Grey Clustering Algorithm for Urban Air Quality Classification: A Case Study in Lima, Peru

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***Abstract*—This study introduces a grey clustering algorithm based on the Central Triangular Whitenization Weight Function (CTWF), designed to classify urban air quality under conditions of limited or uncertain data. Based on Grey Systems Theory (GST), the proposed algorithm facilitates structured multi-criteria assessments using sparse or irregular datasets—a condition frequently encountered in urban environmental monitoring. The algorithm stands out for its low computational complexity, interpretability, and ability to integrate multiple pollutants into a single qualitative classification, making it particularly suitable for smart city applications and real-time decision support systems. To evaluate its performance, the grey clustering algorithm (CTWF) was applied to a case study in Northern Lima, Peru, covering eight semesters between 2011 and 2019 and including four key pollutants: PM_{10}, SO_2, NO_2, and CO. Although all periods were classified as "Good" under national standards, the disaggregated analysis revealed PM_{10} as the most persistent concern, while CO levels remained consistently low, and SO_2 and NO_2 showed moderate fluctuations. These findings validate the algorithm's capacity to extract pollutant-specific insights and spatiotemporal trends even in data-scarce environments. Future enhancements may include meteorological integration, broader pollutant sets (e.g., $PM_{2.5}$, ozone), and satellite data to extend forecasting capabilities and spatial resolution.**

***Keywords*—Grey clustering algorithm; air quality classification; grey systems theory; urban air pollution**

I. Introduction

Urban air pollution remains a critical and escalating environmental concern, particularly in rapidly growing cities in the global South, where population growth often outpaces infrastructure development. Its detrimental effects on respiratory health, mortality, and environmental sustainability are well documented, especially in low- and middle-income countries [1], [2], [3]. In Latin America, cities such as Lima, São Paulo, and Bogotá routinely report air pollutant concentrations that exceed both national regulations and international safety guidelines [4].

In the case of Lima, Peru, the second largest city in South America, unregulated urban sprawl, high vehicular density, and industrial activity have all contributed to significant air quality degradation [5], [6]. This issue is especially pronounced in northern districts such as Comas, Los Olivos, and Carabayllo, where socioeconomically vulnerable populations are exposed to elevated pollution levels but lack access to comprehensive monitoring systems [7]. Despite growing public concern, air quality assessments remain constrained by fragmented datasets, limited monitoring infrastructure, and the absence of tools designed to function under data uncertainty [8], [9].

These challenges have prompted the need for alternative modeling approaches that are capable of generating reliable insights from minimal or limited data. In this context, Grey Systems Theory (GST), introduced by Deng in the early 1980s, offers a mathematical framework tailored for decision-making under conditions of limited or uncertain information [10], [11]. Unlike traditional statistical models, which rely on consistent and extensive datasets, GST is designed to function effectively even when sample sizes are small or measurements are incomplete, conditions commonly found in urban environmental assessments in Latin America [12], [13].

The flexibility and scalability of GST have led to its increasing application in environmental engineering. Studies have used GST to analyze water quality [14], assess environmental impacts [15], or uncertain environments [16]. Among its techniques, grey clustering has gained attention for its capacity to classify environmental conditions using minimal but representative datasets [17]. These models assign qualitative categories (for example, "Good", "Unhealthy") to pollution levels by mapping quantitative indicators through triangular membership functions, allowing intuitive yet rigorous environmental assessments [18].

In recent years, the integration of GST with entropy weighting, fuzzy logic, and multi-criteria decision making (MCDM) has further expanded its utility, particularly in complex scenarios with multiple and overlapping pollution sources [19], [20], [21]. Such hybrid models offer enhanced classification accuracy and robustness, particularly in urban areas where pollution is influenced by dynamic factors such as traffic, industry, and meteorology [22].

Given these advantages, this study develops and applies an algorithm based on the Central Triangular Whitenization Weight Function (CTWF), a specific grey clustering model, to classify historical air quality conditions in Northern Lima over an eight-semesters period (2011 to 2019). Drawing on publicly available concentration data for four primary pollutants (PM_{10}, SO_2, NO_2, and CO), the grey clustering algorithm (CTWF) enables a structured multi-criteria classification aligned with national Environmental Quality Standards (EQS). This methodological framework is particularly valuable in urban settings like Lima, where technical and governance constraints limit the use of conventional monitoring infrastructure.

Therefore, this work pursues two main objectives:

- To develop a grey clustering algorithm (CTWF) to classify urban air quality; and
- To evaluate the performance of the algorithm through a case study in Lima, Peru.

