Prediction of Satellite Motion under the Effects of the Earth's Gravity, Drag Force and Solar Radiation Pressure in terms of the KS-regularized Variables

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***Abstract*— This paper is concerned with an orbit prediction using one of the best regular theories (KS-regularized variables). Perturbations due to the Earth's gravitational field with axial symmetry up to the fourth order zonal harmonic, atmospheric drag (variation in density model with height) and solar radiation pressure are considered. Applications of the problem with a comparison between the perturbations effect will be illustrated by numerical and graphical example.**

Keywords—KS-regularized variables; orbit determination; Numerical Modeling

I. Introduction

It is well known that the solutions of the Classical Newtonian Equations of motion are unstable and these equations are not suitable for long-term integrations. Many transformations have emerged in the literature in the recent past to stabilize the equations of motion either to reduce the accumulation of local numerical errors or allowing of using a larger integration step size, in the transformed space, or both.

Examples of such transformations include the use of a new independent variable-time transformation, transformation to orbital parameter space which tends to decouple fast and slow variables, and the use of integrals as control terms. One of such transformation, known as the KS-transformation, is due to Kustaa-neimo and Stiefel, who regularized the non-linear Kepler motion and reduced it to linear differential equations of a harmonic oscillator of constant frequency. Reference [29] further developed the application of the KS-transformation to problems of perturbed motion, producing a perturbational equations version ([1] ; [3] ; [4] ; [13] ; [14] ; [15] ; [20] ; [21] ; [23] ; [28] ; [30]; [31]; [32] ; and [33]).

Space vehicles (including artificial Earth satellites) are subjected to a number of disturbing forces which are classed as non-gravitational forces. These non-gravitational forces are, for example, atmospheric drag, solar radiation pressure, drag on a charged satellite and meteorite collisions. Aside from the effects of the Earth's imperfect shape, the largest perturbative force on a space vehicle close to the Earth is caused by the atmosphere. Whenever a space vehicle passes within about 800 Km of the Earth's surface, it is subjected to a dissipative force induced by motion through the Earth's atmosphere.

Most of the other non-gravitational forces acting upon a space vehicle are negligible with respect to the effect of the Earth's oblateness and atmosphere when the vehicle is close to the Earth.

Getting high in the atmosphere (above 600 Km) the solar radiation pressure force is more important than atmospheric drag. As the vehicle enters inter-planetary space, the previously neglected perturbations become increasingly more important as the space vehicle leaves the region of the Earth's influence.

The drag acceleration causes a distortion in the shape of the orbit and a continuous loss of the kinetic energy of the satellite, to the atmosphere (e.g., [11]). If the atmosphere were stationary, the orientation angles would have not been affected. But due to the rotation of the atmosphere the velocity of the satellite relative to the atmosphere differs from its initial velocity. Consequently, the drag force vector will not lie in the plane of the unperturbed motion and therefore, all six orbital elements will be affected. The net result is:

1) a secular variation of the orbital elements, and

2) a drop in orbital altitude which increases the potential energy to compensate the drop in kinetic energy.

This effect is largest at perigee where the density of the atmosphere is maximum (along the orbit), and is reflected as a decrease in altitude at the next apogee passage. The result is that apogee altitudes decrease more rapidly than do the perigee altitudes. Thus an elliptic orbit will tend to become circular, while an initially circular orbit with uniform drag over its entire path will tend to remain nearly circular and decays through a nearly spiraling trajectory.

The interest in studying the effects of radiation pressure on the motion of artificial satellites has been initiated by the discrepancies between theory and observations of the balloon-type satellites. The effect due to direct solar radiation pressure exceeds that of atmospheric drag at a height of 800 Km with a force magnitude of 10^{-5} dyne/cm ([27] and [26]) and is particularly emphasized for balloon-type satellites for which the area to mass ratio is large. Certain such satellites changes shape from spherical to spheroidal shapes, producing a component of force at right angles to the Sun-satellite direction ([18]).

The solar radiation pressure force becomes a discontinuous function of time when the satellite enters the Earth's shadow.

Reference [19] derived first order expressions for the rates of change in the osculating elements caused by solar radiation pressure by the method of variation of vector elements, shadow effects were not taken into account. References [5] and [12] used Lagrange's planetary equations to find first order solutions, with the integrations performed between the times of exit and entry into the shadow. The resonance effects produced by the commensurabilities between the different mean motions gave good field for detailed theoretical studies (e.g., [9] and [22]). The effect of solar radiation pressure are analyzed in four very useful and interesting expositions given by [10] , [25] and [26] who discussed it (as one of the non-gravitational forces) from all its different aspects and [17] which analyzed in great detail the effects produced both by the direct and albedo radiation pressures on both spherical satellites as well as those of complex shapes.

