Enhanced Version of Multi-algorithm Genetically Adaptive for Multiobjective optimization

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Abstract—**Multi-objective EAs (MOEAs) are well established population-based techniques for solving various search and optimization problems. MOEAs employ different evolutionary operators to evolve populations of solutions for approximating the set of optimal solutions of the problem at hand in a single simulation run. Different evolutionary operators suite different problems. The use of multiple operators with a self-adaptive capability can further improve the performance of existing MOEAs. This paper suggests an enhanced version of a genetically adaptive multi-algorithm for multi-objective (AMAL-GAM) optimisation which includes differential evolution (DE), particle swarm optimization (PSO), simulated binary crossover (SBX), Pareto archive evolution strategy (PAES) and simplex crossover (SPX) for population evolution during the course of optimization. We examine the performance of this enhanced version of AMALGAM experimentally over two different test suites, the ZDT test problems and the test instances designed recently for the special session on MOEA's competition at the Congress of Evolutionary Computing of 2009 (CEC'09). The suggested algorithm has found better approximate solutions on most test problems in terms of inverted generational distance (IGD) as the metric indicator.**

Keywords—*Multi-objective optimization, Multi-objective Evolutionary algorithms (MOEAs), Pareto Optimality, Multi-objective Memetic Algorithm (MOMAs).*

I. INTRODUCTION

Multi-objective evolutionary optimization is a subject of intense interest in all fields of Science, Engineering, Economics, Logistics and others. Multi-objective optimization problems (MOPs) have more than one conflicting objective function and they have many real-world applications [6], [59]. A general MOP can mathematically be formulated as follows.

$$\text{minimize } F(x) = (f_1(x), \ldots, f_m(x))^T \qquad (1)$$
$$\text{subject to } x \in \Omega$$

where Ω is the decision variable space, $x = (x_1, x_2, \ldots, x_n)^T$ is an individual or solution and $x_i,\ i = 1, \ldots, n$ are their decision variables, $F(x) : \Omega \to R^m$ consists of m real valued objective functions and R^m is called the objective space. If Ω is a closed and connected region in R^n and all

the objective functions in (1) are continuous for x, we call it a continuous MOP. Furthermore, if $m \geq 3$, then problem (1) is said to be a many objectives problem. In single objective optimization, the main focus is on the decision space while in multi-objective optimization, the focus is mainly on the objective space because objective values are used in checking for optimality [43]. In practical applications of optimization, it is very common that the objective functions of the MOP conflict with one another or are mostly incommensurable. One needs a set of optimal solutions to solve these problems.A solution $u = (u_1, u_2, \ldots, u_n) \in \Omega$ is said to be Pareto optimal if there exist no another solution $v = (v_1, v_2, \ldots, v_n) \in \Omega$ such that $f_j(u) \leq f_j(v)$ for all $j = 1, \ldots, m$ and also $f_j(u) < f_j(v)$ for at least index k. An objective vector is said to be Pareto optimal if their corresponding decision vector is Pareto optimal. All Pareto optimal solutions in the decision space of MOP is called Pareto set (PS) and their corresponding image in their objective space is called Pareto front (PF). The idea Pareto optimality was first proposed by Francis Ysidro Edgeworth in 1881 and then later on generalized by Vilfredo Pareto in 1986 as discussed in [12], [10].

MOEAs are highly effective and powerful stochastic techniques which can find a set optimal solutions in a single simulation run due to their population-based nature, unlike traditional mathematical programming.In the past two decades, and since the inception of vector evaluated GA (VEGA) [48], different types of MOEAs have been suggested, the Pareto dominance based MOEAs [11], [13], [61], [60], [44], [19], [18], [9], [27]), the decomposition based MOEAs [21], [20], [54], [8], [7], [30], [56], [58], [55], [1], [34], [32], [39], [42], [41], [26], [25], [35], [37], [33], [41]), and Indicator Based algorithms [63], [5], [3], [22], [4], [2], [14]. They mainly emphasize three conflicting goals: firstly, the final approximate Pareto front (PF) should be as close as possible to the true PF; secondly, the final set of Pareto optimal solutions should be uniformly distributed and diverse over the true PF of the problem (1); thirdly, the approximated PF should capture the whole spectrum of the true PF. Different fitness assignment procedures, elitism and diversity promoting strategies are found in the current literature of evolutionary computing (EC).

The Pareto dominance concept of MOEAs is very common for solving MOPs [28], [12]. To promote diversity, most of these algorithms use different diversity techniques such as fitness sharing, niching, the kernel approach, the nearest neighbour approach, the histogram technique, crowding or clustering, a relaxed form of dominance and restricted mating [11]. Among them, a fast non-dominated sorting algorithm (NSGA-II) [13], SPEA2 or improving the strength Pareto evolutionary algorithm [60], the Pareto archive evolution strategy (PAES) [27], multi-objective genetic algorithm (MOGA) [18], and niched Pareto genetic algorithm (NPGA) [19] are long-familiar and well known approaches. They have shown good behaviors in several comparative analysis.

