

Image Denoising using Adaptive Thresholding in Framelet Transform Domain

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Abstract— Noise will be unavoidable during image acquisition process and denoising is an essential step to improve the image quality. Image denoising involves the manipulation of the image data to produce a visually high quality image. Finding efficient image denoising methods is still valid challenge in image processing. Wavelet denoising attempts to remove the noise present in the imagery while preserving the image characteristics, regardless of its frequency content. Many of the wavelet based denoising algorithms use DWT (Discrete Wavelet Transform) in the decomposition stage which is suffering from shift variance. To overcome this, in this paper we proposed the denoising method which uses Framelet transform to decompose the image and performed shrinkage operation to eliminate the noise. The framework describes a comparative study of different thresholding techniques for image denoising in Framelet transform domain. The idea is to transform the data into the Framelet basis, example shrinkage followed by the inverse transform. In this work different shrinkage rules such as universal shrink(US), Visu shrink (VS), Minmax shrink(MS), Sure shrink(SS), Bayes shrink(BS) and Normal shrink(NS) were incorporated. Results based on different noise such as Gaussian noise, Poisson noise, Salt and pepper noise and Speckle noise at ($\sigma=10,20$) performed in this paper and peak signal to noise ratio (PSNR) and Structural similarity index measure(SSIM) as a measure of the quality of denoising was performed.

Keywords- Discrete Wavelet Transform(DWT); Framelet Transform(FT); Peak signal to noise ratio(PSNR); Structural similarity index measure(SSIM).

I. INTRODUCTION

The quality of image is degraded by various noises in its acquisition and transmission. Image Denoising has remained a fundamental problem in the field of image processing [1]. There are various noise reduction techniques used for removing noise. Most of the standard algorithms use to denoise the noisy image and perform the individual filtering process which reduces the noise level. But the image is either blurred or over smoothed due to the lose of edges. Noise reduction is used to remove the noise without losing detail contained in the images. Wavelet transform[2] has proved to be effective in noise removal and also reduce computational complexity, better noise reduction performance.

Wavelet transform may not require overlapped windows due to the localization property and wavelet filter does not correspond to time domain convolution [3][4]. Apply discrete wavelet transform (DWT) which transforms the discrete data

from time domain into frequency domain. The values of the transformed data in time frequency domain [5]-[10] are called the coefficients where large coefficients correspond to the signal and small ones represent mostly noise. The denoised data is obtained by inverse transforming the suitably threshold coefficients. DWT does not provide shift invariance. Shift variance results from the use of critical sub sampling in the DWT. For this reason every second wavelet coefficient at each decomposition level is discarded. This can lead to small shifts in the input waveform causing large changes in the wavelet coefficients. Large variations in the distribution of energy at different scales introduce many visual artifacts in the denoised output.

To overcome the problem of DWT, Framelet transform which is similar to wavelets but has some differences. Framelets has two or more high frequency filter banks, which produces more subbands in Decomposition. This can achieve better time frequency localization [11] ability in image processing. There is redundancy between the Framelet subbands, which means change in coefficients of one band can be compensated by other subbands coefficients. After framelet decomposition, the coefficient in one subband has correlation with coefficients in the other subband. This means that changes on one coefficient can be compensated by its related coefficient in reconstruction stage which produces less noise in the original image.

In this paper, we combine the Framelet transform and apply it to image denoising. A tight frame filter bank[12][13] provides symmetry and has a redundancy that allows for approximate shift invariance. This leads to clear edges with effective denoising which is lacked in critically sampled discrete wavelet transform. Experimental results show that using Framelet transform, result in high peak signal to noise ratio for all denoised images. The organization of this paper is as follows. In section 2 Mathematical Representation of Framelet transform is presented. Section 3 and 4 Denoising Algorithm and different thresholding techniques explained. Section 5 and 6 Evaluation criteria and experimental results were explained.

II. FRAMELET TRANSFORM

In contrast to wavelets, Framelets have one scaling function $\varphi(t)$ and two wavelet functions $\psi_1(t)$ and $\psi_2(t)$.

