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Approximation Algorithms for Scheduling with Rejection on Two Unrelated Parallel Machines

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***Abstract*—In this paper, we study the scheduling problem with rejection on two unrelated parallel machines. We may choose to reject some jobs, thus incurring the corresponding penalty. The goal is to minimize the makespan plus the sum of the penalties of the rejected jobs. We first formulate this scheduling problem into an integer program, then relax it into a linear program. From the optimal solution to the linear program, we obtain the two algorithms using the technique of linear programming rounding. In conclusion, we present a deterministic 3-approximation algorithm and a randomized 3-approximation algorithm for this problem.**

Keywords—Scheduling; Rejection; Approximation algorithm; Linear programming; Rounding

I. INTRODUCTION

The unrelated parallel machine scheduling problem to minimize makespan, $R\|C_{\max}$ following the notation of Graham et al. [1], is one of the classic NP- hard problems in combinatorial optimization. This problem is mentioned in many works concerning approximation algorithms [2 ,3], and it has received much attention in the past few decades with many extant approximation algorithms, among which the currently best 2-approximation algorithm is due to [4], who also show that the problem does not admit an algorithm with approximation ratio smaller than $3/2$, unless P = NP. One special case for this problem-Each job can only be assigned to a subset of the machine set with the same processing time-which is also known as the restricted assignment problem. Does there exist an approximation algorithm with approximation ratio better than 2? This problem is regarded as one of the ten open problems in combinatorial optimization [3].

The scheduling problem with rejection arises in make-to-order production systems with limited production capacity and tight delivery requirements, where simultaneous job rejection and scheduling decisions have to be made for maximizing the total revenue. In such systems, accepting orders without considering their impact on the whole system may delay some of the orders beyond their due dates. To be able to preserve the high quality of service to customers of accepted orders, the manufacturer has to determine which orders to accept and how to schedule them to maximize total revenue.

Many important results concerning the parallel machine scheduling problems with rejection appear in recent years. In reference [5] Engels et al. develop some techniques to design approximation algorithm for the general problem with rejection. Their main technique is to reduce a problem with rejection to a scheduling problem without rejection based on the linear programming rounding method. Hoogeveen, Skutella and Woeginger [6] consider the preemptive scheduling with rejection, and the goal is to optimize the preemptive makespan on the m parallel machines plus the sum of the penalties of the rejected jobs. They provide a complete classification of these scheduling problems on complexity and approximability. On the variant with an arbitrary number of unrelated machines, which is APX-hard, they propose a 1.58-approximation algorithm for it. Moreover, their results for unrelated machines may be carried over to the corresponding preemptive open shop scheduling problem with rejection. Li and Yuan [7] consider several parallel machine scheduling problems with deteriorating jobs and rejection. The objective is to minimize the scheduling cost of the accepted jobs plus The total penalty of the rejected jobs. They propose two fully polynomial time approximation schemes for the problems under consideration. In reference [8] Gerstl and Mosheiov study scheduling problems with rejection and general position-dependent processing times on identical parallel machines, and they introduce efficient algorithms for the problems, which run in $O(n^{m+3})$ time (which is polynomial for a given number of machines). There are many other important results ([9-13]).

In this paper, we study the following problem: We have two machines $\{M_1, M_2\}$, together with a job set $\{J_1, \cdots, J_n\}$. The processing time of job J_j on machine M_i is $p_{ij}(i=1,2)$. We may choose not to process job J_j, thus incurring a penalty q_j. The objective is to optimize the makespan for the processed jobs plus the sum of the penalties of the rejected jobs. If we denote by J_A the set of processed jobs, and J_R the set of rejected jobs, the problem may be expressed as $R_2|rej|C_{\max}(J_A)+\sum_{j\in J_R} q_j$.

We organize this paper as follows: In section Two we give a deterministic 3-approximation algorithm for the problem under consideration. In section Three a randomized 3-approximation algorithm is presented for this problem. In section Four we give some concluding remarks.