Reference [24] derived the components of the force in the directions of the radius, normal to it in the direction of motion and normal to the orbit plane, the shadow effect is considered and the effect of diffuse radiation pressure were to be about 1/100 of the direct solar radiation pressure.

References [8] and [26] pointed out the practical use of a shadow function is limited by the number of terms we need to take into account which makes the integration process extremely laborious.

Further, numerical integrations show that the shadow functions give inaccurate results outside of the shadow cylinder since in this region the function is no longer equal to one and the effect is as though the satellite is in the shadow.

Reference [7] studied the behavior of a particle moving under the effect of central attraction and perturbed by the constant radiation pressure. He obtained evidence for the existence of a surface of stable circular orbits with centers on an axis through the primary body and derived the necessary & sufficient conditions for the existence of stable circular orbits when taking the primary's shadow into account.

Also, [16] studied the Kepler problem including radiation pressure and drag, the secular and vector integrals of motion are obtained and [8] pointed out the importance of both solar radiation pressure and atmospheric drag in a first order theory of some satellites.

In this paper, we use the method of fourth order Rung-Kutta method to predict the motion of a satellite under the perturbation effects the Earth's gravitational field with axial symmetry up to the fourth order zonal harmonic, atmospheric drag (variation in density model with height) and solar radiation pressure by using KS-regularized differential equation. we compare graphically the influence of each perturbation.

II. FORMULATE THE PROBLEM

The equations of motion of an artificial satellite are given generally as

$$\ddot{\vec{x}} + \frac{\mu}{r^3}\vec{x} = -\frac{\partial V}{\partial \vec{x}} + \vec{P}, \qquad (2.1)$$

where $\vec{x}$ is the position vector in a rectangular frame (the physical frame), $r = |\vec{x}|$ is the distance from the origin, μ is the Earth's gravitational constant, V is the perturbed time independent potential and $\vec{P}$ is the resultant of all non-conservative perturbing forces and forces derivable from a time dependent potential.

The potential of the Earth's gravity with axial symmetry can be written as

$$V = \mu \sum_{i=2}^{\infty} R^i J_i \left(\frac{1}{r}\right)^{i+1} P_i(x_3/r), \qquad (2.2)$$

where R is the Earth's equatorial radius, J_i is the non-dimensional coefficient of the Earth's oblateness and $P_i(x_3/r)$ is the Legendre polynomial of order i. In the present paper we shall assume that the potential of the Earth's gravity of the axial symmetry is taken up to the fourth order zonal harmonics J_4, then Eq.(2.2) rewrite as

$$V = \frac{3}{2}Q_2 x_3^2 r^{-5} - \frac{1}{2}Q_2 r^{-5} + \frac{5}{2}Q_3 x_3^3 r^{-7} - \frac{3}{2}Q_3 x_3 r^{-5} + \frac{35}{8}Q_4 x_3^4 r^{-9} - \frac{15}{4}Q_4 x_3^2 r^{-7} + \frac{3}{8}Q_4 r^{-5}, \qquad (2.3)$$

where $Q_i = \mu R^i J_i$, i = 2(1)4

and $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

Since the perturbing acceleration due to air drag is expressed as

$$\vec{D} = -\frac{1}{2} C_D \frac{A}{M} \rho |\vec{v}| \vec{v} \qquad (2.4)$$

where - C_D is the non-dimensional drag coefficient depending on the satellite geometry and in most cases its value lies between 2.1 & 2.3;

- A is the effective cross-sectional area, M is the satellite mass;

ρ is the density function of the ambient gas (the atmosphere) and depends primarily on the altitude and to a lesser extent on the solar and geomagnetic activity. In this paper we'll take the most famous models of air density which is

$$\rho = \rho_0 \left[\frac{r_0 - \eta}{r - \eta}\right]^{\tau}, \qquad (2.5)$$

where ρ_0 is the value of ρ at the reference level r_0, while η and τ are two adjustable parameters. They can be adapted to the estimated or observed variations of the solar activity and periodically updated so that the dynamics of the atmosphere is taken into account. The value of η is approximately equal to the mean Earth's equatorial radius and τ equals the inverse of gradient of the density scale height and can take values in the range from 3 to 9 ([6]).

- $\vec{v}$ is the velocity of the satellite relative to the atmosphere.

Also, since the perturbing acceleration due to solar radiation pressure can be expressed as ([16])

$$F_{solar} = \beta \frac{\mu}{r^3} \vec{r} \tag{2.6}$$

where $\vec{r}$ is the radius vector and β is a constant associated with the radiation pressure effect. The range of physically possible β, for a repulsive force, is $0<\beta<1$. For $\beta = 0$ the attracting center does not radiate at all. But for $\beta>1$ the resultant of the collinear force turns from attraction to repulsion, with the consequence that the problem is quite different from the initially stated ([16]).

