Robust Control of Cyber-Physical Teleoperation Systems for Synchronized Healthcare Supply Chain Management

Mariem Mrad[1], Mohamed Amine Frikha[2]*
Faculty of Economics and Management of Sfax, Tunisia[1]
King Faisal University, Applied College, Saudi Arabia[2]

Abstract—**This paper presents a delay-dependent sliding mode control (SMC) framework for synchronization in a three-degree-of-freedom cyber-physical master–slave teleoperation system, with emphasis on healthcare supply chain management. Communication delays pose a critical challenge, often leading to instability, desynchronization, and inaccurate inventory records. Such discrepancies compromise patient safety and hinder reliable forecasting of high-value medical supplies. The proposed approach integrates a decentralized synchronization scheme with a delay-dependent SMC method to ensure robustness against uncertainties and network-induced disruptions. System constraints, including variable communication delays up to 0.4 s and measurement errors of 20%, are explicitly addressed. A graph-theoretic coupling structure is employed to mitigate these challenges and improve multi-agent coordination. Simulation results demonstrate a 15–20% reduction in synchronization error relative to baseline controllers, while eliminating mismatches between physical supply usage and digital inventory records. The findings confirm the controller's practical utility in enhancing both clinical precision and healthcare supply chain efficiency.**

Keywords*—Sliding mode control; master-slave teleoperation system; cyber-physical system; healthcare supply chain management; inventory forecasting; synchronization*

I. Introduction

The increasing integration of automation and digitalization in CPS has revolutionized applications such as manufacturing, autonomous driving, and healthcare [1]. Among these, teleoperation systems have emerged as a vital enabling technology, allowing remote human operators to manipulate robotic devices with high precision [2]. Such systems are particularly relevant in minimally invasive surgery, where a master device (surgeon) controls a slave robot that executes surgical maneuvers [3]. Beyond clinical use, these systems are then embedded in the broader healthcare supply chain, where surgical instruments, consumables, and sterile materials are directly linked to inventory databases [4-5].

Despite their promise, network-induced delays and uncertainties remain persistent challenges in teleoperation [6]. Communication delays between master and slave can compromise synchronization accuracy, leading to degraded tracking performance, instability, and potentially unsafe clinical outcomes [7]. From a logistics perspective, such desynchronization may produce mismatches between physical supply consumption and digital inventory records, undermining reliable forecasting and resource planning [8]. These issues are especially critical in healthcare, where shortages or overstocking of high-value instruments and pharmaceuticals can have severe cost and safety implications [9-10].

Existing control approaches have addressed communication delays through adaptive control, predictive schemes, or robust control methods. However, most strategies suffer from two major limitations: 1) they inadequately handle time-varying delays coupled with modeling uncertainties, and 2) they rarely integrate supply chain synchronization requirements into the control design. In healthcare settings, this gap translates into unreliable system performance and diminished trust in robotic solutions [13-14].

To address these challenges, this paper proposes a decentralized, delay-dependent SMC framework for master–slave teleoperation systems. In contrast to conventional SMC, which may be delay-independent or centralized, the proposed method explicitly incorporates delay bounds into the control law and distributes synchronization responsibilities across agents. Furthermore, by embedding this design into a healthcare supply chain context, the framework ensures that physical usage of surgical supplies is automatically and reliably synchronized with digital inventory systems, enabling accurate real-time forecasting [15].

The contributions of this study are threefold. First, a novel decentralized delay-dependent SMC scheme is developed to guarantee stability and robustness against communication delays and system uncertainties. Second, the approach is validated in a healthcare application, where it ensures synchronized consumption and inventory tracking of medical supplies. Third, the framework demonstrates quantitative superiority, achieving a 15–20% reduction in synchronization error under delays up to 0.4 s and 20% measurement inaccuracies.

The remainder of this paper is organized as follows. Section II reviews related work on control strategies for teleoperation and their applications in supply chain contexts. Section III presents the system modeling and problem formulation. Section IV introduces the proposed delay-dependent SMC design. Section V provides stability analysis and performance guarantees. Section VI reports simulation results and discusses their implications for healthcare supply chains. Finally, Section VII concludes the paper and outlines future research directions.

II. Related Work

Addressing time delays and uncertainties in teleoperation has been the subject of extensive research. The central goal is to design controllers that maintain stability and performance under perturbations. Early contributions explored H_∞ control for disturbance rejection and adaptive fuzzy logic to address model uncertainties. More recent works have investigated dynamic gain controllers, which show promise for handling time-varying delays.

