MILP for Multimodal Urban Transport: Formal and Informal Sectors

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***Abstract*—This study presents a mixed-integer linear programming (MILP) formulation for optimizing multimodal urban transportation networks that integrate formal and informal public transport sectors. Whereas prior optimization approaches have predominantly addressed regulated formal services, the proposed model explicitly incorporates informal operators prevalent in many African cities. The objective is to minimize total user travel time, comprising in-vehicle time, waiting time at stops, and transfer time between modes, subject to flow conservation constraints, limits on mode changes, and sector-specific operational rules. Under the assumption of static demand and known arc travel times, computational experiments use realistic synthetic instances calibrated to reflect operating characteristics of the operators providing public transport services in Abidjan, Côte d'Ivoire—informed by close observation of non-public operational patterns and published mobility statistics—with 5 to 24 logical stops and up to 2,487 active arcs. Compared with formal-only system configurations, the integrated formal–informal formulation reduces total travel time by up to 53% on the largest tested instances (24 stops; with smaller gains on smaller networks). All instances were solved with IBM CPLEX 22.11; maximum solve times were 1.63 s on corridor instances ($n \leq 24$) and 0.69 s on an extended set with $n \leq 100$ (Intel Core i7, 16 GB RAM). These findings indicate that coordinated planning of formal and informal transport can materially improve urban mobility in developing cities.**

***Keywords*—Mixed-integer linear programming; multimodal transportation; urban mobility; formal transport; informal transport; operations research; network optimization**

I. Introduction

In less-developed countries, particularly in sub-Saharan Africa, urban transportation systems face unprecedented challenges driven by rapid urbanization, population growth, and increasing mobility demands. Cities in Africa are experiencing rapid urbanization, with many metropolitan areas growing at annual rates of approximately 3–5% [1]. This growth has created complex mobility ecosystems where formal and informal transportation modes coexist in a delicate, often uncoordinated balance. Understanding and optimizing these hybrid multimodal systems represents one of the most critical challenges in contemporary transportation planning and operations research.

Public transportation systems in African cities often exhibit a dual structure, comprising both formal and informal sectors that coexist and complement each other. The formal transport sector comprises government-regulated bus services, rail systems, water transport, and Bus Rapid Transit (BRT) corridors, which operate scheduled services with fixed routes and designated stops. In contrast, the informal sector encompasses unregulated operators such as minibus taxis, shared taxis (matatus, dala dalas, tro tros), motorcycle taxis (boda bodas, okadas), and three-wheeler rickshaws (salonis, kekes), among other modes [2], providing flexible, on-demand services without fixed schedules or designated stops, responding directly to passenger requests.

The dual structure of formal and informal transport sectors is particularly pronounced in sub-Saharan African cities, where informal transportation often dominates the urban mobility landscape. Public transportation plays a crucial role in urban mobility, particularly in large African cities where rapid urbanization and limited formal transport infrastructure have led to the emergence of extensive informal transport networks [3]–[7].

To illustrate the significance of informal transport in this context, Table I presents a comparative analysis of modal shares across five major African cities: Abidjan (Côte d'Ivoire), Dakar (Senegal), Lagos (Nigeria), Kinshasa (Democratic Republic of Congo), and Niamey (Niger). The data, compiled from studies conducted by urban mobility authorities and research institutions, reveals several key patterns. First, informal transportation accounts for a substantial proportion of urban trips, ranging from 20% in Niamey to 60% in Lagos. Second, formal public transportation represents a relatively small share, typically between 5% and 15% of total trips. Third, walking remains a significant mode of transport, particularly in Niamey, where it accounts for 70% of trips. These statistics underscore the critical importance of incorporating informal transport into urban mobility optimization models for African cities.

This study addresses the optimization of multimodal urban transportation networks by explicitly modeling both formal and informal transport sectors within a unified mathematical framework. A mixed-integer linear programming (MILP) model is proposed that minimizes total user travel time, accounting for in-vehicle time, waiting time at stops, and transfer time between modes. The model incorporates operational constraints specific to each sector, including flow conservation, limits on mode changes, and sector-specific infrastructure rules.

The remainder of the study is organized as follows: Section II reviews related work on multimodal transport optimization. Section III describes the problem setting and distinctive features of formal–informal systems. Section IV presents the mathematical formulation (assumptions, parameters, decision variables, objective function, and constraints). Section V

TABLE I. COMPARISON OF MODAL SHARES OF URBAN TRANSPORT IN FIVE AFRICAN CITIES

Transport Mode	Abidjan	Dakar	Lagos	Kinshasa	Niamey
Walking	9%	23%	18%	25%	70%
Informal transportation	45%	52%	60%	50%	20%
Formal transport	14%	12%	10%	15%	5%
Individual taxi	6%	5%	7%	5%	3%
Two-wheelers	3%	2%	4%	5%	2%
Other	23%	6%	1%	–	–

presents the formulation of the model. Section VI provides a theoretical analysis of the model. Section VII to Section X report computational experiments, results, sensitivity analysis, and comparison with related studies. Section XI discusses limitations and scalability. Section XII concludes and outlines future research.

II. LITERATURE REVIEW

For over 50 years, the Transit Network Design Problem (TNDP) [8] has been an important topic of operations research and transportation planning. The main goal of the TNDP is to find the best set of transit routes and their service frequencies that will make the system work, as well as possible, while still meeting operational and resource limits [8], [9].

The mathematical formulation of TNDP usually has decision variables for choosing a route, setting a frequency, and sometimes scheduling vehicles. Some of the limits are the budget, the size of the fleet, the need to meet demand, and the quality of service. Exact techniques [8], [9], heuristic and metaheuristic methods [10]–[13] are some of the ways to solve TNDP.

The optimization of multimodal transportation systems has received substantial attention in the operations research literature. This section offers a critical evaluation of pertinent contributions, emphasizing gaps that motivate the present study.

Drummond et al. [14] examined organizational issues in multimodal air freight systems, concentrating on the dynamics of supply and demand. Their research focuses on forecasting modal distribution and reducing expenses in freight transportation. Nonetheless, the study does not provide a comprehensive analysis of passenger multimodality and arrival time forecasting.

Pawel et al. [15] suggested a mixed linear programming method to improve supply chains from the point of view of logistics providers who use more than one mode of transportation. Their model takes into account resources, inventory, and transportation parameters, which means it has a lot of room for cost savings. But it doesn't include all the many ways that passengers can get about in its full route planning.