II. A 3-APPROXIMATION ALGORITHM FOR $R_2|rej|C_{\max}(J_A)+\sum_{j\in J_R} q_j$

We introduce a decision variable z_j for job $J_j\,(j=1,2,\cdots,n)$, with the following meaning:

$$z_j=\begin{cases}1, if-job-J_j-is-rejected\\ 0, else\end{cases}$$

For each machine-job pair $(i,j)(i=1,2;\ j=1,2,\cdots,n)$, we introduce a decision variable x_{ij} with the following meaning:

$$x_{ij}=\begin{cases}1, if-job-J_j-is-processed-on-M_i\\ 0, otherwise\end{cases}$$

Based on the above notations we formulate the scheduling problem into an integer program.

$$\begin{aligned}\min\ & T+\sum_{j=1}^{n} q_j z_j\\ s.t.\ & x_{1j}+x_{2j}+z_j=1,\ \ j=1,2,\cdots,n\\ & \sum_{j=1}^{n} p_{1j}x_{1j}\le T, \qquad (1)\\ & \sum_{j=1}^{n} p_{2j}x_{2j}\le T,\\ & x_{ij}\in\{0,1\},\ \ i=1,2;\ j=1,2,\cdots,n\\ & z_j\in\{0,1\},\ \ \ j=1,2,\cdots,n.\end{aligned}$$

We first relax integer program (1) into the following linear program.

$$\begin{aligned}\min\ & T+\sum_{j=1}^{n} q_j z_j\\ s.t.\ & x_{1j}+x_{2j}+z_j=1,\ \ j=1,2,\cdots,n\\ & \sum_{j=1}^{n} p_{1j}x_{1j}\le T, \qquad (2)\\ & \sum_{j=1}^{n} p_{2j}x_{2j}\le T,\\ & x_{ij}\ge 0,\ \ i=1,2;\ j=1,2,\cdots,n\\ & z_j\ge 0,\ \ \ j=1,2,\cdots,n.\end{aligned}$$

We denote by $(x_{1j}^*, x_{2j}^*, z_j^*)$ the optimal solution to linear program (2). Obviously we have $T^*+\sum_{j=1}^{n} q_j z_j^*\le OPT$. Here $T^*=\max\left\{\sum_{j=1}^{n} p_{1j}x_{1j}^*, \sum_{j=1}^{n} p_{2j}x_{2j}^*\right\}$, OPT stands for the optimal objective value for the scheduling problem.

We apply the following rounding procedure to $(x_{1j}^*, x_{2j}^*, z_j^*)$.

Rounding Procedure R_1

Step 1: Whenever $z_j^*\ge\alpha$ $(0<\alpha<1)$, *set* $\bar z_j=1$ *(job* J_j *is rejected).*

Step 2: Otherwise set $\bar z_j=0$ *(job* J_j *is processed).*

Step 3: Whenever $x_{1j}^*\ge x_{2j}^*$, *set* $\bar x_{1j}=1$. *(job* J_j *is processed on machine* M_1*).*

Step 4: Else set $\bar x_{2j}=1$. *(job* J_j *is processed on machine* M_2*).*

We now analyze the quality of the solution obtained by Rounding Procedure R_1.

We first take a look at the makespan $\bar T$, here $\bar T=\max\{\bar L_1,\bar L_2\}$.

$$\bar L_1=\sum_{j:\bar x_{1j}=1} p_{1j}=\sum_{j:x_{1j}^*>\frac{1-\alpha}{2}} p_{1j}<\frac{2}{1-\alpha}\sum_{j:x_{1j}^*>\frac{1-\alpha}{2}} p_{1j}x_{1j}^*$$

$$\le\frac{2}{1-\alpha}\sum_{j=1}^{n} p_{1j}x_{1j}^*\le\frac{2}{1-\alpha}T^*.$$

Similarly we have $\bar L_2\le\frac{2}{1-\alpha}T^*$. Therefore we get $\bar T\le\frac{2}{1-\alpha}T^*$.

As for the total penalty for the rejected jobs, we have

$$\sum_{j:\bar z_j=1} q_j=\sum_{j:z_j^*\ge\alpha} q_j\le\frac{1}{\alpha}\sum_{j:z_j^*\ge\alpha} q_j z_j^*\le\frac{1}{\alpha}\sum_{j=1}^{n} q_j z_j^*$$

So we have

$$\bar T+\sum_{j:\bar z_j=1} q_j\le\frac{2}{1-\alpha}T^*+\frac{1}{\alpha}\sum_{j=1}^{n} q_j z_j^*$$

$$\le f(\alpha)\left(T^*+\sum_{j=1}^{n} q_j z_j^*\right)\le f(\alpha)OPT.$$

Here $f(\alpha)=\max\left\{\frac{2}{1-\alpha},\frac{1}{\alpha}\right\}$ is the approximation ratio. The value for $f(\alpha)$ is minimized if and only if $\frac{2}{1-\alpha}=\frac{1}{\alpha}$, i.e., $\alpha=\frac{1}{3}$, the minimum value for $f(\alpha)$ is 3.