A set of functions $\{\psi_1, \psi_2, \dots, \dots, \psi_{N-1}\}$ in a square integrable space L^2 is called a frame if there exist $A > 0, B < \infty$ so that, for any function $f \in L^2$

$$A\|f\|^2 \leq \sum_{i=1}^{N-1} \sum_j \sum_k |(f, \psi^i(2^j - k))|^2 \leq B\|f\|^2 \quad (1)$$

Where A and B are known as frame bounds. The special case of $A = B$ is known as tight frame. In a tight frame we have, for all $f \in L^2$. In order to derive fast wavelet frame, multiresolution analysis is generally used to derive tight wavelet frames from scaling functions

Now we obtain the following spaces,

$$V_j = \text{span}_k \{\varphi(2^j t - k)\} \quad (2)$$

$$W_j = \text{span}_k \{\psi^i(2^j t - k)\} \quad i = 1, 2, \dots, N - 1 \quad (3)$$

With
$$V_j = V_{j-1} U W_{1,j-1} U W_{2,j-1} U \dots U W_{N-1,j-1} \quad (4)$$

The scaling function $\varphi(t)$ and the wavelets $\psi_1(t)$ and $\psi_2(t)$ are defined through these equations by the low pass filter $h_0(n)$ and the two high pass filters $h_1(n)$ and $h_2(n)$

$$\text{Let } \varphi(t) = \sqrt{2} \sum_n h_0(n) \varphi(2t - n) \quad (5)$$

$$\psi_i(t) = \sqrt{2} \sum_n h_i(n) \varphi(2t - n) \quad i = 1, 2, \dots \quad (6)$$

Perfect Reconstruction conditions and Symmetry Conditions

The Perfect Reconstruction (PR) conditions for the three band filter bank can be obtained by the following two equations

$$\sum_{i=0}^2 H_i(z) H_i(z^{-1}) = 2 \quad (7)$$

$$\sum_{i=0}^2 H_i(-z) H_i(z^{-1}) = 0 \quad (8)$$

A wavelet tight frame with only two symmetric or anti symmetric wavelets is generally impossible to obtain with a compactly supported symmetric scaling function (t) . Therefore if $h_0(n)$ is symmetric compactly supported. Then antisymmetric solution $h_1(n)$ and $h_2(n)$ exists if and only if all the roots of

$$2 - H_0(z) H_0(z^{-1}) + H_0(-z) H_0(-z^{-1}) \quad \text{has even multiplicity.}$$

case $H_2(z) = H_2(-z)$: The goal is to design a set of three filters that satisfy the PR conditions in which the low pass filter $h_0(n)$ is symmetric and the filters $h_1(n)$ and $h_2(n)$ are either symmetric or anti symmetric. There are two cases. Case I denotes the case where $h_1(n)$ is symmetric and $h_2(n)$ is anti symmetric. Case II denotes the case where $h_1(n)$ and $h_2(n)$ are both anti symmetric. The symmetric condition for $h_0(n)$ is

$$h_0(n) = h_0(N - 1 - n) \quad (9)$$

Where N is the length of the filter $h_0(n)$. We dealt with case I of even length filters. Solutions for Case I can be obtained from solutions where $h_2(n)$ time reversed version of is $h_1(n)$ and where neither filter is anti symmetric. To show this suppose that $h_0(n)$, $h_1(n)$ and $h_2(n)$ satisfy the PR conditions and that

$$h_2(n) = h_1(N - 1 - n) \quad (10)$$

Then by defining

$$h_1^{new} = \frac{1}{\sqrt{2}} (h_1(n) + h_2(n - 2d)) \quad (11)$$

$$h_2^{new} = \frac{1}{\sqrt{2}} (h_1(n) - h_2(n - 2d)) \quad \text{with } d \in \mathbb{Z} \quad (12)$$

The filters $h_0, h_1^{new}, h_2^{new}$ also satisfy the PR conditions, and h_1^{new} and h_2^{new} are symmetric and symmetric as follows

$$h_1^{new}(n) = h_1^{new}(N_2 - 1 - n) \quad (13)$$

$$h_2^{new}(n) = -h_2^{new}(N_2 - 1 - n) \quad (14)$$

Where $N_2 = N + 2d$

The polyphase components of the filters $h_0(n)$, $h_1(n)$ and $h_2(n)$ are given in [13] with symmetries in Equ(9) And Equ (10) satisfies the PR conditions. The 2D extension of filter bank is illustrated on "Fig 1".

