Relative Position Estimation for Multi-Robots Based on Vertex Distance Between Regular Tetrahedral Units

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***Abstract*—Optimal decentralized cooperative control in multi-robot systems requires simultaneous local sensing and inter-agent communication. Ultra-wide band (UWB) wireless communication has been investigated as a self-contained positioning system capable of supporting both functions within a single device. Conventional UWB-based positioning methods estimate absolute positions using distance measurements relative to fixed anchors in the environment; moreover, while relative position estimation methods based on antenna configurations have been studied, mutual relative position estimation with respect to the reference coordinate frames of the agents themselves has not yet been investigated. To address this gap, this paper proposes a three-dimensional relative position estimation method for distributed cooperative control based on inter-vertex distances sharing. In the proposed system, inter-vertex distances between units are measured, where each unit is equipped with four UWB devices arranged in a regular tetrahedral geometric structure. An optimization-based estimation process is applied and enhanced with a *k*-means clustering method to mitigate convergence to local minima. The estimation accuracy was evaluated through both simulations and real-world experiments. The results demonstrate that the proposed method can accurately estimate relative positions between units and is effective for multi-robot systems operating on planar surfaces.**

***Keywords*—Multi-robot system; distributed cooperative control; relative position estimation; self-contained positioning system; inrter-vertex distance; geometric structure**

I. Introduction

Transportation tasks involving multiple robots, such as picking robots, are increasingly employed in warehouse environments [1]. To control such robot swarms, large-scale management applications—such as swarm coordination frameworks and warehouse management or control systems—are used to supervise individual robots. In these systems, a central server computes control actions based on the current state of the system and distributes appropriate control commands to each robot [2]. This approach, referred to as centralized control [3], allows the central unit to centrally manage environmental perception and motion planning, and the accuracy of the sensor and control timing are crucial for achieving precise robot motion. However, when information is acquired by robots, the accuracy of their motion may degrade, potentially leading to collisions or other risks [4]. To address this issue, a correction has been applied that supplements data from a new sensor [5]. Nevertheless, such approaches become less effective when sensor systems cannot be further extended due to hardware, cost, or scalability constraints. Consequently, there is a growing need for distributed cooperative control strategies that allow robots to directly perceive their mutual states and autonomously avoid collisions without relying on centralized control architectures.

Distributed cooperative control is a control paradigm in which multiple robots operate autonomously while exchanging information with one another [6]. This approach enables highly redundant cooperative behaviors without relying on a central controller [7]. Moreover, it provides robustness against environmental changes, thereby enhancing system flexibility, diversity, and fault tolerance [8]. Ideally, distributed cooperative control requires robots to simultaneously perform local communication and sensing [9]. Thus far, these functionalities have typically been realized through the integration of multiple devices, such as Bluetooth [10] or Wi-Fi [11] for communication, and LiDAR [12], GNSS [13], or RGB-D cameras [14] for sensing. However, such multi-device integration increases onboard system complexity, which may adversely affect control system stability [15] and significantly increase the number of physical system parameters that must be tuned and managed.

A technology that can address these challenges is ultra-wideband (UWB) wireless communication. UWB is a communication system that utilizes ultra-wide frequency bandwidth [16], enabling high-precision positioning and communication [17]. By integrating the local communication and sensing required for distributed cooperative control, UWB wireless communication can simplify robotic systems and improve overall performance. However, in conventional UWB-based systems, the position of a moving node is typically estimated with respect to fixed reference nodes installed in the environment. This reliance on fixed infrastructure limits system scalability and flexibility, particularly in dynamic or infrastructure-free environments. Recently, UWB-based relative positioning systems that estimate relative positions without relying on fixed reference nodes have been developed. These systems estimate the positions of antennas mounted on other units based on distance measured from multiple UWB antennas installed on the unit. Consequently, if an offset exists between the origin of a mobile agent's coordinate frame and the estimated position of its onboard antenna, the agent's position required for distributed cooperative control cannot be utilized with high accuracy.

To address this limitation, it is essential to accurately relate the estimated antenna positions to the agent coordinate frame while preserving the advantages of UWB-based relative positioning. Accordingly, this paper proposes a UWB-based three-dimensional relative positioning method that enables a robot to estimate the arbitrary coordinates of other robots while simultaneously supporting inter-robot communication. The proposed method satisfies both local sensing and communication requirements using only a single type of device. As a result, the proposed approach enhances the robustness and structural simplicity of robotic systems operating under ideal distributed cooperative control.

