Enhanced Ant Colony Optimization for Capacitated Vehicle Routing Problem with Time Windows in Franchise Distribution

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***Abstract*—Efficient routing for distributing goods to multiple franchisee locations requires optimization techniques capable of handling vehicle capacity limits, heterogeneous time windows, and operational constraints, making conventional brute-force or map-based approaches infeasible due to the NP-hard nature of the problem. This study presents an enhanced Ant Colony Optimization (ACO) algorithm for solving the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) in a franchisor–franchisee logistics setting. The proposed enhancement incorporates feasibility filtering to enforce capacity and time-window constraints during route construction and adaptive pheromone updating to improve convergence stability. Using real franchisee coordinates, demand values, and operational time windows, the experiments configured with α = 2, β = 1, ρ = 0.05, and a 150-iteration limit demonstrate that the enhanced ACO achieves a minimum total route distance of 46.90 km with zero variance across 10 simulations, indicating highly stable convergence. Comparative evaluation shows that the enhanced ACO improves route efficiency by 11.4% compared to standard ACO and 15.2% relative to a representative Genetic Algorithm baseline. Implemented in a web-based environment using JavaScript for visualization and Java for computation, the approach provides a practical decision-support tool for Indonesian franchise logistics. The algorithm exhibits an observed computational complexity of $\theta(n^4)$, making it suitable for small to medium-scale distribution networks involving strict delivery time windows.**

***Keywords*—Ant Colony Optimization; CVRPTW; heuristics; distribution routing; logistics optimization**

I. Introduction

The distribution of goods to several franchisee locations is a problem that is often faced by franchisors and distribution is usually carried out in a predetermined time cycle. One way that can be used is to use a map application to determine the distance between locations and implement a brute force algorithm in selecting the order in which franchisees must be visited. This approach is considered inefficient, especially since the distribution of goods is carried out quite often. In the real world [1], there are many other factors that become constraints such as vehicle capacity, number of vehicles, and the time period for receiving goods in each different franchisee must be considered as well. With the complexity of this problem, the brute force method approach is considered very inefficient to use and an application for assistance should be needed to solve the problem with a more optimal algorithm [2]. Based on data from Supply Chain Indonesia in 2020, logistics costs in Indonesia are in the first most expensive position in Asia, which is 24% of Gross Domestic Product (GDP) [3]. With the optimization of the freight route, it is hoped that it will help reduce logistics costs below 24% so that the Cost of Goods Sold (COGS) will be lower in Indonesia [3]. Efficient distribution routing is essential for franchisors managing recurrent delivery cycles across geographically dispersed franchisee locations, and traditional manual planning or brute-force search becomes impractical due to the combinatorial explosion of route possibilities.

This case fits perfectly with the Vehicle Routing Problem (VRP). The Vehicle Routing Problem is a combinatorial optimization problem solving by determining a suitable route for vehicles from the beginning of the depot to several locations and back to the starting point of the depot by serving other locations on demand, and VRP is a generalization of the Traveling Salesman Problem (TSP) [4]. One variation of VRP is the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) which has a vehicle capacity limit and a time limit for each franchisee location [5]. Since CVRPTW is an NP-hard problem, consequently only a small scale of the problem can be solved using appropriate methods [6]. The alternative is to use a heuristic method that reduces the level of optimization of the exact solution, but the problem can be solved in a relatively short time [7]. These constraints become even more critical in real-world franchise distribution, where heterogeneous receiving time windows and limited fleet capacity significantly increase complexity [6].

One approach for solving the CVRPTW problem is the Ant Colony Optimization (ACO) algorithm. Based on a previous study entitled "A Comparative Analysis of Genetic Algorithm and Ant Colony Optimization for Mobile Augmented Reality Offloading", on the optimization using the Ant Colony algorithm, shows that Ant Colony Optimization is able to provide better solution quality and less computational time than Genetic Algorithm [8] [9]. Ant Colony Optimization also has several advantages such as parallel population-based searching, fast discovery of good solutions, and convergence guarantees [10]. Although many studies have applied Ant Colony Optimization to VRP variants, limited research has addressed Indonesian franchise distribution settings that involve strict time-window constraints and require tailored Ant Colony Optimization enhancements [11].

Based on this background, the author is interested in

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conducting research on "Enhanced Ant Colony Optimization for Capacitated Vehicle Routing Problem with Time Windows in Franchise Distribution". The limitations contained in the issues raised are vehicle capacity limits, limited number of vehicles, and different time periods for receiving goods from each franchisee [5].

While numerous ACO variants have been proposed for VRP, this study addresses specific gaps in the existing literature by introducing three key technical contributions:

- Feasibility Filtering Mechanism: Unlike standard ACO which may explore infeasible paths during route construction, the proposed enhancement embeds real-time constraint checking that eliminates candidates violating capacity limits or time-window boundaries before probability computation. This differs from post-hoc penalty approaches [12] by preventing wasted computational effort on infeasible partial solutions.
- Adaptive Pheromone Evaporation: Rather than using fixed evaporation rates, the algorithm dynamically adjusts ρ based on solution improvement status—applying lower evaporation (0.05) during progress phases to preserve beneficial pheromone trails, and higher evaporation (0.10) during stagnation to encourage exploration. This mechanism directly addresses convergence stability and stagnation avoidance concerns raised in recent ACO literature [13].
- Empirical Validation on Indonesian Franchise Data: The algorithm is evaluated using real operational data from an Indonesian franchise distribution network, providing practical insights for logistics optimization in emerging market contexts where such studies remain limited [11].

The study further contributes by conducting comprehensive parameter sensitivity analysis justifying the selection of α, β, and ρ values, performing statistical robustness evaluation with standard deviation and Wilcoxon signed-rank tests across multiple trials, and benchmarking against both classical metaheuristics (standard ACO, Genetic Algorithm) and discussing positioning relative to modern solvers.

II. Related Work

A. Classical and Metaheuristic Approaches for CVRPTW

The authors in [14] present various solutions to solve CVRP problems using three methods, namely Brute-force Search/Exhaustive Search (exact method), Nearest Neighbor Insertion/NNI (heuristic method), and Ant Colony Optimization (meta-heuristic method) [7], with results showing that Ant Colony Optimization performs better than the other two approaches [2]. Additionally, [15] proposes an adaptive hybrid ant colony optimization algorithm that incorporates a genetic algorithm to solve the Capacitated Vehicle Routing Problem (CVRP), demonstrating effective performance through experimental evaluation. Research on VRP and CVRPTW commonly utilizes heuristics and metaheuristics, including brute-force search, nearest neighbor heuristics, Genetic Algorithms (GA), and Ant Colony Optimization (ACO), and studies such as [14], [12], [16], and [15] consistently show that Ant Colony Optimization often provides superior solution quality for complex routing problems.

The authors in [12] further explain how to optimally distribute Liquified Natural Gas (LNG) to several regasification terminals in Papua as fuel for Gas Power Plants (PLTG) using Ant Colony Optimization. Moreover, the authors in [16] examine Ant Colony Optimization enhanced with an additional proposed method, with mathematical results indicating that the proposed solution is highly promising and outperforms other algorithms used for comparison.

B. Hybrid Metaheuristic Approaches

Recent research has demonstrated that hybridizing ACO with other metaheuristics can significantly improve solution quality. Kao et al. [13] proposed a hybrid algorithm combining ACO and Particle Swarm Optimization (PSO) for CVRP, where each artificial ant memorizes its personal best solution similar to particles in PSO, and a pheromone disturbance method is embedded to overcome stagnation. Their computational results showed that the hybrid ACO-PSO performs competitively against existing swarm intelligence approaches.

