

# Error Analysis of Air Temperature Profile Retrievals with Microwave Sounder Data Based on Minimization of Covariance Matrix of Estimation Error

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**Abstract**— Error analysis of air temperature profile retrievals with microwave sounder data based on minimization of covariance matrix of estimation error is conducted. Additive noise is taken into account in the observation data with microwave sounder onboard satellite. Method for air temperature profile retrievals based on minimization of difference of brightness temperature between model driven microwave sounder data and actual microwave sounder data is also proposed. The experimental results shows reasonable air temperature retrieval accuracy can be achieved by the proposed method.

**Keywords**- Error analysis; leastsquare method; microwave sounder; air temperature profile.

## I. INTRODUCTION

Air temperature and water vapor profiles are used to be estimated with Microwave Sounder data [1]. One of the problems on retrieving vertical profiles is its retrieving accuracy. In particular, estimation accuracy of air-temperature and water vapor at tropopause<sup>1</sup> altitude is not good enough because there are gradient changes of air-temperature and water vapor profile in the tropopause due to the fact that observed radiance at the specific channels are not changed by the altitude [2].

In order to estimate air-temperature and water vapor, minimization of covariance matrix of error is typically used. In the process, error covariance matrix<sup>2</sup> which is composed with the covariance of air temperature and water vapor based on prior information and the covariance of observed brightness temperature<sup>3</sup> based on a prior information as well as difference between model driven and the actual brightness temperature. Error analysis<sup>4</sup> is important for design sensitivity and allowable observation noise of microwave sounder. For this reason, error analysis is conducted for the conventional air temperature profile retrieval method. Other than this, this paper propose another air temperature profile retrieval method

based on minimization of brightness temperature difference between model driven and actual brightness temperature acquired with real microwave sounder<sup>5</sup>. Experiment is conducted for the proposed method. Reasonable retrieval accuracy is confirmed.

The following section describes the conventional air temperature and water vapor profile retrieval method followed by experimental results. Then another retrieval method is proposed with some experimental results. Finally, conclusion is followed together with some discussions.

## II. ERROR ANALYSIS

### A. Microwave Sounder

Air temperature profile can be retrieved with the microwave sounder data at absorption wavelength due to oxygen while water vapor profile can be estimated with the microwave sounder data at the absorption wavelength due to water. The microwave sounder which is onboard AQUA satellite<sup>6</sup> as well as NOAA-15, 16, 17 is called Advanced Microwave Sounding Unit: AMSU<sup>7</sup>. Description of AMSU is available in Analytical Theoretical Basis Document: ATBD document<sup>8</sup>. Observation frequency ranges from 23.8 GHz to 89 GHz. 22.235 GHz is the absorption frequency due to water while absorption frequency due to oxygen is situated in 60 GHz frequency bands. At the absorption frequency, observed brightness temperature is influenced by the molecule, oxygen, water. The influence due to molecule depends on the observation altitude as shown in Fig.1 (a). Also absorption due to atmospheric molecules depends on the observation altitudes as shown in Fig.1 (b). Therefore, it is possible to estimate molecule density of oxygen and water at the different altitude results in air temperature and water vapor profiles retrievals.

<sup>5</sup> [http://en.wikipedia.org/wiki/Advanced\\_Microwave\\_Sounding\\_Unit](http://en.wikipedia.org/wiki/Advanced_Microwave_Sounding_Unit)

<sup>6</sup> [http://en.wikipedia.org/wiki/Aqua\\_\(satellite\)](http://en.wikipedia.org/wiki/Aqua_(satellite))

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[http://disc.sci.gsfc.nasa.gov/AIRS/documentation/amsu\\_instrument\\_guide.shtm](http://disc.sci.gsfc.nasa.gov/AIRS/documentation/amsu_instrument_guide.shtm)

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[http://eosps0.gsfc.nasa.gov/eos\\_homepage/for\\_scientists/atbd/docs/AIRS/atbd-airs-L1B\\_microwave.pdf](http://eosps0.gsfc.nasa.gov/eos_homepage/for_scientists/atbd/docs/AIRS/atbd-airs-L1B_microwave.pdf)