The main contribution of this study is the development of a method for associating the positioning coordinates provided by a UWB-based relative localization unit with the relative coordinates of agents used in control, thereby enabling high-accuracy sensing in distributed cooperative control.

The remainder of this paper is organized as follows: Section II reviews related work on UWB-based positioning methods. Section III presents a relative position estimation method based on inter-vertex distances. In Sections IV and V, this paper evaluates the performance of the method through simulations and field trials, along with a discussion of the results. Finally, Section VI presents the conclusions of this study.

II. Related Work

In conventional UWB-based positioning systems, as illustrated in Fig. 1a, reference devices that serve as positioning anchors are placed at the four corners of the operational area [18]. This configuration enables the estimation of the position of any device located within the area. Consequently, the absolute coordinates of multiple robots operating in a shared environment can be determined, provided that each robot is equipped with a UWB device. In this localization framework, the position of a target device, denoted by (x, y, z), is calculated under the assumption that each robot or its surrounding environment satisfies the conditions described below.

$$\left.\begin{aligned}(x-x_1)^2+(y-y_1)^2+(z-z_1)^2-r_1^2=0\\(x-x_2)^2+(y-y_2)^2+(z-z_2)^2-r_2^2=0\\(x-x_3)^2+(y-y_3)^2+(z-z_3)^2-r_3^2=0\\(x-x_4)^2+(y-y_4)^2+(z-z_4)^2-r_4^2=0\end{aligned}\right\} \quad (1)$$

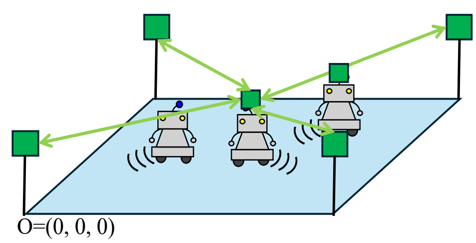
Here, (x_n, y_n, z_n) represent the absolute coordinates of each reference device, and r_n denotes the distance between the target device and the corresponding reference device. Several conventional approaches have been proposed to improve the localization performance. For example, as illustrated in Fig. 1b, adjusting the height of the reference devices can enhance position estimation accuracy [19], while space-division-based methods have been introduced to extend the measurable range [20].

However, as illustrated in Fig. 1c, when relative coordinates between robots are required, such information can only be obtained indirectly through a network or a centralized system. Consequently, acquiring relative positional information between robots, which is essential for distributed cooperative control, remains a significant challenge.

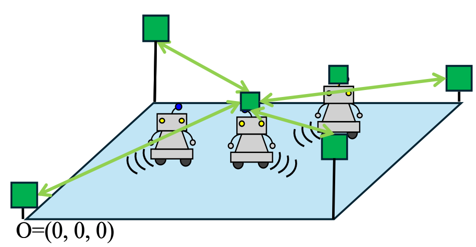
Furthermore, several studies have extended conventional UWB-based localization to relative positioning. As illustrated in Fig. 2, relative positioning methods have been investigated in which the positions of devices mounted on other units are estimated based on multiple devices installed on the unit [21], [22], [23], [24]. These approaches enable peer-to-peer measurement of relative positions between units without relying on anchors installed in the environment.

In such methods, the position of another unit can be obtained either by introducing an additional device that serves as the origin of the coordinate frame [21], [22], [23], or by selecting one of the devices on the unit as a reference when the unit size is sufficiently small [24]. The former approach allows the definition of a reference coordinate frame that is suitable for the robot carrying the unit; however, it increases the number of devices required to compose the unit. In contrast, the latter approach enables relative positioning with a minimal number of devices; however, if the unit is not mounted close to the robot's reference coordinate frame, an offset is introduced into the position estimation.

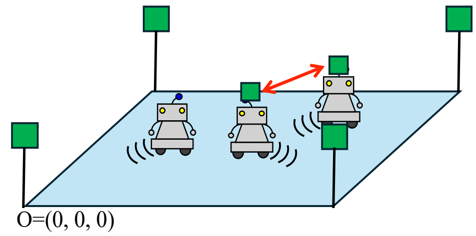
However, existing methods do not adequately address the trade-off between device complexity and agent-based positioning accuracy. Therefore, this study focuses on the capability of UWB communication to support both distance measurement and individual data transmission. By leveraging these features and increasing the amount of measurement data, this paper investigates a method that enables the estimation of arbitrary agent coordinates.



(a) Conventional positioning system.