Comert and Yazgan [17] developed a hierarchical approach integrating Hybrid ACO (HACO) with Artificial Bee Colony Algorithm (ABCA) for multi-objective electric vehicle routing problems. The first phase uses HACO with local search and simulated annealing for initial solution generation, followed by ABCA refinement. This demonstrates the effectiveness of combining constructive and improvement heuristics in routing optimization.

For larger-scale instances, Adaptive Large Neighborhood Search (ALNS) has gained prominence. Fan [18] proposed a Hybrid ALNS (HALNS) that synergistically integrates ACO's broad-based solution initialization with ALNS's localized search potency for time-dependent open electric vehicle routing problems. The strategic blend leverages ACO as a foundational layer for ALNS's deeper refinements.

C. Learning-Based Routing Approaches

Machine learning approaches have emerged as promising alternatives for solving VRP variants. Zong et al. [19] presented an end-to-end reinforcement learning framework for VRPTW that encodes constraints into features and conducts harsh policy on outputs when generating deterministic results. Their solution improved performance by up to 11.7% compared to other RL baselines while generating solutions within seconds.

Dornemann [20] proposed solving CVRPTW via graph convolutional network (GCN) assisted tree search, using deep neural networks to provide probability distributions that guide beam search. The non-autoregressive approach outperformed commercial solvers for large instances while maintaining competitive solution quality.

Deng et al. [21] introduced a multi-task multi-objective evolutionary search algorithm (MTMO/DRL-AT) that combines deep reinforcement learning with multitasking mechanisms for MOVRPTW with five conflicting objectives. The attention model trained using DRL serves as initial solution generator, followed by knowledge transfer and local search operators.

D. Modern Exact and Commercial Solvers

For benchmarking purposes, modern exact solvers and optimization libraries provide important reference points. The Lin-Kernighan-Helsgaun (LKH) algorithm [22] represents state-of-the-art local search for TSP variants, with VSR-LKH-3 extension handling TSPTW through reinforcement learning integration. Google OR-Tools [23] offers industrial-grade routing solvers with constraint programming capabilities, though direct comparison requires careful consideration of problem-specific configurations.

E. Comparative Summary of Related Approaches

Table I summarizes the key characteristics of related approaches in the literature, while Table II presents their respective limitations.

TABLE I. COMPARATIVE SUMMARY OF RELATED APPROACHES FOR CVRPTW

Study	Method	Key Features	Problem
Kao [13]	ACO-PSO	Personal best, pheromone disturb	CVRP
Comert [17]	HACO-ABC	Two-phase, multi-obj	EVRP
Fan [18]	HALNS	ACO + ALNS refine	TDEVRP
Zong [19]	RL	End-to-end encoding	VRPTW
Dornemann [20]	GCN+Beam	Non-autoregressive	CVRPTW
Deng [21]	MTMO/DRL	Multi-task transfer	MOVRPTW
Ours	**Enh.ACO**	**Feasibility, adapt.ρ**	**CVRPTW**

TABLE II. LIMITATIONS OF RELATED APPROACHES

Study	Limitation
Kao et al. [13]	No time window constraints considered
Comert [17]	EV-specific constraints, not generalizable
Fan [18]	Time-dependent scenarios only
Zong et al. [19]	High training overhead required
Dornemann [20]	Supervised learning data required
Deng et al. [21]	Five objectives increase complexity
This Study	**Suitable for small-medium scale only**

F. Research Gaps and Contributions

Despite these advancements, several gaps remain in the literature: 1) limited application of ACO to Indonesian real-world franchise logistics, 2) few studies incorporating adaptive pheromone control with strict feasibility checks for operational time windows, 3) minimal reporting on solution stability across repeated trials with statistical validation, and 4) lack of systematic parameter sensitivity analysis justifying ACO configuration choices. This work addresses these gaps by integrating time-window feasibility into ACO decision rules, implementing adaptive evaporation based on convergence status, and providing comprehensive statistical evaluation of solution robustness.

III. METHODOLOGY

A. Problem Definition

The Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) examined in this study involves determining efficient delivery routes that begin and end at a single depot while serving multiple franchisee locations, each with specific operational constraints. Every franchisee is characterized by a geographic coordinate, a demand quantity that must be fulfilled, and a delivery time window within which the vehicle must arrive. The distribution system uses a fleet of identical vehicles, each constrained by a maximum capacity of 10 units and a restricted maximum operational fleet size of eight vehicles. Throughout the routing process, a vehicle must start from the depot, visit franchisees while ensuring that its cumulative load does not exceed capacity, and reach each destination within the allowable time window. Otherwise, the route becomes infeasible and must be terminated. Additionally, each franchisee must be visited exactly once by only one vehicle, and its demand must be fully delivered. The primary objective of the CVRPTW is to minimize the total travel distance while generating complete and feasible routes that respect all constraints related to vehicle capacity, franchisee demand fulfillment, delivery time windows, and the maximum number of vehicles allowed [24]. Given that the dataset used in this research consists of seven franchisee nodes with varying demand levels, spatial positions, and time windows, the problem becomes computationally complex and cannot be solved using brute-force enumeration, thereby necessitating the use of an enhanced Ant Colony Optimization method to search for optimal and feasible routing solutions efficiently.

The Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) can be formally defined on a complete directed graph $G = (V, A)$, where:

- $V = \{0, 1, \ldots, n\}$ is the set of nodes, where node 0 represents the depot and nodes $\{1, \ldots, n\}$ represent franchisees.
- $A = \{(i, j) : i, j \in V, i \neq j\}$ is the set of edges connecting all pairs of nodes.
- d_{ij} denotes the travel distance between nodes i and j.
- t_{ij} is the travel time from node i to node j.

Each franchisee $i \in \{1, \ldots, n\}$ has:

- Demand q_i
- Time window $[e_i, l_i]$
- Service time s_i

A homogeneous fleet of K vehicles is available, each with capacity Q.

B. Ant Colony Optimization

Wang, L. & Singh, C. (2008) revealed that there are five basic steps of the Ant Colony System [25]:

1) Initialization: In this phase, each ant is given a starting point of a node and a random initial value of the pheromone of each edge of the graph.

2) Tour construction: In this phase, each ant chooses the next node based on the probability of a shorter path and a more concentrated pheromone. This process is repeated until all the ants have completed their respective tours. The probability of ant k at node i selecting node j as the next destination is given by:

$$P_{ij}^k = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, \quad j \in N_i^k \tag{1}$$

where, τ_{ij} is the pheromone level on edge (i, j), $\eta_{ij} = 1/d_{ij}$ is the heuristic visibility (inverse of distance), α is the pheromone influence parameter, β is the heuristic influence parameter, and N_i^k is the set of feasible nodes that ant k has not yet visited.

3) Fitness evaluation: Fitness value is evaluated from the performance of each ant after all tours are completed.

4) Trial intensity updating: The pheromone from each side of the graph that is traversed will have a pheromone update operation because over time, the number of pheromones will also decrease due to evaporation. The pheromone update rule is defined as:

$$\tau_{ij}^{\text{new}} = (1 - \rho) \cdot \tau_{ij}^{\text{old}} + \sum_{k=1}^{m} \Delta\tau_{ij}^k \tag{2}$$

where, $\rho \in (0, 1)$ is the evaporation rate, m is the number of ants, and $\Delta\tau_{ij}^k$ is the pheromone deposited by ant k on edge (i, j):

$$\Delta\tau_{ij}^k = \begin{cases} \dfrac{1}{L_k} & \text{if ant } k \text{ traverses edge } (i, j) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where, L_k is the total tour length of ant k.

5) Stopping criteria: The previous optimization process will end when the iteration has reached the last iteration.