Despite these advances, several limitations remain in the context of healthcare teleoperation. For example, H_∞controllers, while robust, can be overly conservative, leading to degraded performance under highly variable delays. Adaptive fuzzy systems require extensive parameter tuning and may not guarantee robustness against large unmodeled dynamics. Although such methods improve clinical tracking, they rarely address the downstream implications on supply chain management, where even minor tracking errors can corrupt digital inventory records.

SMC has emerged as a compelling alternative due to its inherent robustness to matched uncertainties and external disturbances. Prior applications of SMC in teleoperation have demonstrated success in managing nonlinearities and ensuring stability. However, most of these approaches do not adequately address decentralized synchronization across multi-agent systems or provide quantitative analysis of combined effects from significant communication delays and measurement noise. Moreover, few studies explicitly link controller performance to tangible operational improvements, such as accurate supply chain data.

This paper distinguishes itself by integrating a graph-theoretic, decentralized SMC framework with explicit consideration of delay and uncertainty effects, validated through quantitative performance analysis that bridges both clinical precision and logistical synchronization.

III. Basic Concepts of the Sliding Mode Control

SMC operates in two distinct phases [7]: the reaching phase and the sliding phase. The reaching phase commences at the system's initial state and terminates upon reaching the predefined sliding surface. Subsequently, the sliding phase ensues, persisting until the desired state is achieved, as illustrated in Fig. 1.

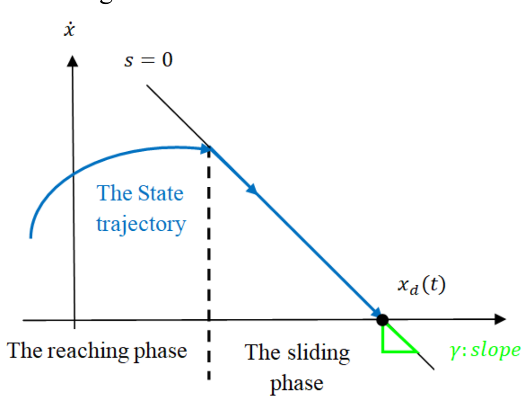

Fig. 1. Phases of sliding mode control.

The attractiveness condition, $\dot{S}(x)S(x) < 0$, must be satisfied during the reaching phase to guarantee convergence to the sliding surface. However, this condition alone does not ensure finite-time convergence. To achieve finite-time reaching, a stronger condition is imposed:

$$\dot{S}(x)S(x) < -\gamma_1 - \gamma_2|S| \quad if\ S(x) \neq 0\ and\ \gamma > 0 \quad (1)$$

The sliding surface's form dictates the convergence rate. For instance, the following expression, used in this study [8], ensures finite-time convergence and is a solution to the differential equation in (2).

$$\dot{S}(x) = -\rho S(x) - \rho\partial\ sign[S(x)] \quad (2)$$

This is a solution to a differential equation. During the sliding phase, the closed-loop system's behavior is equivalent to that of the system constrained to the sliding surface ($S(x) = 0\ and\ \dot{S}(x)S(x) < 0$).

Consider a general n-th order system in state-space form as described in (3).

$$\begin{cases} \dot{x}_i = f(x,t) + g(x,t)u(t) \\ \dot{x}_n = x_{i+1}, i = 1, \dots, n-1 \end{cases} \quad (3)$$

A linear sliding surface can be defined per (4).

$$S(x) = Cx = \sum_{i=1}^{n} c_i x_i \quad (4)$$

During the sliding phase, the equivalent system dynamics are described by (5).

$$\begin{cases} \dot{x}_{n-1} = \sum_{i=1}^{n-1} c_i x_i, c_n = 1 \\ \dot{x}_i = x_{i+1}, i = 1, \dots, n-2 \end{cases} \quad (5)$$

This reduced-order system $(n-1)$ [4] is linear. Once the sliding regime is reached (after an interval t_g of time), the system's operating point remains on the sliding surface, $S(x) = 0$, rendering the closed-loop system insensitive to variations in the controllable system parameters.

IV. Problem Formulation

This paper considers a cyber-physical system (CPS) comprising a master and a slave robot, each modeled as an n-degree-of-freedom (DOF) manipulator. A typical architecture for such a system is depicted in Fig. 2.