Gonzalez et al. [16] proposed a data-driven methodology for assessing multimodal public transport networks, emphasizing travel chaining and spatial similarity analysis. This method gives useful information about how people move around, but it doesn't take into account the limitations of different types of transportation, which makes it less useful for planning multimodal transportation.

Akinlo Mogbojuri et al. [17] used linear programming to set up the schedules for Bus Rapid Transit (BRT) systems in Lagos. This cut down on wait times, but it didn't fully take into account how demand changes from day to day. Their method could help with traffic management, but it doesn't take into account the problems that come with integrating different modes of transportation.

Ma et al. [18] put forward a schedule optimization model that takes into account how many people want to travel at different times and how long it takes to get from one stop to another. The goal is to get as many passengers as possible by using a preference-based passenger selection model. Nonetheless, the study insufficiently tackles obstacles peculiar to urban mobility.

Sezgin et al. [19] used linear goal programming to improve bus frequencies and fleet size in Antalya. This cut down on wait times and kept operating costs in check. Even if it is relevant, the approach does not take into account changes in demand or problems in integrating several modes of transportation.

Das Gupta et al. [20] devised a two-stage linear programming model to make the Shanghai metro system's schedules more energy-efficient. They did this by using braking energy, which improved efficiency by more than 19%. But their approach is based on linearized demand functions and doesn't take into account how to combine different modes of transportation.

Berhan [21] developed a linear programming model for assigning buses to routes in Addis Ababa across four time shifts. The model optimizes fleet use under fixed demand assumptions. Its application is constrained by its neglect of urban mobility-specific limitations and multimodal integration.

In the context of multimodal urban transit coordination, Huang et al. (2022) [22] develop a two-stage framework combining user equilibrium passenger assignment with a bi-objective MILP to synchronize subway and bus timetables across a double-layer network, validated on Beijing's system with real AFC data. Their results demonstrate a 32.6% improvement in intermodal transfer synchronization during peak hours with negligible service quality loss, underscoring the potential of integrated timetable optimization across transit modes.

Addressing operational efficiency within urban rail networks, Eslami et al. (2025) [23] propose a real-time timetable optimization model that jointly considers skip-stop strategies, train speed profile selection, and stochastic passenger demand across multiple synchronized metro lines, solved via a multi-agent deep reinforcement learning algorithm. Applied to Tehran's metro, the approach yields cost reductions of up to 14.9% through speed profile optimization, highlighting the benefits of adaptive, network-level scheduling — though the framework remains limited to rail and does not extend to bus or other feeder modes.

Behiri, Ozturk, and Belmokhtar-Berraf [24] study urban freight transport by rail: they exploit existing suburban rail infrastructure to move goods instead of relying solely on road haulage. The study identifies several urban-freight decision problems and focuses on a commuter line where each station may serve as a loading/unloading platform and commodity flows ("boxes") with known origin–destination pairs must be scheduled over the day. The authors formulate a MILP whose primary objective is to minimise total waiting time of deliveries subject to operational constraints. Reported experiments indicate that moderate-size instances are solved in very short CPU times; the illustrative scale in our table (ten stations, $\sim$2.3 s order of magnitude) reflects that regime. The model is rail-centric and does not integrate road, water, or informal services; it therefore supports our positioning that prior MILP work often emphasises a single mode or formal infrastructure, whereas our formulation explicitly couples formal and informal options in one optimisation model.

Mnif and Bouamama [25] present a multi-objective mixed-integer formulation for multimodal transportation network planning, at the IEEE International Conference on Service Operations and Logistics, and Informatics (SOLI, 2017). The work targets strategic or tactical design of multimodal networks and balances several performance criteria—in line with the "cost & efficiency" positioning in our comparative table—rather than a single user-centric travel-time objective. Their setting is representative of moderate-size synthetic or stylised instances (the companion table cites runs on the order of a few seconds and networks with up to about fifteen stops), which is smaller in scale than the experimental instances evaluated in this study (5 to 24 stops). Methodologically, the study belongs to the stream of multi-objective MILP models for multimodal systems; it therefore offers a useful contrast to our single-objective travel-time minimisation under explicit formal–informal flows, and could be cited when discussing future multi-objective extensions (time, cost, emissions) mentioned in our conclusion.

EDI et al. [26] developed a mixed-integer linear programming (MILP) framework to enhance formal public transportation systems in urban environments. While their approach effectively tackles formal transport optimization, it fails to include the informal sector, which is prevalent across several emerging cities.

A prevalent weakness in these research is their singular emphasis on formal transport systems or single-mode optimization, overlooking the integration of informal transport sectors that are essential to urban mobility in developing countries. The present study addresses this gap through a single MILP formulation that explicitly models both formal and informal transportation sectors.

III. Problem Description

Optimizing multimodal urban transportation systems is critical in developing cities where formal and informal services coexist. The problem addressed here consists of determining a minimum-time itinerary for a user traveling from origin O to destination D on a public transit network that may include both formal and informal modes.

The two types of transportation have different ways of working. The formal sector is made up of legally recognized businesses that offer scheduled services with set routes and stops. People can only get on cars at certain stops that have been set up ahead of time. The informal sector is made up of unregulated businesses that offer flexible, on-demand services without set schedules or stops. They respond directly to passenger requests.

Most public transit systems include three forms of infrastructure: roads, water (lagoons), and rail. This study formulates a comprehensive mathematical model that systematically incorporates both formal and informal sectors into a cohesive optimization framework. The model is validated using empirical data from Abidjan, Côte d'Ivoire, where both transportation sectors are effectively represented and cohabit. There are structured operators providing public transport services (scheduled buses, water buses, and urban trains) in the formal sector. The informal sector offers the same types of transportation (road, water, and rail), but the operators are neither structured or regulated.

The main goal is to cut down on the amount of time users spend in the transportation network, which includes time spent in vehicles, time spent waiting at stops, and time spent switching between modes and sectors.

IV. Mathematical Formulation

This section introduces the mixed-integer linear programming (MILP) model for optimizing multimodal urban transportation networks. The model minimizes total user trip time subject to operational constraints of both formal and informal sectors. Model assumptions, parameters, and decision variables are defined first, followed by the objective function and constraints.

A. Model Assumptions

The following assumptions reflect multimodal public transportation in an urban setting:

- Assumption 1, Multimodality: Users may freely combine formal and informal transport modes.
- Assumption 2, Single choice: At each stop, a user can only select one mode and one route.
- Assumption 3, Imposed waiting: If a user arrives before the next mode's scheduled departure time, they must wait until that time.
- Assumption 4, Single change: At a given time t, only one mode change is allowed per stop.
- Assumption 5, Flexibility: The total number of mode changes depends on the selected route.
- Assumption 6, Unidirectional path: The journey follows a direct path without loops.
- Assumption 7, Infrastructure: Formal modes share designated stops; informal modes operate independently without set stops.