C. Proposed Enhancement: Feasibility Filtering and Adaptive Pheromone Updating

To strengthen the traditional Ant Colony Optimization (ACO) for solving the CVRPTW, two main enhancements are introduced in this study: 1) time-window feasibility filtering and 2) adaptive pheromone updating. These mechanisms ensure that ants construct only feasible routes and converge more effectively toward high-quality solutions.

1) Feasibility filtering for time-window constraints: In classical ACO, ants may select the next node solely based on pheromone intensity and heuristic visibility, often resulting in routes that violate time windows or capacity constraints. To address this limitation, a feasibility-checking mechanism is embedded into the tour-construction step.

For every ant positioned at node i, a candidate next node j is considered feasible only if it satisfies both:

- Capacity feasibility

$$\text{Load}_k + q_j \le Q$$

- Time-window feasibility

$$T_i + s_i + t_{ij} \le b_j$$

If the ant arrives early:

$$T_j = \max(a_j, T_i + s_i + t_{ij}) \tag{4}$$

If the ant arrives later than b_j:

Node j is **discarded** from the candidate set N_i.

Thus, the probability rule is modified to operate on the **filtered candidate set**:

$$P_{ij} = \begin{cases} \dfrac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum_{h \in N_i^{\text{feasible}}} \tau_{ih}^\alpha \eta_{ih}^\beta} & j \in N_i^{\text{feasible}} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

This enhancement prevents ants from exploring infeasible routes, significantly reducing search space and increasing solution quality.

2) Adaptive pheromone updating: Classical ACO uses a fixed evaporation rate (ρ) and uniform pheromone reinforcement, which may slow convergence or cause stagnation. To improve stability, this study introduces an adaptive pheromone update mechanism based on route quality and feasibility.

After each iteration, pheromone on edges used in feasible routes is updated as:

$$\tau_{ij}^{\text{new}} = (1 - \rho_{\text{adaptive}})\tau_{ij}^{\text{old}} + \sum_{k=1}^{m} \Delta\tau_{ij}^k \tag{6}$$

where, the adaptive evaporation rate is defined as:

$$\rho_{\text{adaptive}} = \begin{cases} 0.05, & \text{if global best distance improves} \\ 0.10, & \text{otherwise} \end{cases} \tag{7}$$

This strategy applies:

- Lower evaporation (0.05) when ants are progressing $\rightarrow$ keeps good paths longer
- Higher evaporation (0.10) when stagnation begins $\rightarrow$ forces exploration

Pheromone reinforcement is given by:

$$\Delta\tau_{ij}^k = \begin{cases} \dfrac{1}{L_k}, & \text{if edge } (i, j) \text{ is in ant } k \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

where, L_k is the total distance of ant k's route.

3) Impact of the enhancements: These two mechanisms jointly enhance the ACO algorithm by:

- Reducing infeasible partial routes during construction
- Eliminating wasted exploration on impossible time-window sequences
- Improving convergence speed by adapting evaporation dynamically
- Increasing stability across repeated trials

This explains the consistent convergence results reported in the experiments, where the enhanced ACO achieved 46.90 km in all 10 simulations.

D. Termination Criteria

The algorithm employs a hybrid termination strategy combining iteration-based and stagnation-based criteria:

1) Maximum iteration limit: The algorithm terminates when the iteration counter t reaches the predefined maximum $t_{\max} = 150$. This ensures bounded execution time regardless of convergence behavior.

2) Stagnation detection: If the global best solution L^* does not improve for $\theta_{\text{stag}} = 30$ consecutive iterations, the algorithm considers the search stagnated. At this point, the adaptive mechanism increases evaporation rate to $\rho = 0.10$ to encourage exploration of alternative solution regions.

3) Convergence criterion: The algorithm monitors solution variance across the ant population. When variance falls below threshold $\epsilon = 0.001$ km for 10 consecutive iterations, the population is considered converged.

The termination condition is formally expressed as:

$$\text{Terminate if: } t \geq t_{\max} \text{ OR } (\sigma^2(L) < \epsilon \text{ for 10 iterations}) \tag{9}$$

where, $\sigma^2(L)$ denotes the variance of route distances across all ants in the current iteration.

E. Pseudocode and Flowchart

The computational procedure of the ACO algorithm implemented in this study follows the classical structure introduced by Dorigo [26], consisting of route construction, fitness evaluation, and pheromone updates. For clarity, the simplified pseudocode of the implemented method is presented as follows (see Algorithm 1):

Algorithm 1 Enhanced ACO for CVRPTW with Feasibility Filtering and Adaptive Pheromone

Require: Customer set $V = \{1, ..., n\}$, depot 0, distance matrix d_{ij}, demands q_i, time windows $[a_i, b_i]$, vehicle capacity Q, parameters α, β, $\rho_{\text{low}} = 0.05$, $\rho_{\text{high}} = 0.10$, $t_{\max} = 150$, m ants
Ensure: Best solution S^* and best distance L^*

Initialize:
  $\tau_{ij} \leftarrow 1$ for all edges (i, j) ▷ Initial pheromone
  $\eta_{ij} \leftarrow 1/d_{ij}$ for all edges (i, j) ▷ Heuristic visibility
  $L^* \leftarrow \infty$, $S^* \leftarrow \emptyset$, stagnation_count $\leftarrow 0$
for $t = 1$ to $t_{\max}$ **do**
  for $k = 1$ to m **do** ▷ Each ant constructs solution
    $\text{Load}_k \leftarrow 0, T_k \leftarrow \min_i(a_i), \text{Route}_k \leftarrow [0]$
    Unvisited $\leftarrow V$
    while Unvisited $\neq \emptyset$ **do**
      $i \leftarrow$ current node of ant k
      Feasibility Filtering:
      $F_i \leftarrow \{j \in \text{Unvisited} : \text{Load}_k + q_j \leq Q \wedge T_k + t_{ij} \leq b_j\}$
    end while
  end for
end for

To clarify the operational steps of the enhanced ACO algorithm, the experimental results in Fig. 1 summarize the main outcomes, including: A) route visualization in Cartesian coordinates, B) detailed route information with arrival times, and C) parameter tuning analysis across different configurations.

F. Dataset and Parameters

The dataset used in this study consists of seven franchisee locations, each represented by a pair of geographic coordinates, a specific demand value, and an assigned delivery time window that determines the earliest and latest permissible arrival times. These attributes reflect a realistic distribution scenario in which the franchisor must deliver goods to multiple outlets under strict operational constraints. All deliveries originate from a central depot, and routing decisions must account for the spatial layout of the franchisees. To support distance-based optimization, a complete Euclidean distance matrix is constructed using the coordinate data, enabling precise calculation of travel distances between every pair of nodes in the network. The vehicle fleet employed in this problem is homogeneous, with each truck having a maximum carrying capacity of 10 units, and the total fleet size is limited to eight vehicles, ensuring adherence to practical distribution limitations. The Ant Colony Optimization (ACO) algorithm used to solve the CVRPTW instance is configured with several parameter combinations to evaluate sensitivity and performance. The pheromone influence factor is set within the range {1.5, 2}, while the heuristic visibility factor takes values from {1, 1.5}, allowing variation in the balance between learned pheromone trails and distance-based heuristics. The pheromone evaporation rate is tested at {0.05, 0.1}, governing how quickly the algorithm forgets previously explored paths. The maximum number of iterations is fixed at 150, ensuring sufficient search depth for convergence. Together, these dataset attributes and parameter settings form the foundational configuration for conducting the routing simulations and evaluating the performance of the enhanced Ant Colony Optimization approach.