<sup>1</sup> <http://en.wikipedia.org/wiki/Tropopause>

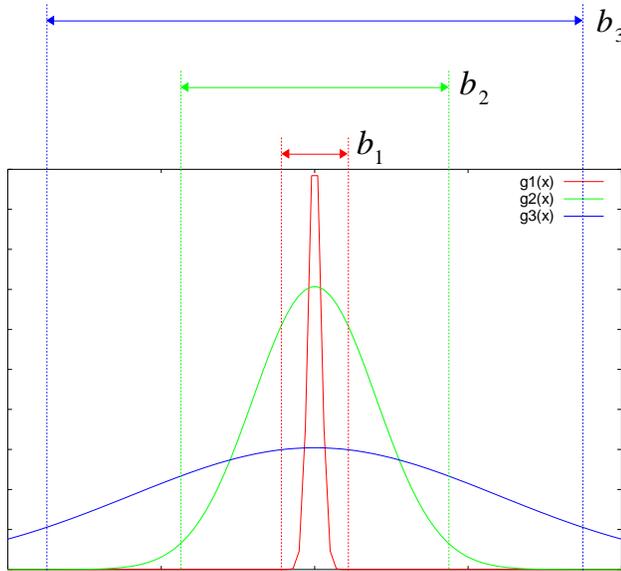
<sup>2</sup> [http://en.wikipedia.org/wiki/Covariance\\_matrix](http://en.wikipedia.org/wiki/Covariance_matrix)

<sup>3</sup> [http://en.wikipedia.org/wiki/Brightness\\_temperature](http://en.wikipedia.org/wiki/Brightness_temperature)

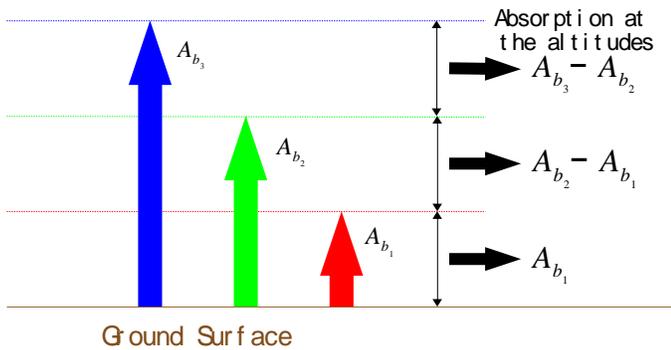
<sup>4</sup> [http://en.wikipedia.org/wiki/Error\\_analysis](http://en.wikipedia.org/wiki/Error_analysis)

Weighting function<sup>9</sup> is defined as the gradient of atmospheric transparency against altitude. The weighting function depends on observation frequency. Observed brightness temperature at the frequency, therefore, is influenced depending on the weighting function. Therefore, the altitude of which peak of weighting function is situated is the most influencing to the observed brightness temperature at the observation frequency. The following observation frequencies are selected for estimation of oxygen absorption (air temperature at the following altitudes,

- 15, 18, 20, 23, 14, 19, 7 km
- 58.7, 59.3, 60.2, 60.5, 61.8, 62.3, 63.7 GHz



(a) Influence due to atmospheric molecule at the different altitudes



(b) Absorption due to atmospheric molecule at the different altitudes

Figure 1 Absorption and influence due to atmospheric molecules at the different altitudes.

The weighting functions for these observation frequencies are shown in Fig.2. Using Millimeter wave Atmospheric Emission Simulator: MAES<sup>10</sup> of radiative transfer calculation software code<sup>11</sup> provided by National Institute for

Communication Technology, Japan, NICT<sup>12</sup>, atmospheric transparency can be calculated at the observation frequency. In this case, Mid. Latitude Summer of atmospheric model<sup>13</sup> is selected. Then gradient of atmospheric transparency against altitude is calculated results in weighting function calculations.

