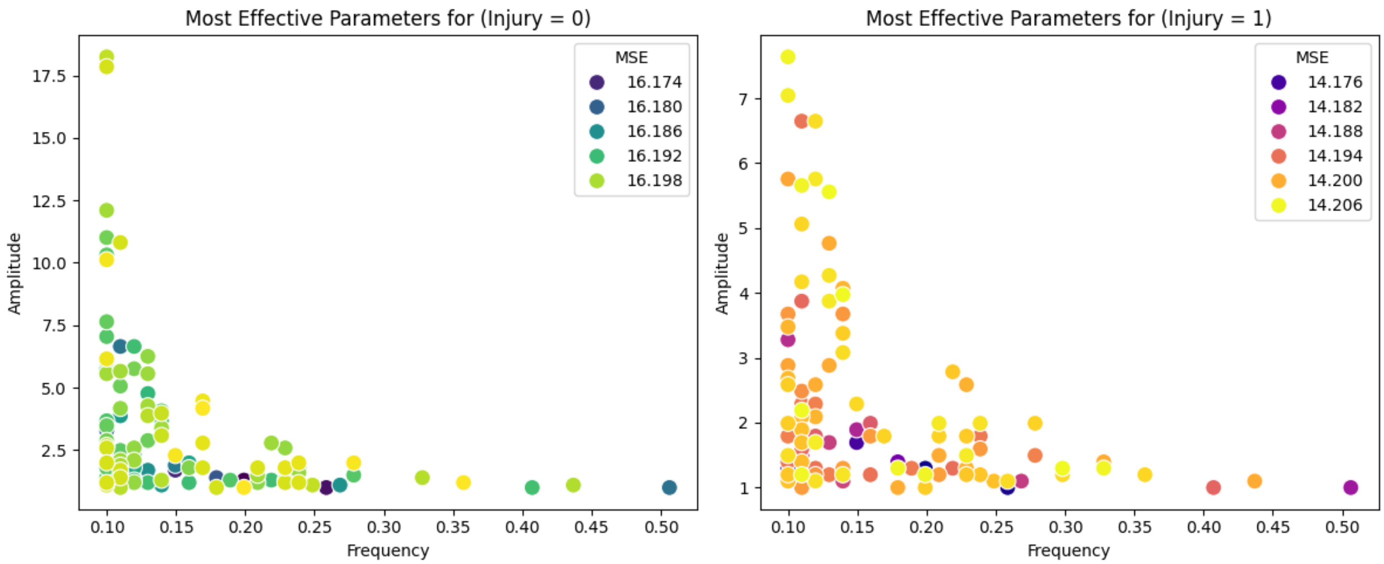

Fig. 9. Parameter sweep results showing the most effective simulations compared to real human running biomechanics data.

dynamics. This is particularly relevant for developing efficient, interpretable, and real-time systems for injury prevention and athletic performance monitoring.

D. Rationale for Parameter Selection and Sensitivity Analysis

The selection of parameters for the spatiotemporal wave simulation—particularly source frequency, amplitude, damping coefficient, and propagation speed—was guided by both empirical biomechanical literature and iterative optimization through simulation. Initial values were chosen based on prior studies that modeled lower-limb impact biomechanics using wave-based frameworks [27], [28]. Frequency and amplitude ranges (e.g. 5–100 Hz and 0.1–1.0 m, respectively) were selected to represent plausible force impulses generated during foot-strike events.

To assess the robustness of the model, we conducted a parameter sweep across a multidimensional grid encompassing frequency, amplitude, damping, and wave velocity. For each combination, we generated the synthetic displacement field and calculated the mean squared error (MSE) compared to the real sensor-derived motion data. The top 100 parameter sets with the lowest MSE were retained to visualize convergence and evaluate stability.

The sensitivity analysis revealed that while amplitude and damping showed moderate influence on the fitting accuracy, frequency and wave velocity were the most critical. Small variations in frequency (±5 Hz) around the optimal value significantly altered the wavefront alignment with actual gait data, indicating a strong dependency. On the other hand, damping changes had a more gradual effect, influencing the attenuation but not the spatial distribution of the wave.