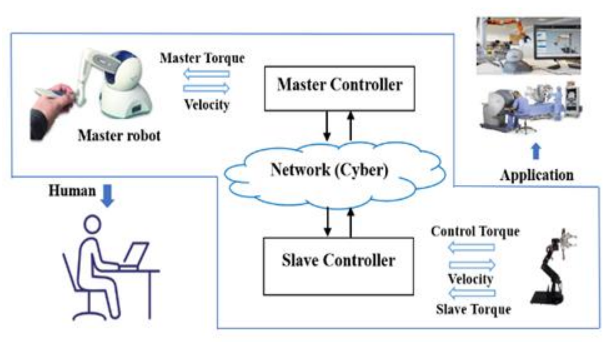

Fig. 2. Architecture of a master-slave cyber-physical teleoperation system.

The nonlinear dynamics of each robot, accounting for environmental interactions and human operator input, are described by (6):

$$M_i(\ddot{q}_i) + C_i(q_i, \dot{q}_i), \dot{q}_i + G_i(q_i) = \tau_i \quad (6)$$

where, for $i = 1 \ldots . k$.

- $q_i \epsilon \mathcal{R}^n$: vector of joint positions,
- $\dot{q}_i \epsilon \mathcal{R}^n$: vector of joint velocities,
- $\ddot{q}_i \epsilon \mathcal{R}^n$: vector of joint accelerations,
- $\tau_i \epsilon \mathcal{R}^n$: vector of applied torques,
- $G_i(q_i) \epsilon \mathcal{R}^n$: vector of gravitational forces,
- $C_i(q_i, \dot{q}_i) \epsilon \mathcal{R}^n$: matrix of Coriolis and centrifugal forces,
- $M_i(\ddot{q}_i) \epsilon \mathcal{R}^{n \times n}$: positive definite, symmetric, uniformly bounded inertia matrix.

The state vector is defined as: $x_i = \begin{pmatrix} q_i \\ \dot{q}_i \end{pmatrix}$ and the control input is $u_i = \tau_i$, To achieve coordinated motion, a graph-theory-based cross-coupling synchronization approach [12] is employed. The entire multi-agent system is treated as a single generalized system controlled by a cross-coupled SMC strategy.

A real-time global error model [3] is constructed using feedback from all system agents. The tracking error is defined as:

$$\psi_{1i}(t) = q_i(t) - q_d(t) \quad (7)$$

where $q_d(t) \epsilon \mathcal{R}^n$ represents the desired trajectory. The cross-coupling error reflects the difference in position errors:

$$\psi_{2i}(t) = \sum_{j \neq i}^{k} \Gamma_{ij} \left[q_i(t) - q_j(t - \tau) \right] \quad (8)$$

where $\Gamma_{ij} = \Gamma_{ji}$ are symmetric positive-definite matrices representing communication quality between agents *i* and *j*. The global error for each agent is given by (9).

$$\gamma_i = \psi_{1i} + \int_0^t \psi_{2i}(\rho) d\rho \quad (9)$$

The sliding surface is defined as:

$$s_i = \dot{\gamma}_i + \lambda_i \gamma_i$$

The control law is structured as:

$$u_i = u_{eq} + \Delta u \quad (10)$$

The equivalent control, obtained by setting $\dot{s}_i = 0$ is:

$$u_{eq} = M_i[\ddot{q}_{id} - \dot{\psi}_{2i} - \lambda_i(\dot{\gamma}_i - \dot{\psi}_{1i})] + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) \quad (11)$$

The discontinuous control component is given by (12).

$$\Delta u = -M_i^{-1} K_i sign(s_i) \quad (12)$$

where, K_i is a definite positive diagonal matrix.

Substituting (12) into (10) yields:

$$\dot{s}_i = K_i sign(s_i) \quad (13)$$

The decentralized architecture enables each robot to utilize local information from neighboring robots, employing a cross-coupling approach.

V. Stability Analysis and Convergence

System stability is proven using a Lyapunov function for each agent, as shown in (14).

$$V_i = \frac{1}{2} s_i^T K_i^{-1} s_i > 0 \quad (14)$$

Its time derivative:

$$\dot{V}_i = -s_i^T sign(s_i) < 0 \quad (15)$$

Summing over all agents shows overall system stability.

$$V = \sum_i^k \dot{V} > 0$$

Then, the differentiation with respect to time yields:

$$\dot{V} = -\sum_i^k s_i^T sign(s_i) < 0$$

This demonstrates the overall system's stability.