Multiobjective evolutionary algorithms (MOEAs) are extremely useful for dealing with MOPs. They evolve their population solutions and provide Pareto optimal solutions in single simulation unlike traditional optimization techniques. In the past two decades, since the inception of vector evaluated genetic algorithm (VEGA) [48], several MOEAs have been suggested and [13], [61], [60], [17], [44], [19], [9], [11], [49], [59], [32], [36], [38]. they have successfully tackled various types of MOPs [16], [25], [35], [37], [33]. In general, classical MOEAs can be divided into three main different classes, namely, the Pareto dominance based MOEAs (e.g., [11], [13], [61], [60], [44], [19], [18], [9], [27]), the decomposition based MOEAs (e.g., [21], [20], [54], [8], [7], [30], [56], [58], [55], [1], [34], [32], [39], [42], [41], [26]), and Indicator Based algorithms (e.g., [63], [5], [3], [22], [4], [2], [14]). All these algorithms try to obtain a set of Pareto optimal solutions with three main features, firstly, It should close as much as possible to the true PF. Secondly, the approximated set should require expand uniformly distributive all over the true PF of the problem (1). Thirdly, final Parerto optimal solutions obtained by particular MOEA should require to desirably capture the whole spectrum of the PF of the problems. The existing algorithms implement different fitness assignment procedures to to evolve their population in order to achieve aforementioned three goals subject to No free Lunch concept [29]. The Pareto dominance concept based MOEAs thoroughly applied for coping with MOPs [28], [12]. To promote diversity, most of these algorithms are utilizing different diversity techniques such as fitness sharing, niching approach, Kernel approach, nearest neighbour approach, histogram technique, crowding or clustering, relaxed form of dominance and restricted mating [11]. Among them, a fast non-dominated sorting algorithm (NSGA-II) [13], SPEA2 or improving the strength Pareto evolutionary algorithm [60], the Pareto archive evolution strategy (PAES) [27], multi-objective genetic algorithm (MOGA) [18], and niched Pareto genetic algorithm (NPGA) [19] have been chosen in several comparative analysis.

Multi-objective memetic algorithms (MOMAs) form a new and attractive area of research in EC. They are inspired by models of adaptation found in nature. They are known as Baldwinian EAs, Lamarckian EAs, cultural algorithms, or genetic local search and hybrid MOEAs [45]. Hybrid MOEAs have been developed with the aim to overcome the shortcomings of stand-alone MOEAs [34], [32], [39], [42], [40].

A genetically adaptive multi-algorithm for multi-objective (AMALGAM) optimisation is recently developed for solving both multi-objective optimization problems [52] and single

optimization problems [53]. It employs multiple search operators for its population evolution. The search operators used include the particle swarm optimizer (PSO) [15], differential evolution (DE) [47] and NSGA-II [13] and allocates resources dynamically to each search operators based on their individual performances. It does not involve any decomposition as in MOEA/D (multio-bjective evolutionary algorithm based on decomposition) [54].

MOEA/D [54] decomposes the approximated PF of the given MOP into a number of different single objective optimization subproblems (SOPs). It then optimizes all SOPs simultaneously using generic evolutionary algorithm. MOEA/D paradigm have tackled diverse benchmark functions and it has several enhanced versions [31], [34], [32], [39], [36], [42]. In this paper, our main objective is to further improve the algorithmic performance of ALMAGAM by employing by employing multiple search operators including the differential evolution (DE) [46], particle swarm optimization (PSO) [15], simulated binary crossover (SBX) [24], Pareto archive evolution strategy (PAES) [23] and simplex crossover (SPX) [50] with self-adaptive alternative procedures for dealing with both CEC'09 test instances [57] and ZDT test problems [62].

The main objective in this paper to develop an enhanced version of ALMAGAM by employing multiple search operators such as differential evolution (DE) [46], particle swarm optimization (PSO) [15], simulated binary crossover (SBX) [24], Pareto archive evolution strategy (PAES) [23] and simplex crossover (SPX) [50] with self-adaptive procedures for dealing with both CEC'09 test instances [57] and ZDT test problems [62].

The rest of this paper is organized as follows. Section II outlines the framework of the enhanced version of the genetically adaptive multi-algorithm multi-objective (AMALGAM)method. Section III presents experimental results obtained with the enhanced AMALGAM on both CEC'09 [57] and five ZDT test problems [62]. Section IV is devoted to a discussion on experimental results. Section V finally concludes this paper and suggest further areas of research on this topic an related ones.

