When θ passes the values $\frac{\pi}{2}$, π, $3\frac{\pi}{2}$, 2π, we will observe that the exteriors r_i become the interiors $r_{i'}$ and inversely.

According to

$$r = x*cos\theta + y*sin\theta \Longleftrightarrow -r = x*cos(\theta+\pi) + y*sin(\theta+\pi) \quad (25)$$

we will work on an interval [0, π], with the tables II, III and the external or internal points of this interval. The others exterior or interior points and the tables IV, V are deductions.

2) Approximation of the dual : Here, we propose an approximation of the area of the dual of a pixel. In computational geometry, algorithms exists to compute the intersection of convex polygons. A lot of methods of approximation possibility could be studied forward.

In our method, we consider a polygonal approximation based on a few points of the dual in the way to obtain some parameters (θ, r) : the idea is that if an object O' is an approximation of an object O then Dual(O') will be also an approximation of Dual(O).

Moreover, if O'⊂O, we will have Dual(O')⊂Dual(O). Some points of the initial pixel are used to obtain its dual approximation.

Let p be a pixel centered in (p_1,p_2). The pixel p has the vertices A($p_1+\frac{1}{2}$, $p_2+\frac{1}{2}$), B($p_1+\frac{1}{2}$, $p_2-\frac{1}{2}$), C($p_1-\frac{1}{2}$, $p_2-\frac{1}{2}$), D($p_1-\frac{1}{2}$, $p_2+\frac{1}{2}$).

Let p^i be a pixel centered in (p_1,p_2) with its vertices $A^i(p_1+\frac{1}{2}-i*\delta,\ p_2+\frac{1}{2})$, $B^i(p_1+\frac{1}{2},\ p_2-\frac{1}{2}+i*\delta)$, $C^i(p_1-\frac{1}{2}+i*\delta,\ p_2-\frac{1}{2})$, $D^i(p_1-\frac{1}{2},\ p_2+\frac{1}{2}-i*\delta)$.

where i∈ ℕ and δ∈[0,1] with the contraints $i*\delta$∈{0,1}. The duals of the vertices of p^i are :

$$Dual(A^i) : r_{A^i} = (p_1+\frac{1}{2}-i*\delta)\cos\theta + (p_2+\frac{1}{2})\sin\theta \quad (26)$$

$$Dual(C^i) : r_{C^i} = (p_1-\frac{1}{2}+i*\delta)\cos\theta + (p_2-\frac{1}{2})\sin\theta \quad (27)$$

$$Dual(B^i) : r_{B^i} = (p_1+\frac{1}{2})\cos\theta + (p_2-\frac{1}{2}+i*\delta)\sin\theta \quad (28)$$

$$Dual(D^i) : r_{D^i} = (p_1-\frac{1}{2})\cos\theta + (p_2+\frac{1}{2}-i*\delta)\sin\theta \quad (29)$$

Then, p^i is an approximative pixel of p where $i*\delta$ is the difference with the real vertice coordinates of p. The maximal value of $i*\delta$ must be inferior to the maximal of the tolerance error.

We use the external points (see the table I) of the dual of the pixels p^0, p^1, p^2,... p^n to get a polygonal approximation of the dual of p with $p^0 = p^{n+1} = p$.

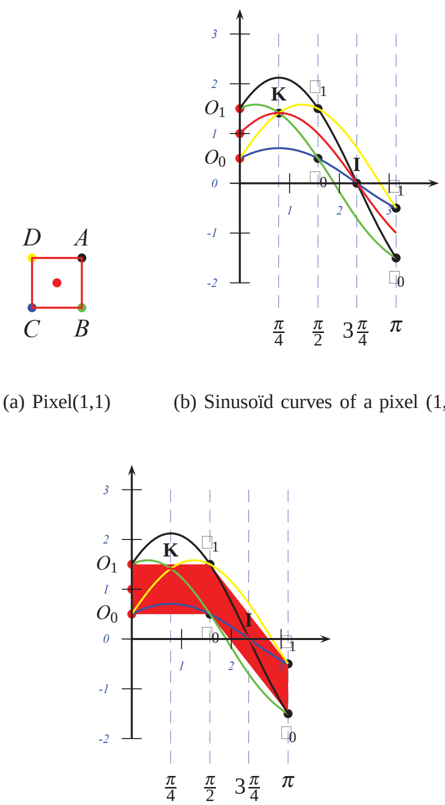
We know that

$$p^{n+1} = p \Longleftrightarrow (n+1)*\delta = 1 \quad (30)$$

Then (30) $\Longleftrightarrow \delta = \frac{1}{n+1}$.

Finally, we need to take $\delta = \frac{1}{n+1}$ to have the pixels p^0, p^1, p^2,...,p^n.