- Assumption 8, Uniform walking speed: All users walk at a constant speed. For access time calculations, all users walk at the same speed (for example, 5 km/h).
- Assumption 9, Cross-sectoral servitude: There is a minimum required distance between formal and informal stops, which varies according to factors such as stop size, capacity, geographic location, and other urban considerations. The model accounts for these differences without explicitly identifying their causes.
- Assumption 10, Transfer points: Connections between sectors happen at the boundaries of easement areas, which means that people have to walk between them.

B. Terminology

In this section, we clarify some key terms essential for the comprehension of the model:

- Stop: A designated point where public transportation vehicles stop so that people can get on or off.
- Mode: A specific type of public transportation that a passenger uses, like a bus, a train, or a water bus.
- Segment: A part of the network that goes from one stop to the next.
- Trip: A continuous journey that a user takes without stopping, using the transportation mode. It can have one or more segments.
- Itinerary: A complete sequence of trips allowing a user to travel from an origin to a destination, possibly using more than one mode of transportation.
- Formal sector: The regulated transportation sector where all operators are officially registered enterprises that are subject to public regulation. Scheduled buses, light rail (metro), and water shuttles are some examples.
- Informal sector: The part of the transportation industry that isn't regulated and where people or unregistered businesses undertake commercial transportation services. Gbaka, Wôrô wôrô, Warren, Trotro, and Danfo are all examples from Africa.

C. Model Parameters

In Table II, we introduce the main parameters of the model.

D. Decision Variables

The following binary decision variables, in Table III, are used to formulate the constraints and objective function.

All decision variables are binary: $X_{ij}^k, Y_i^{k,k'}, W_i^{k,k'} \in \{0,1\}$ for all admissible indices. The variables X_{ij}^k select at most one mode per arc; $Y_i^{k,k'}$ indicate a mode change (and associated waiting) at stop i; and $W_i^{k,k'}$ record a cross-sector transfer when modes k and k' belong to different sectors. Constraint (11), Constraint (12) to Constraint (13), and Constraint (14) couple these variables so that waiting and sector-transfer times are activated only when consecutive arcs use different modes (and, for W, different sectors).

TABLE II. MAIN MODEL PARAMETERS

Symbol	Description
N	Set of stops.
K	Set of modes of transportation.
F	All transport sectors.
o, d	Point of departure and point of arrival.
$Change_{Max}$	Maximum number of mode changes allowed.
$t_{i,j}^k$	Time spent in the vehicle on the journey from stop i to stop j using mode k.
$u_i^{k,k'}$	Waiting time for mode change from mode k to mode k' at stop i.
$\gamma_i^{k,k'}$	Walking time required to transfer from mode k to mode k', and between sectors, at stop i.
$Z^{k,\bar{f}}$	Parameter equal to 1 if mode k belongs to the informal sector.
$Z^{k,f}$	Parameter equal to 1 if mode k belongs to the formal sector.

TABLE III. DECISION VARIABLES

Variable	Description
X_{ij}^k	Binary variable equal to 1 if the edge segment (i, j) is used with transport mode k, and 0 otherwise.
$Y_i^{k,k'}$	Binary variable equal to 1 if the user at stop i has changed mode from k to k', and 0 otherwise.
$W_i^{k,k'}$	Binary variable equal to 1 if there is a change of mode and a change of sector from k to k' at stop i, and 0 otherwise.

V. FORMULATION OF THE MODEL

A. Objective Function

The objective is to minimize the total travel time for users, which comprises three components [see Eq. (1)]:

$$\min Z = Z_1 + Z_2 + Z_3 \tag{1}$$

where, Z_1 represents the total time spent in vehicles, Z_2 represents the total waiting time at stops for connections, and Z_3 represents the total transfer time between sectors, with Eq. (2) and Eq. (3):

$$Z_1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} X_{i,j}^k \cdot t_{i,j}^k \tag{2}$$

$$Z_2 = \sum_{i \in N} \sum_{k \in K} \sum_{k' \in K} Y_i^{k,k'} \cdot u_i^{k,k'} \tag{3}$$

Taking the informal sector into account introduces possible changes between sectors (formal $\leftrightarrow$ informal). These transitions, constrained by formal stops, require walking between stops in different sectors. This additional time is modeled by Eq. (4):

$$Z_3 = \sum_{i \in N} \sum_{k \in K} \sum_{k' \in K} W_i^{k,k'} \cdot \gamma_i^{k,k'} \tag{4}$$

where, $\gamma_i^{k,k'}$ represents the transfer time between modes k and k' from different sectors at stop i.

B. Constraints

The constraints of the model are represented as follows:

- Single mode transportation by segment.

$$\sum_{k \in K} X_{i,j}^k \leq 1 \qquad \forall i, j \in N \times N \tag{5}$$

Constraint (5) limits the user to selecting only one mode of transport $k \in K$ for each segment $[i, j]$ travelled, from among several modes available on that section.

- Changing modes

$$\sum_{k \in K} X_{i,j}^k + \sum_{k \in K} X_{j,h}^k \leq 2 \qquad \forall i, j, h \in N \times N \times N \tag{6}$$

Eq. (6) expresses the possibility for the user to arrive at a stop j using a mode of transport k, then select another mode of transport K' to continue the journey. Typically, the mode used on segment $[i, j]$ may differ from that used on segment $[j, h]$. The upper bound of 2 of the constraint ensures that at most two modes are involved in a user's arrival and departure at a stop.

- Flow conservation

Let o be the departure stop user's and d the destination stop of the user:

$$\sum_{j \in N} \sum_{k \in K} X_{o,j}^k = 1 \tag{7}$$

$$\sum_{i \in N} \sum_{k \in K} X_{i,j}^k = \sum_{i \in N} \sum_{k \in K} X_{j,i}^k \qquad \forall j \in N \tag{8}$$

$$\sum_{j \in N} \sum_{k \in K} X_{j,d}^k = 1 \tag{9}$$

Eq. (7) to Eq. (9) enforce continuity, contiguity, and completeness of the user's journey through classic flow conservation:

- A unit flow departs from the origin o [Eq. (7)], ensuring entry into the network.
- Incoming and outgoing flows balance at all intermediate nodes [Eq. (8)], ensuring progress through the network.
- A unit flow arrives at the destination d [Eq. (9)], ensuring exit from the network.
- Capacity constraints

$$\sum_{i \in N} \sum_{k \in K} \sum_{k' \in K} Y_i^{k,k'} \leq Change_{Max} \tag{10}$$

The capacity constraint in Eq. (10) limits the number of possible mode changes in the model. This number is represented by the system parameter $Change_{\max}$. This parameter can be adjusted as needed depending on the size of the network.