Overall, this study contributes to applied environmental engineering by demonstrating how intelligent clustering tools like CTWF can support monitoring, analysis, and decision-making in urban environments where conventional systems fall short. Its methodological simplicity and replicability also position it as a practical tool for use in similar data-constrained urban contexts globally.

This study is organized as follows: Section I provides the introduction. In Section II, the grey clustering algorithm is presented. In Section III, the case study is developed and in Section IV the results are discussed. Finally, in Section V the conclusions are presented.

II. Grey Clustering Algorithm

In this section the grey clustering algorithm for air quality classification, based on CTWF, is developed.

A. Definition

First, to develop the grey clustering algorithm (CTWF), the following components are defined:

- Study object m study objects are defined O_1, O_2, O_3, ..., O_m
- Criteria n criteria are defined C_1, C_2, C_3, ..., C_n
- Grey classes s grey classes are defined S_1, S_2, S_3, ..., S_s

B. Algorithm Structure and Computational Steps

The steps of the algorithm proposed in this work are detailed below [14]:

1) Step 1 – Center-point determination: The center-points of the s grey classes are determined, the sequence of the center-points is: $\mu_1, \mu_2, \mu_3, ..., \mu_s$.

2) Step 2 – Data normalization: The center-points of the grey classes, for each criterion, are normalized using Eq. (1):

$$\lambda_s = \frac{\mu_s}{\bar{y}_s} \tag{1}$$

where, $\bar{y}_s$ is the mean of the center-points of grey classes for each criterion. Then, the sample data (X_{ij}) are normalized using Eq. (2):

$$x_{ij} = \frac{X_{ij}}{\bar{y}_s} \tag{2}$$

3) Step 3 – Membership function (CTWF): Each normalized value is transformed into a triangular membership function centered at a reference class midpoint, as shown in Fig. 1 and Eq. (3) to Eq. (5):

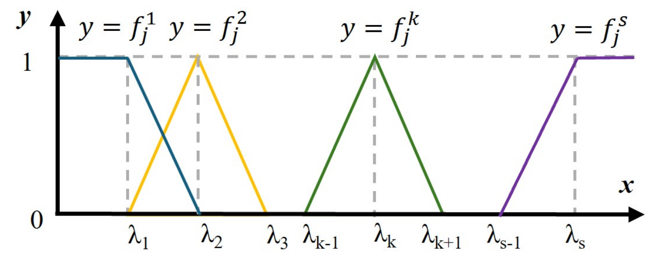

Fig. 1. CTWF graphical representation.

$$f_j^1(x_{ij}) = \begin{cases} 0, & x \notin [0, \lambda_2] \\ 1, & x \in [0, \lambda_1] \\ \dfrac{\lambda_2 - x}{\lambda_2 - \lambda_1}, & x \in [\lambda_1, \lambda_2] \end{cases} \tag{3}$$

$$f_j^k(x_{ij}) = \begin{cases} 0, & x \notin [\lambda_{k-1}, \lambda_{k+1}] \\ \dfrac{x - \lambda_{k-1}}{\lambda_k - \lambda_{k-1}}, & x \in [\lambda_{k-1}, \lambda_k] \\ \dfrac{\lambda_{k+1} - x}{\lambda_{k+1} - \lambda_k}, & x \in [\lambda_k, \lambda_{k+1}] \end{cases} \tag{4}$$

$$f_j^s(x_{ij}) = \begin{cases} 0, & x \notin [\lambda_{s-1}, +\infty) \\ \dfrac{x - \lambda_{s-1}}{\lambda_s - \lambda_{s-1}}, & x \in [\lambda_{s-1}, \lambda_s] \\ 1, & x \in [\lambda_s, +\infty) \end{cases} \tag{5}$$

4) Step 4 – Criteria weight calculation: Relative weights are assigned to each criterion using Eq. (6) [23].

$$\eta_j^k = \frac{\dfrac{1}{\lambda_j^k}}{\sum_{j=1}^{m} \dfrac{1}{\lambda_j^k}} \tag{6}$$

5) Step 5 – Grey clustering coefficient computation: Clustering coefficients are calculated for each study object and grey class, integrating both the CTWF value and the corresponding weight of each criterion by Eq. (7):

$$\sigma_j^k = \sum_{j=1}^{m} f_j^k(x_{ij}) \cdot \eta_j^k \tag{7}$$

6) Step 6 – Final classification: Each study object is assigned to the grey class with the highest clustering coefficient using Eq. (8):

$$\sigma_j^{k^*} = \max_{1 \le k \le s} \left\{ \sigma_j^k \right\} \tag{8}$$

III. Case Study in Lima, Peru

Lima, the capital of Peru, is a coastal mega-city of over ten million inhabitants and is among the most air-polluted cities in South America [24]. Characterized by high vehicular density, poor atmospheric dispersion, and unregulated urban growth, Lima's air pollution is driven largely by emissions from the transportation sector and informal waste burning.