As we know, we work in [0, π]; we determine the approximation of the dual in this interval. We obtain the polygon $O_1□_1□_1□_0□_0O_0$. with the pixel p^0, (see the figure 17 for more details). The following figure 18 shows the polygon (in red color).



(a) Pixel(1,1) (b) Sinusoïd curves of a pixel (1,1)

(c) Sinusoïd curves of p

Figure 18: Approximative pixels Sinusoïd curves

The intersection points of vertices, with i ≥ 1 are Dual(A^i)∩Dual(B^i)={O_1^i}, Dual(A^i)∩Dual(D^i)={$□_1^i$}, Dual(B^i)∩Dual(C^i)={$□_0^i$}, Dual(C^i)∩Dual(D^i)={O_0^i}.

As p^i is close of p, these intersections points are close of O_1, $□_1$, $□_1$, $□_0$, $□_0$, O_0 respectively.

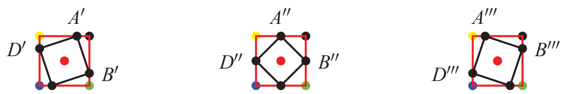
The proposed polygon is then $O_1O_1^1...O_1^n□_1□_1^1...□_1^n□_1$ $□_0□_0^n...□_0^1□_0O_0^n...O_0^1O_0$.

Let S_n be the surface associated to this polygon. According to the tables II, III, we know that the surface of the dual of a pixel is :

$$I = \int_{[0,\frac{\pi}{2}]}(r_A - r_C)dx + \int_{[\frac{\pi}{2},\pi]}(r_D - r_B)dx = 4 \quad (31)$$

According to (31), the extimated error is then $I - S_n = 4 - S_n$ and the percentage will be $\frac{4-S_n}{4}$.

If $\delta = \frac{1}{4}$, we will obtain p^1= p'(p_1,p_2), p^2=p"(p_1,p_2), p^3=p"'(p_1,p_2) that are presented in the following figure 19.

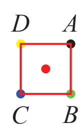


(a) pixel p'   (b) pixel p"   (c) pixel p"'

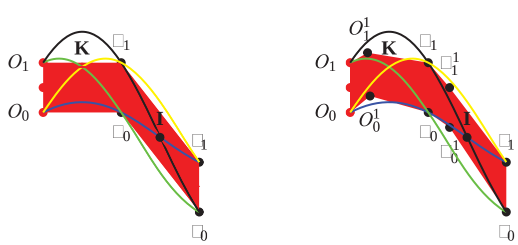
Figure 19: Approximative pixels

.

We illustrates step by step the proposed polygon in the following figures 20, 21

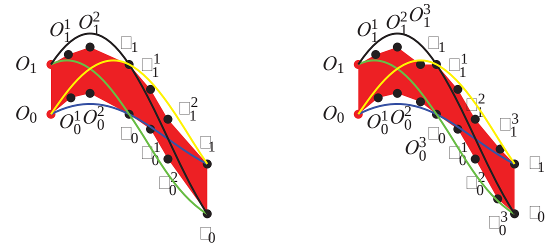


(a) Pixel(1,1)



(b) p   (c) p^1

Figure 20: Sinusoïd curves



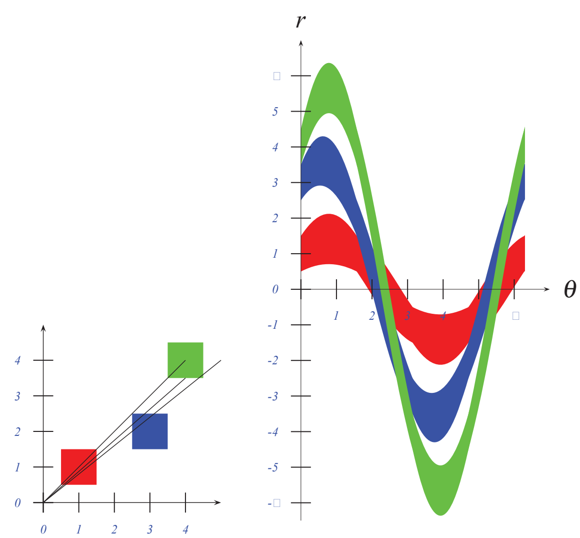
(a) p^2 (b) p^3

Figure 21: Sinusoïd curves

C. Preimage

We present a new definition of preimage of pixels. Here, the term Dual means the Standard Hough transform. The preimage of n pixels is an intersection of n sinusoïd surfaces and differs from the definition introduced in[**?**], by Martine Dexet in her thesis.

Definition □: *Let* $S=\{P_1,P_2,\ldots,P_{n-1},P_n\}$ *be a set of n pixels* P_i *in* ξ_2. *The preimage of S is defined by* $Preimage(S) = \bigcap_{1 \le i \le n} Dual(P_i)$.