- Constraint linking X and Y variables.

$$\sum_{i \in N} X_{i,j}^k + \sum_{h \in N} X_{j,h}^{k'} \leq 1 + Y_j^{k,k'} \qquad \forall j \in N,\ \forall k, k' \in K \times K,\ k \neq k' \tag{11}$$

Constraint (11) links the X variables (travel on arcs) to the Y variables (waiting at stops). A wait at stop j is possible only when arrival and departure segments at j use different modes.

- Cross-sector transfer constraints

$$W_j^{k,k'} \geq Y_j^{k,k'} + Z^{k,\bar{f}} + Z^{k',f} - 2 \qquad \forall j \in N,\ \forall k, k' \in K \times K \tag{12}$$

$$W_j^{k,k'} \geq Y_j^{k,k'} + Z^{k,f} + Z^{k',\bar{f}} - 2 \qquad \forall j \in N,\ \forall k, k' \in K \times K \tag{13}$$

These equations [Constraint (12) and Constraint (13)] model the changes from formal to informal and informal to formal, respectively.

- Constraint linking W and Y variables.

$$\sum_{k \in K} \sum_{k' \in K} W_j^{k,k'} \leq Y_j^{k,k'} \qquad \forall j \in N,\ \forall k, k' \in K \times K \tag{14}$$

Constraint (14) links changes to the corresponding waiting times.

VI. Model Analysis

The suggested model offers a robust method for optimizing multimodal urban transport systems, particularly in developing nations where formal and informal transport coexist. It is founded on a mixed-integer linear programming (MILP) concept, presenting a systematic and rigorous approach to tackling the issues associated with multimodal transportation. The key components of the model are delineated below.

A. MILP Formulation and Flow Constraints

The model's core is its MILP formulation, which is especially adept at addressing the complexity and combinatorial aspects of multimodal transport optimization. The model guarantees continuity within the transportation network through flow conservation constraints. These constraints streamline the mathematical framework and enhance computational efficiency by narrowing the search space to feasible paths.

- Flow conservation: The model enforces flow conservation at the origin, intermediate stops, and destination. This guarantees that the user enters, traverses, and exits the network consistently, mirroring actual trip structure.
- Simplification through structure: The implementation of flow constraints streamlines the problem formulation, rendering it both computationally feasible and conceptually clear. This simplicity facilitates expansion while preserving the necessary flexibility to manage intricate multimodal scenarios.

B. Applicability to Multimodal Transport Systems

The model's primary strength lies in its broad application to various multimodal transport networks. The model, while developed with data from Abidjan, Côte d'Ivoire, is fundamentally adaptable to many metropolitan environments and transportation systems. The primary characteristics are as follows:

- Flexibility for different modes of transport: The model accommodates several transport modes, encompassing both formal and informal sectors, including road and lagoon possibilities. This adaptability guarantees its pertinence across diverse urban landscapes.
- By restricting mode alterations and enhancing transitions, the model can accommodate medium to large networks without considerable degradation in computing performance.
- Structured and formal design: The emphasis on formal and informal transportation modalities facilitates the establishment of a clear and dependable optimization framework. This methodology addresses the demand for systematic solutions in emerging urban areas.

C. Simplicity and Practical Applicability

The model achieves an equilibrium between mathematical precision and practical relevance. The straightforward design, enabled by clearly delineated parameters, variables, and constraints, promotes implementation. The incorporation of waiting times, mode-change limits, and operational constraints increases applicability in practical situations.

- User-centered objective: The emphasis on reducing overall journey time demonstrates a user-centered approach, consistent with the core aim of enhancing public transportation efficiency and user experience.
- Operational feasibility: The incorporation of pragmatic assumptions, including mode change constraints and predetermined stops, guarantees that the model accurately reflects the operational attributes of both formal and informal transportation systems.

D. Potential for Expansion

The model, while currently tailored for transportation systems in developing cities, possesses a modular design that offers a robust basis for future improvements. For instance:

- Incorporation of transportation expenses: Subsequent iterations may enhance the model by integrating both formal and informal transportation cost alternatives, so broadening its scope and relevance to hybrid systems.
- Advanced optimization strategies, including cut generation and boundary tightening, could enhance computational performance, enabling the model to manage larger networks.

In summary, the suggested model, utilizing the MILP approach, offers a flexible, scalable, and computationally efficient framework for optimizing multimodal transportation systems. Utilizing flow constraints, the model streamlines the problem, while preserving its adaptability and relevance across diverse situations. The simplicity, coupled with its robust mathematical base, renders it an invaluable instrument for urban transportation planning, especially in developing cities aiming to enhance the efficiency of their formal transportation systems.

VII. Computational Experiments and Data

A. Experimental Setup

The integrated MILP was implemented in Python and solved with IBM ILOG CPLEX Optimizer version 22.11 [27]. All computational experiments reported in this study use CPLEX on a workstation with an Intel Core i7-8650U CPU (2.11 GHz) and 16 GB of RAM. The model's multimodal characteristics are evidenced by the incorporation of diverse road transport modes and lagoon transport services.

B. Data Sources and Characteristics

Calibration of the computational experiments combined published mobility information for Abidjan [3], [28] with detailed scrutiny of how operators providing public transport services in the city conduct scheduled operations in practice. Access to non-public operational material was facilitated through a confidentiality agreement with the Information and Applications Research Team (ERIA). Observations drawn from that context (e.g., service structure, stop spacing, segment speeds over a multi-month horizon in 2023) were used solely to estimate distributional ranges for synthetic instance generation; no operator-identifying dataset is reproduced here, and the MILP is not solved on a published operational network graph.

The calibration for formal-sector operators providing public transport services reflects five mode types, heterogeneous stop spacing across lines, and urban link speeds up to 60 km/h, as inferred from those observations. Empirical ranges and moments were derived for segment travel times, inter-stop distances, and connection waiting. Informal-sector parameters (flexible stops, sector-specific transfer penalties $\gamma_i^{k,k'}$) were set from the same published Abidjan mobility sources and expert judgment, because comparable operational traces for informal operators were not available under the same ERIA-mediated access regime.