### B. Conventional Air Temperature and Water Vapor Profile Retrieval Method

In order to estimate air-temperature and water vapor, minimization of covariance matrix of error is typically used. In the process, covariance matrix which is composed with the covariance of air temperature and water vapor based on prior information<sup>14</sup> and the covariance of observed brightness temperature based on a prior information as well as difference between model driven and the actual brightness temperature. Covariance matrix of estimation error is defined as follows,

$$X - X_0 = (S_x^{-1} + A^T * S_\epsilon^{-1} * A)^{-1} * A^T * S_\epsilon^{-1} * (G - G_0) \quad (1)$$

where  $X_0$ ,  $S_x$ ,  $A$ ,  $S_\epsilon$ ,  $G$ ,  $G_0$  denote air temperature at each altitude, covariance matrix of air temperature for a prior information, Jacobian matrix<sup>15</sup> for brightness temperature of each frequency band, covariance matrix of observation error for a prior information, model driven brightness temperature, and estimated brightness temperature, respectively.

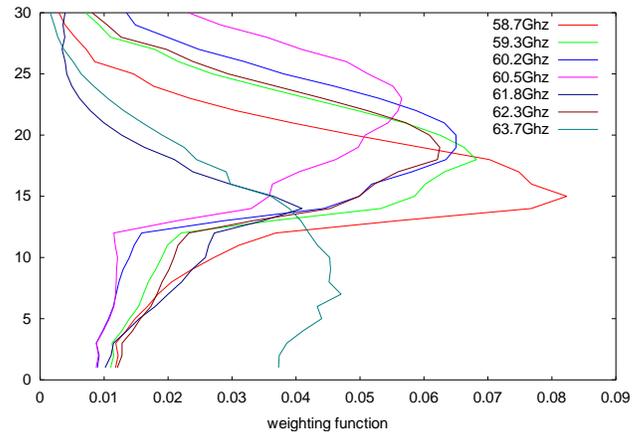


Figure 2 Weighting functions for observation frequencies, 58.7, 59.3, 60.2, 60.5, 61.8, 62.3, 63.7 GHz

$A$  can be determined from equation (2).

$$\begin{matrix} B(T_{\lambda_1}, \lambda_1) \times K_{\lambda_1} & \cdots & B(T_{\lambda_7}, \lambda_1) \times K_{\lambda_1} \\ \vdots & \ddots & \vdots \\ B(T_{\lambda_1}, \lambda_7) \times K_{\lambda_7} & \cdots & B(T_{\lambda_7}, \lambda_7) \times K_{\lambda_7} \end{matrix} \quad (2)$$

where  $B$ ,  $T_\lambda$ ,  $\lambda$ ,  $K_\lambda$  denotes Plank function, air temperature at the peak of weighting function, frequency, and weighting

<sup>9</sup> [http://www.lmd.jussieu.fr/~falmd/TP/results\\_interpret\\_AMSU/AMSU.pdf](http://www.lmd.jussieu.fr/~falmd/TP/results_interpret_AMSU/AMSU.pdf)  
<sup>10</sup> [http://www.sat.ltu.se/workshops/radiative\\_transfer/minutes.php](http://www.sat.ltu.se/workshops/radiative_transfer/minutes.php)  
<sup>11</sup> [http://en.wikipedia.org/wiki/Atmospheric\\_radiative\\_transfer\\_codes](http://en.wikipedia.org/wiki/Atmospheric_radiative_transfer_codes)

<sup>12</sup> <http://www.nict.go.jp/>  
<sup>13</sup> [http://www.arm.gov/publications/proceedings/conf05/extended\\_abs/mlawer\\_e\\_j.pdf](http://www.arm.gov/publications/proceedings/conf05/extended_abs/mlawer_e_j.pdf)  
<sup>14</sup> [http://andrewgelman.com/2011/03/prior\\_informati/](http://andrewgelman.com/2011/03/prior_informati/)  
<sup>15</sup> [http://andrewgelman.com/2011/03/prior\\_informati/](http://andrewgelman.com/2011/03/prior_informati/)

function at the peak altitude, respectively. On the other hand,  $G_0$  can be calculated with equation (3).

$$\begin{aligned} & \sum_{h=1}^H B(T_h, \lambda_1) \times K(\lambda_1, h) \\ & \quad \vdots \\ & \sum_{h=1}^H B(T_h, \lambda_7) \times K(\lambda_7, h) \end{aligned} \quad (3)$$

where  $h$ ,  $H$ ,  $T_h$  denotes altitude, peak altitude at which weighting function is maximum, and air temperature at altitude.