The northern districts of Lima, including Comas, Carabayllo, Los Olivos, San Martín de Porres, and Puente Piedra, exemplify these challenges. These areas combine high residential density with commercial corridors and are under constant urban expansion. Meteorological conditions such as thermal inversion and limited wind flow further exacerbate pollutant accumulation [25]. Urban infrastructure and environmental monitoring are weaker in these districts, which are often socioeconomically marginalized and environmentally underserved [26]. The evaluation of this air quality study was carried out in metropolitan Lima, specifically in northern Lima, in Los Olivos district, as shown in Fig. 2.

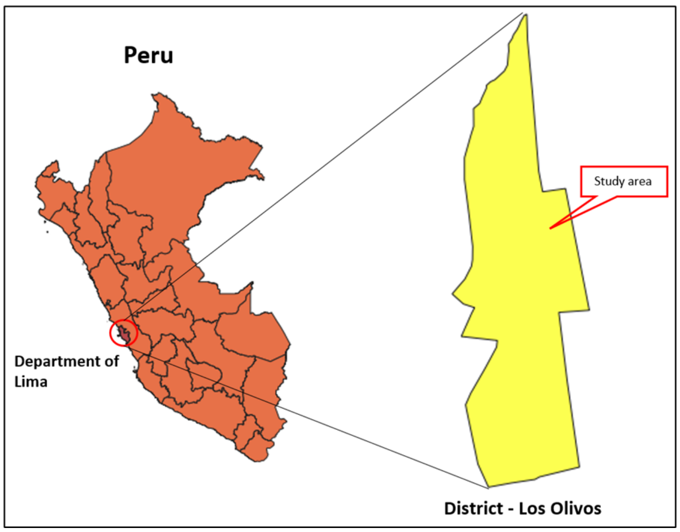

Fig. 2. Location of the study area.

A. Application of the Definition

The components for the case study were defined as follows:

1) Study object: Eight study objects were defined according to the semesters studied from 2001 to 2019 years. The study objects are presented in Table I .

2) Criteria: Four criteria were studied. The definition of these criteria was based on air pollution parameters, considering the following main pollutants: PM_{10}, SO_2, NO_2, and CO. The collected data (in μg/m^3) for each study object are presented in Table I [27].

3) Grey classes: Four grey classes were defined, based on Peru's Environmental Quality Standards (EQS) for PM_{10}, SO_2, NO_2, and CO parameters, as shown in Table II [28].

B. Application of Algorithm steps

The steps of the algorithm proposed in this work were applied to the case study as shown below:

TABLE I. Study Objects, Criteria, and Monitoring Data

Study Object	Semester	PM_{10} (C1)	SO_2 (C2)	NO_2 (C3)	CO (C4)
O1	2011-1	85.6	13.5	3.8	3.1
O2	2012-1	134.2	11.8	3.7	2.6
O3	2013-1	117.8	13.0	156.0	1809
O4	2014-1	143.2	12.15	78.35	600
O5	2015-1	165.4	13.0	45.46	1599
O6	2016-1	100.56	13.0	66.81	600
O7	2018-1	99.12	12.15	8.75	652
O8	2019-1	95.48	13.0	3.33	600

TABLE II. Grey Classes of the Case Study

Air Quality Level	Range (μg/m^3)	Grey Class
PM_{10} (C1)		
Good	0–75	S1
Moderate	76–150	S2
Unhealthy for sensitive groups	151–250	S3
Unhealthy	251–350	S4
SO_2 (C2)		
Good	0–20	S1
Moderate	21–80	S2
Unhealthy for sensitive groups	81–500	S3
Unhealthy	501–920	S4
NO_2 (C3)		
Good	0–100	S1
Moderate	101–200	S2
Unhealthy for sensitive groups	201–300	S3
Unhealthy	301–400	S4
CO (C4)		
Good	0–5000	S1
Moderate	5001–10000	S2
Unhealthy for sensitive groups	10001–15000	S3
Unhealthy	15001–20000	S4

1) Step 1: From Table II, the center-points of the four grey classes, in each criterion, were determined. The results are presented in Table III.