C. Instance Generation

An instance generator based on Python was created to produce test instances that emulate real-world operating characteristics. The generator yields the subsequent parameters:

- `Time_mode`: Duration of in-vehicle travel for a user utilizing a certain mode of transport (minutes)
- `Time_wait`: Duration of waiting at stops for connections (minutes)
- `Number_mode`: Count of available transport modes
- `Number_change`: The maximum permissible mode changes within the network.
- `Number_stop`: The total number of stops permitted for the journey.

The generator draws parameters from uniform or truncated-normal distributions whose support was calibrated to empirically informed statistics (minimum, maximum, mean, and standard deviation of segment times and stop spacing). For each synthetic instance, arc travel times, waiting times, and the number of active arcs are sampled independently subject to network size and $Change_{Max}$. Representative calibrated ranges are:

- Travel time (`Time_mode`): uniform on $[2, 15]$ minutes (consistent with in-vehicle segment durations observed among operators providing public transport services in Abidjan)
- Waiting time (`Time_wait`): uniform on $[0, 30]$ minutes at connection stops
- Number of stops: instances with 5, 10, 15, 20, or 24 stops
- Maximum mode changes ($Change_{Max}$): 5 or 10 per instance family
- Number of modes: fixed at five (formal and informal variants per infrastructure type)

Each row in Table IV corresponds to ten random instances per ($Change_{Max}$, stop count) combination, for a total of 100 solved instances.

Table IV summarizes the characteristics of the instances. The columns denote: (1) #Stops: the number of stops per instance, (2) #Var. Total: total number of decision variables, (3) #Var. Bin.: number of binary variables, (4) #Cont: total number of constraints, (5) #Arcs: number of arcs within the interval [min, max], (6) #Inst. (ch max=5): number of instances with a maximum of 5 mode changes, and (7) #Inst. (ch max=10): number of instances with a maximum of 10 mode changes.

The dataset illustrates the model's scalability across various problem sizes. The complexity of the instance increases polynomially with the number of stops, as seen by the rising quantity of variables and constraints. The existence of cases with two distinct maximum change values (5 and 10) facilitates comparative performance evaluation under differing flexibility limitations.

D. Data Validation

Model validation was performed via a rigorous methodology. A meticulously selected test dataset was produced utilizing statistical techniques that guarantee distributions reflective of real-world scenarios. The preprocessing stages encompassed data normalization and cleansing to guarantee data quality and pertinence. Established performance criteria were utilized to assess the model on the test set, contrasting the results achieved with established optimal values when applicable.

The validation method confirmed the model's robustness and laid the groundwork for subsequent study. The validated model was then exercised on synthetic Abidjan-style network instances, producing findings that illustrate practical applicability for corridor-level urban transportation planning.

VIII. Computational Results

This section analyzes computational results from solving the proposed MILP model, focusing on the relationship between processing time and problem size (number of stops, active arcs, maximum mode transitions, and proportion of forbidden arcs).

A. Effect of Model Complexity

The duration of processing demonstrates a significant positive link with the complexity of the challenge. Instances with a greater number of stops, active arcs, and elevated maximum mode change limitations necessitate extended computation durations. The quantity of binary variables and constraints directly influences computing effort, as the solver is required to navigate an exponentially bigger solution space. This relationship is anticipated in mixed-integer programming, where the magnitude of the problem substantially affects solution time.

B. Influence of Network Size and Mode Change Limits

Fig. 1 depicts the correlation between processing duration and network attributes. The mean calculation duration increases with the quantity of stops for each value of $Change_{Max}$. This tendency is more evident when $Change_{Max} = 10$, suggesting that the increased flexibility in mode changes significantly broadens the solution space compared to $Change_{Max} = 5$.

With $Change_{Max} = 5$, the augmentation in processing time relative to the network size is less pronounced, indicating that restricting the mode changes has computational advantages, while possibly limiting solution quality. Scenarios with an increased number of stops invariably necessitate extended processing durations, irrespective of the maximum mode change limit. These observations underscore the necessity of judiciously choosing $Change_{Max}$ in accordance with problem-specific attributes and computing resource limitations.

TABLE IV. INSTANCE CHARACTERISTICS FOR FORMAL AND INFORMAL TRANSPORT SECTORS

#Stops	#Var. Total	#Var. Bin.	#Cont	#Arcs [min, max]	#Inst. (ch max=5)	#Inst. (ch max=10)
5	375	375	689	[60, 90]	10	10
10	1,000	1,000	2,224	[269, 405]	10	10
15	1,875	1,875	5,359	[637, 947]	10	10
20	3,000	3,000	10,844	[1,137, 1,709]	10	10
24	4,080	4,080	17,428	[1,663, 2,487]	10	10

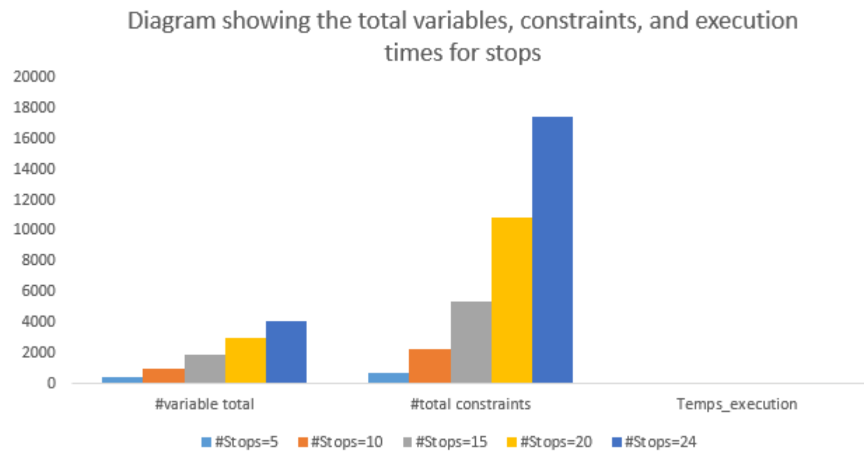

Fig. 1. Execution time and active arcs versus number of stops.

TABLE V. CHARACTERISTICS OF GENERATED INSTANCES AND CORRESPONDING CPLEX SOLVE TIMES (22.11, SAME HARDWARE AS SECTION VII-A)

Instance Family	#Stops	#Modes	#Change_max	%Forbidden Arcs	#Arcs Active	Process Time (seconds)
1	5	5	5	10	90	0.483
2	5	5	10	15	85	0.515
3	10	5	5	69	381	0.918
4	10	5	10	45	405	0.871
5	15	5	5	103	947	1.325
6	15	5	10	158	892	1.437
7	20	5	5	191	1,709	2.409
8	20	5	10	286	1,614	2.681
9	24	5	5	273	2,487	4.449
10	24	5	10	414	2,346	4.068

C. Processing Time Analysis

Table V summarizes computational results for instances of different complexity levels. The data shows clear relationships between the problem characteristics and the processing time.