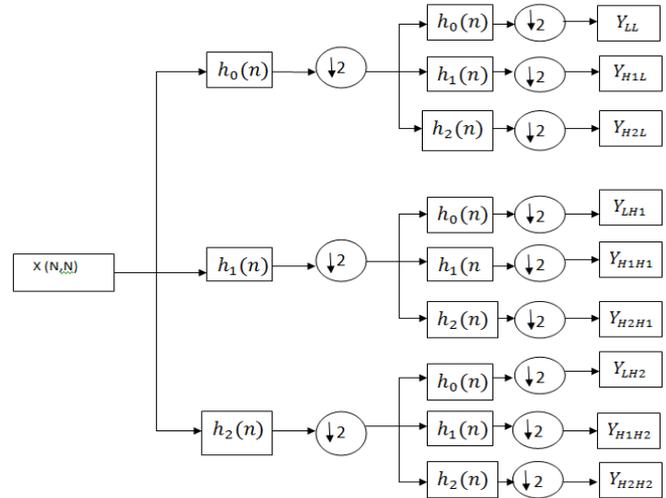


Fig.1. An over sampled filter bank for 2D image

III. IMAGE DENOISING ALGORITHM

Noise is present in an image either additive or multiplicative form. Gaussian noise is most commonly known as additive white Gaussian noise which is evenly distributed over the signal. Each pixel in the noisy image is the sum of the true pixel value and random Gaussian distributed noise vale. Salt and pepper noise is represented as black and white dots in the images. This is caused due to errors in data transmission. Speckle noise is a multiplicative noise which occurs all coherent imaging systems like laser and Synthetic Aperture Radar imagery.

Additive noise satisfies

$$w(x, y) = s(x, y) + n(x, y)$$

Multiplicative noise follows the rule

$$w(x, y) = s(x, y) \times n(x, y)$$

Where $w(x, y)$ is the original image $n(x, y)$ denoted noise and $n(x, y)$ represents pixel location in the image. The image is corrupted when noise is introduced in the images. Depending on the specific sensor there are different types of noises.

The goal is to estimate the image $s(x, y)$ from noisy observations (x, y) . The image denoising algorithm has the following steps.

1. Perform Decomposition using discrete wavelet transform (DWT) and Framelet transform (FTT).
2. Calculate threshold value of detailed parts using shrinkage rules.
3. Apply soft thresholding to the noisy coefficients.
4. Invert the decompositions to reconstruct the denoised image.

IV. THRESHOLDING

Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. Each coefficient is thresholded by comparing against threshold which is calculated using shrinkage techniques. If the coefficient is smaller than the threshold it is set to zero otherwise it is modified. Replacing the small noisy coefficients by zero and inverse transform on the result provide reconstruction with the essential image characteristics without noise with mean squared error (MSE) is minimum.

There are two primary thresholding methods: hard thresholding and soft thresholding [15]. Hard thresholding and soft thresholding are denoted as following

The hard thresholding operator is defined as

$$D(U, \lambda) = U \quad \text{if } |U| > \lambda$$
$$D(U, \lambda) = 0 \quad \text{otherwise}$$

The soft thresholding operator is defined as

$$D(U, \lambda) = (\text{sgn}(U) * \max(0, |U| - \lambda))$$

The soft-thresholding rule is chosen over hard thresholding. Hard thresholding is found to introduce artifacts in the recovered images. But soft thresholding [14] is most efficient and it is also found to yield visually more pleasing images. Different shrinkage rules used in this framework are described below.

V. SHRINKAGE RULES

The choice of a threshold is an important point of interest. It plays a major role in the removal of noise in images because denoising most frequently produces smoothed images, reducing the sharpness of the image. Care should be taken so as to preserve the edges of the denoised image. There exist various methods for wavelet thresholding, which rely on the choice of a threshold value. Some typically used methods for image noise removal as follows.

A. Universal Shrink (US)

The universal threshold can be defined

$$\lambda = \sigma \sqrt{2 \log(N)}$$

N being the signal length, σ is noise variance. Universal threshold give a better estimate for the soft threshold if the number of samples is large. It tends to over smooth the signal, thereby losing some details of the original signal, which result in an increased estimation error.

B. VisuShrink (VS)

Visu shrink is thresholding by applying Universal threshold proposed by Donoho and Johnston [1994].

The threshold is given by

$$\lambda = \sigma \sqrt{2 \log(M)}$$

Where M is the number of pixels in the image. VisuShrink does not deal with minimizing the mean squared error. Another disadvantage is that it cannot remove speckle noise. It can only deal with an additive noise. For the denoising purpose this method is found to give up a smoothed estimate.

C. Minimax Shrink (MS)

The threshold value is calculated using minmax principle. The minimax estimator is the one that realizes the minimum of the maximum MSE obtained for the cost function. The minimax threshold is computed by

$$\lambda = 0.394 + 0.264 \log(M)$$

It has the advantage of giving good predictive performance.