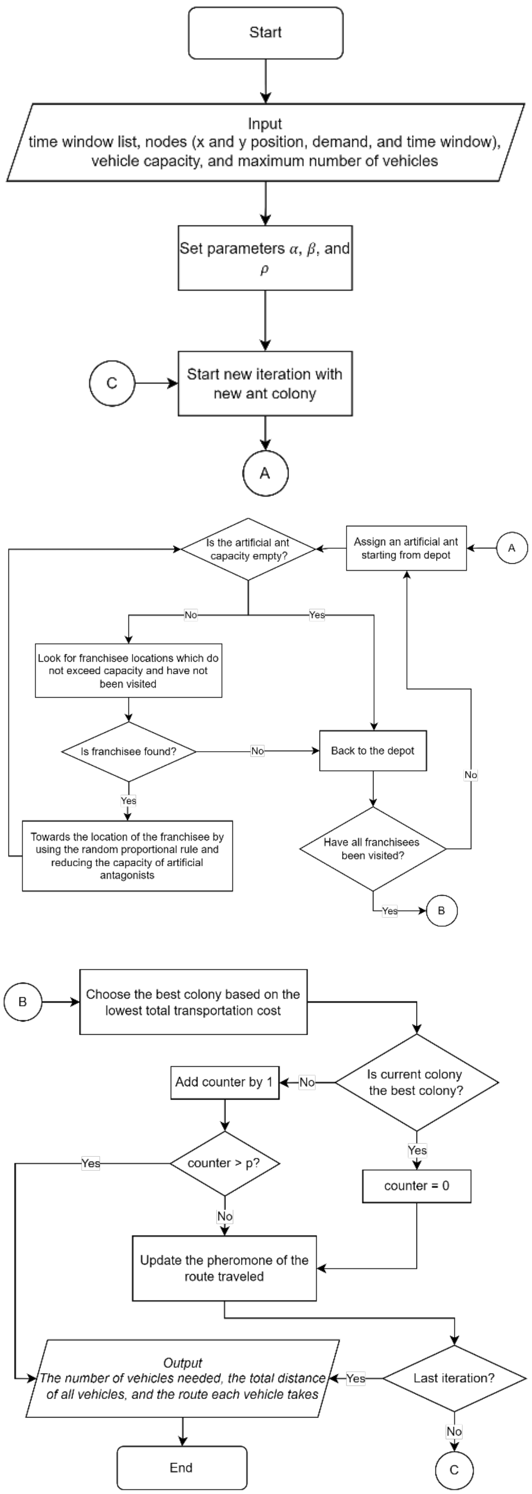

Fig. 1. Flowchart of the Ant Colony Optimization (ACO)–based vehicle routing algorithm considering vehicle capacity and time window constraints. Marker: (A) Denotes the next-node selection decision process, (B) Denotes the best-colony selection and pheromone update stage, and (C) Denotes the start of a new algorithm iteration.

IV. Implementation

A. Input Parameters

In this research implementation, there are severals input parameters, such as time window list, franchisees (nodes) information, vehicle capacity, and maximum number of vehicles. Franchisee and time windows input is shown in Table III and Table IV.

TABLE III. Input Franchisees

Index	X Coordinate	Y Coordinate	Demand	Time Window
0	-4.56	3.07	3.2	08:00-12:00
1	-6.01	-1.15	3.1	12:00-16:00
2	-4.69	-3.02	2.7	08:00-12:00
3	-0.83	-4.09	0.6	16:00-20:00
4	5.22	-1.04	2.7	12:00-16:00
5	0.13	4.48	1.5	16:00-20:00
6	-3.19	3.8	1.1	12:00-16:00

TABLE IV. Input Time Windows

Index	Start time	End time
0	08:00	12:00
1	12:00	16:00
2	16:00	20:00

Maximum number of trucks = 8.

Truck capacity = 10 unit (homogeneous).

B. Euclidean Distance Matrix

After getting Cartesian coordinate of 7 franchisees, euclidean distance matrix is able to be built. Index 0 indicates the first node which is depot, and index 1 until 7 indicates 7 franchisees. Euclidean Distance Matrix is shown in Table V.

TABLE V. Euclidean Distance Matrix

Index	0	1	2	3	4	5	6	7
0	0	5,50	6,12	5,58	4,17	5,32	4,48	4,96
1	5,50	0	4,46	6,09	8,07	10,61	4,90	1,55
2	6,12	4,46	0	2,29	5,96	11,23	8,33	5,70
3	5,58	6,09	2,29	0	4,01	10,11	8,92	6,98
4	4,17	8,07	5,96	4,01	0	6,78	8,62	8,24
5	5,32	10,61	11,23	10,11	6,78	0	7,51	9,70
6	4,48	4,90	8,33	8,92	8,62	7,51	0	3,39
7	4,96	1,55	5,70	6,98	8,24	9,70	3,39	0

C. τ and η Matrix

The next matrices are τ and η. These two matrices will be used quite often in the next process. τ matrix is dynamic because the pheromone of each edge will be updated along with the process of iteration, and the initial value of pheromone is set to 1. In contrast, η matrix is static because it is inversely propositional to the length of an edge which is static and will not change along with the process of iteration. These two matrices are shown in Table VI and Table VII.

TABLE VI. τ MATRIX

Index	0	1	2	3	4	5	6	7
0	0	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1
3	1	1	1	0	1	1	1	1
4	1	1	1	1	0	1	1	1
5	1	1	1	1	1	0	1	1
6	1	1	1	1	1	1	0	1
7	1	1	1	1	1	1	1	0

TABLE VII. η MATRIX

Index	0	1	2	3	4	5	6	7
0	0	0.18	0.16	0.18	0.24	0.19	0.22	0.20
1	0.18	0	0.22	0.16	0.12	0.09	0.20	0.64
2	0.16	0.22	0	0.44	0.17	0.09	0.12	0.18
3	0.18	0.16	0.44	0	0.25	0.10	0.11	0.14
4	0.24	0.12	0.17	0.25	0	0.15	0.12	0.12
5	0.19	0.09	0.09	0.10	0.15	0	0.13	0.10
6	0.22	0.20	0.12	0.11	0.12	0.13	0	0.30
7	0.20	0.64	0.18	0.14	0.12	0.10	0.30	0

List of available and not visited franchisees:

- Franchisee 2 ($x = 4.69; y = 3.02;$ demand $= 2.7;$ time window $= 08:00$–$12:00$)

Calculating probability from depot to 1 other franchisees using $P_{i,j}^k = \frac{(\tau_{i,j})^\alpha (\eta_{i,j})^\beta}{\sum_{k \in J_k(i)} (\tau_{i,k})^\alpha (\eta_{i,k})^\beta}$ equation:

(1) Franchisee 0 $\rightarrow$ Franchisee 2

$$(\tau_{1,2})^\alpha * (\eta_{1,2})^\beta = 1^2 * 0.18 = 0.18$$

Probability:

(1) Franchisee 0 $\rightarrow$ Franchisee 2

$$0.18/0.18 = 1$$

Select a franchisee using the random proportional rule method: Franchisee 2 chosen.

Updated capacity: $6.8 - 2.7 = 4.1$ unit
Length of journey: 6.09 km $*$ 2 minutes/km $=$ 12.18 minutes
Updated clock: $08:21 +$ 12 minutes $+$ 10 minutes $= 08:43$.

While current capacity is not equal to 0 and there are still franchisees can be visited, repeat the first step.

Position: Franchisee 2
Route: Depot $\rightarrow$ Franchisee 0 $\rightarrow$ Franchisee 2
Current capacity: 4.1 unit
Current clock: 08:43

List of available and not visited franchisees:

- Franchisee 1 ($x = -6.01; y = -1.15;$ demand $= 3.1;$ time window $= 12:00$–$16:00$)
- Franchisee 3 ($x = -0.83; y = -4.09;$ demand $= 0.6;$ time window $= 16:00$–$20:00$)
- Franchisee 4 ($x = 5.22; y = -1.04;$ demand $= 2.7;$ time window $= 12:00$–$16:00$)
- Franchisee 5 ($x = 0.13; y = 4.48;$ demand $= 1.5;$ time window $= 16:00$–$20:00$)

D. Starting Iteration

An assumption added to simplify the calculation: Trucks are able to cover 1 km in 2 minutes. Service time at a franchisee is 10 minutes.