Finally, the equations of motion of an artificial satellite in KS-regularized variables are

$$\vec{u}'' + \alpha_k \vec{u} = \frac{r}{2} \vec{\lambda}, \tag{2.7.1}$$

$$\alpha_k = -<\vec{u}',\vec{\lambda}>, \tag{2.7.2}$$

$$t' = r, \tag{2.7.3}$$

$$r'' + 4\alpha_k r = \mu + r <\vec{u},\vec{\lambda}>, \tag{2.7.4}$$

where

- α_k is one-half of the negative Keplerian energy as
$$\alpha_k = \left(\frac{\mu}{2} - <\vec{u}',\vec{u}'>\right)/r;$$
- $$\vec{\lambda} = L^T(\vec{u})\vec{b} = L^T(\vec{u})\left(-\frac{\partial V}{\partial \vec{x}} + \vec{P}\right),$$
- $$L(\vec{u}) = \begin{pmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{pmatrix},$$

and

- $$r = -<\vec{u},\vec{u}> \quad , \quad r' = 2<\vec{u},\vec{u}'>;$$

hence $<\vec{a},\vec{b}>$ is used to denote the scalar product of two vectors $\vec{a}$ and $\vec{b}$. Denoting differentiation with respect to the new time s (knowing as the fictitious time) by a prime ($'$), since the independent variable is changed from time (t) to fictitious time (s) according to ([29])

$$\frac{dt}{ds} = r,$$

then for any variable ζ we have

$$\zeta' = r\dot{\zeta}.$$

III. Equations of Motion

The differential equations of motion for the satellite in KS-regularized variables under the perturbations of the Earth's gravity and air drag are

$$u_1'' = -\alpha_k u_1 + \frac{r}{2}\lambda_1, \tag{3.1}$$

$$u_2'' = -\alpha_k u_2 + \frac{r}{2}\lambda_2, \tag{3.2}$$

$$u_3'' = -\alpha_k u_3 + \frac{r}{2}\lambda_3, \tag{3.3}$$

$$u_4'' = -\alpha_k u_4 + \frac{r}{2}\lambda_4, \tag{3.4}$$

$$\alpha_k' = -u_1'\lambda_1 - u_2'\lambda_2 - u_3'\lambda_3 - u_4'\lambda_4, \tag{3.5}$$

$$t' = r, \tag{3.6}$$

$$r'' = \mu + r\left(-4\alpha_k + u_1\lambda_1 + u_2\lambda_2 + u_3\lambda_3 + u_4\lambda_4\right), \tag{3.7}$$

where

$$\lambda_1 = u_1 b_1 + u_2 b_2 + u_3 b_3,$$
$$\lambda_2 = -u_2 b_1 + u_1 b_2 + u_4 b_3,$$
$$\lambda_3 = -u_3 b_1 - u_4 b_2 + u_1 b_3,$$
$$\lambda_4 = u_4 b_1 - u_3 b_2 + u_2 b_3;$$

and we have two forms of b's; one with drag force only and the second with drag and solar radiation pressure; of course under the Earth's gravity.

The first form of b_i (i=1,3) are

$$b_1 = \frac{15}{2}Q_2 x_1 x_3^2 r^{-4} - \frac{3}{2}Q_2 x_1 r^{-5} + \frac{35}{2}Q_3 x_1 x_3^3 r^{-9} - \frac{15}{2}Q_3 x_1 x_3 r^{-7} + \frac{315}{8}Q_4 x_1 x_3^4 r^{-11} - \frac{105}{4}Q_4 x_1 x_3^2 r^{-9} + \frac{15}{8}Q_4 x_1 r^{-7} - \gamma\rho v v_1,$$

$$b_2 = \frac{15}{2}Q_2 x_2 x_3^2 r^{-4} - \frac{3}{2}Q_2 x_2 r^{-5} + \frac{35}{2}Q_3 x_2 x_3^3 r^{-9} - \frac{15}{2}Q_3 x_2 x_3 r^{-7} + \frac{315}{8}Q_4 x_2 x_3^4 r^{-11} - \frac{105}{4}Q_4 x_2 x_3^2 r^{-9} + \frac{15}{8}Q_4 x_2 r^{-7} - \gamma\rho v v_2,$$

$$b_3 = -\frac{9}{2}Q_2 x_3 r^{-5} + \frac{15}{2}Q_2 x_3^3 r^{-7} - 15 Q_3 x_3^2 r^{-7} + \frac{35}{2}Q_3 x_3^4 r^{-9} + \frac{3}{2}Q_3 r^{-5} - \frac{175}{4}Q_4 x_3^3 r^{-9} + \frac{315}{8}Q_4 x_3^5 r^{-11} + \frac{75}{8}Q_4 x_3 r^{-7} - \gamma\rho v v_3.$$