TABLE III. Center-Points of the Grey Classes

Class	C_1 (PM_{10})	C_2 (SO_2)	C_3 (NO_2)	C_4 (CO)
S1	37.5	10.0	50.0	2500.0
S2	113.0	50.5	150.5	7500.5
S3	200.5	290.5	250.5	12500.5
S4	300.5	710.5	350.5	17500.5

2) Step 2: From Table III, the center-points of the grey classes, for each criterion, were normalized using Eq. (1). The results are presented in Table IV.

TABLE IV. Normalized Values for Grey Classes

Criterion	λ_1	λ_2	λ_3	λ_4
C_1	0.230	0.694	1.231	1.845
C_2	0.038	0.190	1.095	2.677
C_3	0.250	0.751	1.250	1.749
C_4	0.250	0.750	1.250	1.750

Then, from Table I , the sample values, for each criterion, were normalized using Eq. (2). The results are presented in Table V.

3) Step 3: Each normalized value was transformed into a triangular membership function centered at a reference class

TABLE V. NORMALIZED VALUES FOR SAMPLE VALUES

Study Object	C_1	C_2	C_3	C_4
O1	0.526	0.051	0.019	0.000
O2	0.824	0.044	0.018	0.000
O3	0.723	0.049	0.779	0.181
O4	0.879	0.046	0.391	0.060
O5	1.016	0.049	0.227	0.160
O6	0.617	0.049	0.333	0.060
O7	0.609	0.046	0.044	0.065
O8	0.586	0.049	0.017	0.060

midpoint by Eq. (3) to Eq. (5). The equations were then synthesized and adapted to the four grey classes, resulting in the following base Eq. (8) to Eq. (11).

$$f_j^1(x_{ij}) = \begin{cases} 0, & x \notin [0, \lambda_2] \\ 1, & x \in [0, \lambda_1] \\ \dfrac{\lambda_2 - x}{\lambda_2 - \lambda_1}, & x \in [\lambda_1, \lambda_2] \end{cases} \tag{8}$$

$$f_j^2(x_{ij}) = \begin{cases} 0, & x \notin [\lambda_1, \lambda_3] \\ \dfrac{x - \lambda_1}{\lambda_2 - \lambda_1}, & x \in [\lambda_1, \lambda_2) \\ \dfrac{\lambda_3 - x}{\lambda_3 - \lambda_2}, & x \in [\lambda_2, \lambda_3] \end{cases} \tag{9}$$

$$f_j^3(x_{ij}) = \begin{cases} 0, & x \notin [\lambda_2, \lambda_4] \\ \dfrac{x - \lambda_2}{\lambda_3 - \lambda_2}, & x \in [\lambda_2, \lambda_3) \\ \dfrac{\lambda_4 - x}{\lambda_4 - \lambda_3}, & x \in [\lambda_3, \lambda_4] \end{cases} \tag{10}$$

$$f_j^4(x_{ij}) = \begin{cases} 0, & x \notin [\lambda_3, +\infty) \\ \dfrac{x - \lambda_3}{\lambda_4 - \lambda_3}, & x \in [\lambda_3, \lambda_4) \\ 1, & x \in [\lambda_4, +\infty) \end{cases} \tag{11}$$

The values from Table IV and Table V were then substituted into Eq. (8) to Eq. (11). As an example, the results of O_1 study object are presented in Table VI.

TABLE VI. CTWF VALUES FOR STUDY OBJECT O_1

Function	C_1	C_2	C_3	C_4
f_j^1	0.362	0.914	1.000	1.000
f_j^2	0.638	0.086	0.000	0.000
f_j^3	0.000	0.000	0.000	0.000
f_j^4	0.000	0.000	0.000	0.000

4) Step 4: From Table IV, the criteria weights were calculated using Eq. (6). The results obtained are presented in Table VII.

TABLE VII. WEIGHT OF EACH CRITERION

Criterion	λ_1	λ_2	λ_3	λ_4
C_1	0.11	0.15	0.24	0.26
C_2	0.68	0.56	0.27	0.18
C_3	0.10	0.14	0.24	0.28
C_4	0.10	0.14	0.24	0.28

5) Step 5: Clustering coefficients were calculated for each study object and grey class using Eq. (7). As an example, the results obtained for O_1 study object are presented in Table VIII.

