A New Approach of Trust Relationship Measurement Based on Graph Theory

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Abstract—**The certainty trust relationship of network node behavior has been presented based on graph theory, and a measurement method of trusted-degree is proposed. Because of the uncertainty of trust relationship, this paper has put forward the random trusted-order and firstly introduces the construction of trust relations space (TRS) based on trusted order. Based on all those above, the paper describes new method and strategy which monitor and measure the node behavior on the expectancy behavior character for trusted compute of the node. According to manifestation of node behavior and historical information, adjust and predict the trusted-order of node behavior. The paper finally establishes dynamic trust evaluation model based node behavior characters, and then it discusses the trusted measurement method which measures the connection and hyperlink for node behavior of network in trust relationship space.**

Keywords- Trust relations; trust relationship graph; trusted-order; random trust relationship.

I. Introduction

In recent years, with the wide application of trusted computing in the network security area, the studies of trust relationship in the node behavior of network have been made an important point [1, 2, and 3]. However, conventional environment of trust relationship conduction is "man around the model of experience", which is difficult to deal with the trust featured by procedures of brain thinking. Because of the almighty ability of experience model while processing empiricism information, people depend on experience model to a great extent. When the results from experience model are different from the reality significantly, people will doubt it.

This is the reason of the occurrence of the incompatible problems when traditional information of trust relationship conducting ways is applied to process node behavior of network system.

The trust network model is the prerequisite of trusted computing, how to evaluate the trust in the network, there is not a unity and general method up to now. In fact, the trust network is a kind of network with trust relationships, trust relationship networks can be abstracted as a kind of topological relationships in mathematics. Recently, the research around the model of trust networks is come from different angles [1-6]. But their common point is seeking a formal representation method reasonably.

Studies in literature [3] have shown that a trust relationship can be expressed with a graph. In the paper we study a quantitative expression of trust relationship in network system by using the method of graph theory. Then it measures the trusted degree of each node, and it also presents the trusted measurement of the connection and hyper connection for node behavior of network.

The object of this paper is to establishes dynamic trust evaluation model based on node behavior characters, Through the construction of the relationship between practical node behavior characters and on-the-spot model, it sets up a couple of mapping models of trust relationship, and sketches the skeleton of relationship mapping inversion.

The paper is organized as follows. Section 2 introduces the basic concept of trust relationship based on graph theory, and some properties of the evaluate principle of the trusted network. Section 3 studies measurement of trusted relationship. Section 4 we present dynamic trust evaluation model based on random trusted relationship. Section 5, conclusion puts forward the discoveries of this research and future research direction.

II. Basic Concepts And Methods

A. Certainty trusted relational graphs

Suppose that a trusted network T_N can be expressed as the corresponding graph $G = \{V, E\}$, where $V = \{v_1, v_2, \Lambda, v_n\}$ is a node set of T_N, and $E = \{e_1, e_2, \Lambda, e_m\}$ is an edge set. Moreover, $e_k : v_i \to v_j$ ($k = 1,2,\Lambda, m$; $i, j = 1,2,\Lambda, n)$, it represents that there is a trust relationship between v_i and v_j, namely v_i trusts v_j. Therefore, the trusted graph G is called a directed graph. For example, given a trusted network with five vertices, based on the analyzing of network behaviors, the trust relationship of vertices is described as follows:

$v_1 \to v_5$, $v_2 \to v_1$, $v_2 \to v_5$,

$v_3 \to v_2$, $v_3 \to v_4$, $v_3 \to v_5$,

$v_4 \to v_1$, $v_4 \to v_3, v_4 \to v_5$,

$v_5 \to v_1$, $v_5 \to v_3$.

Its adjacency matrix is

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix},$$

and the trusted relational graph G is shown in Figure 1.

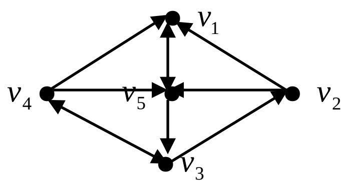

Figure 1. Trusted relational graph

B. Trusted relational trees

Given a trusted network with five vertices (As shown in Figure 1), Let $D(1) = AI = (1, 2, 3, 3, 2)^T$, where $I = (1, 1, 1, 1, 1)^T$, then $D(1)$ is a trusted level vector of each node. Such as the number of trusted vectors about the node v_4 and v_5 is 3 and 2 respectively. According to this a conclusion can be drawn that the trusted level of the node v_4 is taller than the node v_5.

But, the number of trusted vectors about the node v_5 and v_2 are both 2, how to distinguish the difference of v_5 and v_2?