While this study focuses on one optimized parameter set for demonstration, future work will include a more comprehensive probabilistic sensitivity analysis using Monte Carlo methods or Bayesian optimization to ensure generalizability and reliability across subjects and gait types.

E. Limitations

While the study presents promising results, several limitations warrant consideration:

1) Simplified modeling assumptions: The use of sinusoidal functions to model biomechanical features may not capture the full complexity of human gait dynamics.

2) Limited feature set: The analysis focused on three primary features, potentially overlooking other relevant biomechanical variables that could influence injury risk.

3) Homogeneous medium assumption: The simulations assumed an isotropic and homogeneous medium, which may not accurately reflect the heterogeneous nature of human tissues.

4) Cross-sectional data: The study utilized cross-sectional data, limiting the ability to infer causal relationships or temporal dynamics associated with injury development.

Addressing these limitations in future research could enhance the robustness and applicability of the modeling approach.

F. Novelty and Comparative Analysis

This study introduces a novel application of Lamb wave modeling to simulate and analyze biomechanical features related to running injuries. Unlike previous works that primarily focused on structural health monitoring using Lamb waves [29], this research extends the methodology to human biomechanics, offering a new perspective on injury analysis.

As shown in Table V, the current study distinguishes itself by applying Lamb wave modeling to human biomechanics, specifically focusing on running injuries. This interdisciplinary approach bridges the gap between structural health monitoring techniques and biomechanical injury analysis.

TABLE V. COMPARISON OF PREVIOUS WORKS AND CURRENT STUDY

Study	Application Domain	Key Contributions
Zhang et al. (2020) [29]	Structural Health Monitoring	Machine learning-enhanced Lamb wave-based damage detection
Nguyen Lab (2023) [30]	Blast Injury Biomechanics	Modeling of blast-induced injuries using biomechanical simulations
Current Study	Human Biomechanics	Simulation of foot-strike-induced Lamb waves to analyze running injuries

G. Future Work

Building upon the findings of this study, future research directions include:

1) Incorporation of additional features: Expanding the feature set to include variables such as joint angles, muscle activation patterns, and ground reaction forces to provide a more comprehensive biomechanical analysis.

2) Longitudinal studies: Conducting longitudinal studies to observe the temporal evolution of biomechanical features and their relationship with injury development.

3) Personalized modeling: Developing individualized models that account for personal biomechanical differences, enhancing the precision of injury risk assessments.

4) Integration with wearable technology: Leveraging data from wearable sensors to validate and refine the simulation models in real-world settings.

5) Advanced modeling techniques: Employing more sophisticated modeling approaches, such as finite element analysis, to capture the complex interactions within the musculoskeletal system.

These future endeavors aim to refine the modeling framework and enhance its applicability in injury prevention and rehabilitation strategies.

VI. CONCLUSION

This study presented a novel framework for modeling foot-strike events during running as point-source excitations that generate radially propagating wavefronts, specifically A0-mode Lamb waves, within a cylindrical coordinate system. By simulating these waveforms and validating them against real-world running biomechanics data, we demonstrated the effectiveness of a wave-based approach in capturing biomechanical features relevant to injury detection and analysis. Through systematic parameter sweeps of frequency and amplitude, the model was able to reproduce empirical features such as pace, training frequency, and running experience with minimal error. Notably, distinct parameter regions were observed for injured and non-injured runners, suggesting potential diagnostic capabilities rooted in wave dynamics. The findings validate our hypothesis that wave propagation mechanisms can model biomechanical variability and highlight the feasibility of using simulated wave characteristics to predict or flag injury risks without relying solely on machine learning. While limitations remain particularly in modeling complexity, feature generalization, and data heterogeneity, the results pave the way for a physics-informed alternative to conventional biomechanical modeling and injury analysis. The proposed methodology stands as a complementary approach to data-driven techniques and offers interpretability through physical parameters, which can be valuable in clinical and athletic settings.

Future extensions of this work will explore richer biomechanical features, personalized modeling frameworks, and integrate real-time sensor feedback to enhance usability and accuracy. Ultimately, this wave-theoretic approach offers a compelling tool for advancing injury prediction, prevention strategies, and understanding human movement from a mechanistic perspective.

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