TABLE VIII. CLUSTERING COEFFICIENTS FOR O_1

Function	C_1	C_2	C_3	C_4	Coeff.
f_j^1	0.362	0.914	1.000	1.000	0.861
f_j^2	0.638	0.086	0.000	0.000	0.144
f_j^3	0.000	0.000	0.000	0.000	0.000
f_j^4	0.000	0.000	0.000	0.000	0.000

6) Step 6: Each study object was assigned to the class with the highest clustering coefficient, using Eq. (8). The results of maximum value of clustering coefficient for each study object are presented in Table IX.

TABLE IX. MAXIMUM VALUES OF THE CLUSTERING COEFFICIENT

Semester	λ_1	λ_2	λ_3	λ_4	Max.
2011-I	0.861	0.144	0.000	0.000	0.861
2012-I	0.853	0.136	0.058	0.000	0.853
2013-I	0.731	0.314	0.026	0.000	0.731
2014-I	0.816	0.167	0.083	0.000	0.816
2015-I	0.831	0.100	0.144	0.000	0.831
2016-I	0.833	0.189	0.000	0.000	0.833
2018-I	0.864	0.152	0.000	0.000	0.864
2019-I	0.857	0.155	0.000	0.000	0.857

IV. RESULTS AND DISCUSSION

The results and discussion, according to specific objectives in this work, are presented below:

A. *Significance of the Grey Clustering Algorithm*

This work contributes meaningful insights, both in theory and in practice, for classifying air quality in uncertain conditions. By applying the grey clustering algorithm (CTWF), which is based on the grey systems theory, it offers a flexible and adaptable way to classify air quality, especially in places where data are limited, a situation often seen in Latin American cities [24].

In addition, the grey clustering algorithm (CTWF) brings several practical advantages that make it highly applicable for managing air quality, as shown below:

- It delivers consistent and trustworthy results, even when working with small datasets or limited information, which is particularly useful for cities lacking extensive monitoring infrastructure [16].
- Unlike traditional air quality indices that typically reflect only the worst pollutant, the grey clustering algorithm (CTWF) provides a more complete picture by incorporating all measured pollutants into the evaluation [11].
- The use of triangular membership functions produces results that are easier to interpret for both technical experts and policymakers [14].

However, the grey clustering algorithm (CTWF) presents some limitations for considering, as shown below:

- It is focused on classification rather than prediction. Therefore, it needs to be complemented to design for forecasting or proactive environmental planning.
- It relies on predefined categories and thresholds, which can introduce some level of subjectivity into the analysis.
- While this algorithm used a specific method for weighting the importance of criteria, using different weighting schemes could lead to different classification outcomes [29].

Despite these limitations, this algorithm stands out as a robust and adaptable tool for analyzing how air quality changes over space and time. Its simplicity and resilience in contexts of uncertainty or limited information make it a valuable resource for environmental planning in fast-growing urban areas.

B. *Performance of Algorithm in the Case Study*

The grey clustering algorithm (CTWF), was implemented to classify air quality levels in Lima, Peru, over an eight-semester period from 2011 to 2019. The analysis was based on the concentration levels of four primary pollutants: PM_{10}, SO_2, NO_2, and CO. This classification relied on the calculation of grey clustering coefficients for each semester, allowing for an integrated assessment aligned with Peru's national Environmental Quality Standards (EQS).

As shown in Table IX, all semesters analyzed were categorized as "Good", with λ_1 clustering coefficients exceeding the 0.70 threshold. The highest air quality was recorded during the first semester of 2018, with a λ_1 value of 0.864. In contrast, the lowest value within the "Good" range was observed in the first semester of 2013, registering a λ_1 of 0.731. These findings suggest moderate seasonal variation, but an overall trend of compliance with national air quality standards.

Fig. 3 illustrates the temporal evolution of clustering coefficients. A gradual improvement in air quality is evident after 2016, likely reflecting the impact of stricter emissions regulations implemented in Lima's transportation sector. Conversely, the lowest λ_1 scores—ranging from 0.73 to 0.76—were observed during the first three semesters (2011-I to 2013-I), possibly due to the expansion of the vehicle fleet and increased industrial activity during that period [30].

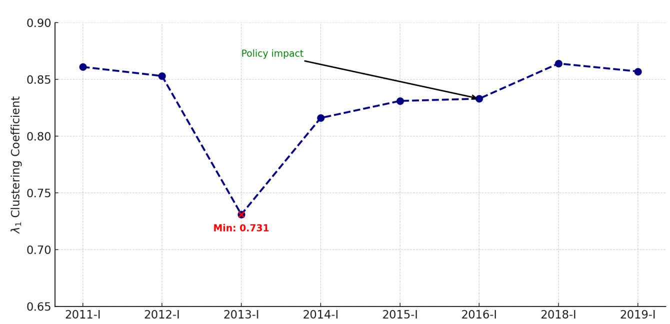

Fig. 3. Temporal evolution of air quality from 2011 to 2019.