D. Sure Shrink (SS)

Sure Shrink is a thresholding by applying sub band adaptive threshold, a separate threshold is computed for each detail sub band based upon SURE (Stein's unbiased estimator for risk), It is a combination of the universal threshold and the SURE threshold. The sure threshold is define as

$$\lambda = \min(t, \sigma \sqrt{2 \log(M)})$$

Where M is number of wavelet coefficients in the particular subbands. t denotes the value that minimizes Stein's Unbiased Risk Estimator. σ is noise variance. n is the size of the image. SURE shrink has yielded good image denoising performance and comes close to the true minimum MSE of the optimal soft-threshold estimator.

E. Bayes Shrink

BayesShrink is an adaptive data-driven threshold for image de-noising via wavelet soft-thresholding. The threshold is driven in a Bayesian framework, and we assume generalized Gaussian distribution for the wavelet coefficients in each detail sub band and try to find the threshold which minimizes the Bayesian Risk.

Bayes shrink is calculated as follows

$$\sigma_y^2 = \sigma_x^2 + \sigma^2$$

σ_y^2 Variance of noisy image
 σ_x^2 Variance of original image
 σ^2 Noise variance

$$\sigma_y^2 = \frac{1}{M} \sum_{m=1}^M B_m^2$$

Where B_m are the coefficients of wavelet in every scale,

M is the total number of subband coefficients.

$$\sigma_x^2 = \sqrt{\max(\sigma_y^2 - \sigma^2, 0)}$$

$$\text{Where } \sigma = \frac{\text{median}(|d_{ij}|)}{0.6745}$$

$$\lambda = \frac{\sigma^2}{\sigma_x}$$

The reconstruction using Bayes Shrink is Smoother and more visually appealing than one obtained using Sure Shrink.

F. Normal Shrink (NS)

Normal shrink method is computationally more efficient and adaptive because the parameters required for estimating the threshold depends on subband data. The threshold value is calculated as

$$\lambda = \beta \frac{\sigma^2}{\sigma_y}$$

Where β is scale parameter.

$$\beta = \sqrt{\log \left[\frac{L_k}{J} \right]}$$

L_k is the length of the subband at k th scale and J is total number of decompositions. Performance of Normal shrink is similar to Bayes shrink. But normal shrink preserves edges better than Bayes shrink.

VI. EVALUATION CRITERIA

Image Quality [19][20] is a characteristic of an image that measures the perceived image degradation Peak signal to noise ratio (PSNR) and structural similarity index measure (SSIM) were used to measure the efficiency of the proposed method.

A - Perfect image, B - Denoised image, i - pixel row index, j - pixel column index

Mean squared error

This parameter carries the most significance as far as noise suppression is concerned

$$MSE = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (A(i, j) - B(i, j))^2$$

Peak Signal to Noise Ratio

$$PSNR = 10 \times \log_{10} \left(\frac{Peak^2}{MSE} \right)$$

PSNR is the peak signal to noise ratio in decibels(DB).The PSNR is measured in terms of bits per sample or bits per pixel.The image with 8 bits per pixel contains from 0 to 255. The greater PSNR value is, the better the image quality and noise suppression.

The structural similarity index measure (SSIM)

The structural similarity index is a method for measuring the similarity between two images .The SSIM index is a full reference metric, measuring of image quality based on an initial noise free image as reference. SSIM is designed to improve on traditional methods like peak signal to noise ratio.

$$SSIM(A, B) = \frac{(2\mu_A\mu_B + C_1)(2\sigma_{AB} + C_2)}{(\mu_A^2 + \mu_B^2 + C_1)(\sigma_A^2 + \sigma_B^2 + C_2)}$$

Where μ_A and μ_B are the estimated mean intensity along A,B directions and σ_A and σ_B are the standard deviation respectively. σ_{AB} Can be estimated

$$\sigma_{AB} = \left(\frac{1}{N-1} \sum_{i=1}^N (A_i - \mu_A)(B_i - \mu_B) \right)$$

C_1 and C_2 are constants and the values are given as

$$C_1 = (K_1L)^2$$

$$C_2 = (K_2L)^2$$

Where $K_1, K_2 \ll 1$ is a small constant and L is the dynamic range of the pixel values (255).The resultant SSIM index is a decimal value between -1 and 1, and value 1 is only reachable in the case of two identical sets of images.