The second form of b_i (i=1,3) are

$$b_1 = \frac{15}{2}Q_2 x_1 x_3^2 r^{-4} - \frac{3}{2}Q_2 x_1 r^{-5} + \frac{35}{2}Q_3 x_1 x_3^3 r^{-9} - \frac{15}{2}Q_3 x_1 x_3 r^{-7} + \frac{315}{8}Q_4 x_1 x_3^4 r^{-11} - \frac{105}{4}Q_4 x_1 x_3^2 r^{-9} + \frac{15}{8}Q_4 x_1 r^{-7} - \gamma\rho v v_1 - \beta\mu x_1 r^{-3},$$

$$b_2 = \frac{15}{2}Q_2 x_2 x_3^2 r^{-4} - \frac{3}{2}Q_2 x_2 r^{-5} + \frac{35}{2}Q_3 x_2 x_3^3 r^{-9} - \frac{15}{2}Q_3 x_2 x_3 r^{-7} + \frac{315}{8}Q_4 x_2 x_3^4 r^{-11} - \frac{105}{4}Q_4 x_2 x_3^2 r^{-9} + \frac{15}{8}Q_4 x_2 r^{-7} - \gamma\rho v v_2 - \beta\mu x_2 r^{-3},$$

$$b_3 = -\frac{9}{2}Q_2 x_3 r^{-5} + \frac{15}{2}Q_2 x_3^3 r^{-7} - 15 Q_3 x_3^2 r^{-7} + \frac{35}{2}Q_3 x_3^4 r^{-9} + \frac{3}{2}Q_3 r^{-5} - \frac{175}{4}Q_4 x_3^3 r^{-9} + \frac{315}{8}Q_4 x_3^5 r^{-11} + \frac{75}{8}Q_4 x_3 r^{-7} - \gamma\rho v v_3 - \beta\mu x_3 r^{-3};$$

and

$$\gamma = \frac{1}{2}C_D\frac{A}{M}.$$

IV. SOLUTION TECHNIQUE

In this section, the solution technique of the formulations of section 3 will be applied by two steps. The first step is to transform Eqs.(3.1) to (3.7) into first order differential equations by the following substitutions

$$y_i = u_i, \qquad y_{i+4} = u_i', \quad \mathrm{i} = 1(1)4,$$

$$y_9 = \alpha_k, \qquad y_{10} = t, \qquad y_{11} = r$$

and $y_{12} = r'$.

Then the first order system of the problem becomes

$$y_1' = y_5, \tag{4.1}$$

$$y_2' = y_6, \tag{4.2}$$

$$y_3' = y_7, \tag{4.3}$$

$$y_4' = y_8, \tag{4.4}$$

$$y_5' = -y_9\, y_1 + \tfrac{1}{2} y_{11}\, b_1, \tag{4.5}$$

$$y_6' = -y_9\, y_2 + \tfrac{1}{2} y_{11}\, b_2, \tag{4.6}$$

$$y_7' = -y_9\, y_3 + \tfrac{1}{2} y_{11}\, b_3, \tag{4.7}$$

$$y_8' = -y_9\, y_4 + \tfrac{1}{2} y_{11}\, b_4, \tag{4.8}$$

$$y_9' = -y_5\, b_1 - y_6\, b_2 - y_7\, b_3 - y_8\, b_4, \tag{4.9}$$

$$y_{10}' = y_{11}, \tag{4.10}$$

$$y_{11}' = y_{12}, \tag{4.11}$$

$$y_{12}' = \mu + y_{11}\left(y_1 b_1 + y_2 b_2 + y_3 b_3 + y_4 b_4 - 4 y_9 \right). \tag{4.12}$$

Also, the accuracy checks were need in the solution could be obtained. The accuracy of the computed values of the *y's* variables at any fictitious time *s* (corresponding to the time *t*) could be checked by the bilinear relation (BI)

$$BI = y_4\, y_1' - y_3\, y_2' + y_2\, y_3' - y_1\, y_4',$$

and it must be equal to zero in excellent accuracy. The second step is solving the above system by using the fourth-order Runge-Kutta method with a fixed step size in the next section.

V. RESULTS AND CONCLUSION

We'll take as the numerical example the Explorer 19 at 750 Km height ([2]). So, the initial position and velocity components are

$$\vec{x}_0 = (3538.646, -2902.799, -5483.478)\ Km,$$

$$\dot{\vec{x}}_0 = (5.842408, -1.772259, 4.707377)\ Km/sec,$$

at epoch 14 February 1976, where one orbital revolution is elapsed in 111 min., it has the ratio *A/m = 13.04E-07 Km/Kg.*

Since the adopted physical constant are

$R = 6378.135\ Km,$ $\qquad \mu = 398600.8\ Km^3/sec^2$,

and the coefficients of the four order zonal harmonic are

$J_2 = 1.0826157 \times 10^{-3}$,

$J_3 = -2.53648 \times 10^{-6}$,

$J_4 = -1.6233000 \times 10^{-6}$,

where $C_D = 2.2$ ([27]), also we'll chose τ equals 4, and finally β equals 0.5 .