II. ENHANCED VERSION OF MULTI-ALGORITHM GENETICALLY ADAPTIVE FOR MULTIOBJECTIVE OPTIMIZATION

Algorithm 1 outlines the framework of the an enhanced version of the AMALGAM. In Sept 1, a population P with size N has been generated uniformly and randomly within the search space of the given MOPs. We then evaluate the fitness values of solution of population P. We calculate the crowding distance of each member of population after categorize them into different layers by using fast non-dominating sorting procedure adopted in NSGA-II [13] framework. After this, an Algorithm 1 divide the whole population according to k number of search operators in order to work each search operator on specified number of sub-populations N_1, N_2, N_3 to generate Q offspring population of sizes whose sum is equal to N. We have used five different search operators such as differential evolution (DE) [46], particle swarm optimization (PSO) [15], simulated binary crossover (SBX) [24] and Pareto archive evolution strategy (PAES) [23] and simplex crossover (SPX) [50] in

the evolutionary process of the suggested algorithm. Each individual search operator is getting resources at population level according to their current individual performance based on self-adaptive procedure as explained in subsection II-A.

Algorithm 1 Enhanced Version of Multi-algorithm Genetically Adaptive for Multiobjective optimization

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1: **Input:**
2: MOP: the multiobjective optimization problem; N: the population size and other main parameters; $\mathbf{F_{eval}}$: maximum function evaluations;
3: **Output:** $\{x^1, \ldots, x^N\}$ and $\{F(x^1), \ldots, F(x^N)\}$;
4: Generate an initial population P of size N uniformly and randomly.
5: Calculate the F-function values of each member of the P population.
6: Assign rank to each member of P using fast non-dominating procedure.
7: Assign sub-populations $P = \{P_1, P_2, \ldots, P_k\}$ to k operators for creating an offspring population $Q = \{Q_1, Q_2, \ldots, Q_k\}$ of size N.
8: Calculate F-function values of Q offspring population.
9: Assign rank to each member of Q using fast non-dominating procedure.
10: Combine the new and old population P and Q, $R = P \cup Q$.
11: Select population P of size N from population R of size $2N$ based on their ranks and crowding distances for next generation.
12: Update N best individuals among C population with high ranks and crowding density.
13: Update $P = \{P_1, P_2, \ldots, P_k\}$ (Explanation can be found in subsection II-A) based on the individual performances of each search operator.

A. Alternative Adaptive Resources Allocation Scheme

- We calculate the number of solutions that successfully enter to the next generation in the evolutionary process of enhanced version of AMALGAM. A successful solution is rewarded by 1 and unsuccessful by 0. An efficient operator gets more resources in the form of subpopulation to be operate on them as compared to weaker one.

- Let δ_k, $k = 1, 2, \ldots, q$ are total number of non-dominated solutions produced by q search operators (i.e, differential evolution (DE) [46], particle swarm optimization (PSO) [15], simulated binary crossover (SBX) [24], Pareto archive evolution strategy (PAES) [23] and simplex crossover (SPX) [50]) that enter successfully to next generation are convert into normalized form to develop probability formula (3)

$$\mathbf{P_k} = \frac{\zeta_\mathbf{k}}{\sum_{\mathbf{k=1}}^{\mathbf{q}} \zeta_\mathbf{k}}, \text{ where } \qquad \zeta_k = \frac{\delta_k}{\sum_{k=1}^{q} \delta_k} \quad (2)$$

$$P_k = \alpha P_{k-1} \times N + (1-\alpha) P_k \times N \quad (3)$$

Where P_k is the current and P_{k-1} is the previous probability of successes of the k search operators. More importantly, the above mentioned dynamic resources allocation did not switch on at every generation of proposed algorithm. It can allocate

resources at every multiple of 5th generation to tackle ZDT test problems [62]. The suggested enhanced AMALGAM allocates resources to each of its embedded search operator at every multiple of 10th generation for dealing with CEC'09 test instances [57].

III. PARAMETERS SETTING AND EXPERIMENTAL RESULTS

We have carried out Experiments using benchmark functions with two and three objectives. The ZDT test [62] were tackled with parameter settings as explained in the subsection III-A while CEC'09 [57] were handled with parameter settings are explained in the subsection III-B, respectively.

A. Parameter Settings for ZDT Problems

- $N = 100$: population size for 2-objective test instances.