Based on the discussions above we propose a 3-approximation algorithm for $R_2|rej|C_{\max}(J_A)+\sum_{j\in J_R} q_j$.

Algorithm 1

Step 1: Formulate the scheduling problem into an integer program (1).

Step 2: Relax integer program (1) to linear program (2).

Step 3: Solve linear program (2) and obtain an optimal solution $\left(x_{1j}^*, x_{2j}^*, z_j^*\right)$.

Step 4: Set $J_R=\left\{j : z_j^* \ge \frac{1}{3}\right\}$, $J_A=\left\{j : z_j^* < \frac{1}{3}\right\}$.

Step 5: Regulate $J_{M_1}=\left\{j : x_{1j}^* \ge x_{2j}^*, z_j^* < \frac{1}{3}\right\}$, $J_{M_2}=\left\{j : x_{2j}^* \ge x_{1j}^*, z_j^* < \frac{1}{3}\right\}$.

Step 6: Process the jobs in $J_{M_i}(i=1,2)$ *continuously on machine* M_i *in an arbitrary order.*

Theorem 1. Algorithm 1 is a 3-approximation algorithm for $R_2|rej|C_{\max}(J_A)+\sum_{j\in J_R} q_j$.

III. A RANDOMIZED 3-APPROXIMATION ALGORITHM FOR $R_2|rej|C_{\max}(J_A)+\sum_{j\in J_R} q_j$

In section Two we have a deterministic principle that determines the set J_R, J_{M_1} and J_{M_2}. In this section the processed jobs are assigned to machine M_1, M_2 with some probability.

We still use program (2) as a linear program relaxation for the scheduling problem. For the optimal solution $\left(x_{1j}^*, x_{2j}^*, z_j^*\right)$ to linear program (2), we apply the following rounding procedure.

Rounding Procedure R_2

Step 1: Whenever $z_j^* \ge \alpha$ $(0<\alpha<1)$, *set* $\bar{z}_j = 1$ *(job* J_j *is rejected)*

Step 2: Otherwise set $\bar{z}_j = 0$ *(job* J_j *is processed). Job* J_j *is assigned to machine* $M_i(i=1,2)$ *with probability* $\tilde{x}_{ij}=\frac{x_{ij}^*}{x_{1j}^*+x_{2j}^*}$.

In rounding procedure R_2, the set of rejected jobs J_R is a deterministic set. While the schedule formed by the jobs in J_A is a randomized one. We denote by $L_i(i=1,2)$ the load on machine M_i in the randomized schedule, and by T the makespan for the schedule. Obviously we have $T=\max\{L_1, L_2\}$.

For processed job J_j, obviously we have $z_j^* < \alpha$, $x_{1j}^*+x_{2j}^* > 1-\alpha$, thereby we get $\tilde{x}_{ij}=\frac{x_{ij}^*}{x_{1j}^*+x_{2j}^*} < \frac{x_{ij}^*}{1-\alpha}$.

$$E[L_1]=\sum_{j:z_j^*<\alpha} p_{1j}\tilde{x}_{1j} < \frac{1}{1-\alpha}\sum_{j:z_j^*<\alpha} p_{1j}x_{1j}^*$$

$$\le \frac{1}{1-\alpha}\sum_{j=1}^{n} p_{1j}x_{1j}^* \le \frac{1}{1-\alpha}T^*$$

Similarly we have $E[L_2] \le \frac{1}{1-\alpha}T^*$.

Lemma 1. $E[T] \le E[L_1]+E[L_2]$.

Proof: $\forall J_{M_1} \subseteq J_A$.

The notation $\left(J_{M_1}, J_A \setminus J_{M_1}\right)$ stands for the random event that jobs in J_{M_1} are assigned to machine M_1, while the jobs in $J_A \setminus J_{M_1}$ are assigned to machine M_2.