We'll use all the above values to compute the position and velocity components, i.e., the six elements; especially (the elements a, e, i) because of these elements are much affected by our studied forces. Also, we'll get the accuracy check (bilinear relation, *BI*) at any time (days); and we get the following figures and supplemented tables. The figures show the variations of the classical orbital elements with the time over one hundred, one thousand and two thousand revolutions (as an example). All the Figures show the effects of the Earth's gravitational field with axial symmetry up to the four order zonal harmonic, air drag and solar radiation force. Also, all the Figures show a significant difference in *a,i*; but in *e* show the slightly difference, that is because the height of satellite about 750 Km. All Tables give the bilinear relation (*BI*) under the studied forces at any time (days), which indicates a good prediction for the numerical solution. The numerical results are just only as an example, since this method could be applied to any orbit. To get more accurate prediction of the motion of the artificial satellite we will be taken into account the whole other forces affecting on the motion.

TABLE I. THE VALUES OF BILINEAR RELATION CORRESPOND TO THEIR PERTURBATION FORCES, OVER ONE HUNDRED REVOLUTIONS.

Time (Days)	The bilinear relation (BI)		
	Only gravity	*With pert. and without SRP*	*With pert. and with SRP*
0.0	9.094947018E-13	9.094947018E-13	9.094947018E-13
0.768729642	-2.119122655E-10	-2.437445801E-10	-2.037268132E-10
1.537459283	-4.147295840E-10	-4.174580681E-10	-4.110916052E-10
2.306188925	-6.075424608E-10	-6.511982065E-10	-6.184563972E-10
3.074918567	-7.621565601E-10	-9.304130799E-10	-8.494680515E-10
3.843648208	-9.813447832E-10	-1.155967766E-09	-1.048647391E-09
4.61237785	-1.190528565E-09	-1.263288141E-09	-1.218722900E-09
5.381107492	-1.396074367E-09	-1.469743438E-09	-1.429725671E-09
6.149837133	-1.651642378E-09	-1.701664587E-09	-1.690750651E-09
6.918566775	-1.876287570E-09	-1.972694008E-09	-1.965418051E-09
7.687296417	-2.097294782E-09	-2.193701221E-09	-2.183696779E-09

TABLE II. THE VALUES OF BILINEAR RELATION CORRESPOND TO THEIR PERTURBATION FORCES, OVER ONE THOUSAND REVOLUTIONS.

Time (Days)	The bilinear relation (BI)		
	Only gravity	*With pert. and without SRP*	*With pert. and with SRP*
69.10879479	-1.300668373E-08	-1.282751327E-08	-1.281568984E-08
69.87752443	-1.291800800E-08	-1.270018402E-08	-1.277567208E-08
70.64625407	-1.278158379E-08	-1.255057214E-08	-1.275566319E-08
71.41498371	-1.259513738E-08	-1.254329618E-08	-1.268517735E-08
72.18371335	-1.251191861E-08	-1.239686753E-08	-1.255466486E-08
72.95244299	-1.232592695E-08	-1.229727786E-08	-1.244961823E-08
73.72117264	-1.223497748E-08	-1.209718903E-08	-1.233775038E-08
74.48990228	-1.212993084E-08	-1.196940502E-08	-1.219359547E-08
75.25863192	-1.194939614E-08	-1.186663212E-08	-1.204853106E-08
76.02736156	-1.177386366E-08	-1.179296305E-08	-1.197349775E-08
76.7960912	-1.170656105E-08	-1.166154107E-08	-1.182388587E-08

TABLE III. THE VALUES OF BILINEAR RELATION CORRESPOND TO THEIR PERTURBATION FORCES, OVER TWO THOUSAND REVOLUTIONS.