The following key trends emerge from the analysis.

- Network size effect: Processing time increases with the number of stops and the number of active arcs. Larger instances with more arcs require greater computational resources, with processing time scaling approximately linearly with network size (see Figure 1).
- Mode change flexibility: Increased maximum permitted mode changes ($Change_{Max}$) broaden the solution space, leading to longer computation times. The effect is more pronounced for larger networks.
- Forbidden arcs: An increased proportion of prohibited arcs diminishes the feasible solution space while augmenting constraint complexity, resulting in variable processing times contingent upon the particular arc arrangement.

Fig. 1 illustrates that the processing time increases almost linearly with the number of stops and active arcs, signifying favorable scalability for medium-sized issue situations. This linear relationship suggests the model remains computationally tractable for practical urban transportation networks.

D. Comparative Performance Analysis

Fig. 2 compares execution times across different model configurations, illustrating how various parameter combinations affect computational performance. The analysis reveals that parameter interactions significantly influence solution time, with certain combinations leading to more efficient problem-solving.

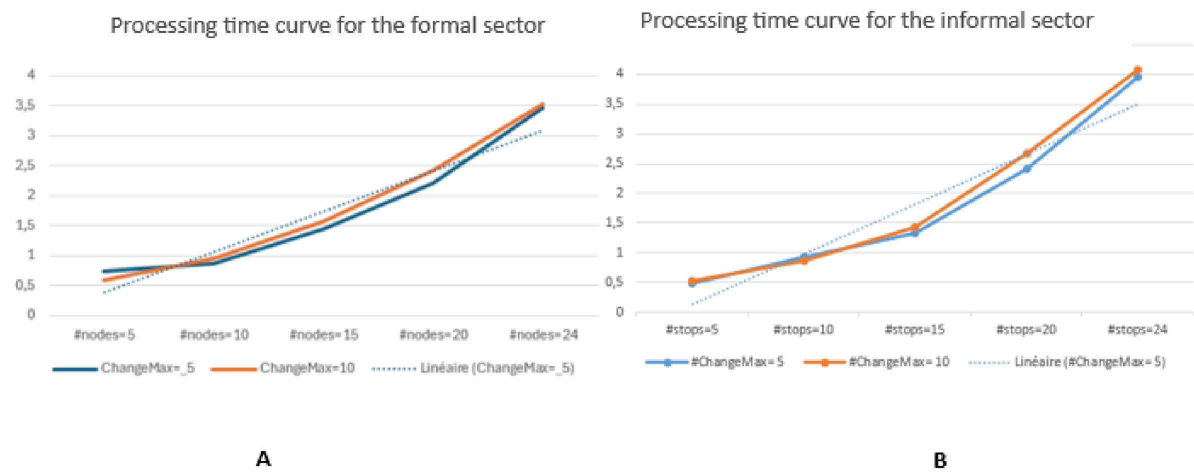

Fig. 2. Comparison of execution times across different model configurations.

IX. Sensitivity Analysis

This section reports sensitivity analysis on the impact of key parameters on MILP solution time, including $Change_{Max}$ (maximum mode changes), the number of modes, the number of stops, the percentage of forbidden arcs, and the number of active arcs.

A. Impact of Maximum Mode Changes

Increased $Change_{Max}$ significantly impacts processing time by broadening the feasible solution space. This effect is observed in instances with 5, 10, 15, and 20 stops. In small instances (5 stops), increasing $Change_{Max}$ has minimal impact on processing time, as the solution space remains tractable. For larger instances (15-24 stops), higher $Change_{Max}$ values resulted in significant increases in computation time, with processing time nearly doubling when $Change_{Max}$ grows from 5 to 10 for 24-stop instances.

This observation indicates that although increased mode changes flexibility improves solution quality, it incurs a computational cost that rises exponentially with network size. Practitioners must balance solution quality demands with available computational resources while setting the value of $Change_{Max}$.

B. Impact of Number of Transport Modes

The number of available transport modes directly influences model complexity by increasing the number of feasible options at each stop. Table V summarizes results for instances with 5 modes. Empirical analysis reveals that doubling the number of modes increases processing time by approximately 30-50% for medium-sized instances (10-15 stops). This moderate increase suggests that the model scales reasonably well with mode diversity, making it suitable for networks with multiple transport options.

C. Effect of Forbidden Arcs

The percentage of forbidden arcs introduces additional constraints that reduce the number of feasible routes while increasing computational complexity. The processing time exhibits variable behavior depending on the forbidden arc percentage. When the percentage of forbidden arcs exceeds 70%, the processing time increases slightly as the solver must navigate a more constrained solution space. However, moderate percentages (30-50%) can actually reduce processing time by eliminating infeasible paths early in the search process.

D. Travel Time Performance: Formal vs. Integrated Systems

Table VI presents a comparative analysis of travel time performance between formal-only and integrated (formal + informal) transport systems. The results demonstrate that combining formal and informal transport systems provides a more effective strategy for meeting urban mobility needs.

The analysis reveals significant structural limitations of formal-only transport systems in large African cities. While formal systems benefit from their organization, planning, and regulatory frameworks, they often fail to adequately serve all urban mobility needs, especially in peripheral regions or places with significant informal transit prevalence. The inflexibility of routes and sporadic service on specific lines diminish their efficacy as independent systems.

In contrast, informal transit provides flexibility, adaptability, and extensive access to underserved metropolitan regions, delivering responsive, locally tailored services. Nonetheless, in the absence of regulation, this sector experiences dependability challenges, safety issues, and insufficient alignment with overarching urban transportation objectives.

