

A Modified Method for Order Reduction of Large Scale Discrete Systems

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Abstract— In this paper, an effective procedure to determine the reduced order model of higher order linear time invariant discrete systems is discussed. A new procedure has been proposed for evaluating Time moments of the original high order system. Numerator and denominator polynomials of reduced order model are obtained by considering first few redefined time moments of the original high order system. The proposed method has been verified using numerical examples.

Keywords- order reduction; eigen values; large scale discrete systems; modeling.

I. INTRODUCTION

The rapid advancements in science and technology led to extreme research in large scale systems. As a result the overall mathematical complexity increases. The computational procedure becomes difficult with increase in dimension. Therefore high-order models are difficult to use for simulation, analysis or controller synthesis. So it is not only desirable but necessary to obtain satisfactory reduced order representation of such higher order models. Here the objective of the model reduction of high order complex systems is to obtain a Reduced Order Model(ROM) that retains and reflects the important characteristics of the original system as closely as possible.

Several methods are available in the literature for large-scale system modeling. Most of the methods based on the original continued fraction expansion technique [1, 2] fail to retain the stability of the original systems in the reduced order models. To overcome this major drawback, alternative methods have been suggested but the common limitation of such extension is that in some cases, they may generate models of order even higher than that of the original system [1-3].

Modal-Padé methods [4] use the concept of the dominant poles and matching the few initial time moments of the original systems. The major disadvantage of such methods is the difficulty in deciding the dominant poles of the original system, which should be retained in the reduced order models. Retaining the poles closest to the imaginary axis need not be always the best choice. Sinha et al, [5] used the clustering technique but the serious limitation of this method is the number and position of zeros of the original system sometimes decides the minimum order of the reduced order model. The major disadvantage of such methods is in deciding the clusters of poles hence cannot generate unique models.

Some methods based on Eigen Spectrum which is the cluster of poles of high order system considered to derive the approximant. Pal *et al* proposed [6] pole-clustering using Inverse Distance Criterion and time-moment matching. Vishwakarma and R.Prasad [7,8] modified the pole clustering by an iterative method. The difficulty with these methods is in selecting poles for the clusters. Mukherjee [9], suggested a method based on Eigen Spectrum Analysis. The Eigen Spectrum consists of all poles of the high order system. The poles of the reduced model are evenly spaced between the first and last poles. Parmar *et al* [10] proposed a mixed method using Eigen Spectrum Analysis with Factor division algorithm to determine the numerator of the reduced model with known denominator. Parmar *et al* [11] proposed another mixed method using Eigen Spectrum Analysis equation and Particle Swarm method to find the reduced model. The methods based on eigen spectrum analysis cannot be applied to high order systems having complex poles. Saraswathi *et al* [17] proposed a method retaining some of the properties of original system based on eigenspectrum. The method can be applied for systems having both real and complex poles unlike the other existing methods [9-11].

Many methods are available for order reduction of high order continuous systems but very few are extended for discrete systems. The proposed method is extended to discrete systems using Tustin approximation [12] to maintain the static error constants identical in both discrete and continuous transfer functions. In the proposed reduced order method the Time moments and Markov parameters are redefined as function of Residues and poles for strictly proper rational transfer functions having real and complex poles. Poles of the Reduced Order Model (ROM) are selected by considering the highest contribution of each pole in redefined Time Moments (RTMs) and lowest contribution in Redefined Markov Parameters (RMPs).

II. PROPOSED METHOD

Let the original high order transfer function of a linear time invariant discrete system of n^{th} order be

$$G(z) = \frac{N(z)}{D(z)} = \frac{c_m z^m + c_{m-1} z^{m-1} + \dots + c_1 z + c_0}{z^n + d_{n-1} z^{n-1} + d_{n-2} z^{n-2} + \dots + d_1 z + d_0}; m < n \quad \dots(1)$$

Let the original high order transfer function of a linear time invariant discrete system of n^{th} order in continuous form using Tustin approximation be

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0}; m < n \dots (2)$$

where $D(s) = \prod_{i=1}^n (s + \lambda_i)$; n is the number of poles and $N(s) = \prod_{i=1}^m (s + \delta_i)$; m is the number of zeros, $\lambda_i = (\alpha_i \pm j\beta_i)$ and $\delta_i = (\varepsilon_i \pm j\gamma_i)$ of high order system $G(s)$. The poles and zeros may be real and/or complex. If they are complex, they occur in conjugate pairs.

The reduced order model of k^{th} order using proposed new algorithm in continuous form is defined as

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{p_r s^r + p_{r-1} s^{r-1} + \dots + p_1 s + p_0}{s^k + q_{k-1} s^{k-1} + q_{k-2} s^{k-2} + \dots + q_1 s + q_0}; k < n \dots (3)$$

where $D_k(s) = \prod_{i=1}^k (s + \lambda_i)$; k is the number of poles and $N_k(s) = \prod_{i=1}^m (s + \delta_i)$; $r \leq m$, r is the number of zeros, $\lambda_i = (\alpha_i \pm j\beta_i)$ and $\delta_i = (\varepsilon_i \pm j\gamma_i)$ of reduced order model $R_k(s)$. In the reduced model poles and zeros may be real and/or complex. If they are complex, they occur in conjugate pairs as mentioned for original system.