(b) Improved conventional positioning system.



(c) Unconsidered direct relative positioning.

Fig. 1. UWB positioning system.

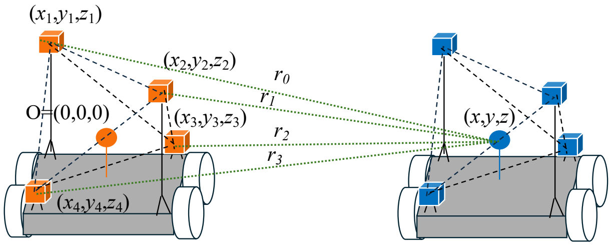


Fig. 2. Systems capable of relative position estimation.

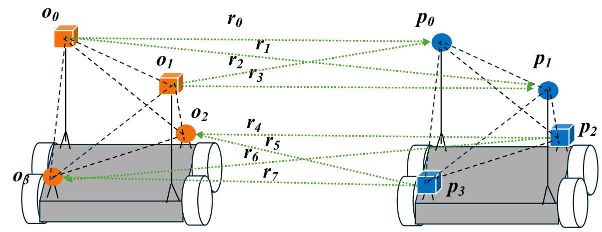


Fig. 3. Three-dimensional relative position estimation system based on vertex-to-vertex ranging.

III. Proposed Method

This paper examines a system composed of multiple units, as illustrated in Fig. 3, designed to measure the relative positions between units. Each unit is equipped with four UWB positioning devices mounted at the vertices of a regular tetrahedron. Among these devices, two function as transmitters and the other two function as receivers.

In UWB wireless communication, distance measurement is performed using receiver devices based on signals transmitted from transmitters. The obtained measurement information is then sent back to the transmitters as a response. Through this mechanism, the two units can determine the inter-device distance r_n shown in Fig. 3. In the figure, the upper devices of the left unit are configured as transmitters, whereas those of the right unit are configured as receivers.

By predefining the coordinate system of each device relative to the origin of the agent's coordinate system, the position of each unit can be estimated based on the acquired distance information and the geometric constraints of a regular tetrahedron, as described below.

$$\left.\begin{array}{l} \left|\|o_i - p_i\| - r_i\right| = 0 \quad ;\forall i \\ \left|\|p_i - p_j\| - r_i\right| = 0 \quad ;i < j \\ \left|\dfrac{1}{6}\det\left(p_1 - p_0, p_2 - p_0, p_3 - p_0\right) - V\right| = 0 \end{array}\right\} \tag{2}$$

where, the position of unit P is estimated relative to that of unit O, and i and j denote the device indices for each unit. As predefined parameters, o_n represents the coordinates of the n-th device, e denotes the edge length of the regular tetrahedron formed by the unit, and V represents the volume of the tetrahedron.

By solving for p_n that satisfies these equations, the relative position can be estimated. Although distance measurements obtained using UWB wireless communication are relatively robust to external disturbances, they still contain measurement errors. Consequently, the geometric constraints cannot always be satisfied exactly, making it difficult to compute a single consistent position estimate. To address this issue, this paper determines the relative position of the other unit by optimizing a solution that minimizes Eq. III.

However, optimization-based position estimation depends on initial values and may converge to local minima as the proposed method estimates a six-degree-of-freedom pose. To address this issue, this paper applies k-means clustering to multiple sets of estimated three-dimensional coordinates obtained from different initializations. Spatially proximate estimates are grouped into k clusters, and the cluster with the lowest variance is selected as the final position estimate. This approach effectively filters dispersed local solutions and guides the estimation toward convergence to the true position.

By applying the k-means clustering method using three-dimensional coordinates as feature values, outliers can be suppressed, resulting in more reliable position estimation. The number of data points provided to the k-means algorithm (i.e., the number of estimation results obtained from different random initializations) is intentionally set to an odd value so that the number of samples in each cluster became uneven. This design helps prevent bias in the clustering process.

IV. Estimated Performance Evaluation through Simulation

A. Simulation Setting

The accuracy of the position estimation obtained through the clustering approach using the k-means method, as described in the proposed method, was evaluated via a simulation. First, the appropriate numbers of clusters and data points were determined. The estimation accuracy was assessed for units formed by the regular tetrahedron, with sizes measuring 0.1, 0.2, 0.5, 0.7, 1, 2, 5, 7, and 10 m. For each unit size, the evaluation was based on an average of 10,000 trials with randomly initialized position estimates.