Time (Days)	**The bilinear relation (BI)**		
	Only gravity	***With pert. and without SRP***	***With pert. and with SRP***
145.981759	3.085688149E-09	3.104332791E-09	2.578872227E-09
146.7504886	3.039986041E-09	3.069544618E-09	2.542719812E-09
147.5192182	2.985871106E-09	3.016566552E-09	2.493834472E-09
148.2879479	2.934029908E-09	2.949946065E-09	2.428350854E-09
149.0566775	2.875594873E-09	2.886963557E-09	2.359001883E-09
149.8254072	2.803062671E-09	2.816250344E-09	2.294882506E-09
150.5941368	2.724959813E-09	2.752358341E-09	2.222009243E-09
151.3628664	2.638671504E-09	2.673573363E-09	2.130605026E-09
152.1315961	2.550564204E-09	2.580577529E-09	2.056481208E-09
152.9003257	2.444039637E-09	2.470642357E-09	1.969851837E-09
153.6690554	2.341835170E-09	2.371393748E-09	1.866283128E-09

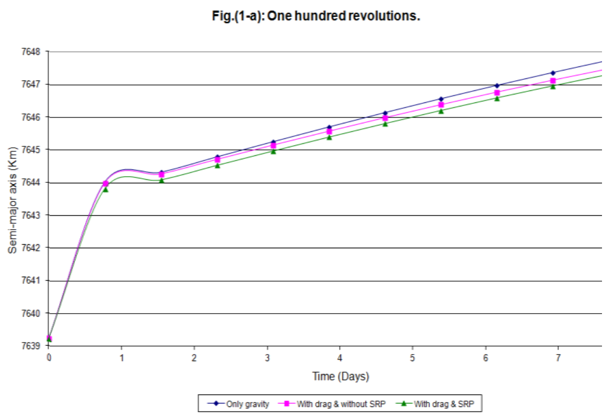

Fig. 1.a Semi-major axis of One hundred revolutions

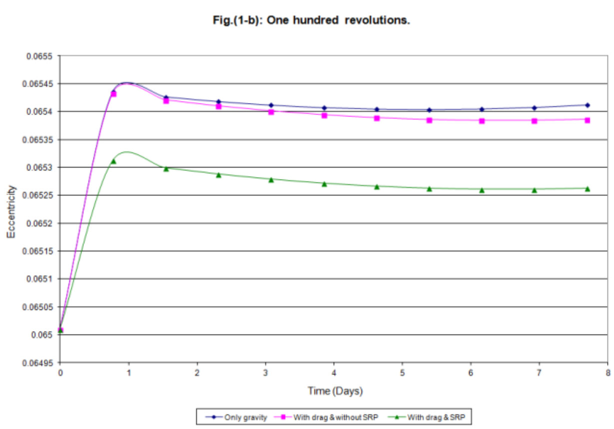

Fig. 1.b Eccentricity of One hundred revolutions

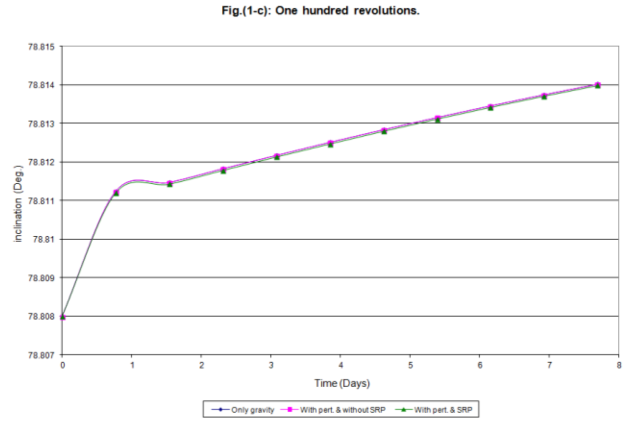

Fig. 1.c Inclination of One hundred revolutions

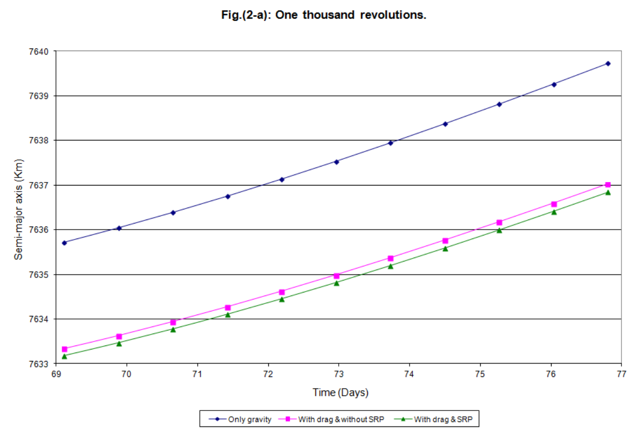

Fig. 2.a Semi-major axis of One thousand revolutions