On the analysis of Figure 1 it was found that v_1 and v_5 have a trusted relationship with v_2, and the number of trusted vectors about the node v_1 and v_5 is 1 and 2 respectively. The Node v_1 and v_3 have trusted relationships with v_5, and the number of trusted vectors about the node v_1 and v_3 is 1 and 3 respectively.

It can be seen that the indirect trusted level of v_2 is 3, and the indirect trusted level of v_5 is 4. Thereby, it may be taken for granted that the trusted level of v_5 is taller than the node v_2.

Based on the graph theory, the trusted path determines the trusted level in trusted networks. The above analysis can be representing by the tree in figure 2.

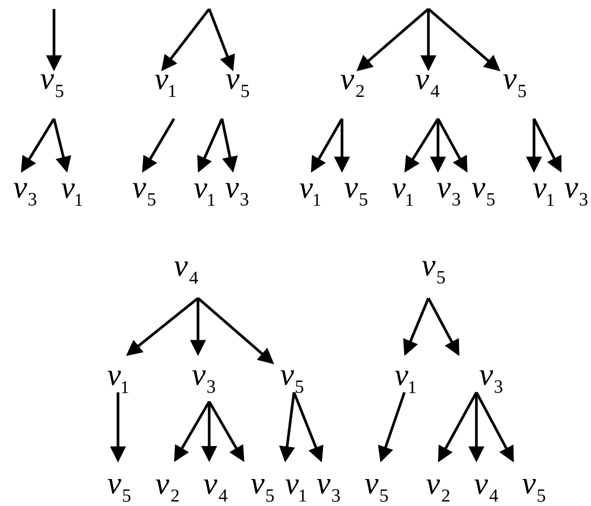

Figure 2. Trusted relational tree

III. Measurement In Trusted Relationship

The indirect trusted vector of each node in the network (when K=2) is as follows: $TD(2) = (2, 4, 7, 6, 4)^T$, then it is certain that $TD(K+1)$ can be measure the trusted level (trusted degree) accurately than $TD(K)$. In most cases, we must think about the limit of $TD(K)$, when $K \to \infty$. In order to ensure the limit convergence, and furthermore, the measuring value of each node should be trusted degree, therefore this paper regard the following limit as the measuring of each node in the trusted relational graph in networks:

$$\lim_{k \to \infty} \frac{TD(K)}{I^T TD(K)}.$$

It will find in fig. 2 that the measurement of the trusted degree about each node is the number of the path in the directed tree, which takes each node for the root. And then the relations are extended to the general case, the definition is as follows:

Definition 3.1 In a network with n nodes, the trusted capability (trusted degree) of a node v_i can be determining by the number of the path, which connects with the K-th path and starts from the node v_i. This number is called the K-th trusted capability, and denoted as $td_k(v_i)$. Vector

$$TD(k) = \{td_k(v_1), td_k(v_2), \Lambda, td_k(v_n)\}$$

is called the *K*-th trusted capability vector of the trusted relational graph G.

Definition 3.2 In a network of n vertices, a limit

$$td(v_i) = \lim_{k \to \infty} \frac{td_k(v_i)}{\sum_{i=1}^{n} td_k(v_i)}$$

is entitled relatively limit trusted degree of the node v_i, it is called a trusted degree for short. For that reason, we have

$$T = (td(v_1), td(v_2), \Lambda, td(v_n))^T = \lim_{k\to\infty} \frac{TD(k)}{I^T TD(k)},$$

which is called a trusted vector of each node in the trusted relational graph, where $I = (1,1,\Lambda,1)^T$.

Theorem 3.1 Let G be a trusted relational graph of n vertices, its adjacency matrix is A, if G is bidirectional connected and $n \geq 4$, α_1 is an eigenvector corresponding to the biggest values of A, then T exists certainly and $T = \alpha_1/(I^T\alpha_1)$, moreover $\|\alpha_1/(I^T\alpha_1)\| = 1$.

It can seem from theorem3.1, the *K*-th trusted degree $TD(k)$ of each node is computable in the trusted relational graph G with *n* vertices, and it can be obtain by the following algorithm:

（1）when $k = 0$，$TD(0) = I$；

（2）when $k = 1$，2，…，$TD(k) = ATD(k-1)$；$T\overline{D}(k) = TD(k)/(I^T TD(k)$；

（3）when given precision $e > 0$，calculated until $k = m$，if it is satisfied：

$$\|T\overline{D}(k) - T\overline{D}(k)\| < e,$$

then stopped calculating to choose $T = T\overline{D}(m)$.