- $F = 0.5$: scaling factor of the DE;

- $CR = 0.5$: crossover probability for DE;

- w is the inertia factor which lies in $[0.8, 1.2]$;

- c_1 and c_2 are the two acceleration constant or acceleration coefficients that usually lies between 1 and 4;

- $u_r \in [-1, 1]$ is a continuous uniform random number

- $w = 0.5 + rand/2$: inertia factor which lies in $[0.8, 1.2]$ and $\xi = 1$;

- $c_1 = c_2 = 1.5$: acceleration constant or acceleration coefficients that usually lies between 1 and 4;

- $F_{eval} = 25000$: maximum function evaluations;

B. Parameter Settings for CEC'09 Test Instances

This subsection explains the parameters setting to validate enhanced AMALGAM on CEC'09 test instances [57].

- $N = 600$: population size for 2-objective test instances;

- $N = 1000$: for 3-objective test instances;

- $F = 0.5$: scaling factor of the DE;

- $CR = 1$: crossover probability for DE;

- $F_{eval} = 300,000$: maximum function evaluations;

C. Performance Indicators

Two main goals for dealing with multiobjective evolutionary optimization are very important: 1) convergence towards the Pareto-optimal front, 2) to find uniform and well-distributive set of multiple solutions that cover the whole true PF of the problem at hand [12]. Several performance metrics are found in the specialized literature of evolutionary computing (EC) [51], [13], [12], [64] which are using to judge which algorithm is better than others and in what aspects. Inverted generational distance (IGD) [64], [57], relative hypervolume [51], [12], Gamma Υ and delta Δ [12], [13] which are commonly in several comparative analysis of different algorithms. The aforementioned performance indicators can

use only if the reference set for the test problems are known in advance or available. In this paper, we have used the Inverted Generational Distance (IGD) as performance indicator to judge the quality of final approximated set of Pareto optimal solutions obtained by proposed algorithm in comparison with other MOEAs.
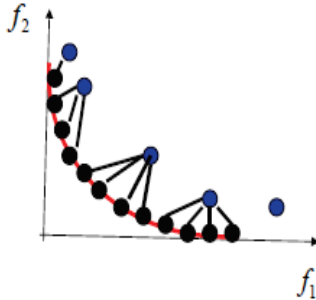

Fig. 1. Inverted Generation Distance (IGD) has been used as a Performance Indicator. The above solutions (blue fill circle) are approximated solution of the algorithm and (black fill circle) are the Pareto solutions uniformly distributive along the Pareto Front (PF).

Let P^* be a set of uniformly distributed points along the PF. Let A be an approximate set to the PF, the average distance from P^* to A is defined as [57]:

$$D(A, P) = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|}$$

where $d(v, A)$ is the minimum Euclidean distance between v and the points in A. If P^* is large enough to represent the PF very well, $D(A, P)$ could measure both the diversity and convergence of A in a sense. The closer the IGD metric values, better is the approximation set. We have used $P^* = 500$ in our carried experiments to tackle 2-objectives test instances and $P^* = 1000$ to solve 3-objectives problems. For two objectives case, the IGD metric is pictorially depicted in the figure 1. It measures both the convergence and the spread of the obtained solutions. Smaller IGD-metric values, better will be approximated set of Pareto optimal of solutions of corresponding MOEAs.

IV. DISCUSSION OF THE EXPERIMENTAL RESULTS OBTAINED ON ZDT AND CEC'09 TEST INSTANCES

The simulation related parameters are as follows.

- Operating system: Windows XP Professional

- Programming language of the algorithms: Matlab

- CPU: Core 2 Quad 2.4 GHz

- RAM: 4 GB DDR2 1066 MHz

- 30 independent runs were performed on each test problem.

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A. Discussion of IGD-metric Values

Table I and Table II record the IGD-metric values in terms of minimum (Best), Median, Mean, and standard deviation(std) and maximum (worst) which are found by enhanced AMALGAM and NSGA-II [13], respectively. These statistics have been collected by executing each algorithm 30 times independently with different random seeds on each ZDT test problem [62] and CEC'09 test instance [57]. The average CPU time spent by each algorithm are also provided in the last columns of I and Table II. It is evident from these Tables, that enhanced AMALGAM has found better approximated solution set with reduced the IGD-metric values as compared to NSGA-II [13] for most test problems. In most test problems, global convergence has been got for both test suites of problems. However, the complete Pareto front for some CEC'09 test instances have not been attained by enhanced AMALGAM in multiobjective optimization context. The primary reasons of this weak performance could be reason of the complicated objective functions profile of some CEC'09 test instances [57] which are mostly multi-modal near the global Pareto-optimal frontier and a slight perturbation in their optimization variables causes their solutions to become dominated.