Using Tustin approximation the reduced order linear time invariant discrete model of k^{th} order is

$$R_k(z) = \frac{N_k(z)}{D_k(z)} = \frac{u_r z^r + u_{r-1} z^{r-1} + \dots + u_1 z + u_0}{s^k + v_{k-1} s^{k-1} + v_{k-2} s^{k-2} + \dots + v_1 s + v_0}; k < n \dots (4)$$

We know that the power series expansion of $G(s)$ about $s = 0$ is

$$G(s) = C_0 + C_1 s + C_2 s^2 + \dots = \sum_{i=0}^{\infty} C_i s^i \dots (5)$$

Where $C_i = \frac{(-1)^i}{i!} RTM_i$; $i = 0, 1, 2, 3, \dots$

The expansion of $G(s)$ about $s = \infty$ is

$$G(s) = \mu_0 s^{-(n-m)} + \mu_1 s^{-(n-m+1)} + \mu_2 s^{-(n-m+2)} + \dots = \sum_{i=0}^{\infty} \mu_i s^{-(n-m+i)} \dots (6)$$

Where $\mu_i = RMP_i$; $i = 0, 1, 2, 3, \dots$

i) Considering the original high order system $G(s)$ with distinct poles

Define the expressions for redefined time moments (RTMs) as

$$RTM_i = \sum_{j=1}^n x_{ij}; i = 0, 1, 2, \dots \dots (7)$$

where $x_{ij} = i! \frac{P_j}{\lambda_j^{(i+1)}}; i = 0, 1, 2, \dots$

Define Redefined Markov Parameters (RMPs) as

$$RMP_i = \sum_{j=1}^n y_{ij} \dots (8)$$

where $y_{ij} = P_j \lambda_j^i; i = 0, 1, 2, \dots$ and P_j are residues.

ii) If $G(s)$ is having 'r' repeated poles

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{(s + \lambda_1)^r (s + \lambda_{r+1}) \dots (s + \lambda_n)}; m < n$$

Define the expressions for redefined time moments (RTMs) as

$$RTM_i = i! \left[\frac{P_1}{\lambda_1^{(i+1)}} + [ET_{ix}] + \sum_{k=r+1}^n \frac{P_k}{\lambda_k^{(i+1)}} \right]; i = 0, 1, 2, \dots (9)$$

where $ET_{ix} = \left[\sum_{x=1}^{r-1} \frac{P_{(x+1)}}{\lambda_1^{(i+x+1)}} \left\{ \prod_{j=1}^x \frac{(-1)^j (i+j)}{x!} \right\} \right]$ and P_j are residues.

Define Redefined Markov Parameters (RMPs) as

$$RMP_i = [P_1 \lambda_1^i + [EM_{ix}] + \sum_{k=r+1}^n P_k \lambda_k^i]; i = 0, 1, 2, \dots (10)$$

where $EM_{ix} = \left[\sum_{x=1}^{r-1} P_{(x+1)} \lambda_1^{(i-x)} \left\{ \prod_{j=1}^x \frac{(i-j+1)}{x!} \right\} \right]; E_{ix} = 0$ if $i \leq x$ and P_j are residues.

The denominator polynomial $D_k(s)$ of the k^{th} order reduced model is obtained by selecting poles with the highest contribution in RTMs and lowest contribution in RMPs according to their contribution weight as shown in Table I.

Table I : Contributions of individual poles

Parameters	λ_1	λ_2	λ_3	...	λ_j	...	λ_n	Sum
RTMs	x_{i1}	x_{i2}	x_{i3}	...	x_{ij}	...	x_{in}	RTM_i
RMPs	y_{i1}	y_{i2}	y_{i3}	...	y_{ij}	...	y_{in}	RMP_i

Where x_{ij} is the contribution of pole λ_j in RTM_i ; and y_{ij} is the contribution of pole λ_j in RMP_i . The numerator polynomial, $N_k(s)$ of the k^{th} order reduced model is obtained by retaining the first few initial RTMs and RMPs of the original system as follows:

$$N_k(s) = \sum_{i=0}^{r_1} p_i s^i + \sum_{i=1}^{r_2} p_{(r_1+i)} s^{(r_1+i)}; r = r_1 + r_2, r \leq m \text{ and } r_1 \geq 1. \dots (11)$$

where $p_i = \sum_{j=0}^i q_j C_{(j-i)}; j = 0, 1, 2, 3, \dots, r_1$; r_1 is number of RTMs

and $p_{(r-j)} = \sum_{i=0}^j q_{(r-i)} \mu_{(j-i)}; j = 0, 1, 2, 3, \dots, r_2$; r_2 is number of RMPs

r_2 , the number of RMPs is zero if r_1 is considered. If $n - m < 1$, naturally $r_2 = 0$.