### C. Inverse Problem Solving Based Method with Microwave Sounder Data

As aforementioned,  $A$  can be calculated in advance for air temperature profile retrievals.  $A$  is square matrix. Therefore, it is easy to calculate inverse matrix of  $A$ . Using inverse matrix  $A$ , air temperature profile can be retrieved as follows,

$$T = T_0 + A^{-1}(G - G_0) \quad (4)$$

where  $T_0$ ,  $G$ ,  $G_0$  denotes air temperature at the designated altitude, brightness temperature derived from the acquired AMSU data, and model derived brightness temperature, respectively. This method is referred to Inverse Matrix Method: IMM hereafter. Fig.3 shows the weighting functions for assumed observation frequencies, 52.8, 55.5, and 57.29 GHz, respectively.

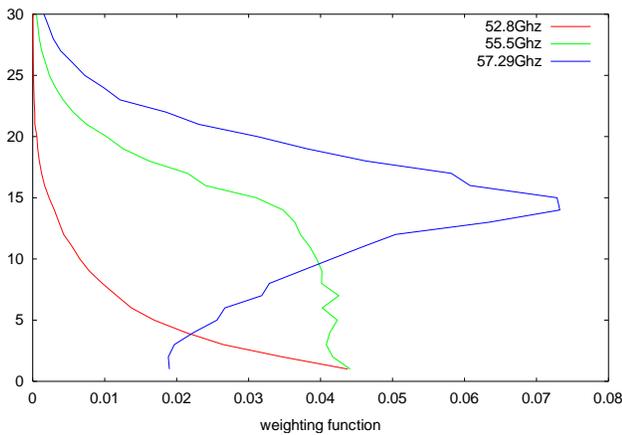


Figure 3 Weighting functions for the designated observation frequencies of 52.8, 55.5, and 57.29 GHz

## III. EXPERIMENTS

### A. Error Analysis on Air Temperature Profile Retrieval Accuracy for the Conventional Error Covariance Based Method

Brightness temperature at the designated observation frequency can be calculated with MAES (Mid. Latitude Summer of atmospheric model). One of the input parameters is air temperature profile. Therefore, error analysis is made through the following procedure,

- (1) Designate air temperature profile
- (2) Calculate observed brightness temperature at the designated observation frequencies

- (3) Estimate air temperature profile based on the conventional error covariance based method
- (4) Compare the designated and estimated air temperature profiles at the altitudes at which weighting function is maximum (peak weighting function altitude)

Table 1 shows estimated air temperature derived from the conventional covariance matrix based method and truth air temperature as well as estimation error. Table 1 (a) shows those for 1K of additive noise while Table 1 (b) shows those for 3K of additive noise. On the other hand, Table 1 (c) shows those for 5K of additive noise. 1, 3, 5K of noises are added to the observed brightness temperature of AMSU data.

TABLE I. AIR TEMPERATURE PROFILE ESTIMATION ACCURACY FOR THE CONVENTIONAL ERROR COVARIANCE BASED METHOD

(a) Additive Noise = 1K

| Altitude(km) | Estimated | Truth | Error |
|--------------|-----------|-------|-------|
| 7            | 256.356   | 254.7 | 1.658 |
| 14           | 217.713   | 215.7 | 2.031 |
| 15           | 217.876   | 215.7 | 2.176 |
| 18           | 219.529   | 216.8 | 2.729 |
| 19           | 219.691   | 217.9 | 1.791 |
| 20           | 220.712   | 219.2 | 1.512 |
| 23           | 224.517   | 222.8 | 1.717 |

(b) Additive Noise=3K

| Altitude(km) | Estimated | Truth | Error |
|--------------|-----------|-------|-------|
| 7            | 258.391   | 254.7 | 3.691 |
| 14           | 219.93    | 215.7 | 4.23  |
| 15           | 219.483   | 215.7 | 3.783 |
| 18           | 220.787   | 216.8 | 3.987 |
| 19           | 221.762   | 217.9 | 3.862 |
| 20           | 223.24    | 219.2 | 4.04  |
| 23           | 226.808   | 222.8 | 4.008 |