To explore the behavior of individual pollutants, Fig. 4 presents a disaggregated analysis derived from the grey clustering results. Among all measured parameters, PM_{10} (C1) consistently recorded the lowest clustering values throughout the study period. In particular, during the first semester of 2013, NO_2 levels dropped below a clustering coefficient of 0.3, indicating poor air quality even though the overall classification remained within the "Good" category. These findings reinforce the characterization of PM_{10} as Lima's most persistent pollutant, primarily due to road dust re-suspension and emissions from diesel-powered vehicles [31].

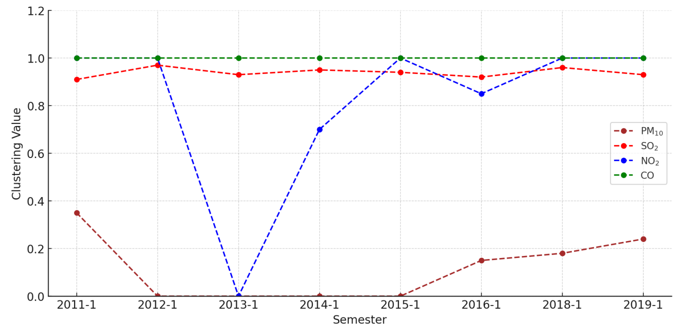

Fig. 4. Disaggregated results trends by pollutant from 2011-1 to 2019-1.

In contrast, CO (C_4) consistently recorded the highest grey clustering values—often above 0.9—indicating minimal health risk. Meanwhile, SO_2 (C_2) and NO_2 (C_3) showed moderate but stable trends. Their lower values during 2012 to 2013 may be linked to increased traffic emissions and reduced atmospheric dispersion capacity during dry seasons [6].

These temporal and pollutant-specific trends confirm the grey clustering algorithm (CTWF) strength in delivering comprehensive multi-criteria air quality assessments, even under conditions of uncertainty and limited information.

V. Conclusion

First, this work demonstrates that the grey clustering algorithm (CTWF) offers a practical and adaptable solution for classifying urban air quality under uncertain and limited information. Its capacity to integrate multiple pollutants highlights its suitability for real-world urban environments with restricted monitoring infrastructure. By enabling structured and interpretable assessments, the algorithm provides a reliable alternative to traditional models, contributing to more informed and inclusive environmental management strategies.

As a second key finding, the grey clustering algorithm not only enabled classification but also uncovered distinct pollutant-specific trends that would be masked by aggregated indices. While all periods met national "Good" air quality standards, disaggregated analysis revealed PM_{10} as the most recurrently critical pollutant, particularly in high-traffic and industrial areas. CO maintained consistently low risk levels, while SO_2 and NO_2 showed moderate but stable variability. Notably, a gradual improvement in air quality after 2016 suggests early effects of emissions control measures, although inconsistent enforcement likely limits their full potential. These insights underscore the value of multi-indicator, temporally sensitive approaches in supporting more nuanced and targeted environmental policy.

Finally, future research could expand the grey clustering algorithm (CTWF) by integrating meteorological variables such as wind speed, temperature, and humidity to better contextualize seasonal pollution trends. The use of predictive grey models like GM(1,1) may strengthen early warning capabilities, while incorporating pollutants such as PM2.5 and ozone would improve health relevance. Combining CTWF with satellite-derived data could further enhance its spatial resolution and support its application in regional planning. As urban centers continue to grow under constrained conditions, grey modeling offers a promising pathway for more resilient and inclusive environmental governance.

REFERENCES

[1] World Health Organization, *Ambient (outdoor) air pollution*. [Online]. Available: https://www.who.int/news-room/fact-sheets/detail/ambient-(outdoor)-air-quality-and-health. Accessed: May 21, 2025.

[2] Y. Wang *et al.*, "The Effect of Long-Term Particulate Matter Exposure on Respiratory Mortality: Cohort Study in China," *JMIR Public Health Surveill*, vol. 10, p. e56059, Sep. 2024, doi: 10.2196/56059.

[3] K. Newell, C. Kartsonaki, K. B. H. Lam, and O. P. Kurmi, "Cardiorespiratory health effects of particulate ambient air pollution exposure in low-income and middle-income countries: a systematic review and meta-analysis," *Lancet Planet Health*, vol. 1, pp. e360–e367, 2017, doi: 10.1016/S2542-5196(17)30166-3.