In this simulation, an ideal environment without distance-measurement errors was used; therefore, the estimation error arose solely from the geometric structure and characteristics of the algorithm. Through the evaluation of the proposed method, the effects of two parameters in the k-means algorithm were analyzed: the number of data points (index) and the number of clusters (cluster). Subsequently, the estimation accuracy was assessed with respect to the distance between units.

The simulation was implemented using Python 3.7.9, with the following libraries: NumPy 1.21.6, SciPy 1.7.3, and PyClustering 0.10.1.2.

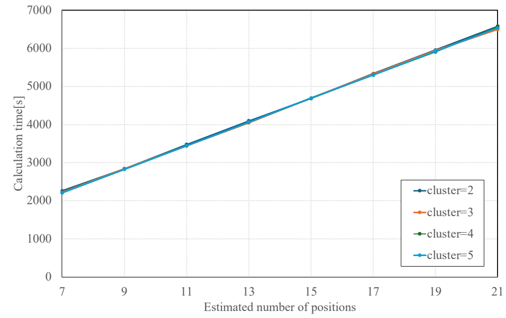
B. Effect of Parameter Settings in the k-means Method on Location Estimation Accuracy

The influence of the k-means parameter settings on the position estimation accuracy was examined. In this evaluation, the unit size was set to 1.0 m, and the distance between the units was 2.0 m. Fig. 4a and 4b show the total computation time and estimation error with respect to the numbers of estimations and clusters, respectively.

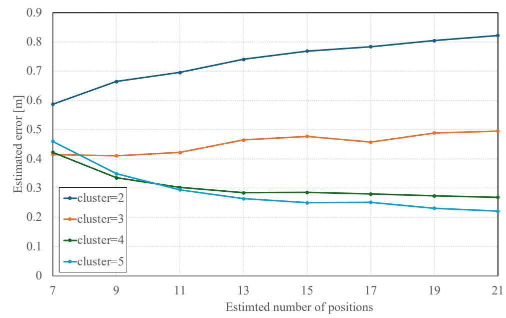
The computation time increased with the number of data points, whereas it slightly decreased with increasing number of clusters. This behavior is attributed to the additional time required to assign data points to clusters and calculate their associated errors as the dataset grows. Conversely, the slight reduction in the computation time with an increasing number of clusters is likely due to the faster convergence of cluster centers achieved using the *k*-means method. Based on these results, it can be concluded that the primary factor affecting the computation time of the *k*-means method is the number of data points.

Next, we focused on the estimation error. Unlike the trend observed in the computation time, the estimation accuracy tended to improve with increasing number of clusters. Furthermore, when examining the effect of the number of data points, different trends emerged depending on the cluster size. With a small number of clusters (two or three), the estimation error increased with increasing number of data points. By contrast, with a larger number of clusters (four or five), the error decreased with increasing number of data points.

This behavior can be attributed to the fact that with a small number of clusters, local solutions may merge with clusters containing the correct estimation results. By contrast, a larger number of clusters allows these solutions to be separated. Consequently, when the number of clusters is large, increasing the number of data points makes it easier to identify clusters containing a majority of estimations that are close to the true coordinates, thereby improving accuracy.



(a) Computation time.



(b) Measurement error.

Fig. 4. Variation in the position estimation accuracy with the number of indices and clusters.

C. Position Estimation Accuracy Relative to Unit Spacing

Based on the results described above, the estimation accuracy relative to the distance between units was evaluated under two conditions: one with a faster computation time (Index: 7, Cluster: 5) and another with a lower estimation error (Index: 21, Cluster: 5). Fig. 5 shows the results, where the horizontal axis represents the inter-unit distance normalized by each unit size, enabling comparisons across the different unit sizes tested.

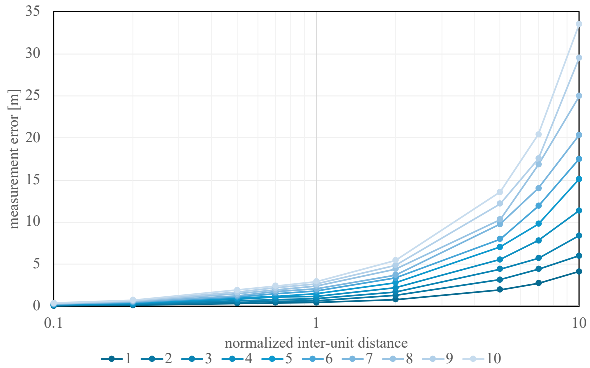
Under both the scenarios, the estimation error tended to increase with increasing unit size. This is because a greater distance between the units results in a larger separation between the correct and local solutions. Although both sets of *k*-means parameters showed an increase in the error with increasing inter-unit distance, the trend was more moderate under the conditions shown in Fig. 5b. This indicates that increasing the number of data points in the *k*-means method enables more reliable estimations over longer ranges. In addition, the results showed a significant accumulation of errors when the inter-unit distance exceeded five times the unit size. Therefore, the proposed system can effectively benefit from the error-reduction effect of the *k*-means method for distances of up to five times the unit size.