B. Discussion of the Pareto Fronts of ZDT and CEC'09 Test Instances.

Table I and Table II record the IGD-metric values in terms of minimum (Best), Median, Mean, and standard deviation (std) and maximum (worst) which are found by enhanced AMALGAM and NSGA-II [13], respectively. These statistics have been collected by executing each algorithm 30 times independently with different random seeds on each ZDT test problem [62] and CEC'09 test instance [57]. The average CPU time spent by each algorithm are also provided in the last columns of I and Table II. It is evident from statistics gathered in these Tables, that enhanced AMALGAM has found a better approximate solution set with reduced IGD-metric values compared to those of NSGA-II [13] for most test problems. In most test problems, global convergence has been achieved for both test suites of problems. However, the complete Pareto front for some CEC'09 test instances has not been attained by the enhanced AMALGAM algorithm, in the multi-objective optimization context. The primary reasons for this weak performance could be the complicated objective functions profile in some CEC'09 test instances [57]; these are mostly multi-modal near the global Pareto-optimal frontier and a slight perturbation in their optimization variables causes their solutions to become dominated.

Figure 2 and Figure 6 depict the approximated Pareto front (PF) against the real PF of ZDT test problems displayed by enhanced AMALGAM and NSGA-II [13], respectively. These figures indicate that both algorithms have found better approximated PF in their best run among 30 independent runs on each ZDT test problem. We have plotted 30 PFs together in Figures 3 and Figures 7 of the enhanced AMALGAM and NSGA-II [13], respectively. These figures indicate that enhanced AMALGAM has displayed all 30 in better distribution ranges in all 30 independent runs as compared to NSGA-II [13].

Figures 4 display the best approximated PF of the CEC'09 test instances as demonstrated by enhanced AMALGAM

TABLE I.   THE IGD-METRIC VALUES OF THE ENHANCED AMALGAM FOR ZDT1-ZDT4 AND ZDT6. AVG-T MEANS AVERAGE CPU TIME IN SECONDS.

ZDT	Best	Median	Mean	St.Dev.	worst	AVG-T
ZDT1	0.004301	0.004521	0.004603	0.000223	0.005065	14.940230
ZDT2	0.004235	0.004794	0.004613	0.000258	0.005643	14.633287
ZDT3	0.005067	0.005498	0.005565	0.000170	0.006134	14.580179
ZDT4	0.004696	0.005175	0.005235	0.000162	0.005575	14.696021
ZDT6	0.003615	0.004037	0.004045	0.000175	0.004691	14.395623

TABLE II.   THE IGD-METRIC VALUES OF THE NSGA-II [13] FOR DEALING WITH ZDT1-ZDT4 AND ZDT6. AVG-T MEANS AVERAGE CPU TIME IN SECONDS.

ZDT	Best	Median	Mean	St.Dev.	worst	AVG-T
ZDT1	0.0042193	0.004472	0.004369	0.000139	0.004258	18.01
ZDT2	0.0043213	0.004649	0.004656	0.000182	0.005011	22.85
ZDT3	0.005132	0.00546	0.00912	0.01388	0.0602182042	17.596
ZDT4	0.00482	0.006421	0.00825	0.009649	0.059017370	22.85
ZDT6	0.005606	0.007045	0.007003	0.0005878	0.0080474634	19.90

TABLE III.   THE IGD-METRIC VALUES OBTAINED BY ENHANCED AMALGAM OVER CEC'09 TEST INSTANCES. AVG-T MEANS AVERAGE CPU TIME IN SECONDS.

CEC'09	best	mean	median	st. dev.	worst	AvG-T
UF1	0.028431	0.058596	0.057886	0.008465	0.070089	286.307532
UF2	0.011235	0.018219	0.013157	0.001349	0.016859	288.558110
UF3	0.091864	0.134375	0.136495	0.022836	0.198629	297.999586
UF4	0.040348	0.041052	0.041039	0.000337	0.041667	286.159044
UF5	0.165346	0.171338	0.171421	0.002796	0.176307	258.823135
UF6	0.068597	0.079037	0.078632	0.005978	0.088905	308.462841
UF7	0.014935	0.017689	0.017787	0.001267	0.020866	290.402351
UF8	0.103734	0.234131	0.230672	0.026091	0.261546	720.849149
UF9	0.056715	0.067789	0.114643	0.085653	0.325885	700.875474
UF10	0.273393	0.327878	0.326937	0.020029	0.360963	686.032888

TABLE IV.   THE IGD-METRIC VALUES GENERATE BY NSGA-II [13] IN 30 INDEPENDENT RUNS FOR CEC'09 TEST INSTANCES. AVG-T MEANS AVERAGE CPU TIME IN SECONDS.