(c) Additive Noise=5K

| Altitude(km) | Estimated | Truth | Error |
|--------------|-----------|-------|-------|
| 7            | 260.309   | 254.7 | 6.609 |
| 14           | 220.009   | 215.7 | 4.309 |
| 15           | 221.181   | 215.7 | 5.481 |
| 18           | 223.253   | 216.8 | 6.453 |
| 19           | 223.553   | 217.9 | 5.653 |
| 20           | 227.823   | 219.2 | 8.612 |
| 23           | 227.258   | 222.8 | 4.458 |

Trend of the estimation error against additive noise shows exponential function as shown in Fig.4. The estimation error at additive noise is zero (without any observation noise is added to brightness temperature) ranges from 1.2 to 2.5 K. It is a reasonable accuracy of air temperature profile.

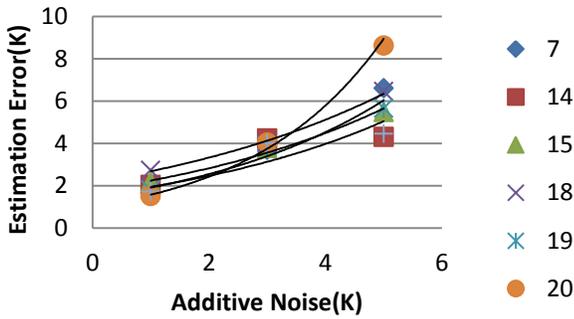


Figure 4 Estimation error trend of air temperature profile as a function of additive noise.

**B. AMSR Data Used**

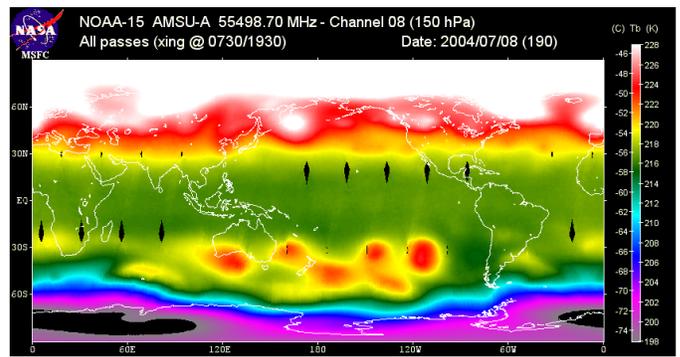
The proposed method which minimizing the difference between model derived and the actual microwave sounder data derived air temperature is validated with AMSU data of suburban of London (Longitude: 0 degree West, Latitude: 51.3 North) which is acquired on July 8 2004.

Fig.5 (a), (b), (c) shows brightness temperature of the AMSU Channel 4, 8, and 9, respectively.

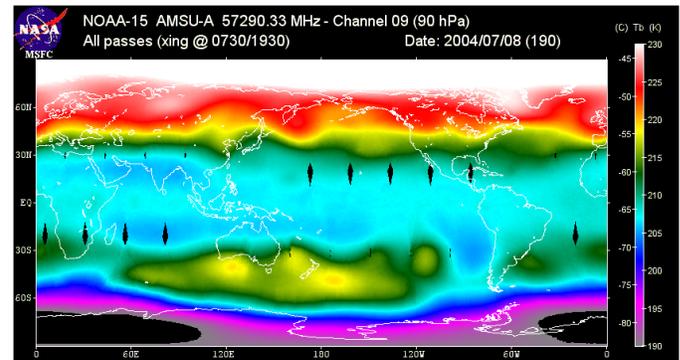
The brightness temperature at the test location for the designated three frequency bands are as follows,

- 52.8GHz (247.2 K),
- 55.5GHz (213.3K), and
- 57.29GHz (210.6K)

Assuming Mid. Latitude Summer of atmospheric model, brightness temperature of these three observation frequency bands is estimated.