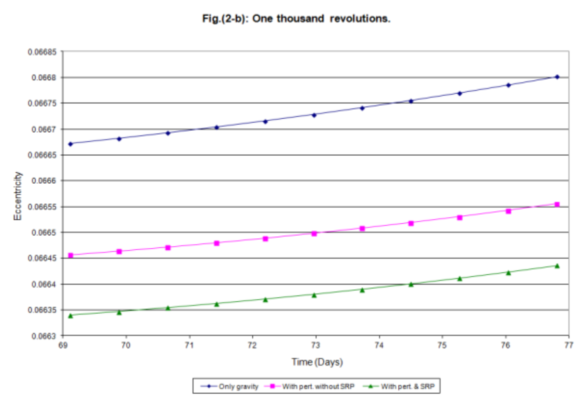

Fig. 2.b Eccentricity of One thousand revolutions

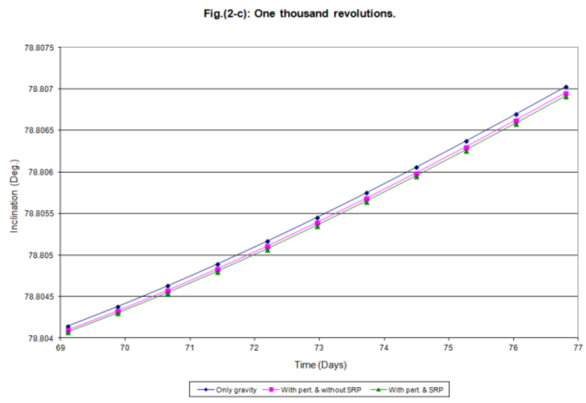

Fig. 2.c Inclination of One thousand revolutions

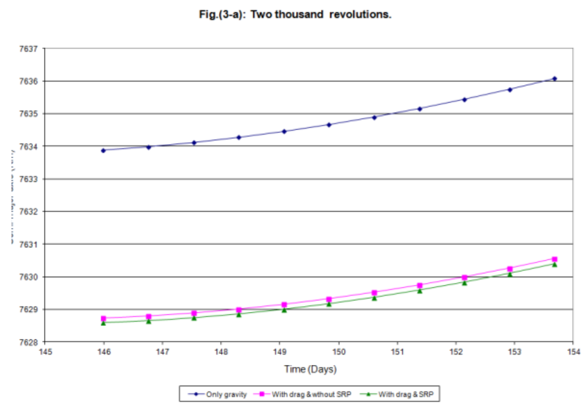

Fig. 3.a Semi-major axis of Two thousand revolutions

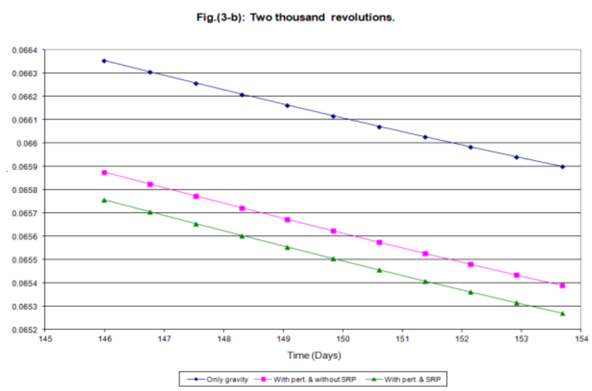

Fig. 3.b Eccentricity of Two thousand revolutions

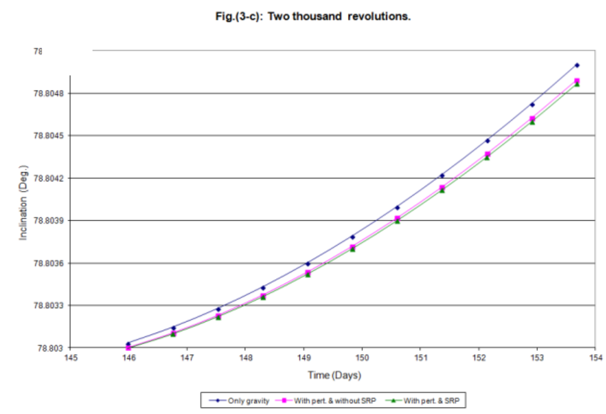

Fig. 3.c Inclination of Two thousand revolutions

REFERENCES

[1] R. F. Arenstorf, "New regularization of the restricted problem of three bodies," Astron. J. , vol. 68, p. 548, Oct 1963.

[2] C. J. Brookes and F. C. E. Rylan, "Perturbation of the orbit of Explorer 19 due to solar radiation," Celestial Mechanics, vol. 27, pp. 339-352, Aug 1982.

[3] R. Broucke, "Solution of the N-Body Problem with Recurrent Power Series," Celestial Mechanics, vol. 4, pp. 110-115, Sep 1971.

[4] C. J. Cohen and R. H. Lyddane, "Radius of convergence of Lie series for some elliptic elements," Celestial Mechanics, vol. 25, pp. 221-234, 1981.

[5] G. E. Cook, "Luni-Solar Perturbations of the Orbit of an Earth Satellite," Geophysical Journal International, vol. 6, pp. 271-291, Apr 1962.