D. Evaluation of Position Estimation Accuracy Based on Position Estimation Success Rate

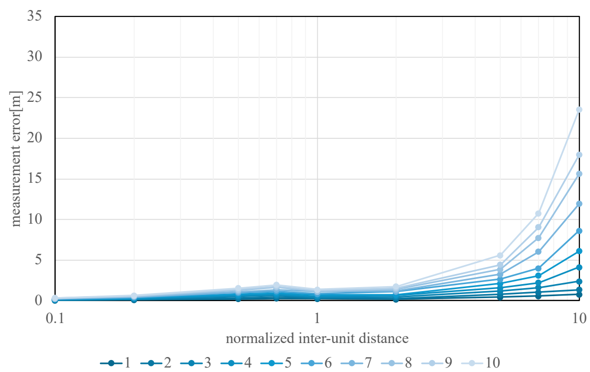
Finally, to assess the effectiveness of combining nonlinear optimization with the *k*-means method in the proposed method, we evaluated the success rate of the position estimation. Table I presents the results. The success rate represents the probability of the estimation error falling below a specified threshold. The evaluation was conducted using a unit size of 1.0 m and an inter-unit distance of 2.0 m.

The results shown in Table I indicate that the proposed method achieved a higher success rate than the approach without the use of the *k*-means method across all threshold values. The success rate was particularly high when a greater number of estimation results was used. These findings demonstrate that clustering with *k*-means significantly improves the estimation accuracy. Although nonlinear optimization in the proposed method often fails to compute the correct position, the use of *k*-means enables the formation of groups of data with similar characteristics, thereby enhancing the overall accuracy.

In Section IV-B, the success rate improves with an increase in the number of data points, indicating that an appropriate selection of both the number of data points and the number of clusters is a crucial design factor. With sufficient data points, the likelihood of a cluster containing the estimated results close to the true coordinates increases, resulting in more stable estimations.



(a) Index: 7, Cluster: 5.



(b) Index: 21, Cluster: 5.

Fig. 5. Evaluation of position estimation accuracy with unit size variation.

TABLE I. Accuracy Evaluation using Location Estimation Success Rate

Threshold [%]	Without k-means	Success rate [%] k-means Index: 7 Cluster: 5	k-means Index: 21 Cluster: 5
0.5	0	68.30	85.67
5	0	68.43	86.50
10	0	69.52	87.61
20	0.31	72.59	88.41
30	1.42	76.36	89.46
40	2.59	78.88	90.28
50	4.72	82.29	91.22

V. Evaluation of Position Estimation Accuracy through Field Experiment

A. Field Experiment Setting

This field experiment aims to validate the practical feasibility and robustness of the proposed relative positioning method under real-world measurement conditions, including ranging noise, device resolution limitations, and orientation-dependent errors. By comparing the experimental results with the simulation-based performance evaluation presented in Section IV, the experiment demonstrates that the proposed method maintains consistent performance trends even in non-ideal environments, thereby highlighting its applicability to real robotic systems.

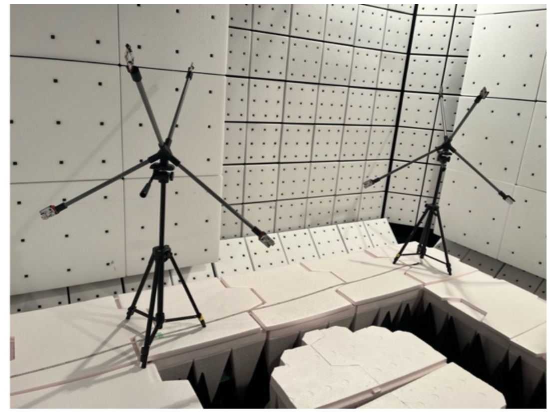


Fig. 6. Experimental setup for relative position estimation in an electromagnetic anechoic chamber.