(b)Channel 8 which corresponds to 150 hPa



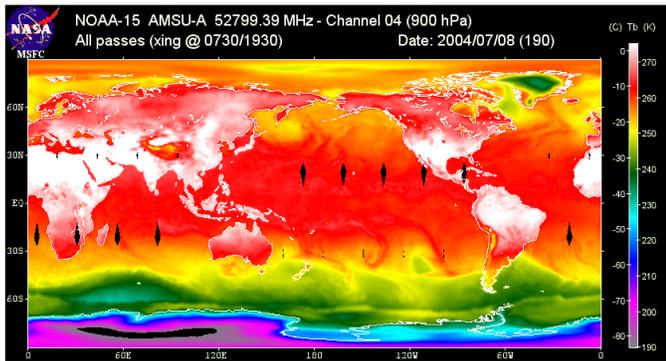
(c)Channel 9 which corresponds to 90 hPa

Figure 5 AMSU data used

**C. Air Temperature Estimation Accuracy**

Using these brightness temperature, air temperature profile is estimated with the proposed method. Fig.6 and Table 2 shows the estimated and model derived air temperature profiles.

The estimation error at the altitudes of 7 and 14 km are common to the conventional method and the proposed method. Therefore, the averaged estimation error at altitude of 7 and 14 km are compared. The result is shown in Table 3.



(a)Channel 4 which corresponds to 900hPa

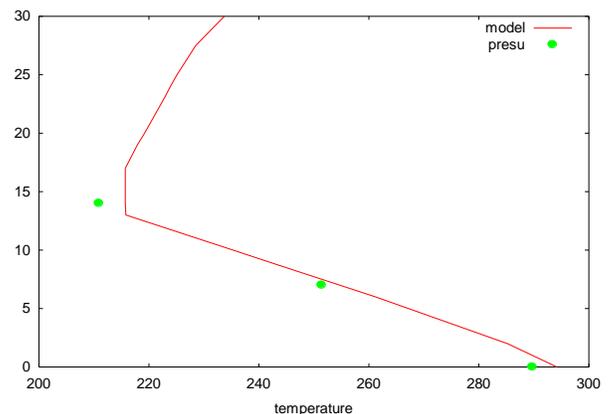


Figure 4 Model derived and the estimated air temperature profiles

TABLE II. AIR TEMPERATURE PROFILE ESTIMATION ACCURACY FOR THE PROPOSED INVERSE MATRIX BAED METHOD

| Altitude(km) | Estimated | Truth | Error |
|--------------|-----------|-------|-------|
| 0            | 289.745   | 294.2 | 4.456 |
| 7            | 251.429   | 254.7 | 3.271 |
| 14           | 210.913   | 215.7 | 4.787 |

TABLE III. AVERAGE AIR TEMPERATURE ESTIMATION ERROR BETWEEN ERROR AT THE ALTITUDE OF 7 AND 14KM FOR BOTH OF THE CONVENTIONAL AND THE PROPOSED MTHEODS

| Additive Noise      | 1K    | 3K    | 5K    |
|---------------------|-------|-------|-------|
| Conventional Method | 1.845 | 3.961 | 5.459 |
| Proposed Method     | 4.029 |       |       |

Even though, the estimation error of the proposed method do not take into account any additive noise, the estimation error is corresponding to the error of the conventional method with 3K of additional noise. Although the proposed method is not so accurate retrieval method for air temperature profile, it is quit fast and does not required huge computer resources because only thing we have to do is to calculate inverse matrix of A. It is 10 times faster than the conventional method.

#### IV. CONCLUSION

Error analysis of air temperature profile retrievals with microwave sounder data based on minimization of covariance matrix of estimation error is conducted. Additive noise is taken into account in the observation data with microwave sounder onboard satellite. Method for air temperature profile retrievals based on minimization of difference of brightness temperature between model driven microwave sounder data and actual microwave sounder data is also proposed.

The experimental results shows reasonable air temperature retrieval accuracy can be achieved by the proposed method. The air temperature estimation error of the proposed Inverse Matrix Based Method is around 4K and is corresponding to that of the conventional method with 3K of observation noise. Also it is found that air temperature estimation error of the conventional error covariance based method ranges from 1.2 to 2.5K and is getting large exponentially in accordance with increasing of observation noise.

#### ACKNOWLEDGMENT (HEADING 5)

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#### AUTHORS PROFILE

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