Obviously we have

$$P_r\left(J_{M_1}, J_A \setminus J_{M_1}\right)=\prod_{j:j\in J_{M_1}} \tilde{x}_{1j} \prod_{j:j\in J_A\setminus J_{M_1}} \tilde{x}_{2j}$$

$$L_1\left(J_{M_1}, J_A \setminus J_{M_1}\right)=\sum_{j:j\in J_{M_1}} p_{1j}$$

$$L_2\left(J_{M_1}, J_A \setminus J_{M_1}\right) = \sum_{j: j\in J_A \setminus J_{M_1}} p_{2j}$$

$$E[L_1] = \sum_{J_{M_1}\subseteq J_A}\left(\sum_{j: j\in J_{M_1}} p_{1j}\right) P_r\left(J_{M_1}, J_A \setminus J_{M_1}\right)$$

$$E[L_2] = \sum_{J_{M_1}\subseteq J_A}\left(\sum_{j: j\in J_A \setminus J_{M_1}} p_{2j}\right) P_r\left(J_{M_1}, J_A \setminus J_{M_1}\right)$$

$$E[T] =$$

$$\sum_{J_{M_1}\subseteq J_A} \max\left\{\sum_{j: j\in J_{M_1}} p_{1j}, \sum_{j: j\in J_A \setminus J_{M_1}} p_{2j}\right\} P_r\left(J_{M_1}, J_A \setminus J_{M_1}\right)$$

Obviously we have $E[T] \le E[L_1] + E[L_2]$. The proof is completed.

From the discussions above we have $E[T] \le \frac{2}{1-\alpha} T^*$.

As mentioned in section 2, we have an upper bound for the total penalty of the rejected jobs

$$\sum_{j:\bar{z}_j=1} q_j = \sum_{j:z_j^*\ge\alpha} q_j \le \frac{1}{\alpha}\sum_{j:z_j^*\ge\alpha} q_j z_j^* \le \frac{1}{\alpha}\sum_{j=1}^{n} q_j z_j^*$$

Hence we have

$$E[T] + \sum_{j:\bar{z}_j=1} q_j \le \frac{2}{1-\alpha} T^* + \frac{1}{\alpha}\sum_{j=1}^{n} q_j z_j^*$$

$$\le \max\left\{\frac{2}{1-\alpha}, \frac{1}{\alpha}\right\} OPT$$

The approximation ratio $\max\left\{\frac{2}{1-\alpha}, \frac{1}{\alpha}\right\}$ is minimized when $\alpha = \frac{1}{3}$, and the minimum value is 3.

We give a randomized 3-approximation algorithm for $R_2|rej|C_{\max}(J_A) + \sum_{j\in J_R} q_j$ in the following way.

Algorithm 2

Step 1: Formulate the scheduling problem into an integer program (1).

Step 2: Relax integer program (1) to linear program (2).

Step 3: Solve linear program (2) and obtain an optimal solution $\left(x_{1j}^*, x_{2j}^*, z_j^*\right)$.

Step 4: Set $J_R = \left\{j : z_j^* \ge \frac{1}{3}\right\}$, $J_A = \left\{j : z_j^* < \frac{1}{3}\right\}$.

Step 5: For job j *satisfying* $z_j^* < \frac{1}{3}$, *assign job* j *to machine* $M_i(i=1,2)$ *with probability* $\tilde{x}_{ij} = \frac{x_{ij}^*}{x_{1j}^* + x_{2j}^*}$.

Step 6: Process the jobs in $J_{M_i}(i=1,2)$ *continuously on machine* M_i *in an arbitrary order.*

Theorem 2. Algorithm 2 is a randomized 3-approximation algorithm for $R_2|rej|C_{\max}(J_A) + \sum_{j\in J_R} q_j$.

IV. CONCLUSION

In this paper, we study the scheduling problem with rejection on two unrelated parallel machines. We may choose not to process some jobs, thus incurring the corresponding penalty. The goal is to minimize the makespan plus the sum of the penalties of the rejected jobs. We present a deterministic 3-approximation algorithm and a randomized 3-approximation algorithm for this problem. We obtain the two algorithms using the technique of linear programming rounding.

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