and

$$\begin{aligned} W_2^* &= -\frac{\mu^6 R_e^4}{2L^{10}} \sum_{\substack{k=-2 \\ j=-2,0,2,4}}^{k=2} \widetilde{C}_{jk}^{*22} \sin(jg + k(h_a - h)) \\ &+ \frac{1}{3} \sum_{\substack{j=-4 \\ k=-3}}^{j=4,k=3} \sum_{n=-2}^{2} \left[Q_{jkn}^{*44} \sin(jg + kh + nh_a) - \widetilde{Q}_{jkn}^{*44} \cos(jg + kh + nh_a) \right] \\ &+ \frac{1}{2} \sum_{\substack{j=-6 \\ k=-6}}^{j=8,k=6} \sum_{n=-4}^{n=4} \left[S_{jkn}^{*16} \sin(jg + kh + nh_a) - \widetilde{S}_{jkn}^{*16} \cos(jg + kh + nh_a) \right] \end{aligned} \tag{74}$$

D. Elements of Long Period transformation

The elements of the transformation and its inverse maybe obtained from the equations of short period transformation, but with the replacement of W_n by W_n^*, (u, U) by $(\grave{u}, \grave{U})$, L_n by L_n^* and $(\grave{u}, \grave{U})$ by $(\grave{\grave{u}}, \grave{\grave{U}})$.

E. Secular perturbations and the computation of position and velocity

The equations of motion are now reduced to

$$\frac{d\grave{\grave{U}}}{dt} = -\frac{\partial \mathcal{H}^{**}}{\partial \grave{\grave{u}}} = 0 \quad , \quad \frac{d\grave{\grave{u}}}{dt} = \frac{\partial \mathcal{H}^{**}}{d\grave{\grave{U}}} = c \tag{75}$$

where c are arbitrary constants so that they admit the solution

$$\grave{\grave{U}} = \grave{\grave{U}}_0 \quad , \quad \grave{\grave{u}} = \grave{\grave{u}}_0 + ct \tag{76}$$

where the constants $(\grave{\grave{U}}_0, \grave{\grave{u}}_0)$ are to be determined from the initial conditions.

Let the elements (u_0, U_0) be known at a given initial epoch t_0 then we can obtain the constants $(\grave{\grave{U}}_0, \grave{\grave{u}}_0)$ as follows:

1- From the elements of the transformation we can compute the initial values $(\grave{U}_0, \grave{u}_0)$ from

$$\grave{u}_0 = u_0 + \sum_{n=1}^{2} \frac{J_2^n}{n!} u_0^{(n)} \quad , \quad \grave{U}_0 = U_0 + \sum_{n=1}^{2} \frac{J_2^n}{n!} U_0^{(n)} \tag{77}$$

2- From the corresponding equations for the elements of the long period transformations

$$\grave{\grave{u}}_0 = \grave{u}_0 + \sum_{n=1}^{2} \frac{J_2^n}{n!} \grave{u}_0^{(n)} \quad , \quad \grave{\grave{U}}_0 = \grave{U}_0 + \sum_{n=1}^{2} \frac{J_2^n}{n!} \grave{U}_0^{(n)} \tag{78}$$

Now having determined $\grave{\grave{u}}_0$ and $\grave{\grave{U}}_0$ we can evaluate $\mathcal{H}^{**} = \mathcal{H}^{**}(\grave{\grave{U}})$, and in turn the constants c are now known.

To compute the position and velocity at any time t we compute

$$\grave{u} = \grave{\grave{u}} + \sum_{n=1}^{2} \frac{J_2^n}{n!} \grave{\grave{u}}^{(n)} \quad , \quad \grave{U} = \grave{\grave{U}} + \sum_{n=1}^{2} \frac{J_2^n}{n!} \grave{\grave{U}}^{(n)} \tag{79}$$

then

$$u = \grave{u} + \sum_{n=1}^{2} \frac{J_2^n}{n!} \grave{u}^{(n)} \quad , \quad U = \grave{U} + \sum_{n=1}^{2} \frac{J_2^n}{n!} \grave{U}^{(n)} \tag{80}$$

Having determined (u, U) at time t, we compute the position, velocity, attitude and attitude motion of the spacecraft.

VI. Conclusion

In this paper we have obtained an analytical solution for the orbit-attitude motion of a charged spacecraft under the effect of Earth oblateness (J_2) and Lorentz force through Hamiltonian framework. The problem is tackled by means of Lie perturbation technique. Two successive canonical transformations were performed in order to eliminate the short and long period terms in succession. secular and periodic terms were retained up to $O(J_2^3)$ and $O(J_2^2)$ respectively.

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