The intentional amalgamation of formal and informal transport elements into a cohesive multimodal framework mitigates these constraints. Computational results show that integrated systems reduce total journey time by up to 53% relative

TABLE VI. Travel Time Comparison: Formal-Only versus Integrated Systems

Instance Type	#Changes Max	#Stops	Objective Value	#Constraints	Reduction (%)
Formal only	5	5	23	308	–
Integrated	5	5	21	689	8.7
Formal only	5	10	16	1,463	–
Integrated	5	10	10	2,224	37.5
Formal only	5	15	16	4,218	–
Integrated	5	15	12	5,359	25.0
Formal only	5	20	19	9,323	–
Integrated	5	20	10	10,844	47.4
Formal only	5	24	19	15,603	–
Integrated	5	24	9	17,428	52.6

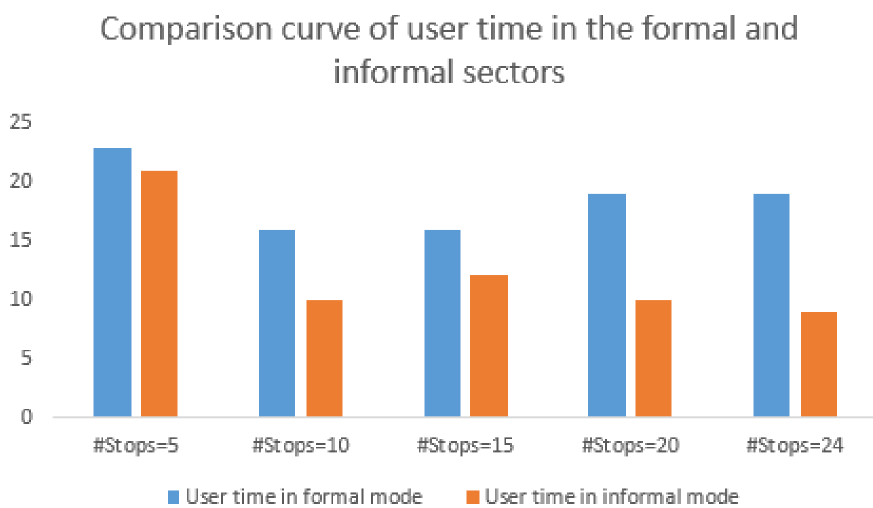

Fig. 3. Comparison of user travel time: Formal-only versus integrated (formal + informal) systems.

to formal-only configurations on the largest tested instances (24 stops; objective values 19 versus 9). Improvements are instance-dependent: for 15-stop cases the reduction is 25% (16 to 12), and Table VI reports gains between 25% and 52.6% across the tested stop counts. The headline figure of 53% therefore denotes the maximum observed improvement, not an average over all instances.

This performance enhancement arises from: 1) augmented network coverage through informal sector integration, 2) diminished waiting periods by adaptive scheduling, 3) improved accessibility to remote regions, and 4) heightened system resilience to demand variability. These findings endorse policy proposals for acknowledging and systematically incorporating informal transit into modern urban mobility planning frameworks.

Fig. 3 clearly demonstrates the performance superiority of integrated systems across various network sizes. The findings indicate steady enhancements in transit time reduction, with the advantage amplifying for more extensive networks. This study advances mathematical modeling and optimization of multimodal transport systems, while also informing broader policy debates on inclusive governance and hybrid management of transport systems in fast growing African cities.

X. Comparative Analysis with Related Studies

This section situates the proposed MILP model relative to similar studies in the literature. The comparative analysis focuses on four key criteria: 1) model type and formulation approach, 2) optimization objective, 3) computational performance (processing time), and 4) case study complexity and scale.

A. Comparative Analysis

Table VII presents a detailed comparison with related studies. The proposed model exhibits several strengths relative to prior work.

- Travel time reduction: The integrated approach reduces total travel time by up to 53% on the largest tested instances (24 stops, 2,487 active arcs) compared with formal-only networks, and by 25–52.6% on smaller instances (Table VI). Edi et al. [26] address a related formal-sector MILP at comparable network size but without informal integration.
- Multimodal adaptability: Unlike Behiri et al. [24], who focus exclusively on rail transport, the present formulation integrates multiple transport modes (road, water, rail) from both formal and informal sectors.

TABLE VII. PERFORMANCE COMPARISON WITH RELATED STUDIES

Reference	Model Type	Optimization Criterion	Average Computation Time	Case Study Size
Proposed model	MILP with flow constraints	Total travel time	1.63 s (24 stops, 2,487 arcs)	Up to 24 stops
Edi et al. (2025) [26]	MILP with flow constraints	Total travel time	N/A (24 stops, 2,487 arcs)	Up to 24 stops
Mnif and Bouamama (2017) [25]	Multi-objective MILP	Cost and operational efficiency	3.47 s	Up to 15 stops
Behiri et al. (2016) [24]	MILP for urban rail transport	Minimized waiting time	2.3 s (10 stations, 150 arcs)	10 stations

Note. CPU times for the proposed model were obtained with CPLEX 22.11 on the hardware described in Section VII-A. Times reported for other studies used different solvers and platforms; cross-paper comparisons should be interpreted with caution.

- Computational efficiency: CPLEX solve times remain below 2 s for corridor instances up to 24 logical stops (maximum 1.63 s on the calibrated testbed), while handling explicit formal–informal sector coupling.
- Problem scale: Experiments cover corridor-level networks with up to 24 stops and 2,487 active arcs; city-wide networks would require further algorithmic development (see Section XI).

However, certain limitations and opportunities for future enhancement are identified:

- Multi-objective optimization: Unlike Mnif and Bouamama [25], our current formulation employs a single-objective function (travel time minimization). Future work could extend the model to multi-objective optimization, simultaneously considering travel time, cost, and environmental impact.
- Real-time data integration: Compared to Gonzalez et al. [16], the present formulation relies on static parameters without real-time data integration. Incorporating dynamic demand and real-time traffic conditions would enhance practical applicability.

B. Key Findings and Insights

The comparative analysis indicates that the proposed formulation offers a balanced combination of modeling transparency, computational tractability on tested instances, and relevance to developing cities where formal and informal sectors coexist.

The analysis of processing time determinants yields the following insights:

- The percentage of forbidden arcs significantly impacts processing time, as additional constraints reduce feasible solutions and increase computational effort. This relationship is consistent with theoretical expectations in constraint programming.
- While individual parameters may not show strong individual correlations, their collective interaction substantially influences computational performance. This highlights the importance of parameter optimization for efficient computation.

TABLE VIII. EXTENDED SCALABILITY EXPERIMENT (CPLEX 22.11): INTEGRATED MILP ON LARGE CALIBRATED INSTANCES (n = 25–100).

n	$\lvert V\rvert$	#Var.	#Constr.	Time (s)	Status
25	50	5,345	24,394	0.31	OPTIMAL
40	80	11,722	28,219	0.11	OPTIMAL
55	110	20,081	32,901	0.16	OPTIMAL
70	140	30,158	38,252	0.20	OPTIMAL
85	170	48,991	47,114	0.47	OPTIMAL
100	200	47,188	51,821	0.40	OPTIMAL

- Processing time grows with network size (Fig. 1) but remains manageable on the tested hardware for instances up to 24 stops; supplementary tests up to $n = 100$ are reported in Section XI.