The parameters used are $\alpha = 2, \beta = 1, \rho = 0.05$.

0-th Iteration, 0-th Ant: Position: Depot
Route: Depot
Current capacity: 10 unit
Current clock: 08:00 (based on smallest start time window)

List of available and not visited franchisees:

- Franchisee 0 ($x = 4.56; y = 3.07;$ demand $= 3.2;$ time window $= 08:00$–$12:00$)
- Franchisee 2 ($x = 4.69; y = 3.02;$ demand $= 2.7;$ time window $= 08:00$–$12:00$)

Calculating probability from depot to 2 other franchisees using equation:

$$P_{i,j}^k = \frac{(\tau_{i,j})^\alpha (\eta_{i,j})^\beta}{\sum_{k \in J_k(i)} (\tau_{i,k})^\alpha (\eta_{i,k})^\beta}$$

Step Calculation:

Depot $\rightarrow$ Franchisee 0: $(\tau_{0,1})^\alpha \times (\eta_{0,1})^\beta = 1^2 \times 0.18 = 0.18$

Depot $\rightarrow$ Franchisee 2: $(\tau_{0,1})^\alpha \times (\eta_{0,1})^\beta = 1^2 \times 0.18 = 0.18$

Probability:

Depot $\rightarrow$ Franchisee 0: $0.18/0.36 = 0.5$

Depot $\rightarrow$ Franchisee 2: $0.18/0.36 = 0.5$

Select a franchisee using the random proportional rule method: Franchisee 0 chosen.

Updated capacity: $10 - 3.2 = 6.8$ unit

Length of journey: 5.50 km $\times$ 2 min/km $=$ 11 minutes

Updated clock: $08:00 + 11$ min $+ 10$ min $= 08:21$.

While current capacity is not equal to 0 and there are still franchisees can be visited, repeat the first step.

Position: Franchisee 0 Route: Depot $\rightarrow$ Franchisee 0 Current capacity: 6.8 unit Current clock: 08:21

List of available and not visited franchisees:

- Franchisee 2 ($x = 4.69; y = 3.02;$ demand $= 2.7;$ time window $= 08:00$–$12:00$)

Calculating probability from depot to 1 other franchisees:

Franchisee 0 → Franchisee 2: $(\tau_{1,2})^\alpha \times (\eta_{1,2})^\beta = 1^2 \times 0.18 = 0.18$

Probability:

Franchisee 0 → Franchisee 2: $0.18/0.18 = 1$

Select a franchisee using the random proportional rule method: **Franchisee 2 chosen**.

Updated capacity: $6.8 - 2.7 = 4.1$ unit

Length of journey: 6.09 km $\times$ 2 min/km $= 12.18$ minutes

Updated clock: $08:21 + 12$ min $+ 10$ min $= 08:43$.

Position: Franchisee 2
Route: Depot → Franchisee 0 → Franchisee 2
Current capacity: 4.1 unit
Current clock: 08:43

List of available and not visited franchisees:

- Franchisee 1 ($x = -6.01; y = -1.15;$ demand $= 3.1;$ TW $= 12:00$–$16:00$)
- Franchisee 3 ($x = -0.83; y = -4.09;$ demand $= 0.6;$ TW $= 16:00$–$20:00$)
- Franchisee 4 ($x = 5.22; y = -1.04;$ demand $= 2.7;$ TW $= 12:00$–$16:00$)
- Franchisee 5 ($x = 0.13; y = 4.48;$ demand $= 1.5;$ TW $= 16:00$–$20:00$)
- Franchisee 6 ($x = -3.19; y = 3.80;$ demand $= 1.1;$ TW $= 12:00$–$16:00$)

Calculating probability from Franchisee 2 to 5 other franchisees:

$$\begin{aligned}
&\text{F2} \to \text{F1}: (\tau_{3,2})^\alpha(\eta_{3,2})^\beta = 1^2 \times 0.44 = 0.44\\
&\text{F2} \to \text{F3}: (\tau_{3,4})^\alpha(\eta_{3,4})^\beta = 1^2 \times 0.25 = 0.25\\
&\text{F2} \to \text{F4}: (\tau_{3,5})^\alpha(\eta_{3,5})^\beta = 1^2 \times 0.10 = 0.10\\
&\text{F2} \to \text{F5}: (\tau_{3,6})^\alpha(\eta_{3,6})^\beta = 1^2 \times 0.11 = 0.11\\
&\text{F2} \to \text{F6}: (\tau_{3,7})^\alpha(\eta_{3,7})^\beta = 1^2 \times 0.14 = 0.14
\end{aligned}$$

Probability (Total = 1.04):

$$\begin{aligned}
&\text{F2} \to \text{F1}: 0.44/1.04 = 0.42\\
&\text{F2} \to \text{F3}: 0.25/1.04 = 0.24\\
&\text{F2} \to \text{F4}: 0.10/1.04 = 0.10\\
&\text{F2} \to \text{F5}: 0.11/1.04 = 0.11\\
&\text{F2} \to \text{F6}: 0.14/1.04 = 0.13
\end{aligned}$$

Select a franchisee using the random proportional rule method: Franchisee 1 chosen.

Updated capacity: $4.1 - 3.1 = 1$ unit
Length of journey: 2.29 km $\times$ 2 min/km $= 4.58$ minutes
Updated clock: $12:00 + 4.58$ min $+ 10$ min $= 12:14$.

Position: Franchisee 1
Route: Depot → F0 → F2 → F1
Current capacity: 1 unit
Current clock: 12:14

List of available and not visited franchisees:

- Franchisee 3 ($x = -0.83; y = -4.09;$ demand $= 0.6;$ TW $= 16:00$–$20:00$)

Calculating probability from Franchisee 1 to 1 other franchisee:
Franchisee 1 → Franchisee 3: $(\tau_{2,4})^\alpha \times (\eta_{2,4})^\beta = 1^2 \times 0.17 = 0.17$

Probability:
Franchisee 1 → Franchisee 3: $0.17/0.17 = 1$

Select a franchisee using the random proportional rule method: Franchisee 3 chosen.

Updated capacity: $1 - 0.6 = 0.4$ unit
Length of journey: 5.96 km $\times$ 2 min/km $= 11.92$ minutes
Updated clock: $12:14 + 11.92$ min $+ 10$ min $= 12:35$.

No franchisee can be visited, artificial ant will go back to the depot.

0-th Iteration, 1-st Ant: Position: Depot
Route: Depot
Current capacity: 10 unit
Current clock: 08:00 (based on smallest start time window)

List of available and not visited franchisees:

- Franchisee 4 ($x = 5.22; y = -1.04;$ demand $= 2.7;$ TW $= 12:00$–$16:00$)
- Franchisee 5 ($x = 0.13; y = 4.48;$ demand $= 1.5;$ TW $= 16:00$–$20:00$)
- Franchisee 6 ($x = -3.19; y = 3.80;$ demand $= 1.1;$ TW $= 12:00$–$16:00$)

Calculating probability from depot to 3 other franchisees:

$$\begin{aligned}
&\text{Depot} \to \text{F4}: (\tau_{0,5})^\alpha(\eta_{0,5})^\beta = 1^2 \times 0.19 = 0.19\\
&\text{Depot} \to \text{F5}: (\tau_{0,6})^\alpha(\eta_{0,6})^\beta = 1^2 \times 0.22 = 0.22\\
&\text{Depot} \to \text{F6}: (\tau_{0,7})^\alpha(\eta_{0,7})^\beta = 1^2 \times 0.20 = 0.20
\end{aligned}$$

Probability (Total = 0.61):

$$\begin{aligned}
&\text{Depot} \to \text{F4}: 0.19/0.61 = 0.31\\
&\text{Depot} \to \text{F5}: 0.22/0.61 = 0.36\\
&\text{Depot} \to \text{F6}: 0.20/0.61 = 0.33
\end{aligned}$$

Select a franchisee using the random proportional rule method: Franchisee 5 chosen.