VII. EXPERIMENTAL RESULTS

The experiments were conducted on gray scale test image LENA of size 512x512 at different noise levels ($\sigma=10,20$) combined with various thresholding such as Universal shrink(US),Visu shrink(VS),Minmax shrink(MS),Sure shrink(SS) ,Bayes shrink(BS),and Normal shrink(NS).The proposed method is compared with Discrete wavlet transform(DWT)[16][17][18] based image denoising.In this experiment ,we choose PSNR and SSIM as evaluated standard.The greater PSNR and SSIM value shows that our proposed method gives better noise suppression without artifacts. PSNR and SSIM values of test image LENA with DWT and FT shown in Table 1&2.

VIII. CONCLUSION

In this work image denoising scheme based on Framelet transform was implemented using MATLAB platform.Various shrinkage rules combined with soft thresholding function were applied to the test image at noise levels ($\sigma=10,20$) with Gaussian noise,poission noise ,Salt &pepper Noise and speckle noise. Experimental results shows that the Framelet transform offers superior performance then discrete wavelet transform (DWT) based denoising techniques both visually and in terms of PSNR&SSIM.

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TABLE.1 PSNR&SSIM of LENA IMAGE USING DWT

Noise Type	Noise Level	Universal Shrink (US)	Visu Shrink (VS)	Min-Max Shrink (MS)	Sure Shrink (SS)	Bayes Shrink (BS)	Normal Shrink (NS)
Gaussian noise	$\sigma=10$	29.685	30.632	28.678	29.856	27.632	30.123
		0.6523	0.7212	0.6932	0.5963	0.7550	0.7189
	$\sigma=20$	27.632	28.952	28.152	27.960	28.123	27.650
0.7453		0.8296	0.7903	0.8523	0.7012	0.6977	
Poisson Noise	$\sigma=10$	26.532	25.960	26.352	26.506	26.921	25.262
		0.7620	0.8632	0.7291	0.8310	0.7320	0.6352
	$\sigma=20$	24.345	24.320	25.312	24.202	25.156	24.260
<i>0.7960</i>		<i>0.8110</i>	<i>0.8001</i>	<i>0.7350</i>	<i>0.7513</i>	<i>0.7125</i>	
Salt & Pepper noise	$\sigma=10$	28.350	27.220	26.650	24.620	23.250	23.960
		0.5450	0.6325	0.7315	0.6320	0.7255	0.6150
	$\sigma=20$	27.601	26.220	25.998	24.890	22.141	23.927
0.6460		0.7125	0.8123	0.7013	0.7556	0.7540	
Speckle noise	$\sigma=10$	27.620	26.652	25.567	26.789	25.127	24.996
		0.5675	0.5963	0.7321	0.7502	0.8123	0.8423
	$\sigma=20$	25.239	24.976	25.134	24.936	24.456	25.789
0.7962		0.8532	0.8532	0.8334	0.8632	0.8706	

TABLE.2 PSNR&SSIM of LENA IMAGE USING FT

Noise Type	Noise Level	Universal Shrink (US)	Visu Shrink (VS)	Min-Max Shrink (MS)	Sure Shrink (SS)	Bayes Shrink (BS)	Normal Shrink (NS)
Gaussian noise	$\sigma=10$	33.654	32.926	33.786	32.110	31.356	31.330
		0.8130	0.8650	0.8023	0.8561	0.8534	0.8943
	$\sigma=20$	32.650	30.332	31.800	30.550	32.113	31.556
0.8250		0.8761	0.8134	0.8672	0.8725	0.8790	
Poisson Noise	$\sigma=10$	30.325	28.663	26.336	27.333	29.332	29.300
		0.8741	0.8650	0.9123	0.8932	0.8790	0.8870
	$\sigma=20$	28.452	26.665	28.320	26.500	26.110	28.215
0.9250		0.8760	0.9250	0.9320	0.8875	0.8960	
Salt & Pepper noise	$\sigma=10$	32.745	30.512	32.630	33.118	30.110	32.127
		0.9614	0.8960	0.9320	0.9415	0.8964	0.9153
	$\sigma=20$	31.342	30.132	31.651	32.160	29.115	30.632
0.8969		0.8967	0.9324	0.9143	0.8976	0.9220	
Speckle noise	$\sigma=10$	31.896	30.360	30.112	31.660	32.632	31.650
		0.9065	0.9156	0.9215	0.9618	0.9715	0.9867
	$\sigma=20$	30.620	29.750	31.632	30.655	30.830	32.632
0.9120		0.9240	0.9354	0.9645	0.8964	0.9743	