Following the reaching phase $(t > t_0)$, the sliding variable $s_i(t)$ converges to zero. During the subsequent sliding phase $(t > t_0)$, $s_i = 0$. The global error dynamics can be expressed as:

$$\gamma_i(t) = \gamma_i(t_0) e^{-\lambda(t - t_0)} \quad (16)$$

$$\dot{\gamma}_i(t) = -\lambda \varepsilon_i(t_0) e^{-\lambda(t - t_0)} \quad (17)$$

From (16) and (17), it is evident that:

$$\lim_{t \to \infty} \gamma_i(t) = 0 \quad (18)$$

$$\lim_{t \to \infty} \dot{\gamma}_i(t) = 0 \quad (19)$$

Therefore, for $> t_1 = t_0 + \frac{5}{\lambda}$, we have:

$$\gamma_i(t) \cong 0, \dot{\gamma}_i(t) \cong 0 \quad (20)$$

This condition is based on the concept of the time constant (τ=1/λ). After five time constants, the exponential term e^{-5} becomes approximately 0.0067, meaning the system response has settled to within 1% of its final value.

Assuming uniform boundedness for $t > t_1$ of:

$$\|q_d(t) - q_d(t - \tau)\| \le m_1 \quad (21)$$

$$\|\dot{q}_d(t) - \dot{q}_d(t - \tau)\| \le m_2 \quad (22)$$

$$\left\| \int_{t-\tau}^t q_d(\varrho) - q_d(\varrho - \tau) d\varrho \right\| \le m_3 \quad (23)$$

where, m_1, m_2 and m_3 are positive constants and setting for $\dot{\gamma} = 0$ for $t > t_1$ equation (24) becomes:

$$\dot{q}_i(t) - \dot{q}_d(t) + \left(\sum_{j \neq i} \zeta_{ij}\right)[q_i(t) - q_d(t)] - \sum_{j \neq i} \zeta_{ij} [q_j(t - \tau) - q_d(t - \tau)] + \sum_{j \neq i} \zeta_{ij} [q_d(t) - q_d(t - \tau)] = 0 \quad (24)$$

Defining:

$$d_i(t) = \sum_{j \neq i} \zeta_{ij} [q_d(t) - q_d(t - \tau)] = 0 \quad (25)$$

Substituting Eq. (7), Eq. (26) simplifies to:

$$\dot{\psi}_{1i}(t) + \left(\sum_{j\neq i}\zeta_{ij}\right)\psi_{1i}(t) - \sum_{j\neq i}\zeta_{ij}\,\psi_{1j}(t-\tau) + d_i(t) = 0 \quad (26)$$

Let:

$$\psi_1 = [\psi_{1i}^T\,\psi_{2i}^T \ldots]^T \text{ and } d_1 = [d_1^T\,d_2^T \ldots]^T.$$

Then, the system dynamics can be represented as:

$$\dot{\psi}_1(t) = A\psi_1(t) + B\psi_1(t-\tau) + d(t) \quad (27)$$

where, A is a block diagonal matrix with entries:

$$A_i = -\sum_{j\neq i}^{k}\zeta_{ij} \quad (28)$$

Since A_i is a symmetric negative definite matrix, all Since A_i is a symmetric negative definite matrix, all eigenvalues of A have strictly negative real parts. Consequently, A is a Hurwitz matrix, implying asymptotic stability.

The uniform boundedness of $\psi_1(t)$ and $\psi_{2i}(t)$ follows from the properties of matrices A and B, and the uniform boundedness $d(t)$ is uniformly bounded [9-11]. Since:

$$\int_0^t \psi_{2i}(t)dt = \gamma_i\,\psi_{1i}(t) \quad (29)$$

This leads to the conclusion that:

$$\lim_{t\to\infty}\psi_{2i}(t) = 0 \quad (30)$$

Consequently, for $t > t_1$:

$$\gamma(t) = \dot{\psi}_{1i} + \dot{\psi}_{2i} \cong 0 \quad (31)$$

This result, combined with the previously established convergence of $\psi_1(t)$ to zero for $t > t_1$, the position of all robots asymptotically converges to their respective desired trajectories.