[4] A. Manukova, K. Krastev, and I. Stoev, "Concentration for monitoring of air pollutants of domestic and industrial nature," in *Proc. 8th Int. Conf. Energy Efficiency and Agricultural Engineering (EE&AE)*, 2022, pp. 1–4, doi: 10.1109/EEAE53789.2022.9831404.

[5] I. C. Fernández, T. Koplow-Villavicencio, and C. Montoya-Tangarife, "Urban environmental inequalities in Latin America: A scoping review," *World Development Sustainability*, vol. 2, Jun. 2023, doi: 10.1016/j.wds.2023.100055.

[6] Y. Romero, N. Chicchon, F. Duarte, J. Noel, C. Ratti, and M. Nyhan, "Quantifying and spatial disaggregation of air pollution emissions from ground transportation in a developing country context: Case study for the Lima Metropolitan Area in Peru," *Science of the Total Environment*, vol. 698, Jan. 2020, doi: 10.1016/j.scitotenv.2019.134313.

[7] N. S. La Colla, S. E. Botté, and J. E. Marcovecchio, "Atmospheric particulate pollution in South American megacities," *Environmental Reviews*, vol. 29, pp. 415–429, 2021, doi: 10.1139/er-2020-0105.

[8] J. E. Johnston *et al.*, "Air quality and wheeze symptoms in a rural children's cohort near a drying saline lake," *Environmental Research*, vol. 263, Dec. 2024, doi: 10.1016/j.envres.2024.120070.

[9] A. A. Romero-Baylón, J. C. Mayorga-Rojas, J. C. Del Valle, W. J. Diaz-Cartagena, J. H. Ccatamayo-Barrios, and L. M. Soto-Juscamayta, "Spatial Localization of Air Pollutants in Lima: Air Quality Monitoring in the Troposphere," *International Journal of Environmental Impacts*, vol. 7, pp. 485–493, Sep. 2024, doi: 10.18280/ijei.070310.

[10] M. Hu and W. Liu, "Grey system theory in sustainable development research—a literature review (2011–2021)," *Grey Systems: Theory and Application*, vol. 12, no. 4, pp. 523–543, Nov. 2022, doi: 10.1108/GS-05-2022-0049.

[11] D. Julong and Y. Lin, "Introduction to grey systems theory," *Understanding Complex Systems*, vol. 68, pp. 1–24, 1988, doi: 10.1007/978-3-642-16158-2_1.

[12] N. Gouveia *et al.*, "Short-term associations between fine particulate air pollution and cardiovascular and respiratory mortality in 337 cities in Latin America," *Science of the Total Environment*, vol. 920, Apr. 2024, doi: 10.1016/j.scitotenv.2024.171073.

[13] N. Gouveia, A. D. Slovic, C. M. Kanai, and L. Soriano, "Air Pollution and Environmental Justice in Latin America: Where Are We and How Can We Move Forward?," Springer Science and Business Media Deutschland GmbH, Jun. 2022, doi: 10.1007/s40572-022-00341-z.

[14] A. Delgado, A. Aguirre, E. Palomino, and G. Salazar, "Applying triangular whitenization weight functions to assess water quality of main affluents of Rimac river," in *Proceedings of the 2017 Electronic Congress, E-CON UNI 2017*, 2018. doi: 10.1109/ECON.2017.8247308.

[15] A. Delgado and I. Romero, "Environmental conflict analysis on a hydrocarbon exploration project using the Shannon entropy," in *Proceedings of the 2017 Electronic Congress, E-CON UNI 2017*, 2018. doi: 10.1109/ECON.2017.8247309.

[16] A. Domenteanu, G. A. Crișan, C. Frăsineanu, and C. Delcea, "Exploring Grey Systems in Uncertain Environments: A Bibliometric Analysis of Global Contributions and Research Themes," *Sustainability (Switzerland)*, vol. 17, Mar. 2025, doi: 10.3390/su17062764.

[17] M. Gerus-Gościewska and D. Gościewski, "Grey systems theory as an effective method for analyzing scarce, incomplete and uncertain data on the example of a survey of public perceptions of safety in urban spaces," *Land*, vol. 10, no. 1, pp. 1–13, Jan. 2021, doi: 10.3390/land10010073.