CEC'09	best	mean	median	st. dev.	worst	AvG-T
UF1	0.051996	0.106873	0.096076	0.024862	0.128739	759.27
UF2	0.016012	0.019849	0.020050	0.001407	0.023589	518.07
UF3	0.066353	0.098234	0.097065	0.017958	0.134235	491.95
UF4	0.052199	0.054388	0.054551	0.001274	0.056679	393.60
UF5	1.523087	1.671735	1.676288	0.099452	1.844279	792.28
UF6	0.705834	0.762023	0.762271	0.028052	0.831784	822.79
UF7	0.067270	0.114403	0.112305	0.012055	0.125719	722.11
UF8	0.095436	0.108548	0.120433	0.030475	0.195112	1443.73
UF9	0.088857	0.188603	0.160832	0.047975	0.218993	1270.73
UF10	0.473865	0.744428	0.781509	0.134987	1.043141	1359.30

within 30 times independent execution for dealing with each problem. The Figure 8 is of the NSGA-II [13] produced in its best run among 30 independent runs for CEC'09 test instances [57]. The PFs displayed by enhanced AMALGAM are more promising than NSGA-II [13] in terms of diversity and proximity.

We have also plotted 30 PFs altogether estimated by enhanced AMALGAM in the figure 5 and the figure 9 exhibited by NSGA-II [13] in all 30times independent runs over the problems UF1-UF4 and UF7-UF10. These figures clearly indicate that enhanced AMALGAM has tackled the most CEC'09 test instances more effectively and spend less CPU time dealing with each test problem as compared to NSGA-II [13].

The problems UF5 and UF6 have not been tackled by both algorithms as per demand and genetic drift has been occurred in their respective population due to the presence of highly multi-modality in these problems like UF5-UF6. The search process of both algorithms are get stuck in the local basin of attraction of these problems and due to this both algorithms have not shown further improvement dealing with these problems.

V.   CONCLUSION AND FURTHER WORK

Different operators suite different optimization and search problems. The dynamic use of multiple operators in the EA framework has exhibited good performance on complicated MOPs, [31], [39], [34], [36], [32], [41], [40]. A multi-algorithm genetically adaptive multi-objective (AMALGAM) has recently been developed for solving both single objective [53] and multi-objective optimization problems [52]. In this paper, we have suggested new adaptive resource allocation procedure and developed a enhanced version of AMALGAM to cope with CEC'09, [57], and ZDT test problems [53]. The suggested algorithm has shown promising results on most test problems compared to NSGA-II [13] in terms of proximity and diversity. Furthermore, the suggested algorithm is more efficient which is desirable when solving real-word problems where time can be an issue. In the future, we intend to examine the performance of enhanced AMALGAM over real-world problems such as:

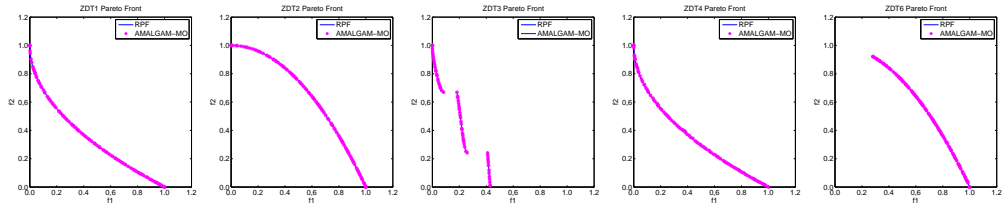• Tubular permanent magnet linear synchronous motor

Fig. 2. Plots of the final non-dominated solutions in the objective space displayed by enhanced AMALGAM in its best run among 30 independent runs over ZDT1-ZDT4 and ZDT6 problems.
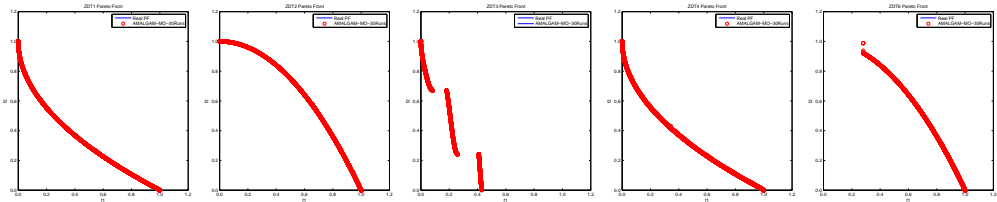