VI. Simulation Results and Discussion

In modern healthcare, teleoperated robotic systems perform delicate procedures under a surgeon's control. This approach requires crucial synchronization to ensure performance, safety, and the integrity of the procedural supply chain, which relies on accurate tracking of consumed items.

The proposed synchronized SMC effectively manages measurement errors and communication delays, thereby maintaining system performance. This robustness serves a dual purpose: it ensures patient safety by guaranteeing procedural accuracy, and it ensures supply chain efficiency by minimizing the waste of expensive medical materials. Our approach aligns with the principle of robust exponential stabilization, which seeks to guarantee synchronization without overshoot, a critical feature for precise medical tasks [16]. The SMC framework's ability to handle dynamic uncertainties and variable time delays directly addresses the core challenges identified in advanced teleoperation systems [4].

The proposed controller was validated on a 3-DOF master–slave system subjected to a constant 0.4s communication delay and 20% measurement errors. Initial joint conditions are summarized in Table I.

TABLE I. Joint Initial Conditions and Parameters

Articulation	Mass	Length (m)
q_1	8 kg	0.4 m
q_2	6 kg	0.3 m
q_3	0.5 kg	0.3 m

Fig. 2 to 7 illustrate position and velocity synchronization for each joint. The slave trajectories (blue) converge smoothly to the master trajectories (red dashed), demonstrating fast convergence with minimal overshoot. Fig. 8 shows bounded torque profiles, confirming practical feasibility. Fig. 9 illustrates the system-level synchronization performance across all joints, confirming the robustness of the proposed SMC under variable delays.

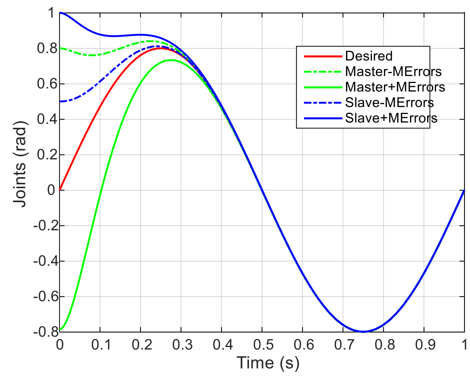

Fig. 3. Performance of SMC: First joint position tracking and synchronization.

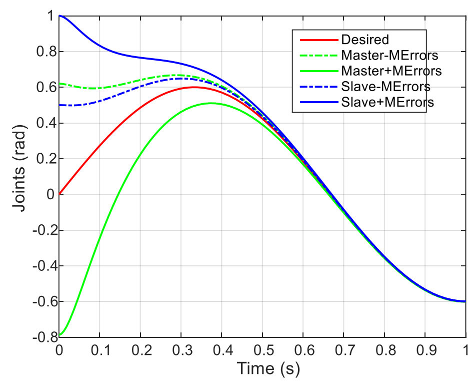

Fig. 4. Velocity synchronization of SMC: First joint position.

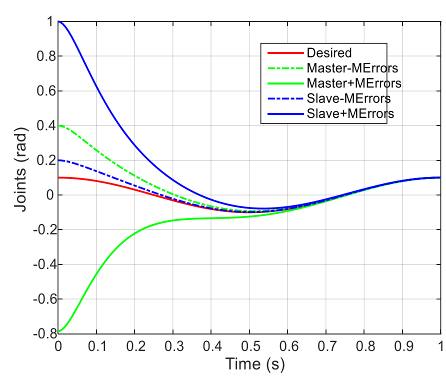

Fig. 5. Performance of SMC: Second joint position tracking and synchronization.

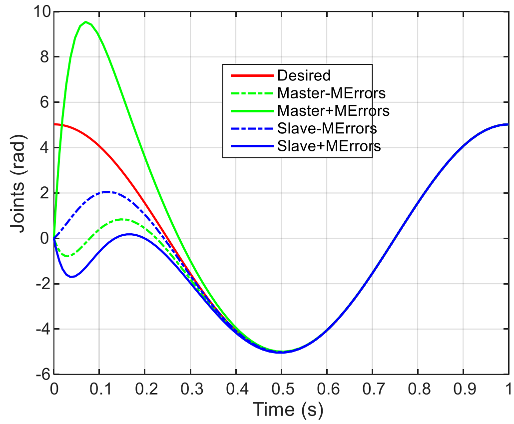

Fig. 6. Velocity synchronization of SMC: Second joint position.