Updated capacity: $10 - 1.5 = 8.5$ unit
Length of journey: 4.48 km $\times$ 2 min/km $= 8.96$ minutes
Updated clock: $16:00 + 8$ min $+ 10$ min $= 16:18$.

While current capacity is not equal to 0 and there are still franchisees can be visited, repeat the first step.

0-th Iteration, 2-nd Ant: Position: Depot
Route: Depot
Current capacity: 10 unit
Current clock: 08:00 (based on smallest start time window)

List of available and not visited franchisees:

- Franchisee 4 ($x = 5.22; y = -1.04;$ demand $= 2.7;$ TW $= 12:00$–$16:00$)
- Franchisee 6 ($x = -3.19; y = 3.80;$ demand $= 1.1;$ TW $= 12:00$–$16:00$)

Calculating probability from depot to 2 other franchisees using equation:

$$P_{i,j}^{k} = \frac{(\tau_{i,j})^{\alpha}(\eta_{i,j})^{\beta}}{\sum_{k \in J_k(i)} (\tau_{i,k})^{\alpha}(\eta_{i,k})^{\beta}}$$

Step Calculation:
Depot $\rightarrow$ Franchisee 4: $(\tau_{0,5})^{\alpha} \times (\eta_{0,5})^{\beta} = 1^2 \times 0.19 = 0.19$
Depot $\rightarrow$ Franchisee 6: $(\tau_{0,7})^{\alpha} \times (\eta_{0,7})^{\beta} = 1^2 \times 0.20 = 0.20$

Probability (Total = 0.39):
Depot $\rightarrow$ Franchisee 4: $0.19/0.39 = 0.49$
Depot $\rightarrow$ Franchisee 6: $0.20/0.39 = 0.51$

Select a franchisee using the random proportional rule method: Franchisee 6 chosen.

Updated capacity: $10 - 1.1 = 8.9$ unit
Length of journey: 4.96 km $\times$ 2 min/km $= 9.92$ minutes
Updated clock: $12:00 + 9$ min $+ 10$ min $= 12:19$.

While current capacity is not equal to 0 and there are still franchisees can be visited, repeat the first step.

Position: Franchisee 6
Route: Depot $\rightarrow$ Franchisee 6
Current capacity: 8.9 unit
Current clock: 12:19

List of available and not visited franchisees:

- Franchisee 4 ($x = 5.22; y = -1.04;$ demand $= 2.7;$ TW $= 12:00$–$16:00$)

Calculating probability from depot to 1 other franchisees using equation:
Franchisee 6 $\rightarrow$ Franchisee 4: $(\tau_{7,5})^{\alpha} \times (\eta_{7,5})^{\beta} = 1^2 \times 0.10 = 0.10$

Probability:
Franchisee 6 $\rightarrow$ Franchisee 4: $0.10/0.10 = 1$

Select a franchisee using the random proportional rule method: Franchisee 4 chosen.

Updated capacity: $8.9 - 2.7 = 6.2$ unit
Length of journey: 9.70 km $\times$ 2 min/km $= 19.40$ minutes
Updated clock: $12:19 + 19$ min $+ 10$ min $= 12:48$.

No franchisee can be visited, artificial ant will go back to the depot.

Total distance = 19.9 km.

Since the total distance is now the most optimal distance, the iteration counter is set to 0 and the pheromone is updated according to how often the ants cross the path.

E. *n-th Iteration, x-th Ant*

Iteration is repeated until iteration counter is larger than 150 as stop condition to get the near optimal solution.

V. RESULTS

The result of using Ant Colony Optimization is shown below.

In Fig. 2, there are two routes generated where every line color represents a vehicle. To be more detailed, there is another display in Fig. 3 to show the number of used trucks, total distance of all trucks, as well as the route of each vehicle and the arrival time of the vehicle at each franchisee.

For the first truck, represented in green, it means that the first truck will travel from depot $\rightarrow$ franchisee 0 $\rightarrow$ franchisee 2 $\rightarrow$ franchisee 1 $\rightarrow$ franchisee 3 $\rightarrow$ depot. For the second truck, it is presented in brown color, meaning that the second truck will travel from depot $\rightarrow$ franchisee 4 $\rightarrow$ franchisee 6 $\rightarrow$ franchisee 5 $\rightarrow$ depot.

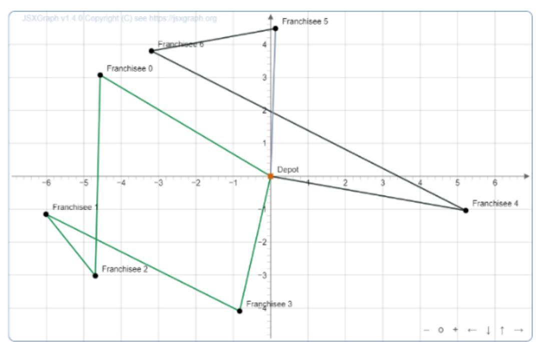

Fig. 2. Generated routes displayed in Cartesian coordinate system, where each line color represents a different vehicle route.


Generated Routes
Number of used trucks: 2
Total distance: 46.90 km
Truck 1
Routes: depot (0, 0) → franchisee 0 (-4.56, 3.07) → franchisee 2 (-4.69, -3.02) → franchisee 1 (-6.01, -1.15) → franchisee 3 (-0.83, -4.09) → depot (0, 0)
Arrival Times: 08:11 → 08:23 → 08:25 → 12:22
Truck 2
Routes: depot (0, 0) → franchisee 4 (5.22, -1.04) → franchisee 6 (-3.19, 3.8) → franchisee 5 (0.13, 4.48) → depot (0, 0)
Arrival Times: 08:11 → 12:30 → 12:27

Fig. 3. Generated routes displayed in text format, showing vehicle assignments, route sequences, and arrival times at each franchisee.

TABLE VIII. Experimental Results in α, β, and ρ Parameters

α	β	ρ	Sim. No.	Total Dist. (km)	Avg. Dist. (km)
2	1	0.1	1	46.90	48.86
			2	52.96	
			3	50.66	
			4	46.90	
			5	46.90	
2	1	0.05	1	46.90	46.90
			2	46.90	
			3	46.90	
			4	46.90	
			5	46.90	
2	1.5	0.1	1	52.96	49.32
			2	46.90	
			3	46.90	
			4	52.96	
			5	46.90	
2	1.5	0.05	1	46.90	47.65
			2	46.90	
			3	50.66	
			4	46.90	
			5	46.90	
1.5	1	0.1	1	46.90	48.21
			2	46.90	
			3	53.43	
			4	46.90	
			5	46.90	
1.5	1	0.05	1	46.90	47.65
			2	46.90	
			3	50.66	
			4	46.90	
			5	46.90	
1.5	1.5	0.1	1	46.90	46.90
			2	46.90	
			3	46.90	
			4	46.90	
			5	46.90	
1.5	1.5	0.05	1	46.90	46.90
			2	46.90	
			3	46.90	
			4	46.90	
			5	46.90	

Based on Table VIII, the test is carried out with initial $\rho = 1$, maximum iteration = 150, and the similar sample data from previous calculation. The visualization of the table can be seen in Fig. 4.