Fig. 3. Plots of the 30 PFs altogether in the objective space displayed by enhanced AMALGAM for ZDT1-ZDT4 and ZD6 problems.
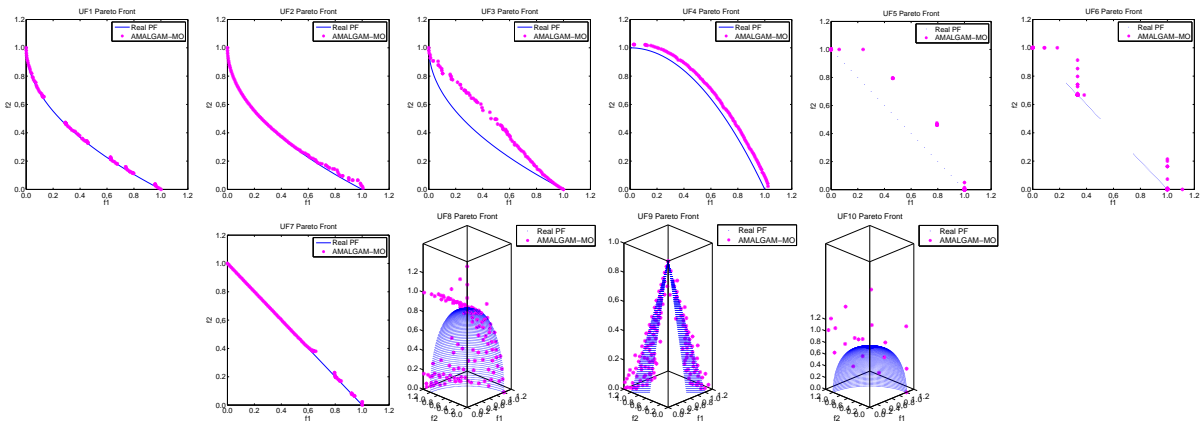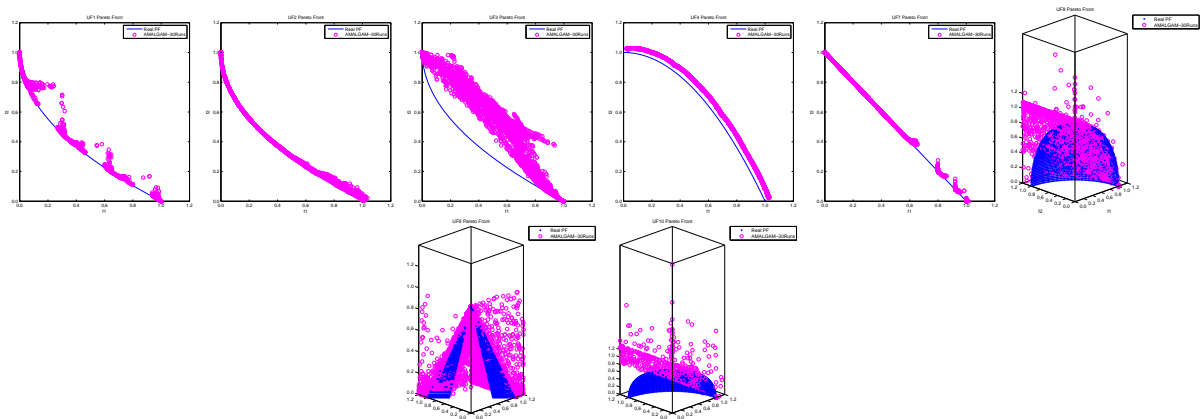

Fig. 4. Plots of the final non-dominated solutions in the objective space displayed by enhanced AMALGAM in its best run among 30 independent runs over UF1-UF10 problems.

Fig. 5. Plots of the 30 PFs altogether in the objective space displayed by enhanced AMALGAM for UF1-UF4 and UF7-UF10 problems.
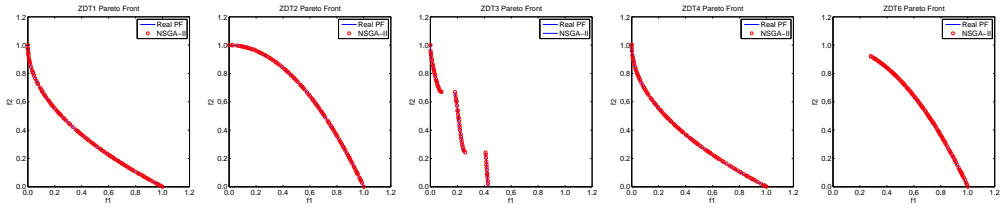
Fig. 6. Plots of the final non-dominated solutions in the objective space displayed by NSGA-II [13] in its best run among 30 independent runs over ZDT1-ZDT4 and ZDT6 problems.
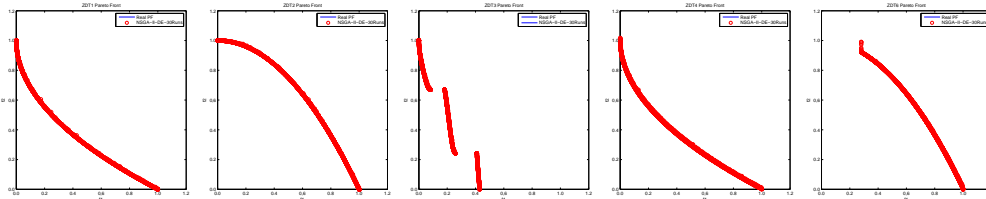