[18] L. Zuo, S. Wang, Y. Sun, S. Cui, J. Fang, X. Ai, B. Li, C. Hao, and J. Wen, "Robustness assessment of wind power generation considering rigorous security constraints for power system: A hybrid RLO-IGDT approach," *CSEE J. Power Energy Syst.*, vol. 10, no. 2, pp. 518–529, 2024, doi: 10.17775/CSEEJPES.2023.05980.

[19] B. A. Alex Tume-Bruce, A. Delgado, and E. L. Huamaní, "Implementation of a Web System for the Improvement in Sales and in the Application of Digital Marketing in the Company Selcom," *Int. J. Recent Innov. Trends Comput. Commun.*, vol. 10, no. 5, pp. 48–59, May 2022, doi: 10.17762/IJRITCC.V10I5.5553.

[20] H. Lin, T. Pan, and S. Chen, "Comprehensive evaluation of urban air quality using the relative entropy theory and improved TOPSIS method," *Air Qual. Atmos. Health*, vol. 14, pp. 251–258, Feb. 2021, doi: 10.1007/s11869-020-00930-7.

[21] Q. Li, M. Ji, Y. Sun, X. Zhai, and Y. Zheng, "Analysis and evaluation of air quality in Shandong province based on AHP," in *IOP Conf. Ser.: Earth Environ. Sci.*, IOP Publishing Ltd, vol. 651, no. 4, Feb. 2021, Art. no. 042033, doi: 10.1088/1755-1315/651/4/042033.

[22] S. A. H. Morales, L. Andrade-Arenas, A. Delgado, and E. L. Huamaní, "Augmented Reality: Prototype for the Teaching-Learning Process in Peru," *Int. J. Adv. Comput. Sci. Appl.*, vol. 13, pp. 806–815, 2022, doi: 10.14569/IJACSA.2022.0130194.

[23] A. Delgado, "Citizen criminality assessment in Lima city using the grey clustering method," in *Proc. 2017 IEEE XXIV Int. Conf. Electron., Electr. Eng. Comput. (INTERCON)*, Lima, Peru, 2017, pp. 1–4, doi: 10.1109/INTERCON.2017.8079662.

[24] J. A. Espinoza-Guillen, M. B. Alderete-Malpartida, F. D. Roncal-Romero, and J. C. Vilcanqui-Sarmiento, "Identification of particulate matter (PM10 and PM2.5) sources using bivariate polar plots and k-means clustering in a South American megacity: Metropolitan Area of Lima-Callao, Peru," *Environ. Monit. Assess.*, vol. 197, Mar. 2025, doi: 10.1007/s10661-025-13696-1.

[25] L. D. L. C. Rojas *et al.*, "Temporal Trend of PM10 and the Associated Risk to Human Health in the Lima Metropolitan Area," *Int. J. Sustain. Dev. Plan.*, vol. 20, pp. 1327–1335, Mar. 2025, doi: 10.18280/ijsdp.200338.

[26] J. J. Soria, A. O. Cardenas, and L. S. Peña, "Linear Regression with PM2.5 and PM10 Concentration for Air Quality in East Lima, Peru," in *Lecture Notes in Networks and Systems*, Springer Science and Business Media Deutschland GmbH, 2024, pp. 519–533. doi: 10.1007/978-3-031-70518-2_45.

[27] Organismo de Evaluación y Fiscalización Ambiental (OEFA), "Informe de Monitoreo," Lima, Peru, May 2024.

[28] MINAM, "Estándares de Calidad Ambiental (ECA) para Aire y establecen Disposiciones Complementarias," *El Peruano*, pp. 6–9, 2017. [Online]. Available: https://www.minam.gob.pe/wp-content/uploads/2017/04/Proyecto-de-DS-ECA-AIRE.pdf

[29] X. X. Xu, H. L. Gong, and X. Q. Ding, "Investigation and comparison of inertia weight control schemes in particle swarm optimization," in *Proc. 15th Int. Conf. Adv. Comput. Intell. (ICACI)*, 2023, pp. 1–8, doi: 10.1109/ICACI58115.2023.10146183.

[30] N. S. La Colla, S. E. Botté, and J. E. Marcovecchio, "Atmospheric particulate pollution in South American megacities," *Environ. Rev.*, vol. 29, pp. 415–429, 2021, doi: 10.1139/er-2020-0105.

[31] M. Bravo-Orellana, C. A. Bravo-Orellana, Á. R. Velázquez Fernández, and J. R. Pérez-Núñez, "Influence between chemical air pollution and cognitive abilities of school-age children in Metropolitan Lima," *Bordón. Revista de Pedagogía*, vol. 76, pp. 29–54, Dec. 2024, doi: 10.13042/Bordon.2024.102421.