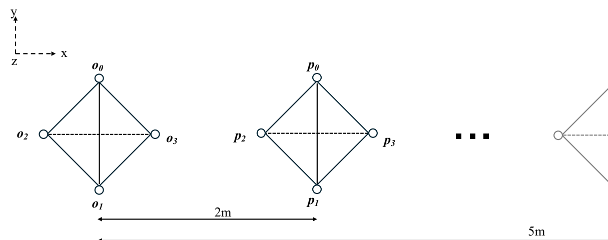


Fig. 7. Experimental overview diagram.

In this experiment, the proposed system was constructed using UWB positioning devices, and the relative position estimation was evaluated in an electromagnetic anechoic chamber, as shown in Fig. 6. The UWB modules used were DWM3001 units (manufactured by Qorvo). For one unit, the two devices on the upper side were configured as transmitters and the two on the lower side were configured as receivers, whereas the opposite configuration was applied to the other unit, with the upper devices set as receivers and the lower devices as transmitters. Two units sizes, 0.64 and 1.02 m, were considered. Using these units, we estimated the relative position on the horizontal plane in terms of the distance and angle.

Fig. 7 shows the coordinate system used in this experiment. The distance between the units was measured along the x-axis, with tests conducted at distances of 2, 3, 4, and 5 m. The relative orientation of the units was evaluated by rotating Unit P counterclockwise from 0° to 0° in 15° increments, where 0° corresponds to the reference orientation, as shown in Fig. 7. Owing to the symmetry of the tetrahedron, measurements up to 180° could capture the characteristics of the entire surrounding direction.

Under the conditions described above, positioning data were obtained using the Qorvo TWR, where each transmitting device measured the distance to the two receiving devices on the other unit. This process provides the eight distance measurements required by the proposed method. Based on these measurements, the relative positions were estimated using the proposed method. For each measurement condition, 100 ranging samples were collected, and the program described in Section IV was applied to compute the estimated coordinates for each sample. The estimation error was calculated by

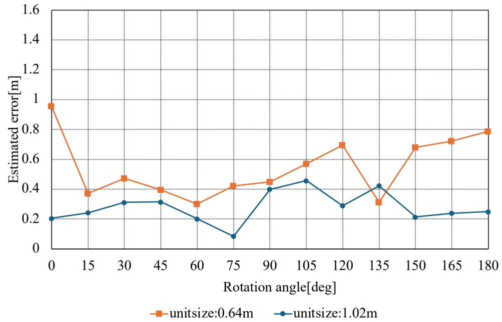
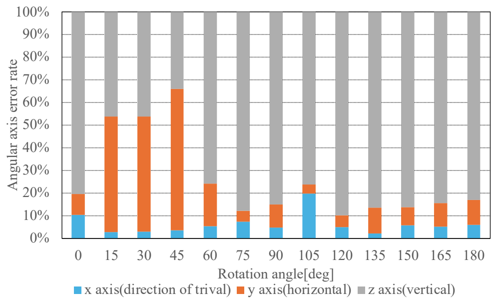
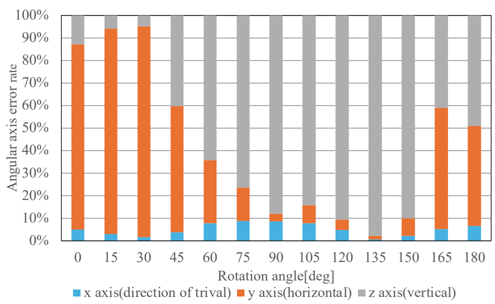


Fig. 8. Estimated distance error.



(a) Unit size: 0.64 m.



(b) Unit size: 1.02 m.

Fig. 9. Error percentage for each axis relative to changes in unit size.

comparing the estimated coordinates with the ground truth.

B. Evaluation Results of Position Estimation Based on Measured Values Relative to Unit Size

For both unit sizes, measurements were performed at a fixed distance of 2 m while varying the orientation angle of the units. Fig. 8 shows the estimation errors between the predicted positions and the ground truth; Fig. 9 shows the contribution of the errors along each axis to the total error. Although the devices used in this experiment had a resolution in the order of centimeters and the measurements included noise, the position was estimated reasonably well for all orientation angles and both unit sizes. Furthermore, focusing on the difference in the unit size, the results were consistent with the analysis described in Section IV-C; at the same relative distance, the smaller unit size produced higher estimation errors.