Fig. 7. Plots of the 30 PFs altogether in the objective space produced by NSGA-II [13] for ZDT1-ZD4 and ZDT6 problems.
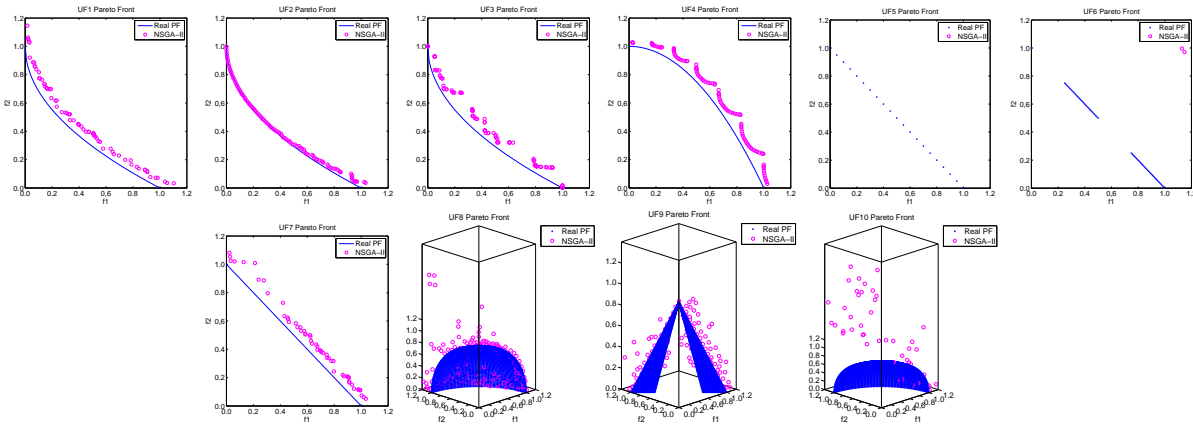

Fig. 8. Plots of the final non-dominated solutions in the objective space display by NSGA-II [13] in its best run among 30 independent runs over UF1-UF10 problems.
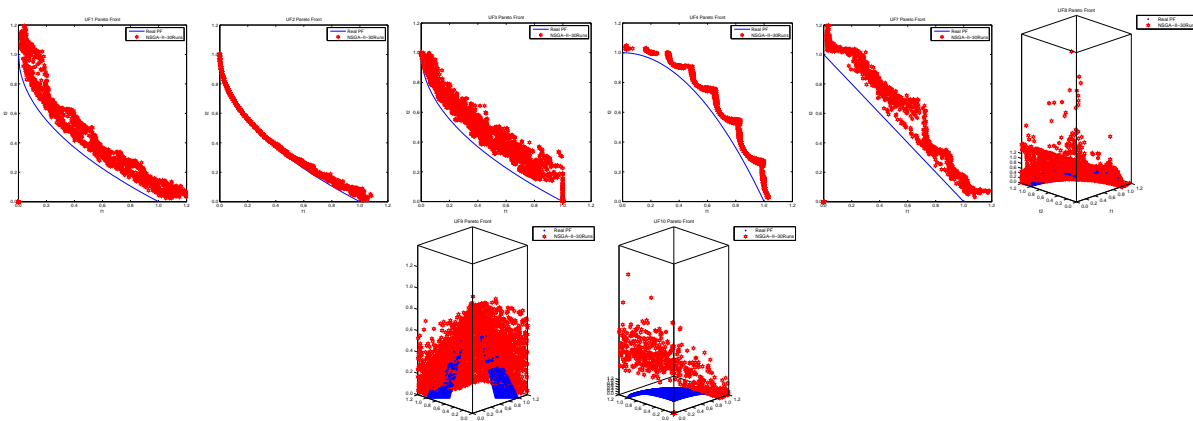

Fig. 9. Plots of the 30 PFs altogether in the objective space display by NSGA-II [13] for UF1-UF4 and UF7-UF10 problems.

(TPMLSM).

- Cancer chemotherapy problems.

- Fuzzy multi-objective reliability redundancy allocation problem.

- Multiobjective Engineering design problems.

- Multi-Objective Capacitated Arc Routing Problem.

- Passive Vehicle Suspension Optimization.

- Multi-objective mobile agent-based Sensor Network Routing.

- Oil and Gas Well Drilling problems.

We will also examine the effect of various enhanced versions of differential evolution [38] with self-adaptive capabilities in our proposed algorithm on the problems mentioned above as well as other complicated test problems.

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