XI. LIMITATIONS AND SCALABILITY

All computational results in this study were obtained with IBM CPLEX 22.11 on the workstation described in Section VII-A. The main study (Section VII to Section X) evaluates synthetic instances with 5 to 24 logical stops and up to 2,487 active arcs; the maximum observed solve time on a 1,000-instance calibrated testbed was 1.63 s. This scale is representative of sub-network or corridor planning in Abidjan but does not, by itself, demonstrate behavior on substantially larger networks. To address scalability beyond the 24-stop cap, an extended set of 100 instances was generated with the same calibration rules applied to operators providing public transport services in Abidjan, with logical stop parameters $n \in [25, 100]$ and network sizes $|V| = 2n$ (50 to 200 nodes).

Table VIII reports representative cases; all 100 runs returned an `OPTIMAL` status. Solution times increased with $|V|$ and model size but remained below 0.69 s on the largest instance ($n = 100$, $|V| = 200$, 47,188 binary variables, 51,821 constraints). Fig. 4 shows that median CPU time grows gradually with n and $|V|$ in this range, without the exponential blow-up feared for city-wide deployment at corridor scale. These results indicate that the present formulation remains tractable for medium-to-large sub-networks under static demand, while full metropolitan systems (hundreds of stops, dynamic demand) would still require decomposition, matheuristics, or rolling-horizon schemes [27].

The formulation relies on static origin–destination demand, deterministic arc travel times, and parameters fixed for the

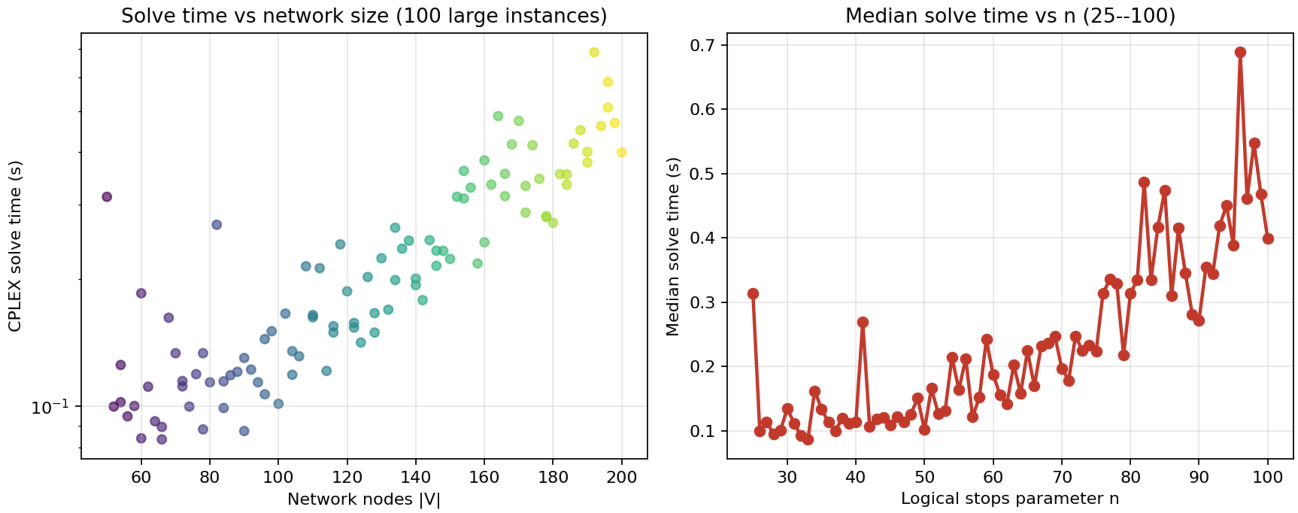

Fig. 4. CPLEX solve time versus logical stop parameter n and network size $|V|$ on 100 large instances ($n = 25$–100).

planning horizon. Extensions incorporating stochastic demand, time-dependent travel times, and real-time traffic or dispatch data would increase modeling fidelity but require additional variables, constraints, or rolling-horizon solution schemes beyond the present static MILP.

XII. Conclusion

This study presents a mixed-integer linear programming (MILP) model for optimizing multimodal urban transportation networks that explicitly accounts for both formal and informal transport sectors. The model was calibrated to operational patterns of operators providing public transport services in Abidjan and evaluated on synthetic instances, demonstrating practical relevance in developing city settings.

A. Main Contributions

The primary contributions of this research are threefold:

- Unified modeling framework: A MILP formulation integrates formal and informal transport sectors within a single optimization model, addressing a gap in the existing literature.
- Performance improvement: On synthetic instances calibrated from operators providing public transport services in Abidjan, integrated systems reduce total travel time by up to 53% relative to formal-only configurations (maximum on 24-stop cases; lower gains on smaller networks).
- Computational efficiency: CPLEX solves corridor instances ($n \leq 24$) in at most 1.63 s and an extended set with $n \in [25, 100]$ and $|V| \leq 200$ in at most 0.69 s (Section XI), while metropolitan-scale dynamic planning remains future work.

B. Key Findings

Sensitivity analysis reveals that processing time is primarily influenced by: 1) the number of stops, 2) the maximum number of mode changes allowed ($Change_{Max}$), and 3) the number of active arcs. Higher values of $Change_{Max}$ and number of modes increase computational complexity by expanding the solution space, while higher percentages of forbidden arcs tighten constraints, adding to computational complexity. Despite these challenges, the model remains solvable within seconds on the tested instance sizes, with processing time increasing with network size as expected for MILP (Section XI).

C. Future Research Directions

Several promising directions for future research are identified:

- Advanced solution techniques: Employing commercial solvers such as CPLEX [27] or Gurobi, along with specialized techniques such as cut generation and bound tightening, could improve computational efficiency for larger problem instances.
- Multi-objective optimization: Extending the model to simultaneously optimize travel time, transportation costs, and environmental impact would enhance practical relevance.
- Dynamic and stochastic elements: Incorporating real-time demand fluctuations, stochastic travel times, and dynamic network conditions would increase model applicability to operational planning.
- Cost integration: Incorporating transportation cost parameters for different routes and modes would enable comprehensive cost-benefit analysis and support policy decision-making.
- Network design extensions: Expanding the model to include network design decisions, such as stop location and route planning, would provide a more comprehensive optimization framework.

The proposed model provides a solid foundation for optimizing multimodal urban transportation systems in developing cities, where formal and informal sectors coexist. The demonstrated performance improvements and computational tractability make it a valuable tool for urban transportation planners and policymakers seeking to enhance mobility while recognizing the complementary roles of both transport sectors.

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