The algorithm given in theorem3.1 can be put to use in network according to different trusted levels. Regardless of the connected meaning of network note, it always measures the trusted degree in the trusted relational graph. Meanwhile, the trusted vector T can be regaled as a weighted vector, which expresses the trusted degree of each node in the trusted relational graph.

IV. Random Trusted Relationship

Thinking about the trusted relational graph of discussion in the previous section, if it has $v_i \to v_j$, then it exists the trusted relationship of completely specified between v_i and v_j

It is called certainty trusted relational graph which has the trusted relationship of completely specified. In fact, trusted relationship is uncertainty in lots of trusted networks. For example, the trusted relationship among people in the Internet, because of the vitality of network activities, the trusted relationship of network is uncertainty, for this reason, the uncertain research methods is used to analyze the trusted relationship in network.

The random trusted relationship is expressed by the random graph for trusted relationship, it has respective trusted relational graph in the basis of different network activities and space-time states. Furthermore, the extent of trusted relationships presents certain probabilistic characteristics with the change of network activities; we can use $P_{ij} (0 \leq P_{ij} \leq 1)$ to express the arisen probability of $v_i \to v_j$. Thereby the trusted relationships in the network consisted of n nodes can be expressed by a family of trusted relational graphs, the family of trusted relational graphs are noted as $G(n,(P_{ij}))$, it is called a random trusted relational graph. When $v_i = v_j$, take for granted, $P_{ij} = 0$; When $i, j = 1,2\Lambda, n$, $i \neq j$, $0 \leq P_{ij} \leq 1$, apparently, $P = [P_{ij}]$ constitutes a square matrix of order n, $G(n,(P_{ij}))$ is called a probability matrix. Suppose the connection of each node be random and independence, then a definition is as follows:

Definition 4.1 A directed and weighted graph, which weighted is a probability matrix P, and it is called a network expression of $G(n,(P_{ij}))$ that is noted as $N(n, P)$.

Definition 4.2 The weighted product of each edge in the directed path L is called a transfer probability in $N(n, P)$. It is called the *k*-th order dispersive degree of the node v_i that the sum of all of transfer probabilities with k connective paths, which starting from the node v_i. Noted as $N_k(v_i)$, and

$$N(k) = (N_k(v_1), N_k(v_2), \Lambda, N_k(v_n))^T.$$

Definition 4.3 The limit $\lim_{k\to\infty} \frac{N_k(v_i)}{I^T N_k(v_i)}$ is called a limit transfer probability of a node v_i.

Based on the probability theory, it is well known that the transfer probability of the path L_{ij} (as dependence), which is from the node v_i to v_j in $N(n, P)$, is the present probability of L_{ij} in $G(n,(P_{ij}))$, that is to say it is a probability of the directed connection (trusted relational chain)between the node v_i and v_j in the random trusted relational graph $G(n,(P_{ij}))$. It is still used $td_k(v_i)$ to express the number of paths, which starting from the node v_i and taking with k paths. We can prove as follows:

Theorem 4.1 Let $N(n, P)$ be disconnected, $n \geq 4$, then the limit transfer probability of each node exists certainly, and equals to the limit

$$\lim_{k\to\infty}\frac{P^kI}{I^TP^kI}.$$

Deduction 4.1 There has $N(k)=P^kI$ in $N(n,P)$.

Theorem 4.2 Let $N(k)$ be the *k*-th order dispersive degree vector of each node in $N(n,P)$, and let $TD(k)$ be the number vector of starting from each node and taking with k paths in $G(n,(P_{ij}))$, then $E(TD(k))=N(k)$ is obtained. This theorem explains that the *k*-th order dispersive degree of the node v_i in $N(n,P)$ is the mathematical expectation of the number of paths, which starting from the node v_i and taking with k paths in $G(n,(P_{ij}))$. Since the measurement of trusted levels for a certain node can be expressed by the number of paths starting from the node. Hereby, we regard the limit transfer probability vector as the weighted vector T of a certain node in the random trusted relational graph. According to theorem 4.2, we can obtain the same arithmetic as theorem 1, so for as changing the adjacency matrix A for the probability matrix P in $N(n,P)$. If and only if $P_{ij}=0$ or $P_{ij}=1$, a random trusted relational graph turns into a certainty trusted relational graph, so the latter is a special case of the former.

V. CONCLUSIONS

In the development of the trusted computing, theoretical research lags behind practical. The trusted measurement is the basic theory of the trusted computing, and is also a key technology in the process of development of the trusted computing. In this paper, a certainty trusted network and a random trusted network were introduced respectively. Then a measurement method of the trusted degree was presented and its arithmetic was described. These theories and methods will help the development of the trusted computing. For future works, the methods will be optimized, which not only depict the fact but also can be used simply and practically.

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