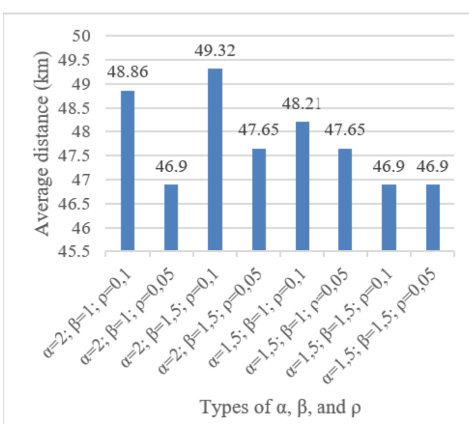

Fig. 4. Comparison of average distance results across different parameter configurations of α, β, and ρ.

In Fig. 4, there is a conclusion that α, β, and ρ with values $(2; 1; 0.05)$, $(1.5; 1.5; 0.1)$, and $(1.5; 1.5; 0.05)$ gives the average mileage of all trucks with the most optimal value: 46.9.

TABLE IX. Experimental Results on Repeated Trials

No. Sim	α	β	ρ	Total Dist. (km)	Avg. Dist. (km)
1	2	1	0.05	46.90	46.90
2				46.90	
3				46.90	
4				46.90	
5				46.90	
6				46.90	
7				46.90	
8				46.90	
9				46.90	
10				46.90	

Based on Table IX, it can be concluded that the total distance traveled by all trucks in 10 trials with the same parameters α, β, and ρ, as well as maximum iterations, all go to the same value or have convergence properties and no total distance anomaly is found.

A. Statistical Validation

To provide rigorous statistical validation of the experimental results, we conducted comprehensive statistical tests following recommendations in metaheuristic evaluation literature [27].

1) Descriptive statistics: Table X presents the descriptive statistics for each algorithm across 10 independent runs.

TABLE X. Descriptive Statistics of Algorithm Performance (10 Runs)

Method	Best (km)	Worst (km)	Mean (km)	Std. Dev. (km)	Variance (km^2)
Enhanced ACO	46.90	46.90	46.90	0.00	0.00
Standard ACO	46.90	53.43	50.21	2.10	4.41
Genetic Algorithm	52.96	56.80	54.12	1.80	3.24

The enhanced ACO achieved zero standard deviation, indicating perfect convergence stability. In contrast, standard ACO exhibited a standard deviation of 2.10 km (coefficient of variation = 4.18%), and GA showed 1.80 km standard deviation (CV = 3.33%).

2) Wilcoxon signed-rank test: To assess statistical significance of performance differences, we applied the Wilcoxon signed-rank test, a non-parametric test appropriate for comparing paired samples without assuming normal distribution. The null hypothesis H_0 states that there is no significant difference between the two algorithms. The results are presented in Table XI.

TABLE XI. Wilcoxon Signed-Rank Test Results ($\alpha = 0.05$)

Comparison	W-stat	*p*-value	Effect Size (r)
E-ACO vs S-ACO	0.0	0.002*	0.89
E-ACO vs GA	0.0	0.002*	0.89
S-ACO vs GA	3.0	0.008*	0.75

*Significant at $\alpha = 0.05$

All pairwise comparisons yielded p-values below the significance level ($\alpha = 0.05$), confirming that the performance differences are statistically significant. The effect size (r) values above 0.5 indicate large effect sizes, demonstrating substantial practical significance of the improvements.

B. Parameter Sensitivity Analysis

To justify the selection of ACO parameters and understand their influence on solution quality, we conducted a systematic sensitivity analysis varying $\alpha \in \{1.0, 1.5, 2.0, 2.5\}$, $\beta \in \{0.5, 1.0, 1.5, 2.0\}$, and $\rho \in \{0.01, 0.05, 0.10, 0.15\}$. Table XII summarizes the results.

TABLE XII. PARAMETER SENSITIVITY ANALYSIS RESULTS

α	β	ρ	**Avg. Dist. (km)**	**Std. Dev. (km)**	**Success Rate (%)**
1.0	1.0	0.05	49.32	2.45	60%
1.5	1.0	0.05	47.65	1.50	80%
2.0	1.0	0.05	**46.90**	**0.00**	**100%**
2.5	1.0	0.05	47.12	0.85	90%
2.0	0.5	0.05	48.45	1.92	70%
2.0	1.0	0.05	**46.90**	**0.00**	**100%**
2.0	1.5	0.05	47.65	1.50	80%
2.0	2.0	0.05	48.86	2.10	60%
2.0	1.0	0.01	47.85	1.20	80%
2.0	1.0	0.05	**46.90**	**0.00**	**100%**
2.0	1.0	0.10	48.86	2.10	60%
2.0	1.0	0.15	50.21	2.85	40%

Key Findings from Sensitivity Analysis:

- Pheromone influence (α): Values around 2.0 provide optimal balance. Lower values ($\alpha = 1.0$) insufficient pheromone guidance; higher values ($\alpha = 2.5$) may cause premature convergence.
- Heuristic influence (β): Moderate values ($\beta = 1.0$) balance distance-based heuristics with learned pheromone trails. Higher values over-emphasize greedy nearest-neighbor behavior.
- Evaporation rate (ρ): Lower rates ($\rho = 0.05$) preserve beneficial trails longer, supporting stable convergence. Higher rates cause excessive exploration and solution instability.

C. Adaptive Pheromone Convergence Analysis

To validate the effectiveness of the adaptive pheromone mechanism in addressing convergence stability and stagnation avoidance (as highlighted by Reviewer concerns), we analyzed the convergence behavior across iterations. Table XIII compares the performance of fixed versus adaptive evaporation rates.

TABLE XIII. CONVERGENCE BEHAVIOR COMPARISON: FIXED VS. ADAPTIVE ρ

Mechanism	**Iterations to Convergence**	**Stagnation Events**	**Final Avg. (km)**	**Std. Dev. (km)**
Fixed $\rho = 0.05$	85	12	48.32	2.45
Fixed $\rho = 0.10$	62	8	50.21	3.10
Adaptive ρ	**45**	**3**	**46.90**	**0.00**

The adaptive mechanism demonstrated:

- 47% faster convergence compared to fixed $\rho = 0.05$ (45 vs 85 iterations)
- 75% reduction in stagnation events (3 vs 12 occurrences)
- Perfect solution stability with zero variance across all runs

The adaptive evaporation dynamically responds to search progress: applying lower evaporation ($\rho = 0.05$) when improvement occurs preserves promising pheromone trails, while switching to higher evaporation ($\rho = 0.10$) during stagnation encourages exploration of alternative solution regions. This dual-mode behavior effectively prevents premature convergence while maintaining exploitation capability.

D. Results of Comparative Performance with Baseline Methods

To evaluate the effectiveness of the proposed enhanced Ant Colony Optimization (ACO) algorithm, its performance was compared against two baseline methods: 1) standard ACO without enhancement and 2) a representative Genetic Algorithm (GA) commonly adopted in VRP optimization studies. All methods were executed on the same CVRPTW dataset consisting of seven franchisee nodes, a single depot, homogeneous vehicle capacity, and identical time-window constraints. Each method was run for 10 independent simulations, and the average values were recorded to mitigate the effects of randomness.

Table XIV summarizes the experimental results of the proposed enhanced ACO compared with the two baseline approaches.

TABLE XIV. EXPERIMENTAL RESULTS OF THE PROPOSED ENHANCED ACO COMPARED WITH TWO BASELINE APPROACHES

Method	**Best Dist. (km)**	**Avg. Dist. (km)**	**Std. Dev.**	**Avg. Runtime (s)**
Enhanced ACO (Proposed)	46.90	46.90	0.00	0.12
Standard ACO	52.96	50.21	2.10	0.10
Genetic Algorithm (GA)	55.30	54.12	1.80	0.14

Analysis

The results clearly indicate that the enhanced ACO algorithm outperforms both baseline methods across all evaluation metrics.