[6] F. Delhaise, "Analytical treatment of air drag and earth oblateness effects upon an artificial satellite," Celestial Mechanics and Dynamical Astronomy, vol. 52, pp. 85-103, Mar 1991.

[7] H. Dankowicz, "Some special orbits in the two-body problem with radiation pressure," Celestial Mechanics and Dynamical Astronomy, vol. 58, pp. 353-370, Apr 1994.

[8] R. V. De Moraes, "Solar radiation pressure and balloon type artificial satellites," in Satellite Dynamics, 1975.

[9] G. Hori, "The Effect of Radiation Pressure On The Motion Of An Artificial Satellite," in Space Mathematics Part III, vol. 5, J. B. Rosser, Ed., Amer. Math. Soc., 1966, p. 167.

[10] B. Kampos, "General Perturbation theory," 1968.

[11] D. King-Hele, Satellite orbits in an atmosphere. Theory and applications., D. King-Hele, Ed., Glasgow, UK.: Blackie and Son Ltd., 1987.

[12] Y. Kozai, "Effects Motion of an Artificial Satellite," SAO Special Report, vol. 56, p. 25, Jan 1961.

[13] E. Leimanis and N. Minorsky, Dynamics and nonlinear mechanics. Some recent advances in the dynamics of rigid bodies and celestial mechanics, New York, USA: Wiley, 1958.

[14] T. Levi-Civita, "Sur la résolution qualitative du problème restreint des trois corps," in Opere Matematiche, vol. 2, Bologna, 1956.

[15] R. H. Lyddane, "Small eccentricities or inclinations in the Brouwer theory of the artificial satellite," Astron. J., vol. 68, p. 555, Oct 1963.

[16] A. G. Mavraganis and D. G. Michalakis, "The two-body problem with drag and radiation pressure," Celestial Mechanics and Dynamical Astronomy, vol. 58, pp. 393-403, Apr 1994.

[17] A. Milani, A. M. Nobili and P. Farinella, Non-gravitational perturbations and satellite geodesy., Bristol,UK.: Adam Hilger Ltd., 1987.

[18] P. Moore, "Perturbations of a spheroidal satellite due to direct solar radiation pressure," Celestial Mechanics, vol. 20, pp. 125-142,Aug 1979.

[19] P. Musen, "The Influence of the Solar Radiation Pressure on the Motion of an Artificial Satellite," J. Geophys. Res., vol. 65, p. 1391, Sep 1960.

[20] D. O'Mathuna, "Satellite Prediction Formulae for Vinti's Model," Celestial Mechanics, vol. 1, pp. 467-478, Sep 1970.

[21] R. Ramnath, "Gravitational Perturbations of Equatorial Orbits," Celestial Mechanics, vol. 8, pp. 85-98, Aug 1973.

[22] M. Roy, Dynamics of satellites, I. U. of Theoretical and A. Mechanics, Eds., Berlin: Springer Heidelberg, 1963.

[23] G. Scheifele, "On Nonclassical Canonical Systems," Celestial Mechanics, vol. 2, pp. 296-310, Sep 1970.

[24] L. Sehnal, "The influence of the re-radiation of the Earth on the motion of the artificial satellites," in The Theory of Orbits in the Solar System and in Stellar Systems, 1966.

[25] L. Sehnal, "Radiation pressure effects in the motion of artificial satellites.," in Dynamics of Satellites, 1970.

[26] L. Sehnal, "Non-gravitational forces in satellite dynamics," in Satellite Dynamics, 1975.

[27] M. A. Sharaf and M. E. Awad, "Prediction of trajectories in Earth's gravitational field with axial symmetry.," Proc. Math. Phys. Soc. Egypt, vol. 60, 1985.

[28] T. E. Sterne, "The gravitational orbit of a satellite of an oblate planet," Astron. J., vol. 63, p. 28, Jan 1958.

[29] E. L. Stiefel and G. Scheifele, Linear and regular celestial mechanics; perturbed two-body motion, numerical methods, canonical theory, Berlin: Springer-Verlag, 1971.

[30] V. Szebehely and G. E. O. Giacaglia, "On the elliptic restricted problem of three bodies," Astron. J., vol. 69, p. 230, Apr 1964.

[31] V. Szebehely, "Selection of regularizing functions," Astron. J., vol. 72, p. 370, Apr 1967.

[32] B. D. Tapley and V. Szebehely, "The regularization of Optimal Trajectories," in JPL Space Programs Summary, vol. 4, Pasadena, Calif., Jet Propulsion Laboratory, 1967, pp. 37-46.

[33] E. T. Whittaker, A Treatise on the Analytical Dynamics, of Particles and Rigid Bodies, Fourth Edition ed., E. T. Whittaker, Ed., London and New York.: Cambridge University Press, 1961.