(a) pixels   (b) Intersection of the Dual of pixels

Figure 22: Preimage of three pixels

In the figure 22, we see in 22a three colored pixels. In 22b, the corresponding colored dual of each pixel. The intersection of these three duals is a set of points representing the parameters (θ, r) of line that cross each pixel.

The parameters (-r, $cos\theta$, $sin\theta$) are equal to the parameters (c_0, c_1, c_2) of the analytical hyperplane in 2D dimension, presented in the section II-B. As you can see in the figure 22 the preimage of n pixels give two areas because

$$r = x * cos\theta + y * sin\theta \iff -r = x * cos(\theta + \pi) + y * sin(\theta + \pi) \quad (32)$$

D. Recognition algorithm

In this part, we recall that the term dual indicates the Extended Standard Hough Transform. We can see that when, the dual of the losange of each pixel is considered, we will be in the case of naive line recognition.

Moreover, according to the definition of the naive and standard hyperplanes introduced in preliminaries and the geometry construction of these models, we see that the preimage contains the parameters of analytical line that crosses the pixels.

We propose the following algorithm :

Algorithm 1 Naive, Standard line recognition

Data : A set S of n pixels P_1,..., P_n.
Begin
  Preimage←Dual(P_1)
  i←2
  while Preimage≠ Ø and i≤ n **do**
    Preimage←Preimage ∩ Dual(P_i)
    i←i+1
  End while
  if Preimage ≠ Ø **then**
    S belongs to a digital line
  else
    S does not belong to a digital line
  End if
End

The Dual(P_i) could be replaced by an approximation (polygon for example) in order to increase the performance of the algorithm.

The Extended Standard Hough Transform has its particularity to answer the question how can we recognize Naive and Standard by the standard Hough Transform. It conserves the properties of the Standard Hough Transform [6] as the size of the parameter space is limited [6]:$\theta \in$ [0, π] and r $\in$[0,$\sqrt{col^2 + row^2}$} with an image col x row. This leads to the vertical line detection.

IV. Dual of a triangle

We are interested in how to compute the dual of a triangle. In case of triangle grid, we need to know how to determine the dual of a triangle. We establish the following theorem 4.

Theorem 4: The dual of a triangle is the union of the dual of one of its two adjacent(consecutif) sides

Proof: Let [AC] and [AB] be two adjacents sides of a triangle (ABC) as we see in the following figure 23. Let N be a point in the triangle (if N is a vertex the proof is realized). There exists two point I and J such that I $\in$[AC], J$\in$[AB] and N $\in$[IJ]. By analogy with the proof of the theorem 2. We have two consecutif sides [AC] and [AB] that are secants on the vertex A of a triangle (ABC). Then, the surface between the dual of [AC] and [AB] does not contains empty space. Morover, we knows that the dual of I is between the dual of A and C like the dual of J is between the dual of A and B. As we know the dual of N is between the dual of I and the dual of J, then the dual of N is in the union of the dual of [AC] and [AB] . ■

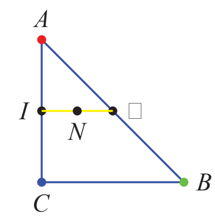
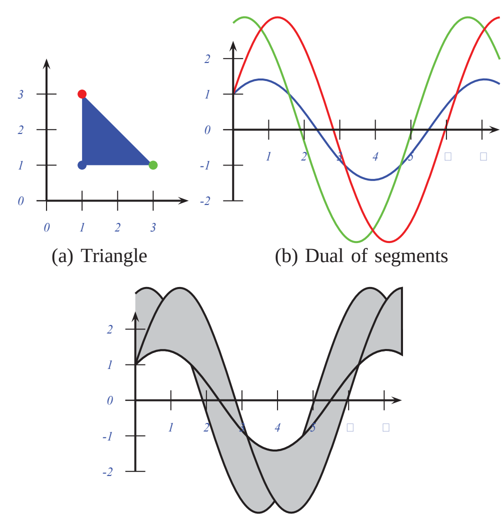


Figure 23: Triangle

The below figure 24 shows the dual of a triangle, it contains the dual of each point of the triangle : In 24b, the dual of the vertex of a triangle in 24a, and in 24c, the dual of a triangle.



(a) Triangle (b) Dual of segments (c) Union of duals

Figure 24: Extended standard hough transform

V. Conclusions

This paper establishes a new method based on the Standard Hough Transform for the recognition of naive or standard lines in a noisy picture. We introduced a new definition of preimage that allows us to obtain an algorithm recognition. We obtained that the dual of a pixel is an non polygonal area. That leads to propose an approximation. Others approximative methods could be studied deeply forward in the objectif to improve the algorithm. Some works still left to be done, particularly the implemention on concrete pictures. The dual of a triangle has been proposed. That could give interested perpectives in the others grids. One of the remaining question is to extend the method in 3D.

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