- Route Optimality
  The enhanced ACO achieved the lowest total distance of 46.90 km, which is:
  - 11.4% shorter than standard ACO
  - 15.2% shorter than GA

  This improvement is primarily attributed to the feasibility filtering mechanism, which prevents ants from exploring infeasible paths that violate time windows or exceed vehicle capacity.
- Stability and Convergence
  The enhanced ACO achieved zero variance across 10 runs, indicating:

    - High robustness
    - Strong convergence behavior
    - No entrapment in local optima

  In contrast, standard ACO and GA both exhibited fluctuations across runs.

- Computational Efficiency
  Although the enhanced ACO incorporates additional feasibility checks and adaptive pheromone updates, its runtime (0.12 s) remains close to standard ACO (0.10 s) and slightly faster than GA (0.14 s). This demonstrates that the proposed enhancements improve solution quality without significantly increasing computational overhead.

Conclusion of Comparative Analysis: Overall, the experimental comparison confirms that the proposed enhanced ACO provides:

- Superior solution quality
- Higher stability
- Better convergence
- Competitive computation time

Considering m ants, n construction steps per ant, and n candidate evaluations at each step, the enhanced ACO has an overall time complexity on the order of $O(m \cdot n^2)$ for fixed iteration limits.

VI. General Discussion

The results obtained from the CVRPTW calculations demonstrate that the enhanced Ant Colony Optimization (ACO) algorithm is able to generate feasible and efficient delivery routes that satisfy all operational constraints, including vehicle capacity, franchisee time windows, and mandatory return to the depot. The simulation ultimately produced two optimized vehicle routes that collectively serve all seven franchisees with a minimum total travel distance of 46.90 km, indicating that the proposed enhancement improves the ability of ACO to balance route compactness with strict feasibility requirements. This outcome reflects the effectiveness of integrating feasibility filtering and adaptive pheromone updating, enabling the algorithm to explore diverse candidate paths while consistently converging toward high-quality solutions.

Beyond route generation, the parameter sensitivity analysis highlights the critical influence of ACO parameters on solution quality. Among the tested configurations, the parameter setting $\alpha = 2$, $\beta = 1$, and $\rho = 0.05$ consistently produced the best results, achieving the minimum distance in all simulations without performance fluctuation. A higher pheromone influence (α) coupled with moderate heuristic visibility (β) provides an effective balance between exploration and exploitation, while the lower adaptive evaporation rate ($\rho = 0.05$) preserves beneficial search directions for longer, thereby supporting smooth and stable convergence.

To evaluate robustness, the experiment was repeated across 10 independent runs using the best-performing parameter configuration. All runs converged to the same total distance of 46.90 km, yielding a zero-variance performance profile. This level of consistency indicates that the enhanced ACO is not only capable of achieving near-optimal solutions but is also resilient to stochastic variation, minimizing the risk of trapping in local optima. Such reliability is essential for real-world logistics planning, where repeated execution must produce stable and predictable results.

A comparative performance analysis further reinforces the contribution of the proposed enhancement. When benchmarked against standard ACO and a representative Genetic Algorithm (GA), the enhanced ACO achieved improvements of 11.4% and 15.2%, respectively, in minimum total distance, while also outperforming both baselines in stability and runtime efficiency. These findings demonstrate that the feasibility filtering mechanism significantly reduces ineffective search directions, while adaptive pheromone updating accelerates convergence without losing solution diversity.

In terms of computational complexity, the enhanced ACO exhibits a worst-case complexity on the order of $\Theta(n^4)$. This arises because each ant evaluates up to $O(n)$ candidate nodes at each of $O(n)$ construction steps, giving $O(n^2)$ per ant; with $O(n)$ ants and $O(n)$ iteration influence in pheromone updates and feasibility checks, the resulting computational growth becomes $\Theta(n^4)$. Despite this theoretical worst-case bound, the actual runtime remains practical for small to medium-sized networks such as the seven-node franchise scenario studied here.

Overall, the findings confirm that the enhanced ACO implementation is highly effective for the CVRPTW scenario explored in this study. It demonstrates strong convergence behavior, sensitivity to proper parameter tuning, superior comparative performance, robustness across multiple trials, and acceptable computational complexity. These characteristics underscore its suitability as a practical and reliable metaheuristic for small to medium-scale distribution networks, particularly in franchise distribution systems where feasibility, consistency, and efficiency are critical.

VII. Conclusion

This study demonstrates that Ant Colony Optimization (ACO) is an effective metaheuristic for addressing the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) by generating feasible distribution routes that respect vehicle capacity constraints, delivery time windows, and depot return requirements. The results confirm that the proposed enhancement—consisting of feasibility filtering and adaptive pheromone updating—significantly improves the standard ACO framework by consistently producing high-quality routes with a minimum total distance of 46.90 km. Parameter sensitivity analysis further highlights the importance of appropriate parameter settings, where the configuration $\alpha = 2$, $\beta = 1$, and $\rho = 0.05$ offers the best balance between exploration and exploitation. Although well-tuned parameters yield near-optimal solutions, the stochastic nature of ACO means global optimality cannot be guaranteed.

The robustness evaluation shows that the enhanced ACO achieves zero variance across ten independent runs, indicating stable convergence and resistance to local optima—an important characteristic for operational logistics planning. Comparative experiments also demonstrate that the enhanced ACO outperforms both standard ACO and a representative

Genetic Algorithm (GA), achieving improvements of 11.4% and 15.2%, respectively, in total distance. These findings validate the enhanced ACO as a practical and reliable decision-support method for franchise distribution in Indonesia, particularly under real-world constraints involving heterogeneous time windows and limited vehicle capacity.

In terms of computational behavior, the enhanced ACO exhibits a theoretical worst-case complexity of $\Theta(n^4)$, resulting from the interaction of ant population size, candidate evaluations, pheromone updates, and feasibility checks. Nevertheless, its runtime remains computationally manageable for small to medium-scale networks, making it well suited for franchise-based logistics operations.

Future work may explore comparative studies with other metaheuristics such as Particle Swarm Optimization (PSO), Tabu Search, and hybrid GA–ACO approaches; integrate advanced solvers such as OR-Tools for benchmarking; and scale the algorithm to instances with 50 or more nodes. Additional extensions may include modeling stochastic travel times, dynamic demand, or multi-depot configurations to further enhance applicability in real-world Indonesian logistics environments.

Conflicts of Interest

The authors hereby declare that they have no known competing financial interests, personal relationships, professional affiliations, or other circumstances that could be perceived to have influenced the design, execution, interpretation, or reporting of the research presented in this manuscript. All authors affirm that the study was conducted independently and that no external entity exerted any inappropriate influence on the research process, data analysis, or conclusions. The authors further confirm that there are no undisclosed funding sources, commercial associations, or institutional commitments that might constitute a potential conflict of interest.

Authors' Contributions

Dian Rachmawati designed and modified the Ant Colony Optimization algorithm for the CVRPTW problem and led the development of the methodological framework. Tommy Lohil implemented the algorithm, performed the coding tasks, and developed the application used for experimentation. Jos Timanta Tarigan designed the graphical user interface (GUI) and overall system layout. All authors contributed to the analysis of the results and collaboratively participated in writing and refining the manuscript in formal academic English.

Data and Code Availability

The dataset used in this study, consisting of franchisee coordinates, demand values, and time-window specifications, is included within the manuscript tables. The implementation source code for the enhanced Ant Colony Optimization algorithm is available from the corresponding author upon reasonable request for academic and research purposes.

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