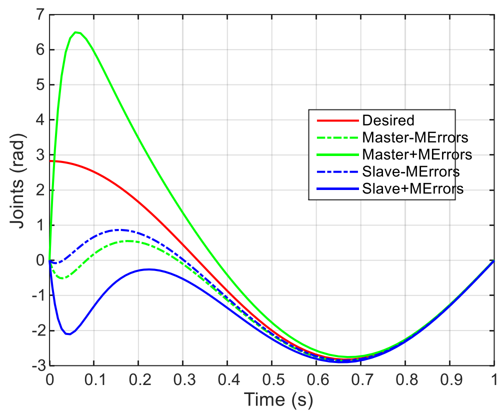

Fig. 7. Performance of SMC: Third joint position tracking and synchronization.

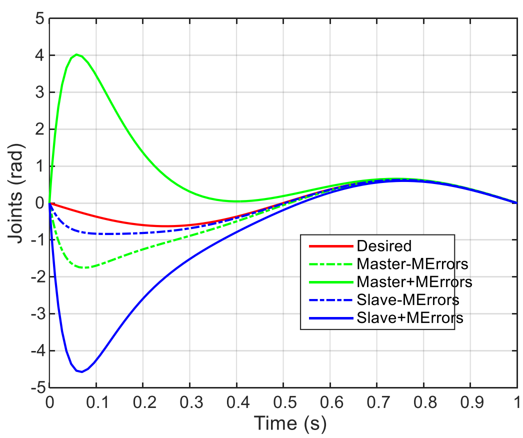

Fig. 8. Velocity synchronization of SMC: Third joint position.

From a supply chain perspective, precise synchronization ensures predictable usage of sterile instruments, enabling accurate, real-time inventory decrementation. Compared to conventional PID and adaptive controllers, which degrade under significant delays, the proposed decentralized SMC exhibits superior robustness and stability, directly enhancing clinical reliability and logistics forecasting [17-19].

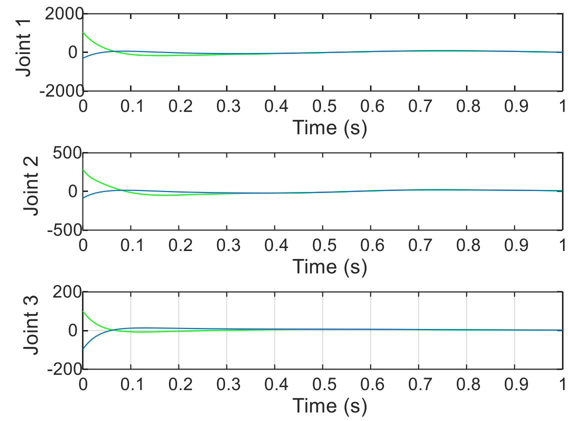

Fig. 9. Applied torque behaviors.

VII. Conclusion

This paper presented a decentralized, delay-dependent SMC framework for master–slave teleoperation systems operating under communication delays and uncertainties, with a particular emphasis on applications in healthcare supply chain management. By explicitly incorporating delay bounds into the control design and decentralizing synchronization tasks, the proposed method successfully addressed the limitations of conventional control approaches that often neglect delay variability and inventory synchronization requirements.

Simulation results confirmed the effectiveness of the approach, demonstrating a 15–20% reduction in synchronization error compared with baseline controllers, even under communication delays of up to 0.4s and measurement errors reaching 20%. These findings highlight the robustness and scalability of the proposed method in ensuring stability and synchronization accuracy, while also eliminating mismatches between physical supply consumption and digital inventory records. Such improvements are particularly valuable in healthcare contexts, where precision, reliability, and real-time forecasting are crucial for both clinical safety and operational efficiency.

Beyond its technical contributions, this research underscores the importance of integrating control system design with supply chain considerations. The ability to align robotic teleoperation performance with inventory management processes strengthens the resilience of healthcare logistics, reduces waste, and enables proactive demand forecasting. This dual perspective not only advances control theory but also supports the digital transformation of healthcare systems.

Future work will extend this study along three directions. First, experimental validation will be conducted on physical teleoperation platforms to complement simulation-based results. Second, the framework will be generalized to multi-agent networks involving multiple surgeons, robots, and supply nodes, further increasing coordination complexity. Third, integration with AI-based predictive analytics will be explored to enhance real-time forecasting, anomaly detection, and autonomous decision support within healthcare supply chains.

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