III. EXAMPLE

Considering a sixth order discrete system described by the transfer function given as

$$G_1(z) = \frac{0.3277z^6 + 0.9195z^5 + 1.038z^4 + 0.5962z^3 + 0.1618z^2 + 0.006986z - 0.005308}{z^6 + 1.129z^5 + 0.2889z^4 - 0.08251z^3 - 0.04444z^2 - 0.00476z}$$

Using the Tustin transformation $G_1(z)$ is transformed to $G_1(s)$ with sampling time $T = 1$.

$$G_1(s) = \frac{s^5 + 15.6s^4 + 124.2s^3 + 510.3s^2 + 1166s + 959.3}{s^6 + 21s^5 + 175s^4 + 735s^3 + 1624s^2 + 1764s + 720}$$

The poles of $G_1(s)$ are $-\lambda_1 = -1, -\lambda_2 = -2, -\lambda_3 = -3, -\lambda_4 = -4, -\lambda_5 = -5$ and $-\lambda_6 = -6$. The contributions of individual poles are derived from equ.(7) for RTMs and from equ.(8) (non zero terms) for RMPs. These contributions are tabulated in Tables II and III. Poles having highest contribution in RTMs and

lowest contribution in RMPs according to their contribution weight age are $-\lambda_{z_1} = -1$ and $-\lambda_{z_3} = -4$.

Table II Contribution of poles in RTMs and RMPs of HOS

s	RTMs/RMP	λ_1	λ_2	λ_3
24	RTM ₀ =1.33	1.616	2.2396	-7.7444
48	RTM ₁ =1.64	1.616	1.1198	-2.5815
00	RMP ₀ =1.00	1.616	4.4792	-23.233
	RMP ₁ = -5.4	1.6167	8.9583	69.700

Table III Contribution of poles in RTMs and RMPs of HOS

λ_4	λ_5	λ_6
10.8146	-8.4433	2.8493
2.7036	-1.6887	0.4749
43.2583	-42.2167	17.0958
173.0333	211.0833	-102.575

For the second order model with two poles in ESZ, Denominator polynomial is

$$D_2(s) = (s + 1)(s + 4) = s^2 + 5s + 4$$

Numerator of the ROM is obtained using equation (11) matching first two RTMs of the original system from Table II.

$$N_2(s) = (1.3324 \times 4) + \{(1.3324 \times 5) + (-1.6448 \times 4)\}s = 5.329 + 0.8244s$$

The transfer function of the second order reduced model is

$$IV. R_{1-1}(s) = \frac{0.8244s + 5.329}{s^2 + 5s + 4}$$

The conversion of continuous transfer function to discrete $R_{1-1}(s)$ to $R_{1-1}(z)$ is done using the Tustin transformation with Sampling time 1. The second order reduced discrete model is

$$R_{1-1}(z) = \frac{0.3052z^2 + 0.5922z + 0.2869}{z^2 - 0.1111}$$

The second order model by Farsi et al [13] is

$$R_{1-1}^F(z) = \frac{0.589z - 0.4495}{z^2 - 1.428z + 0.5329}$$

The step response of the proposed second order discrete model $R_{2-1}(z)$ and second order discrete model of Farsi *et al* [13] $R_{2-1}^F(z)$ are compared with original discrete system $G_1(z)$ in Fig.1. The step response by proposed method is following the original very closely when compared Farsi *et al*[13].

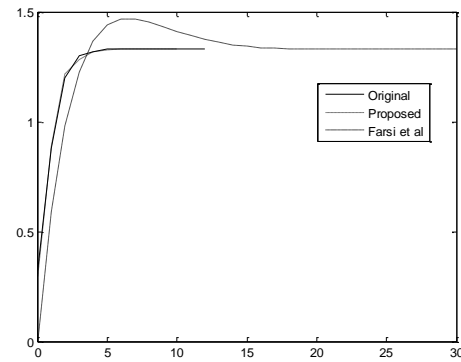


Fig.1 Comparison of step responses of $G_1(z)$, $R_{2-1}^F(z)$ and $R_{2-1}(z)$

V. CONCLUSIONS

Conclusions and Future Research: In this paper, an effective procedure to determine the reduced order model of higher order linear time invariant discrete systems is presented. Numerator and denominator polynomials of reduced order model are obtained by redefining the time moments of the original high order system. The stability of the original system is preserved in the reduced order model as the poles are taken from the original system. The method produces a good approximation when compared with other methods. The method is applied for real, complex and repeated poles of continuous [14] and discrete systems and the work is in progress to make it generalize for interval systems.

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