In addition, examining the proportion of errors along each axis revealed that errors tended to accumulate more significantly on the y- and z-axes than on the x-axis along which the units were arranged. This pattern is likely due to limitations in the device resolution and the influence of ranging noise, which complicates the orientation estimation.

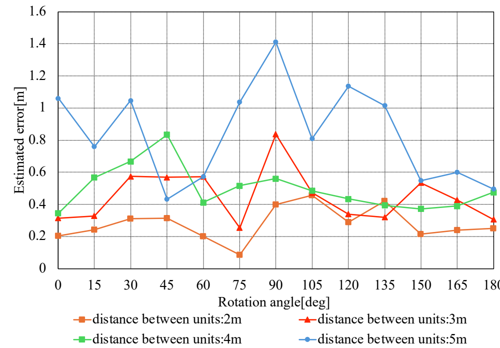


Fig. 10. Estimated error due to distance variation.

These results indicate that stable position estimation can be achieved in real environments using larger unit sizes. Conversely, when a compact unit size is required, the accuracy may be improved using devices with a higher ranging resolution or by employing antenna designs that account for directional characteristics.

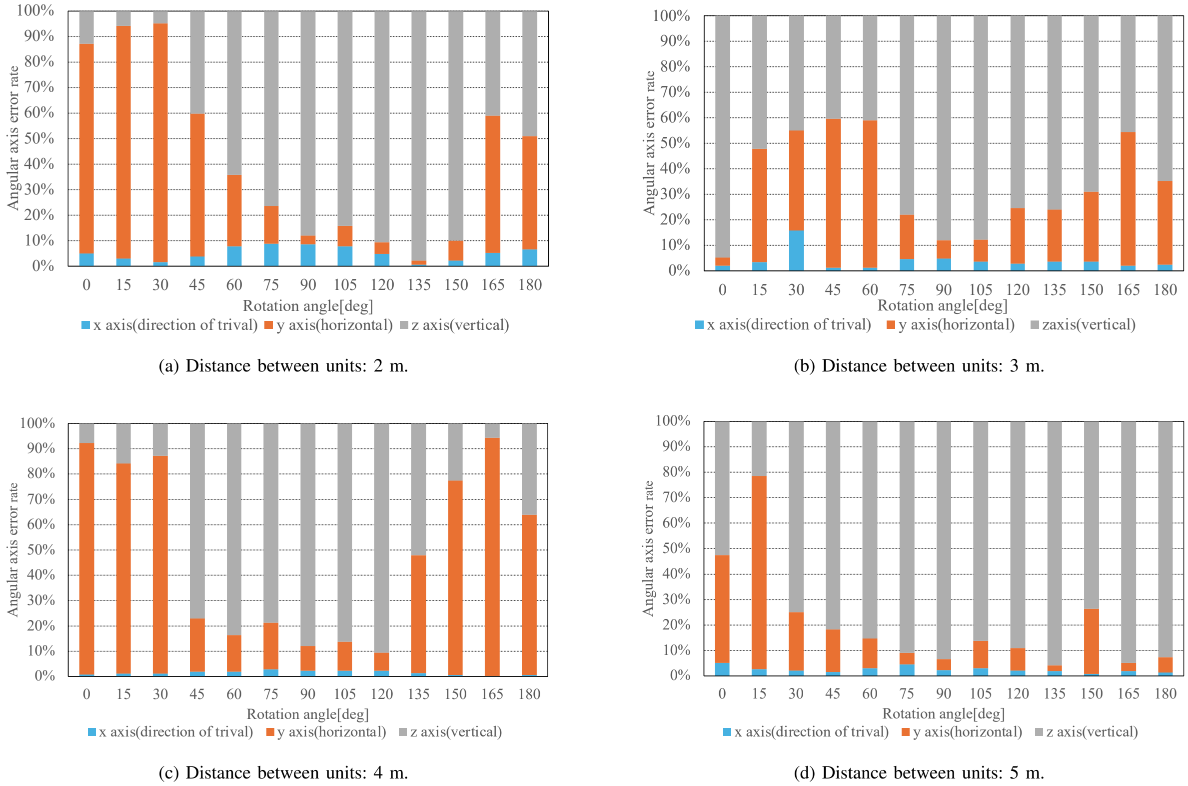
C. Evaluation of Position Estimation based on Actual Measurement Data for Unit Size

Fig. 10 shows the estimation errors for the orientation angle at each inter-unit distance; Fig. 11 shows the error proportions along each axis. Experiments were conducted under a unit size of 1.02 m.

Examination of the distance estimation error revealed that the error increased with increasing inter-unit distance. This trend aligns with the simulation results discussed in Section IV-C. At an inter-unit distance of 2 m, the estimation error varied only slightly with respect to the orientation angle and remained stable. However, as the distance was increased to 4 and 5 m, the variation in the error became more pronounced.

The primary cause of this increased variability is that at certain orientation angles, the accumulation of errors makes it difficult for the optimization process to satisfy the geometric constraints. These resulting inaccuracies lead to changes in the estimated orientations of the units. Moreover, with increasing distance between the units, changes in the orientation angle were less clearly reflected in the ranging measurements, which further contributed to an increase in the estimation error.

Furthermore, as shown in Fig. 11, the distribution of errors along each axis indicates that regardless of the inter-unit distance, errors tended to accumulate more significantly on the y- and z-axes than on the x-axis, along which the units were aligned. This behavior can be attributed to changes in the geometrically dominant axes of rotation caused by the orientation of the unit. As the unit rotates, the axis about which the geometric rotation is the most sensitive shifts, resulting in a periodic change in the direction of error accumulation.



(a) Distance between units: 2 m.

(b) Distance between units: 3 m.

(c) Distance between units: 4 m.

(d) Distance between units: 5 m.

Fig. 11. Percentage of axis error relative to distance change.

D. Limitation of the Proposed Method

The simulation results confirm that the proposed system can estimate the relative positions up to approximately five times the device size. Therefore, this method provides an effective positioning range for applications requiring local sensing and individual communication, such as collective behavior in swarm robotic systems [25]. However, as demonstrated in hardware experiments, when the method is implemented by integrating multiple commercially available devices, the positioning performance is limited by the differences in the ranging schemes of each device, such as the time of arrival or angle of arrival, as well as by their output resolutions. This makes it necessary to consider approaches that integrate all the required functions into a single device or to develop advanced optimization techniques suitable for implementation using heterogeneous composite devices.

VI. Conclusion

This study investigated a three-dimensional relative positioning method using UWB wireless communication as a step toward ideal decentralized cooperative control. We proposed a relative position estimation method based on inter-vertex distances, along with a strategy to improve the estimation accuracy by eliminating local solutions using the k-means clustering algorithm. Simulation results demonstrated that the proposed method achieves accurate positioning at inter-unit distances of up to approximately five times the device size. Furthermore, experiments conducted with actual devices confirmed that the observed performance trends were consistent with the simulation results. These findings suggest that the proposed method is a practical and effective solution for robotic systems (such as swarm robotic systems) that rely on local information.

This study presents a three-dimensional relative position estimation based on a geometrically minimal regular tetrahedral structure. The proposed approach provides a design guideline that simplifies the hardware configuration of robotic systems while preserving redundancy and scalability. It enables relative localization that is inherently aligned with each robot's local coordinate frame, facilitating seamless integration with motion control and cooperative behaviors. These features are particularly advantageous in real-world environments, such as warehouse transportation systems—where robots are frequently added or replaced—as well as in disaster response and infrastructure inspection scenarios, where prior environmental preparation is difficult.

In addition, the quantitative evaluation of the effective localization range relative to unit size provides a practical metric for designing swarm robotic systems based on relative positioning. This study establishes a framework that enables mobile agents to autonomously estimate their relative positions without relying on fixed anchors or centralized management—assumptions traditionally made in UWB-based localization systems—while simultaneously achieving both local sensing and individual communication using a single UWB device configuration.

Future work will address several challenges toward the practical deployment of the proposed three-dimensional relative positioning method in swarm robotic systems. First, the influence of ranging errors and device resolution on positioning accuracy must be considered, as experimental results revealed performance degradation compared with ideal simulation conditions. Appropriate parameter design tailored to the resolution characteristics of UWB devices is therefore required, and error-aware weighting or sensor fusion approaches are expected to mitigate the accumulation of errors along specific axes.

Second, the integration of relative position estimation with orientation estimation remains an important issue. Although the proposed method primarily estimates translational components, experimental results suggest that changes in unit orientation significantly affect ranging measurements and contribute to increased errors, particularly along the y- and z-axes. Extending the method to jointly estimate position and orientation is thus a promising direction.

Finally, real-time implementation is essential for practical swarm robotic applications. While this study focused on offline accuracy evaluation, future work will investigate computational optimization and real-time performance to enable stable cooperative behaviors in real-world environments.

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